2004

Impure Public Goods, Imperfect Labor Mobility, and Matching Grants in a Federation with Decentralized Leadership

Arthur J. Caplan
Utah State University

Emilson C.D. Silva

Follow this and additional works at: https://digitalcommons.usu.edu/eri

Recommended Citation
https://digitalcommons.usu.edu/eri/285

This Article is brought to you for free and open access by the Economics and Finance at DigitalCommons@USU. It has been accepted for inclusion in Economic Research Institute Study Papers by an authorized administrator of DigitalCommons@USU. For more information, please contact dylan.burns@usu.edu.
IMPURE PUBLIC GOODS, IMPERFECT LABOR MOBILITY, AND MATCHING GRANTS IN A FEDERATION WITH DECENTRALIZED LEADERSHIP

by

ARTHUR J. CAPLAN

Department of Economics
Utah State University
3530 Old Main Hill
Logan, UT 84322-3530

EMILSON C. D. SILVA

Department of Economics
Tulane University
New Orleans, LA 70118-5698

May 2004
IMPURE PUBLIC GOODS, IMPERFECT LABOR MOBILITY,  
AND MATCHING GRANTS IN A FEDERATION  
WITH DECENTRALIZED LEADERSHIP

Arthur J. Caplan, Assistant Professor  
Department of Economics  
Utah State University  
3530 Old Main Hill  
Logan, UT  84322-3530

Emilson C.D. Silva  
Department of Economics  
Tulane University  
New Orleans, LA  70118-5698

The analyses and views reported in this paper are those of the author(s). They are not necessarily endorsed by the Department of Economics or by Utah State University.

Utah State University is committed to the policy that all persons shall have equal access to its programs and employment without regard to race, color, creed, religion, national origin, sex, age, marital status, disability, public assistance status, veteran status, or sexual orientation.

Information on other titles in this series may be obtained from: Department of Economics, Utah State University, 3530 Old Main Hill, Logan, UT  84322-3530.

Copyright © 2004 by Arthur J. Caplan and Emilson C.D. Silva. All rights reserved. Readers may make verbatim copies of this document for noncommercial purposes by any means, provided that this copyright notice appears on all such copies.
IMPURE PUBLIC GOODS, IMPERFECT LABOR MOBILITY, AND MATCHING GRANTS IN A FEDERATION WITH DECENTRALIZED LEADERSHIP

Arthur J. Caplan and Emilson C.D. Silva

ABSTRACT

We examine the noncooperative provision of an impure public good by regional governments in a federation similar to the European Union, where regional governments are Stackelberg leaders and the central government is a Stackelberg follower—a federation with decentralized leadership. The center redistributes income and provides budget-balanced lump-sum matching grants after it observes the regions’ contributions to the impure public good. Imperfectly mobile workers react to regional and central governments’ policies by establishing residence in their most preferred region. Despite the degree of labor mobility, we show that the allocation of the impure public good and the interregional income redistribution policy are generally efficiently in a federation with decentralized leadership.

Key words: decentralized leadership; federation; redistribution; labor mobility; matching grants
I. Introduction

In what was the first model of a federation with imperfect labor mobility (or regional attachment benefits) and spillover-inducing public good provision, Wellisch (1994) shows that at least one region under-supplies the public good. As Wellisch himself says: “In the case of imperfect mobility, regions disagree about their objectives since the migration equilibrium can no longer be characterized by equal utilities in each region. At least one (of two) regions cannot achieve its desired resource distribution within the federation, and this region has no incentives to provide a socially efficient level of public goods generating spillovers. The decentralized Nash equilibrium is inefficient.” This inefficiency result is important mostly because it has extended to the case of imperfect labor mobility Oates’ (1972) seminal argument that central government intervention can enhance efficiency in the face of spillovers. However, it leaves unanswered the questions of which policy instruments and what type of hierarchical structures are capable of achieving a socially efficient allocation of public good provision within such a federation.

Caplan et al. (2000) have provided one possible answer for the case of pure public goods. They show that, irrespective of the degree of labor mobility, a federation characterized by “decentralized leadership” generally implements efficient public good policies. In this framework, decentralized leadership means that the central government makes interregional income transfers after it observes the contributions to the pure public good made by the regional governments. By pre-committing to their public good contributions in anticipation of the center’s interregional income policy and subsequent labor mobility, each region therefore has no
incentive to deviate from the socially efficient allocation in the resulting sub-game perfect equilibrium.¹ This result shows that the transfers implemented by the center induce both regional governments to face their personalized Lindahl prices, and that the center is therefore able to completely nullify the incentives of the regional governments to neglect each other’s contribution to the pure public good. Left unanswered by Caplan et al., however, is the question of whether or not this type of decentralized leadership is strong enough to implement efficient policies for impure public goods. The present paper answers this question in the affirmative, but only when the central government is provided with an additional policy instrument - lump-sum subsidies in the form of matching grants.

