Feedback Control For Robot Formation Maneuvers

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Abstract

This paper develops control strategies for moving multiple-agents in formation, using a virtual structure. The controls are specifically applied to robots. By introducing feedback from the followers to the coordinating mechanism, the robots are shown to better coordinate their motion. Hardware results are presented.

1 Background for Formation Maneuvers

Moving a group of agents in formation has received a fair amount of attention in the control literature. Coordinated formations can be used to accomplish various tasks. For example, spacecraft formation maneuvers can be used to synthesize a space based interferometer [3]. As another example, planetary rovers can be used to navigate and explore asteroids or planets. Current schemes for coordinating formation maneuvers can be categorized under either leader-follower, virtual structure, or emergent behavior.

In order to understand the issues involved with coordinating multiple agents, we will look at coordinating motion on multiple robots. Robots were chosen as agents since they are cheaper to build than spacecraft, easy to maintain, and illustrate many of the control problems associated with coordinating a group of agents.

The categories used for coordinating group formations have been applied to mobile robots, i.e., leader-follower [9], virtual structure [8], and emergent behavior control [1]. Leader-follower designates one robot as the leader and the other robots follow the leader. Virtual structure generates a trajectory which the robots (agents) follow. For virtual structure, we shall extend the ideas used in [2] in which a virtual structure moves along a trajectory with the following spacecraft tracking a corresponding position on the structure. Most of the emergent behavior controls have not been analyzed from a dynamical systems perspective. A rigorous treatment from a dynamic systems perspective of a particular behavioral approach for robots is presented in [6].

The purpose of the work presented in this paper is to focus on ways to improve coordinating robot formation maneuvers through virtual structure. The coordination problem could be greatly simplified if all of the robots could be turned on at exactly the same time, with exactly the same gains. Using a control law where each robot regulates to a desired goal, the robots would move in a coordinated fashion. This would be like a race where all the runners start at the same time and run at exactly the same speed. However, this approach lacks robustness since it does not account for communication latency, differences in timing, and manufacturer variability on each robot. To overcome such problems, there needs to be some kind of coordinating mechanism.

With this is mind, the formation control problem can be cast into the general architecture shown in Figure 1. The equations governing the blocks $\mathcal{R}_i$, $\mathcal{K}_i^{(j)}$, $\mathcal{F}^{(k)}$ and $\mathcal{G}$ shall be defined throughout this pa-

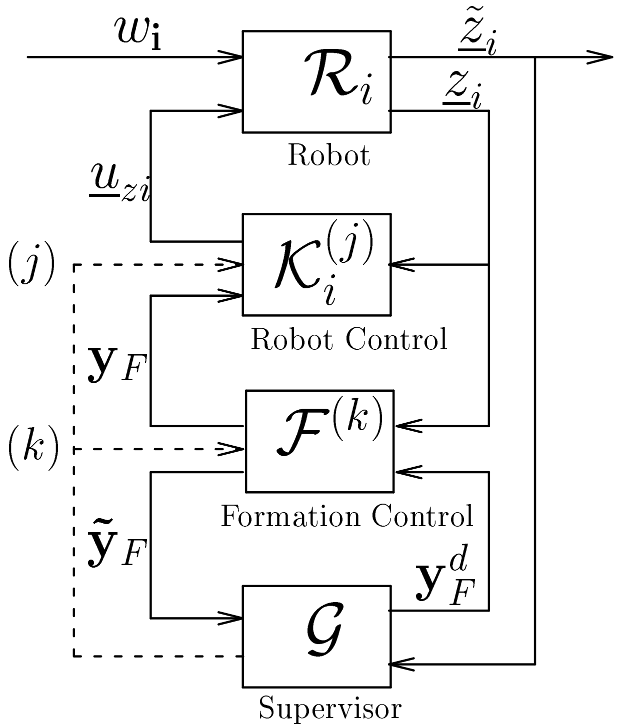

Figure 1: Multi-agent control architecture.

