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Optimization of Torquer Coil Design for Use With the Small Satellite Attitude Control Simulator

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OPTIMIZATION OF TORQUER COIL DESIGN
FOR USE WITH THE
SMALL SATELLITE ATTITUDE CONTROL SIMULATOR

by

David Deloyd Anderson

Thesis submitted in partial fulfillment
of the requirements for the degree

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UTAH STATE UNIVERSITY
Logan, UT
1996
Abstract: This paper presents a procedure used to optimize the performance of a ferromagnetic core magnetic torquer coil design for use on the Space Dynamics Laboratory (Logan, UT) Small Satellite Attitude Control Simulator. The items of optimization include the primary goal of maximizing the coil's magnetic moment while reducing power consumption and system mass within given power, mass, and dimensional constraints. The optimization process makes use of several simple equations to determine a few starting points for design, after which an iterative approach based on experimentation is used to produce the final design. It is found that optimal magnetic moment performance requires the wise use of as much mass as is available. It is also found that power consumption can be greatly reduced by increasing the length of wire used, at a negligible cost to magnetic moment performance and a small increase in total mass. General governing equations are also compiled to help the reader understand the interplay between the wire windings and the ferromagnetic core in determining performance, and may also serve as starting point in developing performance predicting computer models.
1 Background

A torquer coil is essentially a long electromagnet comprised of wire wound on either on a hollow tube (air/vacuum core) or on a ferromagnetic core. (See illustration on cover.) Torquer coils produce torque by acting against an external magnetic field. Their operation can be thought of as being like a very large electromagnetic compass needle which torques to point northward. By utilizing three orthogonally oriented torquer coils and drivers that can provide direct current in either direction a nearly continuous range of satellite attitude adjusting torques can be created.

Torquer coils have several advantages over other attitude control systems. They do not use "consumables" such as compressed gas or combustible propellants. So long as electrical power is available\(^1\), torquer coils can function. They are fairly easy to control, generating torque when current is applied and remaining functionless and unobtrusive when power is off so long as the core can be demagnetized. This means that unlike momentum flywheels (which resist any change in orientation of the axis of rotation), torquer coils can easily be severally. Torquer coils also require no moving parts. Their simple design greatly reduces cost of design, manufacture, testing, and space certification, and virtually eliminates the possibility of failure during launch or while in service in space.

Some drawbacks include an effectiveness that is dependent on their orientation with respect to the external magnetic field. Torquer coils produce a maximum torque when positioned at right angles to the earth’s magnetic field and no torque whatsoever when aligned parallel with the earth’s magnetic field. They are also relatively weak compared to other attitude control actuators. Finally, torquer coils produce high strength local magnetic fields. Special care must be taken to insure that satellite components and instrumentation are not damaged or misled. This is usually accomplished with a carefully prepared satellite layout.

\(^{1}\) Electrical power can be obtained almost indefinitely through solar cells or some other technique.
2 Governing Equations -- Air Core

The simplest torque coil layout to consider is one with an air or vacuum core. The absence of a ferromagnetic core makes for a very weak coil, but the governing equations for this type of system are straight-forward and offer some insight into tradeoff considerations necessary when working with the much more complex ferromagnetic core system. Torque produced by an air/vacuum core coil is given by the following equation (Serway, 1992):

\[ \tau = N \cdot I \cdot A \times B \]

where \( \tau \) is torque, \( N \) is the number of turns or windings, \( I \) is current, \( A \) is a vector whose magnitude represents the cross-sectional area inside the windings and is directed normal to the area, and \( B \) is the external magnetic field vector. The cross product of the area vector and the magnetic field vector indicates that torque is at a maximum when the two are at a right angle, and decrease as the two vectors come together in alignment. Because the external magnetic field provided by the earth varies with position, attitude, and elevation, the first three terms, \( N \), \( I \), and \( A \) are grouped together into one term, \( \mu \), or magnetic moment, which serves as the standard measure of torquer coil performance. Magnetic moment values have the units of A·m\(^2\) or ampere-meter\(^2\).