The previous literature on impure public goods (or what is commonly referred to as “impure altruism”) provides a precedent for matching-grant subsidies. However, this literature has been primarily concerned with the issue of neutrality, not efficiency per se. Thus, the main question motivating this vein of research is which set of policies might alter the regions’ (or individuals’, if we are talking about charitable donations) Nash behavior when it comes to the provision of public goods? In the case of pure public good provision, where individuals are affected only by the aggregate level of the public good and each individual provides a positive level of the public good, Warr (1982) showed that the level of provision is unaffected by a lump-sum redistribution of income. In other words, the public good is neutral with respect to lump-sum taxes and transfers that flow from the central government.² However, as Warr shows, per-unit consumption taxes and subsidies on public good contributions will raise the total level of provision by affecting the individuals’ marginal incentives to donate, and could in fact lead to a

¹ As long as the regional governments each provide positive amounts of the public good.
² Bergstrom et al. (1986) later showed that this neutrality result does not in general hold when at least one individual provides nothing to the public good.
Pareto optimum if the tax and subsidy rates are set with full information of the necessary condition for Pareto optimality, an exceedingly difficult task.  

Boadway et al. (1989), and Andreoni (1989, 1990) have extended the work of Warr in two important respects. Boadway et al. show that Warr's neutrality result is robust to an economy with pre-existing tax and subsidy distortions. Further, they find that differential per-unit subsidies across agents (rather than Warr's uniform rates) are not only non-neutral, but also have the surprising effect of decreasing the welfare of the agent who receives an increased subsidy, while increasing the welfare of the agent whose tax is increased. Andreoni (1989, 1990) is the first to analyse the issue of neutrality for the class of impure public goods—goods which have both "altruistic" and "egoistic" (or "warm glow") components. He finds that neutrality generally does not hold for the case of impure public goods, a result that is consistent with findings from the empirical literature on public goods. The reasoning behind this result is that the warm glow effect makes private contributions imperfect substitutes for contributions from other sources, such as public good provision from other contributors or taxes and subsidies from the central government. As Andreoni shows, perfect substitutability is sufficient for neutrality. Without perfect substitutability of contributions, "transfers of income to the more altruistic from the less altruistic will increase the equilibrium supply of the public good" (Andreoni, 1990). These income transfers may take the form of direct or matching grants financed by lump-sum taxation; however, matching grants Pareto dominate direct grants.

---

3Bernheim (1986) was able to reverse Warr's non-neutrality result for per-unit taxes and subsidies in a sequential equilibrium setting similar to ours. However, in Bernheim's model, the central government provides the public good in the final stage, after the individuals have made their private-good choices. In our case, the individuals (i.e., regions) make their public good choices prior to the central government's choice of taxes and subsidies.

4Boadway et al. (1989) are the first to adopt a federation, rather than a collection of individuals, as their framework for analysis in this vein of research.
While this more recent literature on impure public goods has clearly identified the properties and behavioral assumptions underlying the neutrality hypothesis, it has not adequately answered the question of which policy instruments and hierarchical structures are necessary to completely reverse the neutrality result. In other words, how do we move beyond Pareto dominant outcomes for impure public good provision to the complete set of Pareto efficient solutions?

This paper analyses the class of impure public goods, as modelled by Andreoni (1989, 1990), within the framework of a federation, as in Boadway et al. (1989). However, the federation’s hierarchical structure follows that of Caplan et al. (2000). In general, federations are complex hierarchical organizations that are typically characterized by the coexistence of governments at the central, regional (i.e., state or province) and local levels. In each level of the federal hierarchy, a government is usually endowed with a set of policy instruments, which is utilized to foster its own objectives. Regional and local governments provide many public services and goods. In many instances, however, the economic jurisdiction of a public good provided regionally or locally exceeds the political jurisdiction of a regional or local government. Impure public goods, such as abatement of a pollutant with both intra- and transboundary properties and infrastructure projects, provide good examples of commodities whose economic jurisdiction, being the entire federation, encompasses all regional political jurisdictions. Income redistribution, another important type of service which is often provided regionally, may also generate interregional externalities.

In the United States, the central government’s income-redistribution policy typically redistributes income across individuals regardless their regions of residence. In the European Union (E.U.), on the other hand, the center redistributes income amongst the national
governments that constitute the federation. The member nations subsequently redistribute income amongst their own residents. Wildasin (1991) shows that "when households are mobile among jurisdictions, income redistribution by individual jurisdictions create fiscal externalities" (p. 757). Similarly, Epple and Romer (1991) find that local redistribution induces sorting of the population, "with the poorest households located in the communities that provide most redistribution" (p. 828).

We consider three governmental regimes. We first examine a completely decentralized regime à la Wellisch (1994), where each region independently controls the level of the transfer that it makes to the other region. Next, we examine a regime where the central authority controls all policy instruments. This provides a useful benchmark for future comparison. Given our assumption about the center’s objective function—a weighted sum of the regional governments’ objective functions—we will be able, in the benchmark situation, to derive the entire set of Pareto efficient allocations. The remaining regime, denoted decentralized leadership, is characterized by the assumption that the central authority is solely endowed with an instrument to make interregional income transfers, and, later, lump-sum matching grants. This will represent our characterization of a federal regime in this paper. Both levels of government share responsibility over federal policy making, namely, the central government controls the federal income-redistribution and subsidy policies and the regional governments, behaving noncooperatively, jointly determine the public good policy.

In this federal regime, the regional authorities are Stackelberg leaders and the central authority is a Stackelberg follower. This seems to be a fair representation of the E.U. federation, where the governments of the member nations have historically precommitted to their own public good policies—the environmental policies of the member nations provide good
examples—and the central government, through the Maastricht Treaty, has been endowed with significant power to redistribute income amongst the member nations after it observes these nations’ public good policies—i.e., the interregional income transfers implemented with resources of the E.U.’s Structural Funds. The allocation of the Structural Funds obeys the “additionality” principle, whereby the center’s role is restricted to the provision of additional resources to help member nations to execute their own policies—such as infrastructure development and pollution abatement.

