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Hydraulics of Large Bed Element Channels

Harl E. Judd
Dean F. Peterson

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Hydraulics of Large Bed Element Channels

Utah Water Research Laboratory/College of Engineering
By Hull E. Judd and Dean F. Peterson
August 1969
HYDRAULICS
OF
LARGE BED ELEMENT
CHANNELS

by
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Utah Water Research Laboratory
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Logan, Utah 84321

August 1969

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ACKNOWLEDGMENTS

Experimental work on *Large Bed Element* streams began in 1958 with Dean F. Peterson as project leader under grants EC-15 of the Utah State University Engineering Experiment Station and U-80 of the University Division of Research. The latter grant continued through 1963. In 1962, a grant from the Intermountain Forest and Range Experiment Station of the United States Forest Service permitted field work to begin and a National Science Foundation grant in late 1962 permitted the work to be expanded significantly. During the period of study, dissertations and theses based on laboratory studies were produced by the following: P. K. Mohanty, PhD; A. G. Mirajgaoker, PhD; A. O. Attieh, MS; A. N. Al-Khafaji, PhD; N. S. Kharrufa, PhD; Davoud Hariri, PhD; and M. W. Abdelsalam, PhD; and Julian Anderson, PhD.

The field study was conducted by Harl E. Judd and the first phase resulted in a PhD dissertation in 1963. The field work continued through 1965 and forms the basis for this publication, which has drawn heavily on the laboratory studies in interpreting and generalizing the field results. All of the work has been conducted as a project of the Utah Water Research Laboratory.

Harl E. Judd
Dean F. Peterson
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NOTATION

The following symbols are used in this paper:

- \( a \) = a constant;
- \( a, b, c \) = lengths of principal axes of a particle (L);
- \( A_v \) = area (L²);
- \( A \) = vertically projected area of bed elements (L²);
- \( b, f, f^1, m \) = exponents in Leopold-Maddock equation (1);
- \( B \) = a constant;
- \( B_o \) = a coefficient in the bed element spacing equation accounting for shape and size distribution (1);
- \( c, c^1, p \) = constants in Leopold-Maddock equation (1);
- \( C \) = constant in Chezy's equation (L¹/²T⁻¹);
- \( C_D \) = coefficient of drag (1);
- \( C_2, C_1 \) = constants (1);
- \( C_{sf} \) = channel shape factor (1);
- \( d \) = depth of flow, pipe diameter, (L);
- \( d_e \) = effective stream depth (L);
- \( D \) = statistical depth of flow (L);
- \( EWA \) = effective wetted area (L²);
- \( f, f_1, f_2, \ldots \) = indicates a functional relationship;
- \( F \) = force (F);
- \( F_D \) = force due to form drag (F);
- \( F_f \) = boundary force resisting flow (F);
- \( F_i \) = \((i)th\ force\ (F)\);
- \( F_p \) = propelling force (F);
- \( F_s \) = force due to shear resistance (F);
- \( g \) = acceleration of gravity (LT²);
- \( h \) = an exponent (1);
- \( H, H_s, H_d, H_{d/k} \) = a function, a function of \( s, \theta, d/k \) respectively;
- \( i \) = a number in series;
- \( l_{min} \) = bed element spacing parameter (1);
- \( I \) = minimum value of spacing parameter \( I \) (1);
- \( k \) = roughness or bed element height or diameter (L);
- \( k_n \) = minimum height of bed elements larger than the percentile \( n \) (L);
- \( k_1 \) = a constant;
- \( \zeta \) = length longitudinal to bed (L);
- \( L \) = step spacing (L);
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<tr>
<td>( n )</td>
<td>number;</td>
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<tr>
<td>( N )</td>
<td>number of bed elements;</td>
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<td>( N_F )</td>
<td>Froude number (1);</td>
</tr>
<tr>
<td>( N_R )</td>
<td>Reynolds number (1);</td>
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<tr>
<td>( P )</td>
<td>total pressure (F or FL(^{-1}));</td>
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<tr>
<td>( Q )</td>
<td>stream discharge (L(^3)T(^{-1}));</td>
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<tr>
<td>( R )</td>
<td>hydraulic radius (L);</td>
</tr>
<tr>
<td>( S )</td>
<td>hydraulic slope (1);</td>
</tr>
<tr>
<td>( t )</td>
<td>an exponent (1);</td>
</tr>
<tr>
<td>( u )</td>
<td>an exponent, uniformity coefficient (1);</td>
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<tr>
<td>( V )</td>
<td>velocity (LT(^{-1}));</td>
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<td>( w )</td>
<td>width transverse to bed (L);</td>
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<td>( W )</td>
<td>weight of a prism of water (F or FL(^{-1}));</td>
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<td>( W_A )</td>
<td>actual wetted area (L(^2));</td>
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<td>( \Gamma_o )</td>
<td>unit tractive force (FL(^{-2}));</td>
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<td>( \mu )</td>
<td>dynamic viscosity (ML(^{-1})T(^{-1}));</td>
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<tr>
<td>( \rho )</td>
<td>density (ML(^3));</td>
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<tr>
<td>( \tau_o )</td>
<td>unit shear force (FL(^2));</td>
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<td>( \chi )</td>
<td>a roughness parameter (L).</td>
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INTRODUCTION

Engineers and geologists have long been concerned with the shapes and hydraul­ic resistance of streams which form their own channels. This interest extends also to artificial channels which may erode their beds and their banks to different con­figurations than their designers intended. All streams transmit both water and sedi­ment and the shape of the channel must reflect a balance between the power of the stream, its sediment load, and its shape. Most attention of hydraulic engineers has been given to streams in relatively fine sediments. This publication deals with phenomena observed in streams flowing in sediments containing large elements, derived from valley walls or ancient fluvial or glacial action, which are moved, if at all, only under conditions of extreme flood. Unlike those in finer sediments, the beds of such streams remain relatively fixed between major flood events and, most of the time, transport of sediments is small.

Hydraulically, for wide channels, the important variables are slope, depth of flow (hydraulic radius approximately) and roughness. Using these variables, channels may be characterized into four broad groups: 1) mild slopes, large depth relative to roughness; 2) steep slopes, large relative depth; 3) mild slopes, depth of the same order of magnitude as roughness elements (small relative depth); 4) steep slopes, small relative depth. Streams with relatively small bed elements in the first two classes may appear rough when dunes form.