per. The block $\mathcal{R}_i$ contains the dynamics for each of the robots. The block $\mathcal{K}_i^{(j)}$ corresponds to the j^{th} local control on the i^{th} robot. $\mathcal{F}^{(k)}$ represents the k^{th} formation control block or the coordinating mechanism for the local control, $\mathcal{K}_i^{(j)}$. By exploring the use of feedback to the formation control block, $\mathcal{F}^{(k)}$, we will be able to add robustness to the formation control. The output of the formation control block, $\mathbf{y}_F$, depend on variables in the formation control block to coordinate the formation maneuvers. We will hereafter refer to these variables as the coordination variables. Using feedback to the coordination variables distinguishes the controls presented here from previous work on leader-follower such as [9]. To show improved coordination using feedback to the coordination variables, we will look at hardware results and show the feedback reduces formation error[1].

[1]Formation error will be explicitly defined in section 4.

In order to demonstrate these concepts, the paper has been organized as follows. Section 2 introduces the robot model. Section 3 introduces the type of formation maneuvers to be considered. Section 4 presents the virtual structure control schemes. Section 5 presents hardware results for the virtual structure schemes both with and without feedback from the followers to the coordination variables and discusses these results in the context of coordinating multiple robots. Section 6 contains conclusions and discussions.

2 Robot Dynamics

The equations of motion for differentially driven mobile robots are given below:

$$\dot{x} = v\cos(\theta), \tag{1}$$
$$\dot{y} = vsin(\theta), \tag{2}$$
$$\dot{\theta} = \omega, \tag{3}$$
$$m\dot{v} = F - F_s \text{sign}(v), \tag{4}$$
$$J\dot{\omega} = \tau - rF_s \text{sign}(\omega), \tag{5}$$

where m is the mass, J the inertia, F is force, τ the torque, $\frac{1}{2}F_s$ is the coefficient of friction on each wheel, and r is the radius of the robot. The mapping $\text{sign}(v)$ is defined by:

$$\text{sign}(v) = \begin{cases} 1 & \text{if } v > 0 \\ -1 & \text{if } v < 0 \\ \alpha & \text{if } v = 0, \end{cases} \tag{6}$$

where $\alpha \in [-1, 1]$. To focus attention on the formation control problem, we simplify the dynamics by feedback linearizing about a point off the wheel axis of the i^{th} robot which will be denoted as $\underline{z}_i \triangleq (x_{hi}, y_{hi})^T$, where the underline will denote vectors in $\mathbb{R}^2$ and all other vectors will be in bold. The disadvantage of feedback linearizing a point off the center is that angular information about the robot is lost. The idea of controlling a point off the center of the robot is not new. It has been done for the robot regulation problem [7] and for open-loop formation maneuvers [4]. Consider the feedback linearization of a point off the wheel axis of the robot whose position and orientation is given by the triple (x, y, θ). The components of the i^{th} off-center point $\underline{z}_i$ may be stated as:

$$\underline{z}_i = \begin{pmatrix} x_{hi} \\ y_{hi} \end{pmatrix} = \begin{pmatrix} x_i + L\cos(\theta_i) \\ y_i + L\sin(\theta_i) \end{pmatrix}, \tag{7}$$

where $\ddot{\underline{z}}_i$ is given by:

$$\ddot{\underline{z}}_i = \begin{pmatrix} \ddot{x}_{hi} \\ \ddot{y}_{hi} \end{pmatrix} = R(\theta_i) \begin{bmatrix} \frac{F_i - F_s \text{sign}(v)}{M_i} - L\omega^2 \\ \frac{\tau L_i - rF_s \text{sign}(\omega)}{J_i} + v_i\omega_i \end{bmatrix}. \tag{8}$$

Setting F_i, and τ_i to:

$$\begin{pmatrix} \frac{F_i}{M_i} \\ \frac{\tau L_i}{J_i} \end{pmatrix} = \begin{pmatrix} L_i\omega^2 + \frac{F_s}{M_i}\text{sign}(v) \\ -v_i\omega + \frac{rF_s}{M_i}\text{sign}(\omega) \end{pmatrix} + R(-\theta_i) \begin{pmatrix} u_x \\ u_y \end{pmatrix}, \tag{9}$$