The paper is organized as follows. Section 2 describes the economy in the context of a completely decentralized federation, where regions play a Nash game and have control over both provision of the public good and interregional transfers. Similar to Wellisch (1994) we show that while the Nash solution is efficient with respect to population distribution, it is inefficient with respect to the allocation of public good provision. As mentioned above, we examine and compare two other governmental regimes: a fully centralized (our benchmark allocation) regime and a federal regime with decentralized leadership. Section 3 is devoted to the fully centralized, Pareto efficient solution. Section 4 considers the decentralized leadership regime where the central government retains control over income redistribution policy. In this section we show that the center’s income redistributive authority is not sufficient to restore the Pareto efficient allocation of public good provision across the two regions. Section 5 considers the decentralized leadership game where the center retains control over both income redistribution and lump-sum matching grants. In this section we show that lump-sum matching grants, financed by budget-balanced lump-sum taxation, restore the efficient allocation of public good provision that was lost in the Nash equilibrium. In each section, labor will be assumed to be imperfectly mobile, a characterization of the E.U.’s common labor market. Section 6 concludes the paper.
2. The Decentralized Nash Equilibrium

Imagine a federation with two regions, indexed by \( j, j = 1, 2 \), two regional governments and one central government. There are \( N \) individuals in the federation. The population of region \( j \) is denoted \( n_j \). Assume that \( 0 < n_j < 1 \) and \( N = \sum n_j = 1 \), implying that \( n_2 = 1 - n_1 \). The utility of the representative resident of region \( j, j = 1, 2 \), is assumed to be

\[
V^j = \begin{cases} 
  u^1(x_1, q_1, Q) + a(N - n), & \text{if the household lives in 1,} \\
  u^2(x_2, q_2, Q) + an, & \text{if the household lives in 2.}
\end{cases}
\]  

where \( u^j, j = 1, 2 \), is strictly concave and increasing in all arguments, where \( x_j \) is the consumption of the private (or numéraire) good, \( q_j \) denotes the amount of public good provided by region \( j \) (i.e., the “egoistic” component of welfare), and \( Q = \sum q_j \) represents the aggregate amount of the public good, or the “altruistic” component of welfare.\(^5\) As in Wellisch (1994), the parameter \( n \) measures the psychic benefit a household derives from living in region 2 and the parameter \( (N - n) \) the benefit from living in region 1. Thus, households with relatively small \( n \)'s are at home in region 1, while households with relatively large \( n \)'s are at home in region 2. The parameter \( a \) (\( a > 0 \)) expresses the degree of heterogeneity in tastes for a region. The larger is \( a \), the greater the intensity of psychic attachment to a respective region.

Each resident of region \( j \) is endowed with one unit of labor which is supplied at region \( j \). Workers are assumed to be identically productive and are employed in the production of the numéraire good. The production possibilities for the numéraire good are represented by a strictly concave production function \( F^j(n_j; L_j) = f^j(n_j) \), where \( L_j \) denotes the fixed resource endowment of region \( j \), say land. Since workers are identically productive and are all employed in the

\(^5\)The curvature conditions on \( u^j \), as well as on \( f \) discussed below, ensure concave programming problems for each of the cases considered below. Thus, sufficient second-order conditions are satisfied for each of the maximization problems considered below.
production of the numéraire good, we assume that each individual’s total return from the productive activity in region $j$ is $f'(n_j)/n_j$.

The numéraire good is not only used for consumption, but also as an input in the production of the impure public good, at a constant marginal cost of $1$. Regional government $j$ can therefore produce $q_j$ units of the public good at a total cost of $q_j$. Further, we assume that relative prices do not change, and that units of all commodities are chosen so that their prices are equal to $1$. Each resident of region $j, j = 1, 2$, faces the following budget constraint:

$$\frac{f'(n_j)}{n_j} = \begin{cases} \frac{x_1 + q_1 - \tau_1}{n_1} & \text{if the household lives in 1,} \\ \frac{x_2 + q_2 - \tau_2}{n_2} & \text{if the household lives in 2.} \end{cases} \quad (2)$$

where for each $j$, the right-hand side shows the resident’s total expenditure and the left-hand side gives his total income. The representative resident of region $j$ pays an amount of $\tau_j/n_j$ to region $-j$, and receives an amount of $\tau_{-j}/n_j$ from region $-j$.

Households are free to choose where they will reside. Since they differ in their attachment to a region, the migration equilibrium is characterized by the marginal household, identified by $n_1$, which is indifferent between locating in either region. Substituting (2) into (1), and recalling $n_2 = 1 - n_1$, the migration equilibrium may therefore be expressed as:

$$u_1\left(\frac{f'(n_1)}{n_1} - q_1 - \tau_1 + \tau_2, q_1, Q\right) + a(1 - n_1) = u_2\left(\frac{f'(1 - n_1)}{1 - n_1} - q_2 - \tau_2 + \tau_1, q_2, Q\right) + an_1 \quad (3)$$

Thus, $n_1$ also represents the number of households residing in region $1$.