The streams discussed herein fall into the last two categories, principally the latter one. A complete bibliography and suggested flow formula for flow in channels having comparatively large relative roughness has been published by the Task Force on Friction Factors in Open Channels of the Committee on Hydromechanics of the Hydraulics Division, American Society of Civil Engineers (1963). With large relative roughness heights the commonly accepted logarithmic velocity distribution does not apply. Though relatively little attention has been given to flow characteristics in streams in the last two groups, the existence of such streams is common. Boulder and cobble bed streams exist in mountainous areas throughout the world (see Fig. 1).

In 1958, Utah State University began studies of channels with steep slopes where the roughness elements were large with respect to depth. Peterson and Mohanty (1961), using large regularly spaced bar and cube roughness elements with slopes up to 8.5 percent in a flume, classified flow into three different regimes based on general observations of natural streams and laboratory studies (Mohanty, 1959). In the tranquil and rapid regimes, roughness spacing was clearly an important factor, and discharge seemed to be a power function of slope. In the tumbling regime, flow was accelerated to supercritical velocity near the large bed elements and formed hydraulic jumps downstream thus creating energy sinks or losses of head due to spills and consequently decreasing dependence on slope.

Attieh (1960) and Mirajgaoker (1961), continuing Mohanty's work, made detail­ed studies of flow patterns, pressure distribution, and drag on single elements. Attieh
Figure 1. Large bed element channels of Logan River and Providence Creek.
used a cubical element and Mirajgaoker used cubical, hemispherical, and circular disk elements.

Al-Khafaji (1961) further studied the flow regimes, using bar elements. Additional criteria for flow regimes were proposed, and an unstable regime was studied in which large traveling waves formed. Detailed information on velocity and pressure distribution in the stream was collected.

Until 1961 the work at Utah State University used geometrically shaped roughness elements. In an attempt to relate the previous research to natural channels, Kharrufa (1962) cemented large graded gravel elements to removable beds installed in the flume. A mean velocity formula for the tranquil and tumbling regimes and a formula for the rapid or skimming regime was obtained.

Beginning in 1962 with a small grant from the Intermountain Forest and Range Experiment Station of the United States Forest Service and a later grant from the National Science Foundation, the work was carried to the field by Judd.

Judd (1963) reported an initial study describing the size and spacing of elements occurring in natural streams and has since conducted additional extensive field studies. Hariri (1964) studied the bed characteristics of channels formed in alluvial material in a laboratory flume. Abdelsalam (1965) made further flume studies of flow over beds similar to those of Kharrufa. Anderson (1968) also using beds similar to Abdelsalam's, conducted studies using wind tunnel techniques to study Reynolds effects and to eliminate the free surface.

This paper discusses theoretical means of characterizing size, size distribution, spacing, and shape of bed elements in hydraulically significant terms for large bed element streams; these are related to flume experiments using simulated beds. Observed bed and flow characteristics are reported for a number of natural-stream study sites and the effect of roughness, depth, and slope on flow characteristics of natural streams are analyzed. Finally, possibilities for use of large bed elements in open channel design are suggested.

General description of flow characteristics

For relatively small roughness elements, boundary-layer theory provides a well known picture of skin resistance effects in turbulent flow. If the roughness height does not exceed the thickness of the laminar sublayer, the wall is hydraulically smooth and the velocity depends on viscosity and varies logarithmically with shear velocity and the distance from the wall, Fig. 2a. If the roughness extends through the laminar sublayer, the velocity varies logarithmically with $y/k$, where $y$ is the distance from the wall and $k$ is the roughness height, Fig. 2b. For larger elements there are resistances due to internal distortion in both pipes and open channels. In open channels additional losses occur because of spills at local obstructions (Leopold, Wolman, and Miller, 1964). Large bed elements may cause internal distortion losses that exceed skin resistance losses and under extreme tumbling flow conditions spill losses dominate. Morris (1955) suggested concepts particularly pertinent to consideration of both internal distortion and spill resistances. Isolated roughness flow occurs when the roughness elements are sufficiently far apart that wakes are dissipated before the next element is encountered. In this case, the height of the element is
Figure 2. Hydraulic boundary descriptions.
of primary importance. In wake interference flow the spacing is such that wakes from the elements interfere and spacing becomes more important. In skimming flow the elements are sufficiently close that the flow skims over the spaces between them and a hydraulically smooth condition is approached.

Under conditions of uniform flow with rough boundaries, the velocity distribution is the same, within limits of measurement, for each successive cross section. As elements become increasingly large this is no longer the case. For closed conduit flow as the roughness height increases in relation to conduit size, the degree of mixing becomes more pronounced, Fig. 2c. Zones of separation, acceleration, and deceleration occur and the flow becomes observably nonuniform and unsteady, that is, internal distortion resistance is evident. However, the energy loss along this type of conduit has "on the average" a linear energy grade line.

For open channel flow, where there is a free surface, the variation in velocity distribution becomes even more apparent as bed element sizes increase, Fig. 2d. While nonuniform and unsteady on a small scale, the flow may be macroscopically uniform within a reach providing the roughness pattern has uniform statistical characteristics within that reach. Even though the bed is what is normally considered "very rough," statistical means can be used to examine and define both the boundary and the flow itself. The degree to which practical "statistically uniform" channels may exist in the field depends upon local conditions and the extent to which significant statistical descriptions may be identified and simplified.

Definitions

Large Bed Element (LBE) channels are those channels in which the "bed elements" extend through a major portion of the flow depth. High-gradient LBE channels have sufficiently large slopes so that significant surface disturbances occur at some stages of discharge. The term "bed element" is used rather than roughness since the action causes local accelerations and decelerations more nearly like channel section changes than like the skin resistance associated with smaller elements. From a physical point of view, however, the channel boundaries are "rough." Thus the term roughness will be used interchangeably where convenient to the description. The term "high-gradient" is used rather than "steep" since hydraulically "steep" channels are associated with supercritical flow, which may not be the case for the channels under discussion.

Bed elements are defined as the individual elements which collectively constitute the beds of streams. Analysis of flow in high-gradient LBE channels requires a statistical sample and hence the term macroscopic uniform flow is justified.