Equation (8) reduces to:

$$\ddot{\underline{z}}_i = \begin{pmatrix} \ddot{x}_{hi} \\ \ddot{y}_{hi} \end{pmatrix} = \begin{pmatrix} u_{xi} \\ u_{yi} \end{pmatrix}. \tag{10}$$

This may be stated in terms of $\underline{z}_i$ as:

$$\ddot{\underline{z}}_i = \underline{u}_{zi}, \tag{11}$$

where $\underline{u}_{zi} = (u_{xi}, u_{yi})^T$. $\ddot{\underline{z}}_i = \underline{u}_{zi}$ are classical double integrator dynamics[2]. Placing the dynamics in the context of Figure 1, the output of block $\mathcal{F}^{(k)}$ is given by $\mathbf{y}_F = (\underline{z}_i^T, \dot{\underline{z}}_i^T)^T$, and we have the state space form for $\mathcal{R}_i$ given by:

$$\dot{\underline{z}}_i = \begin{bmatrix} \mathbf{0} & \mathbf{I}_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \underline{z}_i + \begin{pmatrix} \mathbf{0} \\ \underline{u}_{zi} \end{pmatrix},$$

where I_2 is the 2×2 identity matrix. The other output of $\mathcal{R}_i$ is $\tilde{\underline{z}}_i = \underline{z}_i - \underline{z}_{id}$, where $\underline{z}_{id}$ is the desired i^{th} position for the i^{th} robot. $\tilde{\underline{z}}_i$ will be used in measuring formation error.

[2]For single agent control, double integrator dynamics are well understood. See for example, *Feedback Control of Dynamic Systems* [5].

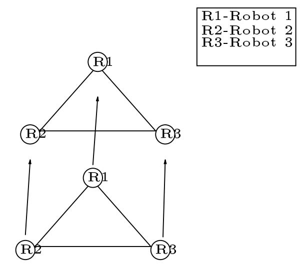

Figure 2: Translations.

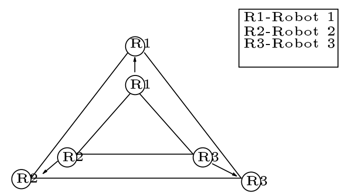

Figure 3: Expansions.

3 Elementary Formation Maneuvers

The ideas for elementary formation maneuvers stem from those used in spacecraft [6]. With just a few simple maneuvers, almost any desired form of group maneuvers which preserve formation shape can be achieved.

For a virtual structure, one way to visualize the maneuvers is to think of the structure as a center with rigid arms attached to it. The end of each arm is the desired i^{th} robot location. Thus, the coordinates of the end of each arm describes an equation for $\underline{z}_{id}$, the

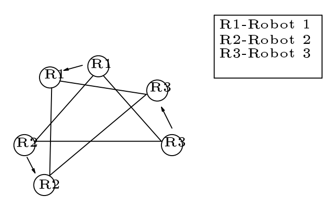

Figure 4: Rotations.

desired position for $\underline{z}_i$. The virtual structure at a specific time can be described by a center and orientation $(x_c(t), y_c(t), \theta_c(t))$, $\mathbf{D}(t) = (D_1(t), \ldots, D_N(t))$ a vector which contains the length of each arm, and $\theta_0(t)$ a vector which contains the angle of the arm relative to the center position. Therefore the parameters which describe a virtual structure can be stacked up into a vector of parameters given by $\zeta = (x_c, y_c, \theta_c, \mathbf{D}, \theta_\mathbf{0})$. For example, increasing the values in $\mathbf{D}$ expands the structure, changing $\theta_c(t)$ rotates the structure, and changing the center, $(x_c(t), y_c(t))$, translates the structure. Using these parameters, the components of $\underline{z}_{id}$ may be expressed mathematically as:

$$\underline{z}_{id} = \begin{pmatrix} x_c(t) + D_i(t)\cos(\theta_c(t) + \theta_{0i}) \\ y_c(t) + D_i(t)\sin(\theta_c(t) + \theta_{0i}) \end{pmatrix}. \qquad (12)$$