---

$^6$As noted in Boadway et al. (1989), linear production technology assures both constant marginal costs and prices.
Equation (3) determines $n_j$ as an implicit function of the regional control variables $\{q_j, \tau_j\}$ for $j = 1, 2$. Thus, total differentiation of (3) enables a full cataloging of the various migration-response functions that will prove useful in the remaining analysis of this section:

$$\frac{\partial n_1}{\partial \tau_1} = \frac{n_1 n_2}{D} < 0, \quad (4a)$$

$$\frac{\partial n_1}{\partial \tau_2} = -\frac{n_1 n_2}{D} > 0, \quad (4b)$$

$$\frac{\partial n_1}{\partial q_1} = \frac{n_1 n_2 - u_1^1 - u_2^1}{D}, \quad (4c)$$

$$\frac{\partial n_1}{\partial q_2} = \frac{n_2 n_2 - u_2^2 - u_2^1}{D}, \quad (4d)$$

where $u_1^1 + u_2^2 - 2a = D < 0$ and $u_i^j = \frac{u_i^j}{n_j}(f_n^j - x_j)$. The inequality $D < 0$ ensures a stable migration equilibrium. It says that starting at the equilibrium, any possible gains in the value of net social marginal product that might be realized with an additional individual residing in region 1 are more than offset by lost attachment benefits. Further, the inequalities in (4a) and (4d) demonstrate that the number of residents moving to region 1 is decreasing in the level of that region’s transfer to region 2, and increasing in the level of region 2’s transfer to region 1. In addition, migration responses to the levels of $q_j$ depend upon the relative strengths of the respective egoistic and altruistic components for each respective region.

\footnote{The subscripts on functions $u^i$ and $f^j$ represent the partial derivatives with respect to the variable indicated.}
Region $j$'s problem is to choose $\{q_j, \tau_j\}$ to maximize its welfare (1) subject to (2) and response functions (4a)-(4d), taking as given $\{q_{-j}, \tau_{-j}\}$. The first-order conditions for these problems are:

$$-\frac{u_j^1}{n_j} + u^1_{s11} \frac{\partial n_1}{\partial \tau_j} \leq 0, \quad \tau_j \geq 0 \quad \text{and} \quad \tau_j \frac{\partial u_j^1}{\partial \tau_j} = 0 \quad j = 1, 2, \quad (5)$$

$$-\frac{u_x^1}{n_1} + u^1_{q1} + u^1_Q + u^1_{xx1} \frac{\partial n_1}{\partial q_1} = 0, \quad (6)$$

$$-\frac{u_x^2}{n_2} + u^2_{q2} + u^2_Q - u^2_{xx2} \frac{\partial n_2}{\partial q_2} = 0. \quad (7)$$

As shown in Appendix 1, combining equations (5) yields for the socially efficient population distribution:

$$-\frac{2an_2}{u_x} \leq (f_{n1}^1 - x_1) - (f_{n2}^2 - x_2) \leq \frac{2an_1}{u_x}. \quad (8)$$

To see that (8) implies the efficient population distribution, consider first the perfect mobility case ($a = 0$). In this case, $f_{n1}^1 - x_1 = f_{n2}^2 - x_2$, which is a traditional result indicating that net benefits of additional households to regions must be equalized across regions (Wildasin, 1986). If households are imperfectly mobile ($a > 0$), then there is a range of efficient allocations that vary with respect to the weights of a utilitarian social welfare function, as will be shown in Section 3. The population distribution indicated by (8) is a direct result of the equal-utility migration equilibrium (3), which induces each region to maximize the common utility level of all households in the federation.
Equations (6) and (7) yield for the following inefficient allocation of the public goods:

\[
n_j \left( \frac{u_q^j}{u_x^j} + \frac{u_Q^j}{u_x^j} \right) = 1 + u_Q^j \left( \frac{f_n^j - x_j}{2a - u_x^j \left( f_n^j - x_j \right)} \right) \quad j = 1, 2 .
\]  

(9)

Equations (9) imply

\[
n_1 u_1^1 \neq n_2 u_2^2 ,
\]

(10)
or that the regions do not provide the public good up to the point where their respective private marginal rates of substitution (MRS) are equated. As will be shown in the next section, equal private MRS’s is a necessary condition for the Pareto efficient provision of the public good. Otherwise, a reallocation of the public good could occur which would make both regions better off. Thus, the decentralized Nash solution is inefficient.

3. Pareto Efficiency

We assume the following objective function for the center:

\[
W (x_1, x_2, q_1, q_2) = \theta u^1 (x_1, q_1, Q) + (1 - \theta) u^2 (x_2, q_2, Q) .
\]

(11)

We further assume that \( \theta \in (0, 1) \). The parameter \( \theta \) corresponds to the weight put on region 1’s welfare by the center. We assume that this parameter is determined exogenously by egalitarian, institutional or political considerations. For a fixed \( \theta \), we can obtain an efficient allocation by choosing \( \{x_j, q_{ij}\}_{j=1,2} \) to maximize (11) subject to (3) and the aggregate resource constraint for the federation:

---

\(^8\)Even though (11) ignores individual attachment benefits, its maximization subject to (3), which includes individual attachment benefits, characterizes a Pareto efficient allocation for a given weight \( \theta \). Any change in location must be accompanied by an increase in either \( u'^1 \) or \( u'^2 \); otherwise it would not be made. This straightforward revealed preference argument explains why a Pareto efficient allocation must maximize (11) (see, e.g., Wellisch (1994, p. 171)).
The set of Pareto efficient allocations, excluding the efficient allocations associated with \( \theta = 0 \) and \( \theta = 1 \), can then be derived by straightforward application of the envelope theorem, namely, by varying \( \theta \) between 0 and 1 and computing the efficient allocation associated with each particular \( \theta \) value.