In nature, high-gradient, LBE streams usually become paved with large boulders and such channels may approach a "fixed bed" condition insofar as the large bed elements are concerned, although there may be some "relaxation" effect at the boundary due to motion of the smaller particles. Only at infrequent extreme discharges will the dominant elements be moved. Primarily this paper will deal with the hydraulics of these streams at less-than-channel-forming discharges.
CHANNEL BED PARAMETERS

In rough conduit flow a detailed description of the boundary is of primary importance. This should include height, spacing, shape, pattern, and size gradation of the elements. Frequently only the height is considered and the other variables are either held constant or assumed not to affect the flow resistance.

Relative height of the elements is normally expressed as the relative roughness \( \frac{d}{k} \), where \( d \) is the depth of flow or the pipe diameter and \( k \) is the roughness height. Several approaches have been made toward characterizing spacing. Longitudinal spacing, particularly in the case of two-dimensional flow, has been expressed in several ways including the ratio of the plan area of the elements to the flume bed area and the volume of the bed elements to the volume of the bed compared with the volume of the elements at maximum areal compaction. The volume ratio is quite satisfactory in the case of cubes as it varies from zero with no cubes to 1.0 with a maximum number of cubes. At both extremes the resistance to flow is the same.

Perhaps more applicable from a drag resistance point of view is the ratio \( \theta \) of the vertical projected area of the boundary elements \( \Sigma A_v \) to the plan area \( A \) in which they occur. This seems a logical approach since resistance to flow includes both shear friction and pressure or form drag on the larger elements. Thus

\[
\theta = \frac{\sum A_v}{A} \tag{1}
\]

The nature of parameter, \( \theta \), may be examined first assuming basic geometric shapes and then extending definitions and results to natural channels. In nearly all laboratory work to date only geometric elements have been used. \( \theta \) includes spacing, shape, and gradation. One cube of size \( k \) is associated with a boundary area \( w \ell \). For \( w \) and \( \ell \) equal to \( 3k \), \( \theta \) equals 1/9 as shown in Fig. 3. For four cubes in the same area \( \theta \) equals 4/9.

In a natural stream \( \theta \) is more difficult to determine. Suppose a large tray is covered with randomly spaced spheres all having the same diameter. Select an area and count all the spheres in this area. Repeat this process for several areas of different sizes on the tray. The slope of the curve resulting from plotting the area against the number of elements counted, i.e. the area per single bed element, is a measure of spacing. Extending this same idea to stream beds, the number of bed elements with heights greater than a certain amount contained within a certain area could be counted. This number divided by the area would be the cumulative spacing; cumulative in the sense that all of those elements of the size indicated as well as all those larger would be included.

Where there is a gradation of sizes, the spacing parameter must include the minimum size \( k_n \) of the bed elements counted as well as the number \( N \) of such bed elements in the area \( A \). The subscript \( n \) is the percentile in the distribution curve.
for the set having minimum size \( k_n \). For example, \( k_{g0} \) is the size such that 90 percent of the elements are larger than \( k_{g0} \). Area and size may be combined in the form \( (A)^{1/2}/k_n \). As was shown by Judd (1963, ch. 4) a power relationship results in the form

\[
A^{1/2}/k_n = IN^u
\]

(2)

where \( I \) is the spacing and \( u \) is the uniformity coefficient. \( I \) is the value of \( (A)^{1/2}/k_n \) when \( N = 1 \) and \( I \) is therefore a measure of the relative area associated with one bed element. The larger the value of \( I \) the greater the spacing between bed elements.

The uniformity coefficient measures the uniformity with which the bed elements are distributed in the area of bed or reach of stream observed. Consider again the tray with randomly spaced spheres and plot \( A \) against \( N \). If the sample is sufficiently large and the distribution is indeed random, the number \( N \) should be in direct proportion to the area, i.e. \( A \sim N \). Also as \( k_n \) is constant for any particular \( n \), then

\[
A^{1/2}/k_n \sim N^{1/2}
\]

(3)

Thus, the theoretical value of \( u \) should be 1/2. If \( u \) is different than 1/2, the distribution of the bed elements is not statistically uniform over the entire area considered; or, as might be in the case of natural channels, some bed elements were missed when measurements were taken; or the sample taken may be too small. The observed value of \( u \) is helpful in practical application of the results; one would expect best results to occur at sites where \( u \) is nearly 1/2. With \( u = 1/2 \)

\[
1/I^2 = Nk_n^2/A
\]

(4)
Both Judd (1963), using data from natural LBE streams, and Abdelsalam (1965), using data from simulated stream beds consisting of graded gravels glued to a flume floor, showed that the reference size n chosen doesn't significantly affect the value of I. $Nk_n^2$ is a measure of the summation of the vertical projected area as defined in Eq. 1 so

$$\sum A_v = B_o k_n^2 N \ldots \ldots \ldots \ldots \ldots$$

where $B_o$ is determined by the shape of the bed elements and the gradation of the sample. Combining Eqs. 1, 4, and 5 gives

$$\theta = B_o / I^2 \ldots \ldots \ldots \ldots \ldots$$

**Shape of the bed elements**

$I$ is independent of bed element shape, i.e., with the same heights and spacing pattern, the value of $I$ will be the same regardless of geometric shape. To illustrate, consider for example, pattern I of the flume experiments of Herbich and Shulits (1964). In this experiment staggered transverse rows of 6-inch cubes on 12-inch centers were spaced at 12 inches. Successively replace each cube with spheres of 6-inch diameter, hemispheres with 6-inch radius, and rectangles 12 inches wide and 6 inches high, as shown in Fig. 4.