Using Equation (12) and by continuously varying the parameters of the virtual structure, ζ, we can obtain equations for translations, rotations and expansions. For translations, the only parameters which change in $\zeta(t)$ are $x_c(t)$ and $y_c(t)$. We let $x_c(t)$ be parameterized by the coordination variable $\xi_x(t)$ with $\xi_x(0) = x_c(0)$ and with $\xi_x(t) \to \xi_{dx}$ as $t \to \infty$, where ξ_{dx} is the final x desired position for the translation. Similarly we let $y_c(t)$ be parameterized by the coordination variable $\xi_y(t)$ with $\xi_y(0) = y_c(0)$ and with $\xi_y(t) \to \xi_{dy}$ as $t \to \infty$, where ξ_{dy} is the final y desired position for the translation. Using these parameterizations, the virtual structure for a translation can be described by:

$$\zeta(t) = (\xi_x(t), \xi_y(t), \theta_c(0), \mathbf{D}(0), \theta_0(0)). \qquad (13)$$

For expansions/contractions, the only parameters which change in $\zeta(t)$ are $\mathbf{D}(t)$. We let $D_i(t)$ be parameterized by the coordination variable $\xi_{Di}(t)$ with $\xi_{Di}(0) = D_i(0)$ and with $\xi_{Di}(t) \to \xi_{Dif}$ as $t \to \infty$, where ξ_{Dif} is the final amount the i^{th} arm of the virtual structure must expand/contract. Using these parameterizations, the virtual structure for an expansion can be described by:

$$\zeta(t) = (x_c(0), y_c(0), \theta_c(0), \xi_D(t), \theta_0(0)). \qquad (14)$$

For rotations, the only parameter which changes in $\zeta(t)$ is $\theta_c(t)$. We let $\theta_c(t)$ be parameterized by the coordination variable $\xi_\theta(t)$ with $\xi_\theta(0) = \theta_c(0)$ and with $\xi_\theta(t) \to \xi_{\theta f}$ as $t \to \infty$, where $\xi_{\theta f}$ is the final amount virtual structure must rotate. Using these parameterizations, the virtual structure for a rotation is described by:

$$\zeta(t) = (x_c(0), y_c(0), \xi_\theta(t), \mathbf{D}(0), \theta_0(0)). \qquad (15)$$

4 Virtual Structure

Of the three categories for coordinated control, a virtual structure scheme was chosen for several reasons. First, the parameters of a virtual structure are not restricted to double integrator dynamics as are the leader's in leader-follower. Second, a virtual structure accurately knows its position whereas in leader-follower and emergent behavior, the coordinating mechanism depends on positions which are corrupted by noisy sensors. In addition, the virtual structure parameters do not have to evolve according to the feedback linearized dynamics.

Hardware considerations are a good reason for choosing different dynamics for a virtual structure. For robots which are driven by DC motors, the voltage input saturates. This in turn implies that the robot's velocity saturates. For a virtual structure, we need to put velocity saturation explicitly into the evolution of the virtual structure's parameters. This motivates having the virtual structure's parameters be given by first order systems. In a first order system, velocity saturation can be put in explicitly by using saturation functions for the velocity. Another problem with a virtual structure is that the followers may not be able to track their desired position very well. One solution is to have the followers use PD tracking with the poles placed at ten times those of the virtual structure's evolving parameters. However, this may cause the virtual structure to perform the desired maneuver very slowly. An alternative solution is to use feedback from the followers to the formation control block as in Figure 1. Such feedback should reduce the formation error and allow the maneuver to be achieved at a faster rate.