As shown in Appendix 2, for a fixed \( \theta \) an efficient allocation is characterized by (3), (8), (12) and the following equations, provided the solution is interior:

\[
\frac{u_q^j}{u_x^j} + \left( n_1 \frac{u_q^1}{u_x^1} + n_2 \frac{u_q^2}{u_x^2} \right) = 1, \quad j = 1, 2.
\]  

(13)

Equations (13) are the familiar Samuelson condition for efficient provision of the pure public good. It says that the sum of the MRS’s between the public and private goods in consumption should equal the marginal rate of transformation between these two goods in production. Unlike for the Nash equilibrium, equations (13) imply

\[
n_1 \frac{u_q^1}{u_x^1} = n_2 \frac{u_q^2}{u_x^2},
\]  

(14)

or that the regions provide the public good up to the point where their respective private MRS’s are equated. Thus, equations (8) and (14) are the benchmark conditions for Pareto efficiency.

4. Decentralized Leadership with Central Control of Income Redistribution

As in Caplan et al. (2000), we initially consider a three-stage game whereby the regional governments, acting as Stackelberg leaders, precommit by selecting their contributions to the public good prior to the interregional income redistribution policy of the center. The center determines its interregional income redistribution policy in the second stage of the game, after observing the public good contributions chosen by the regional governments and in anticipation
of location choices made by residents. Residents make their location choices after they observe both contributions to the public good and interregional redistributions of income.

Formally, the timing for the game is as follows.\(^9\)

**Stage 1:** Regional government 1 chooses \(q_1\) to maximize \(u_1(x_1, q_1, q_1 + q_2) + a(1 - n_1)\) subject to \(x_1 = x_1(q_1, q_2)\) and \(n_1 = n_1^2(x_1(q_1, q_2), x_2(q_1, q_2), q_1, q_2)\). Regional government 2 chooses \(q_2\) to maximize \(u_2(x_2, q_2, q_1 + q_2) + a n_1\) subject to \(x_2 = x_2(q_1, q_2)\) and \(n_1 = n_1^2(x_1(q_1, q_2), x_2(q_1, q_2), q_1, q_2)\). Each regional government takes the other regional government’s choice as given.

**Stage 2:** The center observes \(\{q_1, q_2\}\) and chooses \(\{x_j\}j=1,2\) to maximize (10) subject to (11) and \(n_1 = n_1^2(x_1(q_1, q_2), x_2(q_1, q_2), q_1, q_2)\).

**Stage 3:** After observing the choices made in stages 1 and 2, residents select their preferred residential locations.

The migration equilibrium equation (3) enables us to define the implicit function for stage three:\(^10\)

\[
n_1 = n_1^2(x_1, x_2, q_1, q_2). \tag{15}
\]

Assuming an interior solution, we obtain the following first order conditions in the center’s maximization problem:\(^11\)

\[
\begin{align*}
\theta u_x^1 + \lambda \left( f_n^1 - f_n^2 - x_1 + x_2 \right) \frac{u_x^1}{2a} - n_1 &= 0, \tag{16a} \\
(1 - \theta) u_x^2 + \lambda \left( - f_n^1 - f_n^2 - x_1 + x_2 \right) \frac{u_x^2}{2a} - n_2 &= 0, \tag{16b}
\end{align*}
\]

where \(\lambda > 0\) is the shadow value of an additional unit of aggregate income for the federation.

Combining (16a) with (16b) yields the efficient unit of aggregate income or the federation. We next utilize (12) to define the implicit function:

\(^9\)The implicit migration-response function for \(n_j\) displays a superscript “2” in order to distinguish it from the migration-response function utilized in the Nash game of Section 2.

\(^10\)The corresponding explicit functions for (15), along with all other derivations for this section are included in Appendix 3.

\(^11\)We assume that the local sufficient second order condition is satisfied in the maximization problem of the second stage.
In the first stage of the game, the regional governments choose their contributions to the public good taking into account the implicit functions (15) and (17). Assuming that \( \{q_j > 0\}_{j=1,2} \), the first order conditions that characterize the Nash equilibrium in the first stage for regions 1 and 2, respectively, are as follows:\(^{12}\)

\[
u^1_x \frac{\partial x_1}{\partial q_1} + u^1_q + u^1_Q - a\Omega_1 = 0, \tag{18a}
\]

\[
u^2_x \frac{\partial x_2}{\partial q_2} + u^2_q + u^2_Q + a\Omega_2 = 0, \tag{18b}
\]

where \( \Omega_1 = \left( \frac{\partial n_1}{\partial x_1} \frac{\partial x_1}{\partial q_1} + \frac{\partial n_1}{\partial x_1} \frac{\partial x_1}{\partial Q} + \frac{\partial n_1}{\partial x_2} \frac{\partial x_2}{\partial q_1} + \frac{\partial n_1}{\partial x_2} \frac{\partial x_2}{\partial Q} + \frac{\partial n_1}{\partial q_1} + \frac{\partial n_1}{\partial Q} \right) \) and

\[ \Omega_2 = \left( \frac{\partial n_1}{\partial x_1} \frac{\partial x_1}{\partial q_2} + \frac{\partial n_1}{\partial x_1} \frac{\partial x_1}{\partial Q} + \frac{\partial n_1}{\partial x_2} \frac{\partial x_2}{\partial q_2} + \frac{\partial n_1}{\partial x_2} \frac{\partial x_2}{\partial Q} + \frac{\partial n_1}{\partial q_2} + \frac{\partial n_1}{\partial Q} \right). \]