Several noteworthy tradeoffs are indicated by the vacuum core magnetic moment equations. If the area, power supply's voltage, and wire material and gage are held constant, it can be shown that performance contributions resulting from changes in the number of turns are canceled out by equal and opposite changes in performance due to a current change, leaving coil performance unaffected. For example, if the number of turns is doubled, the wire length used to wind the coil must also double. The wire resistivity equation (given later in this paper) indicates that total wire resistance will also double, cutting current in half and leaving the magnetic moment unchanged. However, power consumption
(voltage times current) will be reduced by a factor of two. This key method for improving power efficiency without harming performance will prove to have relevance to ferromagnetic core coils also.

Another effective tradeoff occurs when area is varied at the expense of the number of turns, with wire type, current, and voltage being held constant. Because the latter three conditions fix the length of wire through the resistivity equation, increasing the coil area decreases the number of turns possible. However area increases faster than the number of turns decreases. Increasing area by a factor of $x$ increases performance by a factor of $x^{0.5}$ with no change in power consumption. This implies that a larger diameter coil is a more powerful coil.

3 Ferromagnetic Core Coils and Reaching for Optimization

In adding a ferromagnetic core, the problem at hand to optimize performance drastically increases in difficulty. The magnetic moment is now a function of the number of turns, current, coil area, core length, and core material magnetic susceptibility. Amid all these interdependent variables the goal for this project remains, primarily to optimize performance, striving for $600 \text{ A} \cdot \text{m}^2$ magnetic moment per coil. Secondary goals are to minimize coil mass and power consumption, as these are both precious resources in space flight systems. Constraint given by the systems supervisor for this project include a mass limit of 4 kilograms per coil, a power limit of 56 watts (2 amps @ 28 volts), and a size limit of 0.05 meter outside diameter by 0.6 meter length.

Governing equations for ferromagnetic core coils are very unwieldy and require core material magnetic property testing and a difficult numerical integration through the core volume. Consequently an iterative experimental approach was used to find an optimization of design within the given constraints, as guided by various more simple equations. Even so, the governing equations do offer support for the optimization assumptions made and give important insights into torquer coil mechanics, so an overview of them is given in appendix A.
Due to the many variables in this design, several assumptions were made to reduce the experimentation and expense required to obtain an optimal design. The first assumption made was that the coil should be as long as the constraints allow. This was justified by noting that as length approaches zero, the core becomes increasingly useless as it vanishes. Long, slender geometries are the norm among coils commercially available. The governing equations also attest to the validity of this assumption (See appendix A). A second assumption is that the maximum voltage available (28 volts) should be used. This is justified by the observation that raising the voltage gives more current which yields a greater magnetic moment in the air core configuration. Putting more power into a system usually gets more performance in return. Finally, copper wire and steel cores were assumed as the materials of choice. Wire conductivity can be only slightly improved beyond copper's performance at great expense (for example by using silver wire), and was thus ruled out. Also other materials exist with greater magnetic susceptibility than steel, but again at a much greater cost. By using steel, core material could be easily purchased and employed with minimal machining time. The greater demands on time and cost required by exotic core materials was not justified by slight performance increases, especially in the optimization stage, where several different core geometries are required. Once the coil design is optimized, the steel core could be replaced by something more exotic should the added expense be justified. In the testing stage, steel cores are sufficient to obtain an optimal design. Again the governing equations in appendix A clarify the tradeoffs involved.

With these assumptions made, the remaining parameters requiring consideration are the wire gage to be used, the number of turns to be wound onto the core, and the core diameter. Rather than blindly iterate on these three variables, spreadsheet calculations were performed and tables were made to help detail the interactions between them and the design constraints such as mass, power, and size. The most useful equations are presented below.
First the previously mentioned resistivity equation (Serway, 1992):

\[ R = \frac{\rho \cdot L_{\text{wire}}}{A_{\text{gage}}} \]

where \( R \) is resistance, \( \rho \) is resistivity (about \( 1.7 \cdot 10^{-8} \) ohm \cdot meters for copper at room temperature), \( L_{\text{wire}} \) is the length of wire used, and \( A_{\text{wire}} \) is the cross-sectional area of the wire excluding insulation. Given Ohm’s law that voltage equals current times resistance and solving the above equation for \( L_{\text{wire}} \), a wire length can be calculated given voltage, current, and a gage length with the following equation:

\[ L_{\text{wire}} = \frac{V \cdot A_{\text{gage}}}{I \cdot \rho} \]

Maximum voltage and current values are given by design constraints, and wire gauge is arbitrarily chosen.