A first order system, $\xi = (\xi_1, \ldots, \xi_M)$, which takes into account velocity saturation is:

$$\dot{\xi} = -k_1 K \tanh\left(\frac{1}{K}(\xi - \xi_d)\right), \quad (16)$$

where ξ_d is a constant vector, and where $\tanh(\cdot)$ is applied element by element to the vector $(\xi - \xi_d)$. We will also need $\ddot{\xi}$ to exist. ξ, $\dot{\xi}$ and $\ddot{\xi}$ must exist for the Lyapunov function candidate to be valid. $\ddot{\xi}$ is given by:

$$\ddot{\xi} = -k_1 \left(\operatorname{sech}\left(\frac{1}{K}(\xi - \xi_d)\right)^2 \dot{\xi}\right), \quad (17)$$

where the function $\operatorname{sech}\left(\frac{1}{K}(\xi - \xi_d)\right)^2 \dot{\xi}$ is applied element by element to the vector $(\xi - \xi_d)$. We have seen that by changing parameters of the virtual structure, ξ, the virtual structure can translate, rotate, or expand. These parameters will be used in the formation control as a vector of coordination variables. This makes the formation control only a function of the coordination vector, ξ. For the elementary formation maneuvers with virtual structure, the local control block $\mathcal{K}_i^{(j)}$ will need to compute $\underline{z}_{id}$ which is a function of the coordination variable, ξ. This may be stated as:

$$\underline{z}_{id}(\xi) = \begin{pmatrix} x_{hid}(\xi) \\ y_{hid}(\xi) \end{pmatrix}, \quad (18)$$

where $\underline{z}_{id}(\xi)$, the i^{th} desired location of each robot, depends on the vector of coordination variables ξ.

One way to have each robot track its desired position is to use PD control on the tracking error. Thus, stacking up the control laws in the control block $\mathcal{K}_{(i)}^{(j)}$, into a vector we have:

$$\mathbf{u}_z = \ddot{\mathbf{z}} - A\tilde{\mathbf{z}} - B\dot{\tilde{\mathbf{z}}}, \quad (19)$$

where $\tilde{\mathbf{z}} = (\underline{z}_1^T - \underline{z}_{d1}^T, \ldots, \underline{z}_N^T - \underline{z}_{dN}^T)^T$, $\dot{\tilde{\mathbf{z}}} = (\dot{\underline{z}}_1^T - \dot{\underline{z}}_{d1}^T, \ldots, \dot{\underline{z}}_N^T - \dot{\underline{z}}_{dN}^T)^T$, $\ddot{\mathbf{z}} = (\ddot{\underline{z}}_1^T, \ldots, \ddot{\underline{z}}_N^T)^T$, $A = diag(k_{px}, k_{py}, \ldots, k_{px}, k_{py})$, and $B = diag(k_{vx}, k_{vy}, \ldots, k_{vx}, k_{vy})$. Using the control laws in Equations (16) and (19), it can be shown that a virtual structure scheme asymptotically achieves formation maneuvers [10]. Under ideal conditions, the proposed control law can be used to obtain tight bounds on formation error. One way to define formation error is by considering the normed square of the vector difference between $(\tilde{\underline{z}}_i - \tilde{\underline{z}}_{i+1})$. This measure for formation error is proposed in [6]. The error may be stated mathematically as:

$$FE(\mathbf{z}(t), \xi(t)) = \sum_{i=1}^{N} (\tilde{\underline{z}}_i - \tilde{\underline{z}}_{i+1})^T (\tilde{\underline{z}}_i - \tilde{\underline{z}}_{i+1}), \quad (20)$$

where $FE(t)$ is the formation error and where the indices are defined modulo N, i.e., $N+1=1$. Using a measure for formation error leads to a natural definition of formation stability. Let $FE(t)$ denote the formation error and $\xi(t)$ the vector of coordination variables, then we have two definitions for formation stability.

- Definition - A control scheme is **formation stable** if $\forall \epsilon > 0$, $\exists \delta > 0$ such that $FE(0) < \delta$ implies that $FE(t) < \epsilon \; \forall t > 0$ and both $FE(t) \to 0$ and $\xi(t) \to 0$ as $t \to \infty$.
- Definition - A control scheme is **strictly formation stable** if $\forall \epsilon > 0$, $\exists \delta > 0$ such that if $FE(0) = \delta$, then $FE(t) \leq \delta \; \forall t$ and both $FE(t) \to 0$ and $\xi(t) \to 0$ as $t \to \infty$.