Combining (18a) with (18b) results in following conditions governing the allocations of the impure public good in regions 1 and 2, respectively:

\[
n_1 \frac{u^1_q}{u^1_x} + n_1 \frac{u^1_Q}{u^1_x} = -n_1 \frac{\partial x_1}{\partial q_1} + \frac{an_1\Omega_1}{u^1_x}, \tag{19a}
\]

\[
n_2 \frac{u^2_q}{u^2_x} + n_2 \frac{u^2_Q}{u^2_x} = -n_2 \frac{\partial x_2}{\partial q_2} - \frac{an_2\Omega_2}{u^2_x}. \tag{19b}
\]

Combining (19a) with (19b) yields (10). The results for the decentralized leadership game with central control over income redistribution policy are summarized in Proposition 1:

**Proposition 1:** Given our modelling assumptions, the subgame perfect equilibria for the decentralized leadership game are inefficient, provided that \( \{q_j > 0\}_{j=1,2} \).

\(^{12}\)We assume that the sufficient second-order conditions are satisfied in the maximization problems of the first stage.
Proposition 1 departs from the result for pure public goods in Caplan et al. (2000), where it was found that the central government’s income-redistribution policy under decentralized leadership is adequate to ensure the efficient allocation of public good provision across regions. Here, the center’s income redistribution policy—associated, for example, with the implementation of structural and cohesion policies in the European Union—does not obey the principle of complete “additionality,” whereby the central government offers enough additional resources to advance to a social optimum the structural and cohesion policies designed and executed by national governments. Proposition 1 says that, when the center’s sole policy instrument is income redistribution in response to regional policies—such as the structural and cohesion policies—imperfect labor mobility and the impureness of the public good, inherent characteristics of the E.U.’s economy, are impediments for efficient policy making at the regional governmental level.

Intuitively, Proposition 1 holds because the egoistic components of the impure good are not fully internalized in each region prior to location decisions. As we discussed in the Introduction, Wellisch (1994) teaches us that household attachment matters for efficiency only if location choices are made in the presence of transboundary externalities. In our decentralized leadership game, the interregional redistribution policy of the center does not have enough power to completely nullify the regional governments’ incentives to underprovide the public good. Therefore, when it comes time for individuals to decide where they should establish their residences, in the third stage of the game, the externalities have not been fully internalized. An immediate implication of this intuitive argument is that any subgame perfect equilibrium with positive contributions is inefficient regardless the degree of labor mobility:
Theorem 1: For the decentralized leadership game, any subgame perfect equilibrium with positive contributions is inefficient regardless of the level at which residents are attached to regions.

Proof: In light of Proposition 1 we need only prove that any subgame perfect equilibrium, with positive contributions, for the decentralized leadership game is inefficient when \( a = 0 \). If \( a = 0 \), the first order conditions of the first stage reduce to:

\[
\begin{align*}
    n_1 \frac{u^1_{x_1}}{u^1_x} + n_1 \frac{u^1_{Q_1}}{u^1_x} &= -n_1 \frac{\partial x_1}{\partial q_1}, \\
    n_2 \frac{u^2_{x_2}}{u^2_x} + n_2 \frac{u^2_{Q_2}}{u^2_x} &= -n_2 \frac{\partial x_2}{\partial q_2}.
\end{align*}
\]

Equations (20a) and (20b) imply (10). \( \blacksquare \)

5. Decentralized Leadership with Central Control of Income Redistribution and Lump Sum Matching Grants

As in Cornes and Silva (2000), assume that the central government can now make lump-sum matching grants to each region, denoted \( m_j, M = \sum m_j, j=1,2 \). Each region funds the matching grants program with budget-balanced lump-sum taxes to the central government, denoted \( c_j = m_j, \sum c_j = M \). Thus, the aggregate resource constraint may now be rewritten as:

\[
n_1x_1 + n_2x_2 + p_1 + p_2 = f'(n_1) + f'(n_2),
\]

where \( p_j = q_j + m_j \) and \( P = \sum p_j, j=1,2 \). Similar to the previous game in Section 4, we consider a three-stage game whereby the regional governments, acting as Stackelberg leaders, precommit by selecting their contributions to the public good prior to the interregional income redistribution and matching grant policies of the center. The center determines its policies in the second stage of the game, after observing the public good contributions chosen by the regional governments and in anticipation of location choices made by residents. Residents make their location choices...
after they observe contributions to the public good, interregional redistributions of income, and matching grants.

The migration equilibrium condition with matching grants in now expressed as $u^1(x_1, p_1, P) + a(1 - n_1) = u^2(x_2, p_2, P) + an_1$, which, after substitution of (21) for $x_1$, may be re-expressed as:

$$u^1\left(\frac{f^1_n(n_1) + f^2_n(1-n_1) - (1-n_1)x_2 - P}{n_1}, p_1, P\right) + a(1 - n_1) = u^2(x_2, p_2, P) + an_1. \quad (22)$$

Equation (22) defines the implicit migration response functions: $n_1 = n_1(x_2, p_1, p_2). \quad (23)$

In stage two, the central government chooses both $x_j$ and $m_j, j=1,2$ to maximize its utilitarian social welfare function. Note that when the central government chooses $m_j, j=1,2$, each region's contribution to the public good is already pre-determined. Since the central government takes $q_j, j=1,2$, as given and chooses $m_j, j=1,2$, it in fact chooses $p_j, j=1,2$. Hence, the problem faced by the central government in the second stage of the game is to choose $\{x_2, p_{j=1,2}\}$ to maximize:

$$\theta u^1\left(\frac{f^1_n(n_1) + f^2_n(1-n_1) - (1-n_1)x_2 - P}{n_1}, p_1, P\right) + (1-\theta)u^2(x_2, p_2, P), \quad (24)$$

subject to (23). As Appendix 4 shows, this problem yields conditions (8) and (13) directly, which implies (14). Since this result holds for any $a \geq 0$, it may be re-stated in the following theorem.