The next equation was derived for this paper and relates wire length to coil geometry:

\[ L_{\text{wire}} \approx \frac{\pi \cdot L_{\text{core}}}{r_{\text{wire}}} \left( n \cdot r_{\text{core}} + n^2 \cdot r_{\text{wire}} \right) \]

where \( L_{\text{core}} \) is the length of the core, \( r_{\text{wire}} \) is the wire radius including insulation, and \( n \) is the number of layers of windings where each layer completely covers the core and any preceding layers. Finally the total mass is monitored with the following equations:

\[ \text{Mass} = \text{Mass}_{\text{core}} + \text{Mass}_{\text{wire}} \]
\[ = \rho_{\text{Fe}} \cdot L_{\text{core}} \cdot \pi \cdot r_{\text{core}}^2 + \rho_{\text{Cu}} \cdot L_{\text{wire}} \cdot A_{\text{gage}} \]

where \( \rho_{\text{Fe}} \) and \( \rho_{\text{Cu}} \) are the densities of iron and copper respectively.

Typical usage of the above three equations is as follows: The first equation is used to calculate the length of wire necessary to provide a resistance such that the current will be limited to 2 amps with 28 volts applied, thus keeping the coil within the given power constraints. This is calculated for several different gauges of wire. With the wire length thus calculated, the second equation is used to calculate the
core radii required for various numbers of winding layers, with the radius of the wire dependent on the
gage used. The third equation is then used to calculate the total weight of the wire and the core. An
example of the results obtained is shown below in figure 1 which is based on 28 volts, a 2 amp. limit, an
iron core with a length of 0.584 meters (slightly less than the length limit to leave room for end pieces),
and copper wire. As shown, the larger the wire (the lower the gage number), the longer the wire needed
to be to provide the proper resistance. To take up this longer wire, larger cores wound with more layers
are needed, with the mass limit eliminating configurations with too few winding layers and too large of
cores.

<table>
<thead>
<tr>
<th>Winding Layers</th>
<th>Radius of core (mm)</th>
<th>Weight of Core (Kg)</th>
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<td>0.20</td>
</tr>
</tbody>
</table>

Figure 1: Number of winding layers and corresponding core radii and total masses for various wire
gage. Below each wire gage is the wire length necessary to limit current to 2 amps with 28 volts
applied.

One lesson learned from this exercise is that wire gage selection is similar to transmission gear
selection in an automobile. In a car most any speed can theoretically be obtained in most any gear, but
given the situation at hand, one gear usually proves to be the most ideal. And so it is with wire gage selection. 24 gage wire was selected due to the flexibility it provided as far as providing a selection of possible core radii wound with relatively few layers. Maximizing torquer coil performance is the primary goal, but tradeoffs with power and mass must also be taken into consideration. Shown below is an "optimization field" based on 24 gage copper wire, 28 volt power supply, and an iron core with a length of 0.584 meters. Lines of constant mass and constant power are visible with the maximum mass and power limits shown as bold solid lines. The region in which the maximum magnetic moment performance is sought lies in the upper left region, with a secondary aim to reduce power consumption by moving up, and mass usage by moving to the left. To simplify the optimization process and eliminate machining, only core diameters that could be easily purchased were considered. The number of layers wound was also limited to integer amounts.

Figure 2: Optimization field showing lines of constant power consumption and constant mass. Design constraints are shown as the two darker lines. The region to the upper left represents the torquer coil design possibilities in which optimal performance sought. Opportunities to minimize mass and power consumption are also sought.
4 Testing and Iteration

The testing was performed using a setup as shown in the figure 3 below. The torquer coil is placed in a collar that is free to rotate on a vertical plane. After measuring the torque produced (or force times the distance from the axis of rotation) and the earth’s magnetic field component in the vertical direction, the magnetic moment can be backed out using a vector form of the definition equation of magnetic moment solved for $\mu$. By orienting the axes with the $y$ axis going along the coil and the $x$ axis along the rod axis of rotation and simplifying, the following equation can be obtained:

$$\mu_j = \frac{\tau_i}{B_k} = \frac{L_{\text{wood}} \cdot M_{\text{scale}} \cdot g}{B_k}$$

where $\mu_j$ is the magnetic moment of the coil, $\tau_i$ is the torque about the test fixture pivot, $B_k$ is the vertical component of the external magnetic field provided by the earth, $L_{\text{wood}}$ is the length of the wooden moment arm, $M_{\text{scale}}$ is the mass force recorded by the scale, and $g$ is the acceleration of gravity.