The vertical projected area of a single bed element for the different shapes is

<table>
<thead>
<tr>
<th>Shape</th>
<th>$A_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td>$k^2$</td>
</tr>
<tr>
<td>Sphere</td>
<td>$\pi k^2 /4$</td>
</tr>
<tr>
<td>Hemisphere</td>
<td>$\pi k^2 /2$</td>
</tr>
<tr>
<td>Rectangle</td>
<td>$2k^2$</td>
</tr>
</tbody>
</table>

![Figure 4. Different shapes of bed element of height k.](image-url)
Calculating and \( l \) gives \( l = 2.0 \) for all shapes, and \( \theta \) as follows:

\[
\begin{array}{cccc}
\text{Spheres} & \text{Cubes} & \text{Hemispheres} & \text{Rectangles} \\
0.196 & 0.250 & 0.393 & 0.500 \\
\end{array}
\]

Knowing that \( \theta = B_0 l^2 \) and plotting, (Fig. 5) the equations resulting between \( \theta \) and \( l \) for pattern I are:

\[
\begin{align*}
\theta &= 0.784 l^2 & \text{Spheres} \\
\theta &= 1.000 l^2 & \text{Cubes} \\
\theta &= 1.570 l^2 & \text{Hemispheres} \\
\theta &= 2.000 l^2 & \text{Rectangles, } b = 2k
\end{align*}
\]

For uniform-sized elements the values of \( B_0 \) are the ratios of the projected area of a single element to \( k^2 \).

Gradation

To examine the effect of gradation on the magnitude of \( B_0 \), a size-graded set of bed elements all having the same shape may be considered, for example, cubes. Data of Herbich and Shulits (1964), Mohanty (1959), and Abdelsalam (1965) are plotted in Fig. 6. For the regular patterns of Herbich and Shulits, spacing between blocks varied from \( k \) to \( 4k \).

Table 1. \( l \) and \( \theta \) for Herbich and Shulits cubes

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Cube Size</th>
<th>( l )</th>
<th>( \theta )</th>
<th>Remarks on Distribution and Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>6&quot;</td>
<td>2.0</td>
<td>0.250</td>
<td>Uniform (Regular Pattern)</td>
</tr>
<tr>
<td>II</td>
<td>6&quot;</td>
<td>2.5</td>
<td>0.160</td>
<td>Uniform (Regular Pattern)</td>
</tr>
<tr>
<td>III</td>
<td>6&quot;</td>
<td>4.0</td>
<td>0.062</td>
<td>Uniform (Regular Pattern)</td>
</tr>
<tr>
<td>IV</td>
<td>6&quot;</td>
<td>5.0</td>
<td>0.040</td>
<td>Uniform (Regular Pattern)</td>
</tr>
<tr>
<td>V</td>
<td>6&quot;</td>
<td>3.0</td>
<td>0.111</td>
<td>Uniform (Regular Pattern)</td>
</tr>
<tr>
<td>IX</td>
<td>6&quot;</td>
<td>2.90</td>
<td>0.117</td>
<td>Uniform (Random Space)</td>
</tr>
<tr>
<td>X</td>
<td>6&quot;</td>
<td>2.37</td>
<td>0.174</td>
<td>Uniform (Random Space)</td>
</tr>
<tr>
<td>XI</td>
<td>3.75 &amp; 6&quot;</td>
<td>3.60</td>
<td>0.116</td>
<td>Two sizes (Random Spacing)</td>
</tr>
</tbody>
</table>

Table 2. \( l \) and \( \theta \) for Mohanty's 0.3 ft. cubes

<table>
<thead>
<tr>
<th>Longitudinal Spacing</th>
<th>Transverse Spacing</th>
<th>( l )</th>
<th>( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75 ft.</td>
<td>0.6 ft.</td>
<td>2.74</td>
<td>0.133</td>
</tr>
<tr>
<td>1.50 ft.</td>
<td>0.6 ft.</td>
<td>3.87</td>
<td>0.067</td>
</tr>
<tr>
<td>2.25 ft.</td>
<td>0.6 ft.</td>
<td>4.74</td>
<td>0.044</td>
</tr>
</tbody>
</table>
Figure 5. Effect of shape on relation between $I$ and $\theta$. 

- Spheres
- Cubes
- Hemispheres
- Rectangles $b = 2k$

The graph shows four lines representing different equations:
- $\theta = 2.0 I^{-2}$
- $\theta = 1.57 I^{-2}$
- $\theta = 1.0 I^{-2}$
- $\theta = 0.784 I^{-2}$
Figure 6. Effect of gradation on relation between I and $\theta$. 

CUBES

<table>
<thead>
<tr>
<th>Size</th>
<th>Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>6&quot;</td>
<td>uniform - regular</td>
</tr>
<tr>
<td>0.3&quot;</td>
<td>uniform - regular</td>
</tr>
<tr>
<td>6&quot;</td>
<td>uniform - random</td>
</tr>
<tr>
<td>8.75&quot; &amp; 6&quot;</td>
<td>two sizes - random</td>
</tr>
</tbody>
</table>

$\theta = 2.5 \, I^{-2}$

$\theta = 1.5 \, I^{-2}$

$\theta = 1.0 \, I^{-2}$
Table 3. Abdelsalam's bed elements as cubes

<table>
<thead>
<tr>
<th>Bed No.</th>
<th>201</th>
<th>203</th>
<th>210</th>
<th>401</th>
<th>403</th>
<th>410</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2.33</td>
<td>4.02</td>
<td>7.36</td>
<td>2.20</td>
<td>3.82</td>
<td>7.09</td>
</tr>
<tr>
<td>$$\theta$$</td>
<td>0.471</td>
<td>0.157</td>
<td>0.048</td>
<td>0.551</td>
<td>0.183</td>
<td>0.054</td>
</tr>
</tbody>
</table>

The first digit represents the maximum size and the last digit the inverse relative areal compaction of the bed elements; for example 203 had 2” maximum size elements, but the maximum amount of gravel which could be spread on one square foot one element deep is spread on 3 square feet.

Using the same size distribution as Abdelsalam (normal distribution of numbers of size elements), but assuming the bed elements to be cubes of sizes equal to the Abdelsalam sieve openings, gives values for I and $$\theta$$ shown in Table 3. Values of $$\theta$$ as a function of I are plotted in Fig. 6. The resulting curves show the influence of gradation on the constant $$B_0$$ as follows.

$$\theta = I^{-2}$$ Uniform distribution ............................................. (7)

$$\theta = 1.5 \cdot I^{-2}$$ Two sizes ............................................. (8)

$$\theta = 2.5 \cdot I^{-2}$$ Normal distribution ............................................. (9)

Changing from a uniform distribution to a normal distribution causes a 2.5 fold increase in $$B_0$$. The effect of gradation can be extended to other shapes by multiplying the shape factors by the above distribution factors since the parameters are purely geometric.