**Theorem 2:** For the decentralized leadership game with lump-sum matching grants, any subgame perfect equilibrium with positive contributions is Pareto efficient regardless of the level at which residents are attached to regions.

---

13 The corresponding explicit functions, along with all other derivations for this section, are included in Appendix 4.
The intuition behind Theorem 2 is as follows. First, the center’s choice of $x_j$ ensures the efficient population distribution by removing any incentives that residents might have in relocating to obtain a higher income level. In other words, because they realize that the center’s income redistribution policy has endogenized their responses, the residents have no better option than to distribute themselves efficiently across the two regions. Second, by choosing its matching-grant policy after the regions’ choices of public good provision, the center effectively controls both the aggregate and regional amounts of the public good that will ultimately be produced. Knowing this, the regions’ choices in the initial stage are essentially redundant. Similar to their choices of how to distribute themselves with respect to population, the regions have no better option than to choose the efficient allocation of the public good, even though the public good provides ‘impure’ benefits. Thus, lump-sum matching grants promote the correct incentives, and thereby induce the regions to equalize their private MRS’s between the numeraire and public goods.\(^{14}\)

This result is striking for yet another reason. Previous work by Boadway et al. (1989) concluded that lump-sum matching grants are neutral, in that they leave unchanged the level of the public good in the new Nash equilibrium. Andreoni (1990) also found that lump-sum grants, funded by lump-sum taxation, are neutral for impure public goods. Only with per-unit subsidies was he able to find non-neutrality. These results are therefore in direct contrast to our’s. Not only are lump-sum grants non-neutral, but they are capable of restoring Pareto efficiency in a federation with decentralized leadership and imperfect labor mobility. Thus, we have also found

\(^{14}\)The comparative static solutions behind Theorem 2 also yield additional insight into this general result. For the case of $M \neq 0$, $\partial M/\partial Q = -1$, $\partial c_i/\partial q_i = -1$, $\partial c_i/\partial q_j = 0$, $i=1,2$. Thus, the central government’s optimal matching grants policy reduces each region’s lump-sum tax one-for-one for each additional unit of public good provision by the regions.
a corrective policy for the type of inefficiency first encountered by Wellisch (1994) in his model of a federation.

6. Conclusions

We have shown that in a federation such as the European Union, characterized by decentralized leadership and imperfect labor mobility, regional provision of impure public goods may be Pareto efficient. Theorem 2, indeed, tells us that a decentralized leadership game, whereby regional governments precommit to contributions to an impure public good in anticipation of the center’s interregional income and matching grant policies and subsequent labor mobility, has efficient subgame perfect equilibria as long as the regional governments provide positive amounts of the public good. This important result does not depend on the regional attachment benefits derived by the residents of the federation.

This result extends several branches of the public goods literature. For instance, it extends the Cornes and Silva (2000) result for matching grants to the case of imperfect labor mobility. It extends the Wellisch (1994) and Caplan et al. (2000) results to the case of impure public goods. It also extends the work on neutrality by Boadway et al. (1989) and Andreoni (1989, 1990) by qualifying the role that a federation’s structural hierarchy can play in ‘neutralizing the effects of neutrality’ by inducing an efficient provision of public goods in a decentralized leadership setting.

The standard assumption that all income generated within a region accrues only to residents of that region, on an equal per-capita basis, is employed here to derive all results. An interesting avenue for future work would be to consider situations whereby migrants retain possession of their non-human wealth when they move from one place to another, since this appears to be ubiquitous. In such an extension, migrants and non-migrants would typically be
heterogeneous, a factor that may generate pecuniary externalities and diversity of incentives within regions. Our intuition, however, tells us that our main result—Theorem 2—would remain valid in this more complex setting, since the interregional policies of the center would induce the regional governments to adopt policies that maximize the federation’s total income. Therefore, all externalities would be completely internalized and individual wealth levels would not be sensitive to migration decisions.
Appendix 1

To derive (8), begin with (5) for region 1. Substitute (4a) into (5) and rearrange to get:

\[-\frac{2an_2}{u_x^2} \leq (f_n^i - x_1) - (f_n^2 - x_2).\]  

(A1)

Similarly for region 2, substitute (4b) into (5) and rearrange to get:

\[-\frac{2an_1}{u_x^1} \geq (f_n^i - x_1) - (f_n^2 - x_2).\]  

(A2)

Bring (A1) and (A2) together to get (8). Substituting (4c) into (6) and rearranging yields (9) for region 1, and substituting (4d) into (7) and rearranging likewise yields (9) for region 2.