With these definitions it can be shown that given the proposed control laws and initial conditions, then the control schemes is either **formation stable** or **strictly formation stable**.

This result states that if the following robots are turned on at exactly the same time, with exactly the same gains, they will track the leader in exactly the same manner – maintaining formation. However, from hardware considerations, this is difficult to achieve. Another difficulty is that the control can saturate, which means that the formation error is not necessarily decaying exponentially. To overcome these problems, the control needs to be made robust with respect to synchronization issues, saturation and manufacturer variability. Increasing feedback to the formation block is one way to help overcome some of these difficulties. Such feedback should slow down the coordination variable if the robots are lagging behind their desired position. Consequently, the robots can put more control effort into tracking a slower moving coordination variable, thus reducing formation error. One way to slow down the coordination variable is to make the gain a function of $1/(\text{tracking error}+\frac{1}{\epsilon})$. This has the affect of making the coordination variable slow down to zero as the tracking error increases. It also allows the coordination variable's dynamics to evolve at the rate ϵ if the followers are keeping up. One such gain which has this property is:

$$\gamma(\mathbf{z}-\mathbf{z}_d) = \gamma(\tilde{\mathbf{z}}) = \left(\frac{1}{\frac{K_F}{N}\tilde{\mathbf{z}}^T\tilde{\mathbf{z}} + \frac{1}{k_1}}\right). \tag{21}$$

The constant K_F in γ determines how much the virtual leader slows down if the following robots lag behind their desired targets. A larger K_F will slow down the coordination variable due to tracking error. If the robots lag infinitely behind their desired position, $\dot{\xi}$ will go to zero and ξ converges to a constant. At the other extreme, if the robots are perfectly tracking their desired positions, the coordination variable moves to its final goal with maximum rate k_1 as desired. The average tracking error was chosen to make the corresponding Lyapunov function candidate continuously differentiable. If formation control could be dependent on the robot which is maximally out of formation, then a guarantee on maximum formation error might be possible. However, such a control would not be continuously differentiable. With this in mind, we can modify the dynamics of the coordination variable as follows:

$$\dot{\xi} = -\gamma(\tilde{\mathbf{z}})K\tanh\left(\frac{1}{K}(\xi-\xi_d)\right), \tag{22}$$

where ξ_d is again a constant vector and tanh is applied element by element to the vector $(\xi-\xi_d)$. The acceleration, $\ddot{\xi}$, is given by:

$$\ddot{\xi} = -\gamma(\tilde{\mathbf{z}})\left(\operatorname{sech}\left(\frac{1}{K}(\tilde{\xi})\right)^2\dot{\xi}\right) + \tag{23}$$

$$\frac{1}{\gamma^2}\frac{k_F}{N}\left(\tilde{\mathbf{z}}^T\dot{\tilde{\mathbf{z}}}\right)K\tanh\left(\frac{1}{K}(\tilde{\xi})\right), \tag{24}$$

which can be shown to be continuous. As before, consider the problem of tracking a position on a virtual structure. The formation control will now include feedback. In the formation control block, $\mathcal{F}^k$, we have that $\dot{\xi} = g(\tilde{\mathbf{z}}, \tilde{\xi})$, i.e., the coordination variable ξ uses feedback from the followers to the formation control. The expressions for $\underline{z}_{id}$ are the same as before with the only difference being that $\dot{\xi}$ depends on $\tilde{\mathbf{z}}$:

$$\underline{z}_{id}(\xi) = \begin{pmatrix} x_{hid}(\xi) \\ y_{hid}(\xi) \end{pmatrix}. \tag{25}$$

As before, using the control laws in Equations (22) and (19), it can be shown that a virtual structure scheme asymptotically achieves formation maneuvers [10]. However, this scheme has feedback from the followers to the virtual structure.