Simulated natural channels

Values of $$\theta$$ and I as listed in Table 4 were obtained from Abdelsalam’s ten simulated natural beds, Fig. 7. $$\theta$$ was determined by assuming that the bed elements are spheres, calculating the projected area of one element representing each size range and multiplying by the number of elements of that size. The projected areas of each size range were summed and divided by the area of the bed. I was calculated from the equation

$$I = A^{1/2} / N_k n$$ ............................................. (10)

Fig. 8 shows a plot of I and $$\theta$$ for Abdelsalam’s beds. The resulting equation is

$$\theta = 2.1 \cdot I^{-2}$$ ............................................. (11)

which gives an increase of 2.1/0.784 = 2.5 times in $$B_0$$ for the change from uniform to a normal distribution. A complete analysis of spacing parameters for beds of natural channels is given in Natural Channels section.
Table 4. Spacing parameters computed from Abdelsalam's beds

<table>
<thead>
<tr>
<th>Bed No.</th>
<th>θ</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>201</td>
<td>0.392</td>
<td>2.33</td>
</tr>
<tr>
<td>202</td>
<td>0.198</td>
<td>3.28</td>
</tr>
<tr>
<td>203</td>
<td>0.133</td>
<td>4.02</td>
</tr>
<tr>
<td>205</td>
<td>0.078</td>
<td>5.19</td>
</tr>
<tr>
<td>200</td>
<td>0.041</td>
<td>7.36</td>
</tr>
<tr>
<td>401</td>
<td>0.455</td>
<td>2.20</td>
</tr>
<tr>
<td>402</td>
<td>0.229</td>
<td>3.13</td>
</tr>
<tr>
<td>403</td>
<td>0.151</td>
<td>3.82</td>
</tr>
<tr>
<td>405</td>
<td>0.088</td>
<td>5.00</td>
</tr>
<tr>
<td>410</td>
<td>0.044</td>
<td>7.09</td>
</tr>
</tbody>
</table>
Figure 7. Abdelsalam’s simulated natural beds.
Figure 8. $\theta$ versus $I$ for Abdelsalam's beds as spheres.
FLOW RESISTANCE IN FLUMES

As previously mentioned, resistance to flow in LBE channels depends both on the skin friction or shear drag and the form drag resulting from differential pressure distribution on the roughness elements. The latter arises both from flow deformation and spills and is influenced to the degree by which the roughness elements are mutually screened by the wakes of other elements. In addition to skin friction and form drag theory, dimensional analysis may shed some light on the nature of flow resistance for the case at hand.

Wakes

Wakes formed behind the roughness elements are of interest since they may shadow downstream elements. If the elements are spaced sufficiently far apart each will act as an isolated roughness and its wake will not influence other elements. As the spacing decreases, resistance to flow increases up to a point where the wake from the upstream elements begins to have a reducing effect on the drag produced by the downstream elements. When spacing decreases further, resistance to flow decreases.

Goncharov (1963) states that when \( L/k = 11 \), where \( L \) is the distance in the direction of flow between large elements, interaction between elements no longer occurs. When this occurs the bed area occupied by the wake and the bed element is about \( 7k^2 \). For subcritical flow, Attieh (1960) showed that the ratio of the length of the wake to the height of the roughness is 8.2 while the corresponding area occupied by the element and the wake is \( 22k^2 \). For supercritical flow the spacing ratio was 12.5 while the area was \( 53k^2 \).

While the spacing ratios found by Goncharov and Attieh are of the same order of magnitude, the wake areas of Attieh are much greater. The difference in the two is basically the shape of the wake, and is probably caused by the difference in the shape of the element. Goncharov shows the wake becoming smaller as the distance from the element increases while Attieh shows it diverging. (See Fig. 9.) When maximum resistance occurs, the wake area should be influenced by relative roughness and whether the flow is subcritical or supercritical. The shape of the elements also influences the size of the wake.

![Figure 9](image_url)  
Wake areas for different shape elements.
Analysis of the results of Abdelsalam (1965) and Kharrufa (1962), who also used graded gravel elements cemented to the flume bed, shows that the maximum resistance occurred when $\theta = 0.10$ which is near $L/k = 4.7$. The relation between wake areas and the interaction effect of several elements is quite complex, however, the resulting spacing of 4.7 is about one-half the wake lengths mentioned by Goncharov and Attieh. Natural channels might tend to form beds such that the spacing parameter gives approximately maximum resistance or values of $\theta$ near 0.10. As shown later such a tendency is born out; however, $\theta$ is not independent of slope.

**Flow regimes**

As observed by Peterson and Mohanty, flow in large bed element channels can be classified as tranquil, tumbling, and rapid. The classification of flow depends on the discharge and slope and the height and spacing of the bed elements.

Tranquil flow is characterized by subcritical velocity throughout, and occurs only at slopes of about 3 percent or less. The depth of flow is always greater than critical depth. The free surface will be generally smooth and transparent but some capillary waves may form.

Tumbling flow is characterized over two-dimensional roughness elements in a flume, by alternate zones of subcritical and supercritical velocities. Flow is supercritical over each element (bar) and then changes, through a hydraulic jump, to subcritical giving the flow a tumbling appearance and hence its name. In flow over a bed with three-dimensional roughness elements distributed randomly, the cyclic order of subcritical to supercritical is not obvious and the definition must be extended as was done by Kharrufa (1962). Regions of supercritical flow may occur only over bed elements as in two-dimensional flow; or may occur at higher velocity regions of supercritical flow next to local hydraulic jumps. Kharrufa called the latter case “transitional tumbling flow” to distinguish it from ordinary tumbling flow.

Rapid flow has supercritical velocity throughout and appears in channels with steep slopes and high discharges. This type of flow is characterized by a streamy, opaque surface and the active depth is everywhere less than critical. The water tends to skim over the elements and horizontal vortices may form between them. In the case of three-dimensional roughness, flow first tends to “hug” the elements, but as discharge increases, zones of separation tend to form behind the elements. The condition of no separation under conditions of supercritical flow was observed by Mirajgaoker (1961) using a single semicircular disk element. As expected the drag is much less than for the case with separation.