Figure 3: Test setup
The magnetic field components in the horizontal planes do not enter into the equation because the torque measured is due only to the vertical component. Torques about the other axes are constrained by the bearings. Extra care must be taken to insure that the scale reads forces induced by the torquer coil’s magnetic moment only, and not from magnetic attraction. To this end the test setup was isolated from the floor by setting it on large Styrofoam blocks. A wooden stick was used to transmit the torque to a high-precision scale a short distance away. The whole test fixture was set up away from walls, furniture, and other possible sources of interference.

Several sources of error needed to be considered. Wires had to be connected to the coils to supply them with power. These wires could hinder the torquer coil from freely torquing against the scale. The core could become magnetized to an uncertain degree, causing an uncertain reference point from which the coil’s torque would be measured. The measurement of the earth’s magnetic field enters into the magnetic moment calculation directly, along with any measurement errors. Bearing friction, scale error, voltage error, and other measurement errors also contribute to the degree of uncertainty. A slight error is incurred by using the acceleration of gravity at sea level instead of at the elevation where the torquer coils were tested. These errors do create a degree of uncertainty in each coil’s magnetic moment performance, but the wire and core magnetization problems were handled in a consistent manner, and the other errors were of sufficiently small magnitudes that relative comparisons between the coils tested can made with a high degree of certainty. Systematic errors bias all tests in a fairly consistent manner, while all error sources of a random nature were too small to undermine these relative comparisons. The above point cannot be overemphasized: An in-depth error analysis was not necessary for optimization, while consistency in testing was imperative. An in-depth error analysis would be needed to quantitatively evaluate the coil’s performance, however.
In testing the coils, capacitors were used to protect against arcing when the power to the torquer coil is turned off. These coils have very large inductances from hundredths to ones of henries. After the coil is in place and balanced in the test fixture, the voltage would be reversed through the coil at -28 volts. This is done to reverse-magnetize the core. Next the power would be turned off and the scale would be nulled to zero. Then +28 volts would be applied and the force created on the scale would be recorded. The power would then be turned off and the force measured by the scale would again be recorded. These forces would be the result of residual magnetism in the core. Generally these left-over forces would be about 10% of the force created when the full +28 volts is applied. The force with +28 volts applied minus half of the residual force was used in the magnetic moment calculations. This method provides a performance measurement with respect to a point halfway between the forward bias residual magnetism and reverse bias residual magnetism, and hopefully near the state the steel core was in before current had ever been applied.

Test results are illustrated below in figure 4. Testing was performed on two different days. On the first day a one inch diameter core wound with two layers was found to produce a force of 1.56 grams (using the +28 volt force minus half of the residual force) giving a magnetic field of 348 A·m². With one additional layer the torquer coil produced 1.45 grams of force or 323 A·m². Next a coil with a 5/8 inch diameter core wound with 3 layers was tested. It produced a force of 0.65 grams indicating a magnetic moment of 145 A·m². The vertical component of the earth's magnetic field was measured to be 37.1 µT². These tests indicate that using a smaller core (moving to the left on the optimization field) adversely affects performance. Moving upward with more windings also appeared to harm performance but to a much lesser degree. Following these tests it was reasonable to explore the consequences of adding more winding layers to see if power savings might be worth a slight cut in performance.

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2 T stands for tesla, the SI unit of a magnetic field. 100 µT = 1 G (gauss).
On the second day of testing, one week later, the vertical component of the earth's magnetic field was measured to be 40.6 μT, an increase of 9.4% over the previous week. It was later determined that this second magnetic field measurement is very likely to be in error. The one inch diameter three layer coil was re-tested, having undergone no modifications. It produced a force of 1.47 grams, consistent with the previous test. However with the stronger external field measurement, the magnetic moment evaluates to 300 A·m². Because the coil was unchanged and the same test procedures were used, it seems likely that the magnetic field was in error and was probably elevated artificially by the presence of magnetized test fixture hardware. The forces created by 4, 5, 6, and 7 layers on the one inch core was 1.48 grams, 1.43 grams, 1.44 grams, and 1.42 grams respectively, giving magnetic moments as shown in the figure below.