5 Hardware Results

As mentioned, the hardware results show a couple of interesting features about the control. First, they show that virtual structure is able to perform elementary formation maneuvers asymptotically. Second, the hardware results show how feedback from the followers to the virtual structure can reduce formation error. The measure for formation error was defined in Equation (20). For rotations, the supervisor block, $\mathcal{G}$, outputs $\mathbf{y}_F^d = (\xi_{dx}, \xi_{dy}, \theta_c(0), \mathbf{D}(0)\theta_0(0))^T$. For translations the supervisor block, $\mathcal{G}$, outputs $\mathbf{y}_F^d = (\xi_{dx}, \xi_{dy}, \theta_c(0), \mathbf{D}(0), \theta_0(0))^T$. For expansions/contractions the supervisor block, $\mathcal{G}$, outputs $\mathbf{y}_F^d = (x_c(0), y_c(0), \theta_c(0), \xi_{Df}, \theta_0(0))^T$.

Both control schemes were implemented on a robot testbed. Both schemes were able to asymptotically perform translation maneuvers. The results are summarized in Table 1, which show that for a given gain k_1, increasing the weighting of the feedback to the coordination variables by increasing k_F, reduces formation error.

Expansion maneuvers were also run on the testbed with various values for k_1 and k_F. Both control schemes asymptotically performed expansion maneuvers. Additionally, from the data on expansions, for a given k_1, increasing k_F lead to reduced formation error. The results are summarized in Table 2.

Table 1: Table of translation results.

k_1	k_F	Maximum Formation Error(m)
1	20	0.01
1	5	0.02
1	1	0.02
1	0	0.05
3	20	0.05
3	5	0.1
3	1	0.101
3	0	0.45

Table 2: Table of expansion results.

k_1	k_F	Maximum Formation Error(m)
1	20	0.05
1	5	0.12
1	1	.15
1	0	.19
3	20	.30
3	5	0.42
3	1	> 0.7
3	0	> 0.7

Rotation maneuvers were run on the hardware testbed. The results are summarized in Table 3., which shows that for a given gain k_1, increasing k_F, which is the weighting on the feedback from the followers to the coordination variables, reduces formation error.

Table 3: Table of rotation results.

k_1	k_F	Maximum Formation Error(m)
1	20	0.1
1	5	0.12
1	1	.2
1	0	.19
3	20	.3
3	5	0.45
3	1	> 0.7
3	0	> 0.7

The hardware results discussed thus far have shown that feedback from the robots to the virtual structure reduces formation error. Of course, decreasing the gain on k_1 can have a similar affect. However, this is not the only added advantage of using feedback from the followers to the formation control block. We ran both controls with an initial error formation error of 0.2 m. With no feedback to the coordination variables, the formation error initially got worse – reaching a maximum of 0.3m. The coordination variables "assume" their is no initial formation error. The virtual structure does not take this into account, moving towards its goal without regards for the following robots. In contrast, when using the same initial conditions and feedback from the followers to the coordination variables, the formation error only slightly increased – reaching a maximum of .22m. The feedback allows the virtual structure to "consider" the initial formation error and slows down so the followers can get back in formation. Follower to coordination variable feedback allows the virtual structure to take into account un-modeled and un-predictable problems like initial formation error, saturation, and poor tracking performance. Thus, follower to coordination variable feedback adds to the robustness of formation keeping by closing a feedback loop. The price paid is the time to convergence is slower.

6 Conclusions and Discussion

This paper has shown how to perform certain coordination formation maneuvers using a virtual structure. By increasing feedback from the followers to the coordination variables, we have shown it is possible to reduce formation error, but the controls take longer to converge. It also adds more robustness to formation keeping than does not having follower to coordination variable feedback. Without such feedback the virtual structure does not compensate for difficulties that cannot be easily modeled or predicted. Introducing feedback from the followers to the virtual structure is an important step in obtaining a formation control which is able to practically obtain coordinated maneuvers for multiple agent.

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