**Velocity formulas**

*Momentum derivation.* The principle of conservation of momentum may be applied to large bed element channels. Equating the change of momentum to the external forces acting on the water between sections 1 and 2, Fig. 10, gives the general form of the momentum equation

$$ (Q \gamma / g) (\bar{\beta}_2 V_2 - \bar{\beta}_1 V_1) = P_1 - P_2 + W \sin \alpha - F_t \quad \ldots \ldots \ldots (12) $$
where \( P_1 \) and \( P_2 \) are the resultant pressures acting at each section, \( \gamma \) is the unit weight of water, \( Q \) is the discharge, \( V \) is the mean velocity, and \( g \) is the acceleration of gravity. \( W \sin \alpha \) is the component of the gravity force acting in the direction of motion, and \( F_i \) is the external resistance force acting on the contact between the prism of water and its boundary. This could also include free surface resistance such as surface tension and drag at the atmospheric interface.

Assuming macroscopic uniform flow, and that on the average \( \frac{\delta_1}{\delta_2} V_1 = \frac{\delta_2}{\delta_2} V_2 \), and \( P_1 = P_2 \) gives

\[
W \sin \alpha = F_f \tag{13}
\]

with all of the external resisting forces included in \( F_f \). These include \( F_s \), those due to shear resistance, and \( F_D \), those due to form drag. The force resulting from shear on the bed is

\[
F_s = \tau_o \cdot \text{(effective wetted area)} \tag{14}
\]

and that resulting from pressure or form is

\[
F_D = \sum_{i=1}^{n} F_i \tag{15}
\]

where \( F_i \) is the force due to one element and \( n \) is the number of elements in a bed area of width \( w \) and length \( l \).

---

Figure 10. Definition sketch for momentum derivation.
The bed elements are shown as hemispheres in Fig. 11. They could just as well be other shapes. The form drag is given by

\[ F_i = C_D A_i \rho V^2 / 2 \]  \hspace{1cm} (16)

in which \( C_D \) is the coefficient of drag, \( A_i \) is the projected area of the bed element in a plane perpendicular to the direction of flow, \( \rho \) is the density, and \( V \) is the mean velocity. Any free surface resistance is included in \( F_i \).

The driving force from Eq. 13 can be written as

\[ W \sin \alpha = w \ell \overline{d} \gamma S \]  \hspace{1cm} (17)

where \( \sin \alpha = S \) for small \( \alpha \) and \( \overline{d} \) is an appropriate value of depth.

Eq. 17 is correct providing an acceptable value for \( \overline{d} \) can be found. In the range of depths and bed element sizes of interest the volume of the bed elements may be of the same order of magnitude as the volume of water.

Equating the driving forces with the resisting forces gives

\[ w \ell \overline{d} \gamma S = F_s + F_d \]  \hspace{1cm} (18)

or

\[ w \ell \overline{d} \gamma S = \tau_o (EWA) + \sum_{i=1}^{n} C_D A_i \rho V^2 / 2 \]  \hspace{1cm} (19)

where EWA is the "effective wetted area."

The wetted area can be considered as follows. The total wetted area is a function of the height, shape, and spacing of the bed elements. For example let the bed be composed of hemispheres of diameter \( k \) and arranged as shown in Fig. 11.

![Figure 11. Definition sketch of hemispherical bed elements.](image-url)
The bed area is $10k^2$ but the wetted area is the gross bed area less the area occupied by the elements plus the surface area of the elements

\[ WA = 10k^2 - 10 \pi k^2/4 + 10 \pi k^2/2 \quad \ldots \quad \ldots \quad (20) \]
\[ = 10 \left(1 + \pi/4\right) k^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21) \]

or surface area per unit area is $\left(1 + \pi/4\right)$.

If the elements are uniform in size, hemispheres, and there are $n$ of them in area $w \ell$ (see Fig. 10) the wetted area is

\[ WA = w \ell - n \pi k^2/4 + n \pi k^2/2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22) \]

If the elements are hemispheres of different sizes, then

\[ WA = w \ell + \sum_{i=1}^{n} \pi k_i^2 / 4 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23) \]

where $k_i$ is the size of the individual elements.

The wetted area is the surface area of the bed and the elements. The effective wetted area is the wetted area less some allowance for the area covered by the wake zones.

\[ EWA = WA - \xi w \ell \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24) \]

where $\xi$ is the effective area of the wake zones per unit area.

Substituting into Eq. 19 gives

\[ w\ell \gamma S = \tau_o \left(w \ell + \pi \sum_{i=1}^{n} k_i^2 / 4 - \xi w \ell + \sum_{i=1}^{n} C_D A_i \rho V^2 / 2\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25) \]

Dividing both sides of Eq. 25 by $w \ell \gamma S$, using Eq. 1 and assuming that $D_o$, $\rho$, and $V$ are constant for each bed element gives

\[ 1 = \left(\tau_o/\ell \gamma S\right) \left(1 + a \theta - \xi\right) + C_D V^2 \theta / 2 g \ell \gamma S \quad \ldots \quad \ldots \quad (26) \]

where $a$ is a constant that depends on the shape of the bed elements and $\xi$ is a function of $\theta$ and relative roughness. The first term on the right hand side is the fractional portion of the resistance force due to shear; and the second term, that due to form drag.
A further examination of $\bar{d}$ is worth while. Referring to Figs. 10 and 11 gives

$$d \omega \xi = \bar{d} \omega \xi + (2n \pi / 3) (k/2)^3 \ldots \ldots \ldots \ldots \ldots (27)$$

or if $\theta = n \pi k^2/8 \omega \xi$ then

$$\bar{d} = d (1 - 2k \theta / 3d) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (28)$$

Defining the hydraulic radius as the cross-sectional area/wetted perimeter gives

$$R = \bar{d} \omega \xi / (2 \omega \xi d + \omega \xi + n \pi k^2/4) \ldots \ldots \ldots \ldots \ldots (29)$$

which gives

$$R = \bar{d} / (2d/w + 1 + \theta) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (30)$$

Combining Eqs. 27 and 30 gives

$$R = d (1 - 2k \theta / 3d) / (2d/w + 1 + \theta) \ldots \ldots \ldots \ldots \ldots (31)$$

If the channel is relatively smooth, i.e. $\theta$ and $k$ are near zero, Eq. 32 reduces to

$$R = d w / (2d + w) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (32)$$

which is the standard equation for $R$ for a rectangular channel. If the channel is infinitely wide, Eq. 32 becomes

$$R = d (1 - 2k \theta / 3d) / (1 + \theta) =$$

$$d \left[ 1 - (1 + 2k/3d) (\theta - \theta^2 + \theta^3 - \theta^4 + \ldots \ldots) \right] \ldots \ldots \ldots (33)$$

Again if $k$ and $\theta$ are equal to zero then Eq. 33 becomes $R = \bar{d}$ as expected. Also $d \geq \bar{d} \geq R$ for all channels.