Appendix 2

The center chooses \(\{x_1, x_2, q_1, q_2, n_1\}\) to maximize the Lagrangian:

\[
\theta u^1(x_1, q_1, Q) + (1-\theta) u^2(x_2, q_2, Q) + \lambda \left( f^1(n_1) + f^2(n_2) - n_1 x_1 - n_2 x_2 - Q \right) \\
+ \Psi \left( u^1(x_1, q_1) + a(1-n_1) - u^2(x_2, q_2) - an_1 \right).
\]

The first-order conditions for this problem are:

\[
\theta u_x^1 - n_1 \lambda + \Psi u_x^1 = 0, \tag{B1}
\]

\[
(1-\theta) u_x^2 - n_2 \lambda - \Psi u_x^2 = 0, \tag{B2}
\]

\[
\theta u_q^1 + \theta u_Q^2 + (1-\theta) u_Q^2 - \lambda + \Psi \left( u_q^1 + u_Q^2 - u_Q^2 \right) = 0, \tag{B3}
\]

\[
\theta u_Q^1 + (1-\theta) u_q^2 + (1-\theta) u_q^2 - \lambda + \Psi \left( u_Q^1 - u_q^2 - u_Q^2 \right) = 0, \tag{B4}
\]

\[
\lambda \left( f_n^i - f_n^2 - x_1 + x_2 \right) - 2a \Psi = 0. \tag{B5}
\]

Equations (B1) and (B2) imply

\[
\frac{u_x^1}{n_1} (\theta + \Psi) = \frac{u_x^2}{n_2} (1-\theta - \Psi). \tag{B6}
\]
Successive substitution of (B1) into (B3) and then (B6) results in (13) for region 1. Similar manipulations of (B2), (B4), and (B6) results in (13) for region 2. Solving (B1) and (B2) for $\lambda$ and $\Psi$ yields, respectively,

\[
\lambda = \frac{u_x^1 u_x^2}{n_x u_x^1 + n_i u_x^2}, \quad \text{(B7)}
\]

\[
\Psi = \frac{n_i (1 - \theta) u_x^2 - n_x \theta u_x^1}{n_x u_x^1 + n_i u_x^2}. \quad \text{(B8)}
\]

Substituting (B7) and (B8) into (B5) yields (8).

**Appendix 3**

A straightforward exercise in comparative statics yields the following migration response functions from (3):

\[
\frac{\partial n_i^2}{\partial x_i} = \frac{u_x^1}{2a} > 0, \quad \text{(C1)}
\]

\[
\frac{\partial n_i^2}{\partial x_2} = -\frac{u_x^2}{2a} < 0, \quad \text{(C2)}
\]

\[
\frac{\partial n_i^2}{\partial q_1} = \frac{u_q^1 + u_{Q_i}^1 - u_{Q_i}^2}{2a}, \quad \text{(C3)}
\]

\[
\frac{\partial n_i^2}{\partial q_2} = \frac{u_{Q_i}^1 - u_x^2 - u_{Q_i}^2}{2a}. \quad \text{(C4)}
\]

Similarly, total differentiation of (12), accounting for (15), yields the following income-redistribution response functions:

\[
\frac{\partial x_i}{\partial q_1} = -\left(\frac{u_q^1 + u_{Q_i}^1 - u_{Q_i}^2}{2a}\right)\left(x_i - x_2 + f_n^2 - f_n^1\right) + \frac{u_x^1}{2a}\left(x_i - x_2 + f_n^2 - f_n^1\right) + n_i. \quad \text{(C5)}
\]
\[
\frac{\partial x_1}{\partial q_1} = \frac{(u_1^0 - u_1^2 - u_2^2)}{2a} (x_1 - x_2 + f_n^2 - f_n^1) + 1
\]
\[
\frac{\partial x_1}{\partial q_2} = \frac{u_1^1}{2a} (x_1 - x_2 + f_n^2 - f_n^1) + n_1
\]
\[
\frac{\partial x_2}{\partial q_1} = \frac{(u_1^1 + u_1^1 - u_2^2)}{2a} (x_1 - x_2 + f_n^2 - f_n^1) + 1
\]
\[
\frac{\partial x_2}{\partial q_2} = \frac{-u_2^2}{2a} (x_1 - x_2 + f_n^2 - f_n^1) + n_2
\]
\[
\frac{\partial x_1}{\partial Q} = \frac{u_1^1}{2a} (x_1 - x_2 + f_n^2 - f_n^1) + n_1
\]
\[
\frac{\partial x_2}{\partial Q} = \frac{-u_2^2}{2a} (x_1 - x_2 + f_n^2 - f_n^1) + n_2
\]

Substituting (C5)-(C10) into the expressions for \(Q_j, j=1,2\) proves that (19a) and (19b) yield (10).

\[\blacksquare\]

**Appendix 4**

A straightforward exercise in comparative statics yields the following migration response functions from (22):

\[
\frac{\partial n_1}{\partial x_2} = \frac{n_2 u_1^1 + n_1 u_2^2}{D} < 0,
\]

(D1)
\[
\frac{\partial n_1}{\partial p_1} = \frac{u_x^1 + n_1 u_p^2 - n_1 (u_p^1 + u_p^2)}{D}, \tag{D2}
\]
\[
\frac{\partial n_2}{\partial p_2} = \frac{u_x^1 - n_1 u_p^1 + n_1 (u_p^2 + u_p^2)}{D}, \tag{D3}
\]

where \( D = u_x^1 \left( f_n^1 - f_n^2 + x_2 - x_1 \right) - 2an_1 < 0 \) for a stable migration equilibrium. In the second stage, maximization of (24) with respect to \( \{x_2, p_{j}\}_{j=1,2} \) and subject to (22) and (D1) - (D3) yields (13) and thus (14).
References