![Figure 4: Test results](image-url)
Power consumption for the 7 layer coil was reduced by a factor of two over the 2 layer coil, with very little loss of performance, and was thus chosen as the most optimal design. The governing equations (appendix A) support this conclusion as being reasonable. The final design calls for a steel core with a length of 23 inches, and a diameter of 1.0 inch. 24 gage copper wire is wound on the core in seven complete layers totaling 7100 turns. Resistance is about $55.6 \, \Omega$ at room temperature, mass is 3.7 kilograms, and outside diameter is 1.31 inches.

Qualitatively the best design within the given constraints is now specified. However a qualitative value for this coil design is not yet satisfactorily obtained. At present it is planned to wind two additional torquer coils using the design specified above. With these three torquer coils in hand, an effort will be made to test them and include an error analysis to establish an accurate magnetic moment performance value.

Estimated performance of the design given above is compared with performance values of some commercially available ITHACO TORQROD(tm) in appendix B.
APPENDIX A

The torque equation (adapted from a permanent magnet equation by Weeks, (1964 p. 645)) for a torquer coil operating with a magnetizable core is as follows:

\[ \tau = V \cdot (H + M) \times B_{\text{ext}} \]

where \( \tau \) is torque, \( V \) is volume of the core, \( H \) is the magnetic field strength (which represents the torque contribution from wire windings), \( M \) is the magnetization factor (which represents the contribution from core magnetization), and \( B_{\text{ext}} \) is the external magnetic field supplied by the earth. As before, the external magnetic field is omitted to calculate the torquer coil's magnetic moment (\( \mu \)). The relationship between the magnetic field strength and the magnetization factor of the core is given as follows:

\[ M = \chi \cdot H \]

where \( \chi \) is the core materials magnetic susceptibility. Herein lies the first difficulty with actually using these equations to predict or verify torquer coil performance. While magnetic susceptibility is constant (and too low to be useful for this type of application) for paramagnetic and diamagnetic substances, it is \textit{nonlinear} for ferromagnetic substances used in torquer coils. \( M \) increases linearly with \( H \) for low values, but as \( H \) increases, saturation starts to occur and \( M \) starts to level off, thus \( \chi \) a function of \( H \). See figure 5 below for an approximation of \( \chi \) versus \( H \), which was produced by recording the magnetic moment produced by a rod as different voltages are applied. The magnetic field strength \( H \) is proportional to voltage, but is not constant across the length of the rod. By using the magnetic moment to back out the magnetic susceptibility an average value for the entire rod is obtained and was plotted against the magnetic field strength at the center of the rod.
Figure 5: Approximate values of magnetic susceptibility verses magnetic field strength obtained from the testing of a torquer coil with 3 layers wound on a 1 inch diameter core. Results are approximate because the magnetic field strength was assumed to be constant and end effects were neglected. Given 28 volts, the average magnetic field strength $H$ for the torquer coils using the 1 inch diameter core is about 6500 A/m.

Functional relationships for $\chi$ are not available for reference because magnetic susceptibility can vary significantly between ferromagnetic metals that are nominally the same. Consequently, accurate relationships between $H$ and $\chi$ must be determined by testing samples from each lot used. Testing could be performed by placing a small sample of the ferromagnetic core material to be used in the midsection of a much longer hollow-core winding and then measuring the change in torque produced. The winding length must be much longer than the core sample to minimize end effects and provide a constant magnetic field strength to the core sample.

The second difficulty is the end effects. The magnetic field strength $H$ at a given point inside the coil windings is dependent on position, being greatest at the center and midsection of a core and decreasing at the coil ends. See figure 6. Vacuum core solenoid equations found in any good physics or electromagnetics book (See Serway (1992) pp. 846-847) can be used to calculate the magnetic field strength with the help of the following relationship (hinted at in Serway (1990) but never stated):

$$H = \frac{B_{\text{solenoid}}}{\mu_0}.$$
where $B_{\text{solenoid}}$ is the magnetic field vector calculated with the solenoid equations and $\mu_0$ is the permeability of free space. The solenoid equations will not be covered here, but it will be noted that the strength of the magnetic field at a given point essentially depends on the view factor it has of the surrounding wire windings. At the center of an infinitely long coil the view factor is unity, while at the end of a very long coil the view factor is one-half.