Assuming in Eq. 26 that $\tau_o = R \gamma S$, and using Eq. 31 applied to a wide channel, expanding in a series and neglecting the product terms of $\theta$ gives

$$V^2 / \bar{d} S = (2g / C_D) [(1 - a + \xi / \theta) / (1 + \theta)] \ldots \ldots \ldots \ldots \ldots (34)$$

or

$$V / (\bar{d} S)^{1/2} = (2g / C_D)^{1/2} f(\theta) \ldots \ldots \ldots \ldots \ldots (35)$$
Dimensional analysis

In the momentum derivation the effects of many of the assumptions made are not known. Dimensional analysis however adds refinement to the mechanical solution.

Nine pertinent variables are considered. These are:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Mean velocity</td>
<td>LT⁻¹</td>
</tr>
<tr>
<td>ρ</td>
<td>Mass density</td>
<td>ML⁻³</td>
</tr>
<tr>
<td>μ</td>
<td>Dynamic viscosity</td>
<td>ML⁻¹ T⁻¹</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration of gravity</td>
<td>LT⁻²</td>
</tr>
<tr>
<td>d</td>
<td>Depth of flow</td>
<td>L</td>
</tr>
<tr>
<td>kₙ</td>
<td>Height of bed element</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>subscript indicates percentile</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Slope</td>
<td>1</td>
</tr>
<tr>
<td>Cₛₙ</td>
<td>Channel shape factor</td>
<td>1</td>
</tr>
<tr>
<td>θ</td>
<td>Spacing parameter</td>
<td>1</td>
</tr>
</tbody>
</table>

In natural channels, another pertinent variable may be sediment transport, but in applying dimensional analysis to LBE channels the bed is assumed fixed. The writers recognize that some of the smaller bed elements may be in motion at less than extreme flows and that even the larger ones may be transported at times of high flow. Nevertheless, except for extreme discharges, the fixed-bed condition is believed to be essentially valid. The physical characteristics and drift of the minor bed elements under ordinary flow conditions would certainly bear some relationship to a valid bed description and to the discharge and depth, and thus would not be fully independent. Choosing V, ρ, and d as repeating variables, one possible relationship is

$$\frac{V}{(gd)^{1/2}} = \ell \{ S, C_{sf}, \theta, d,k_{n}, \rho V d/\mu \} \ldots$$ (36)

where V/(gd)¹⁄₂ is a Froude number Nᵢ and Vd/μ is a Reynolds number Nᵢ. Another arrangement is

$$\tau_o/R \gamma \approx S = \ell \{ N_F, N_R, C_{sf}, \theta, d/k_{n} \} \ldots$$ (37)

Robinson and Albertson (1952), for flow in a flume with strip roughness elements, have shown that, for Reynolds number exceeding certain values for particular relative roughness values, the frictional resistance remains constant. Their limits were for d/k = 12,
\[ N_R > 3 \times 10^4 < N_R \] and for \( d/k = 3, N_R > 0.23 \times 10^4 \). This argues that \( N_R \) might be eliminated from Eq. 37.

Before eliminating Reynolds number, however, further discussion is warranted. Reynolds number in open channel flow is usually calculated using the depth of flow as the characteristic length. The result is, in a sense, a bulk Reynolds number and is not independent of gravitational effect. Morris (1955), using the longitudinal spacing of the roughness elements as the characteristic length, defined a "wall" Reynolds number. Probably a Reynolds number using some characteristic length based on the roughness size or spacing would best describe the viscous effects for the present studies. Robinson and Albertson's elements were all of the same size, were sharp edged, and there was a definite point of separation. The bed elements studied herein are rounded. There is a distribution of sizes, and each size could have a different point of separation for any flow depth. Separation depends on many things, including the wake interference of other elements and the size of the element in relation to flow depth. With a large range of element sizes, there would be a distribution of elements with separation in various stages so that the effects of separation might be relatively constant.

Preliminary plots of friction factor as a function of bulk Reynolds number indicate that the flow observed in the flumes is in the region of hydrodynamically rough flow of the Moody Diagram. Assuming that Reynolds number is not significant, the resulting functional relationship becomes

\[
\frac{V}{(gd)^{1/2}} = f_3(S, C_{sf}, \theta, d/k) \quad \ldots \quad (38)
\]

**Experiments with regular roughness elements**

The first significant study of open channel flow was that of Bazin whose channels were roughened with wooden strips of various spacings, gravel, and other material. Nikuradse (1933), studied rough pipes, using uniform sand grains cemented to the inside of the pipes. The value for spacing used in these experiments is not reported but in all likelihood it was only one value. If the bed elements were aligned and as closely spaced as possible and assuming the full height of the grain to be effective, the value of \( \theta \) could be estimated as 0.785. If they were staggered and as close as possible \( \theta \approx 0.910 \). However, Goncharov (1964) states that the effective height should be only 0.5\( k \) which would make \( \theta \) half the preceding values.

Goncharov (1935) studied the mean velocity of flow and pressure distribution on 25 mm cubes in open channel flow. Various spacings of several patterns were used. Tests were made using two sizes of sand between the test cubes and with one cube surrounded by pebble gravel with a mean diameter of 35 mm. His spacing parameter was the volume of cubes per unit area and varied from 0.0069 to 1.00. Because the elements used were cubes, \( \theta \) would have the same values. Goncharov concluded, among other things, that for a longitudinal spacing 1/\( k \) of between 1.0 and 1.5 the upstream cube completely protects or shields the downstream cube. This protecting influence decreases as 1/\( k \) increases and finally disappears at 1/\( k = 11 \).