![Figure 6: Normalized magnetic field strength ($H*L/N*I$) along the centerline of a torquer coil with a diameter of 1 inch and a length of 23 inches, the same geometry as is used in the optimal torquer coil design. The gray horizontal line indicates the extent of the core.](image)

One last key point arising from the solenoid equations regards wire gage selection. The solenoid equations indicate that the winding density, or number of turns per unit core length directly affect magnetic field strength. This means that while using a larger wire size gives more layers of windings, the winding density will be less due to the larger wire diameter. The former effect is a function of wire area, the latter on diameter, so the these effects aren’t equal in magnitude. Still, larger wire isn’t necessarily better, as thick windings comprised of many layers may be less efficient in magnetizing the core and more susceptible to secondary electromagnetic effects such as the Hall effect.

In order to utilize the governing equations to predict or verify torquer coil performance the following must be done. First, the magnetic susceptibility $\chi$ of the core material verses magnetic field strength $H$ must be determined by experimentation. This allows one to find the magnetization factor of

$^3 \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \text{ (exact)}$
the core given \( H \). The total magnetic moment is then calculated by numerically integrating \((H - M) dV\) or \(H \cdot (1 + \chi(H)) dV\) over the core volume using a double integral over radius and length (assuming radial symmetry). The magnetic field strength just beyond the core ends has no core to magnetize and therefore it's contribution may be neglected as being negligible. The magnetic field strength for a given point is calculated using the solenoid equations which involving another double integral. Then the \( \chi \) corresponding to the computed magnetic field strength must be looked up from the experimental magnetic susceptibility data. That totals four nested integrals plus some method to relate the core magnetic susceptibility to the magnetic field strength to compute the magnetic moment for a simple cylindrical rod.

It seems likely that through experimentation one could develop a relatively simple equation to approximate torquer coil magnetic moment performance where the core magnetic susceptibility is fairly constant and effects are not dominant. A possible form is given below:

\[
\mu = H_{\text{max}} \cdot \left(1 + C_L \cdot C_D \cdot \chi_{\text{max}}\right) \cdot V_{\text{core}}
\]

where \( H_{\text{max}} \) is the maximum magnetic field strength located at the center of the core and is calculated from the solenoid equations, \( \chi_{\text{max}} \) is the maximum core magnetic susceptibility (from the linear portion of the core's magnetization), \( V_{\text{core}} \) is the volume of the core, and \( C_L \) and \( C_D \) are constants used to adjust the core's magnetic moment contribution based on it's length and diameter respectively. End effect loses lose consequence with increased length, therefore the \( C_L \) should start at zero and approach unity as length increases, perhaps being of the form \( C_L = 1 - e^{-al} \) where \( a \) is some constant. End effects also disappear as core diameter approaches zero, suggesting a modifying constant of the form \( C_D = e^{-bD} \) where \( b \) is some constant. An attempt to find values for \( a \) and \( b \) has thus far been hampered by a lack of data. Tests must be performed in which length and core diameter are varied, and all coils use the same core material and
are tested in the same manner. Other factors may need to be added to the model given above to obtain a useful performance predicting tool.

At present research is being conducted on the equations above using the computer math package Mathematica. Using this software package and a few numerical simplifications it has been shown that magnetic moments values calculated from the equations in this appendix for an air core torquer coil match values calculated for the simple air core equation in the paper. Work is in progress to fit an assumed core magnetization factor versus magnetic field strength equation \((A \cdot (1-B \cdot e^{-C}))\), where \(A\), \(B\), and \(C\) are unknown) to what little experimental data is available. Using this fit and Mathematica’s numerical integration capabilities, performance for other torquer coil geometries can be predicted and compared to experimentally determined values. If successful, this work may lead to the ability to optimize torquer coil design without experimentation beyond obtaining core magnetic susceptibility data and verifying coil performance.
APPENDIX B

(The following is taken from Ithaco’s homepage on the World Wide Web)

The ITHACO TORQROD(tm) is an electromagnet designed to provide complete momentum management on an Earth orbiting spacecraft. Dipole moments developed by the TORQRODS interact with the Earth’s magnetic field to generate gentle torques on the spacecraft. This torque can be used to stabilize tumbling spacecraft, control the spin of spinning spacecraft or manage the momentum in 3-axis stabilized spacecraft. ITHACO is the foremost supplier of these devices worldwide. Conical Earth Sensor (CES) The Conical Earth Sensor (CES) is a scanning horizon sensor used to gather attitude information on orbiting spacecraft. The most common application utilizes one or more CESs to provide pitch and roll attitude data for 3-axis stabilized orbiting spacecraft. The CES, a robust and a versatile sensor, can be used from low Earth orbit to above geosynchronous orbits, as well as during transfer orbits. The CES has been used on such spacecraft as LANDSAT, UARS, TOPEX and DOD programs.

Flight History
ITHACO TORQRODs have been used on over 100 satellites during the past 20 years with no known failures. Among the many programs for which ITHACO has supplied TORQRODs are HCMM, SAGE, NIMBUS, LANDSAT, SATCOM, STEP, and SMEX programs.

Catalog TORQROD(tm) Design Specifications
TORQRODs(tm) like the TR30CFR have been successfully used on over 100 satellites during the past 20 years.

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<th>Linear Voltage (V)</th>
<th>Saturated Voltage (V)</th>
<th>Resistance at 25 deg C (ohm)</th>
<th>Scale Factor (Am^2/mA)</th>
<th>Mass [kg/lb]</th>
<th>Length [cm/in]</th>
<th>Diameter [cm/in]</th>
<th>No. of Coils</th>
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<tbody>
<tr>
<td>TR10CFN</td>
<td>13</td>
<td>15</td>
<td>11.0</td>
<td>13.9</td>
<td>150</td>
<td>0.18</td>
<td>0.40 (0.9)</td>
<td>40 (16)</td>
<td>1.8 (0.7)</td>
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<td>TR16CFR</td>
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<td>15</td>
<td>17.0</td>
<td>20.0</td>
<td>270</td>
<td>0.21</td>
<td>0.45 (1.0)</td>
<td>39 (15)</td>
<td>1.8 (0.7)</td>
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<tr>
<td>TR30CFR</td>
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<td>40</td>
<td>24.0</td>
<td>28.0</td>
<td>132</td>
<td>0.19</td>
<td>0.95 (2.1)</td>
<td>50 (19.5)</td>
<td>2.3 (0.9)</td>
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<tr>
<td>TR60CFR</td>
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<td>70</td>
<td>10.3</td>
<td>12.6</td>
<td>40</td>
<td>0.25</td>
<td>1.7 (3.8)</td>
<td>64 (25.1)</td>
<td>2.6 (1.1)</td>
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<tr>
<td>TR65CAR</td>
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<td>80</td>
<td>9.2</td>
<td>12.3</td>
<td>39</td>
<td>0.28</td>
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<td>64 (25)</td>
<td>2.7 (1.1)</td>
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</tr>
<tr>
<td>TR100CFN</td>
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<td>130</td>
<td>10.4</td>
<td>13.0</td>
<td>20</td>
<td>0.21</td>
<td>4.5 (9.8)</td>
<td>41 (17)</td>
<td>4.9 (2.0)</td>
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<tr>
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<td>130</td>
<td>9.6</td>
<td>12.3</td>
<td>106</td>
<td>1.21</td>
<td>3.2 (7.0)</td>
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<td>3.6 (7.8)</td>
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</table>

Note: Custom designed TORQRODs can be readily provided to meet unique customer requirements. Reliability (1 year operating). Ps of 0.99997 (3 failures/10^9 hours)

[Based upon a parts count analysis per MIL-SPEC-217]
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In rough comparison, the optimal design obtained in this work is a coil with a magnetic moment of about 325 A·m^2 @ 28 vdc, resistance at room temperature is 55.6 Ω, scale factor is about 0.65 A·m^2/mA, mass is 3.7 kg (8.2 lb), length is 58.42 cm (23 in), outside diameter is 3.33 cm (1.31 in), and number of coils is 7.
Source Material