Following the work of Nikuradse, Schlichting (1936), realizing that spacing and shape as well as relative roughness influenced the flow, continued to study the effects of roughness
by soldering spheres, hemispheres, cones, and angles to one side of a small rectangular closed conduit. He used 21 geometric patterns and converted his roughness heights to “equivalent sand roughnesses.” He concluded: (1) that the resistance to flow in rough conduits depends on relative roughness and also on roughness density, (2) that the greatest resistance did not occur with the maximum density but at a smaller value, (3) that for small values of roughness density the resistance coefficient was independent of roughness density and that the resistance coefficient decreased sharply at higher values of density. He also states that the greatest resistance occurs at $\theta = 0.4$ for spheres and $\theta = 0.1$ for angles. A value of $\theta = 0.2$ would approximate both cases as the curve at the point of maximum resistance is quite level.

Johnson (1944) examined open channel flow roughness using rectangular strips secured to the bottom of a flume. He showed that the maximum resistance occurred at $\theta = 0.06$ which is some less than the value of 0.10 found by Schlichting for his angles. Johnson’s work was all at one slope (0.00245) but relative roughness $d/k$ varied from 14 to 29.

To examine the hydrodynamic forces acting on the particles in the bed of a stream, El Samni (1949) studied flow on a rough channel using 2.7-inch diameter hemispheres and gravel (3/4 to 3-inch diameter) placed on the bed of a flume. He showed that the effective “bottom” of the channel was 0.2k below the extreme protrusions where k is the diameter of spherical roughness units for which 67.5 percent is finer. He states that for relative roughness $R/k > 2$ the logarithmic velocity formula holds but for values of $R/k < 1.5$ there is a free surface effect. This free surface effect undoubtedly results from the roughness of the bed. Spacing is not discussed, but for hemispheres $\theta$ should equal 0.455. There is no indication of the spacing of the gravel used but the value of $\theta$ should be approximately the same as that of spheres.

Robinson and Albertson (1952) used baffles for roughness. Their basic relationship states

$$C/(g)^{1/2} = f_4 (R/k, s_f, N_F, N_R)$$

where $k$ represents the roughness of the channel and is a function of height, thickness and length, and longitudinal and transverse spacing, and $s_f$ is a roughness shape factor.

The height of the baffles was 1/2 inch and 1 inch, slope varied from 0.001 to 0.04, relative roughness $d/k$ from 4 to 17 and $\theta$ for both heights of baffles was 0.067. This is about the value that Johnson found gave maximum resistance, somewhat less than $\theta = 0.10$ as found by Schlichting.

Sayre and Albertson (1961), using Karman’s logarithmic velocity distribution law, postulated that Chezy’s $C$ might be expressed by

$$C/(g)^{1/2} = (2.30/k) \log (d_n/x)$$

where $d_n$ is the normal depth of flow and $x$ is a roughness parameter, depending on size, shape, and spacing.
Koloesus and Davidian (1961) examined the flow in a channel roughened with 3/16-inch cubes at various spacing. $\theta$ varied from 0.25 to 0.00195. Herbich and Shulits (1964) studied flow where the depth of flow was comparable to the size of the bed elements and the slope was mild. They used cubical elements of two sizes arranged in various patterns. $\theta$ varied from 0.046 to 0.353.

**Simulated natural channels**

Flow conditions in natural LBE channels cannot be controlled and pertinent variables cannot be changed as desired. Using a simulated natural channel in a laboratory flume variables can be controlled, measured, and observed quite readily.

Kharrufa used beds roughened with graded gravel of 4-inch, 3-inch, and 2-inch maximum size. The gravel had a straight-line size distribution by weight on a regular scale. The entire bed sample was retained on the 1/4-inch screen and 40 percent of the sample was retained on the largest screen. Abdelsalam (1965) used beds composed of 2-inch and 4-inch maximum size gravel using normal probability size distribution by number of elements rather than by weight. Kharrufa used three different spacings with each size; Abdelsalam used five.

**Effect of slope and relative roughness**

Since the channel shape will be constant for any set of runs, Eq. 38 can be written

$$N_F = f_5(S, \theta, d/k_n)$$  \hspace{1cm} (41)

in which $N_F$ is regarded as a channel conductance coefficient. A preliminary plot of $N_F$ and shape at various spacings and relative roughness indicates that the four terms of Eq. 39 could combine as a product of the function H, i.e.,

$$N_F = k_1 H_s H_\theta H_{d/k}$$  \hspace{1cm} (42)

where the H’s indicate functions of $S$, $\theta$, and $d/k$ respectively. In this event a multifactorial analysis as described by Schenck (1961) was helpful. The “experiment” is set up as follows. Let $X, Y, Z$, be the three independent $P_i$ terms, then the data could be collected at the following levels.

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$Y_1$</th>
<th>$Z_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_2$</td>
<td>$Y_2$</td>
<td>$Z_2$</td>
</tr>
<tr>
<td>$X_3$</td>
<td>$Y_3$</td>
<td>$Z_3$</td>
</tr>
<tr>
<td>$X_4$</td>
<td>$Y_4$</td>
<td>$Z_4$</td>
</tr>
</tbody>
</table>

This method provides a means for averaging out any two of the variables and examining the effect of the third on the dependent variable, in this case $N_F$. The functional relationship can also be obtained as well as 16 values for $k_1$, which should all be the same if the data are accurate and the assumption that the variables may combine as a product is correct. The variation in $k_1$ indicates the accuracy of the data.
Five multifactorial “experiments” were conducted on Abdelsalam’s data using this method with the following results.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Mean (k_1)</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.05</td>
<td>0.20</td>
</tr>
<tr>
<td>B</td>
<td>0.87</td>
<td>0.06</td>
</tr>
<tr>
<td>C</td>
<td>1.04</td>
<td>0.17</td>
</tr>
<tr>
<td>D</td>
<td>1.90</td>
<td>0.20</td>
</tr>
<tr>
<td>E</td>
<td>1.04</td>
<td>0.18</td>
</tr>
</tbody>
</table>

The results are plotted in Figs. 12, 13, and 14 and lead to the following equations

A. \(N_F = 4.2 S^{0.52} (d/k_{25})^{33} f(\theta)\)

B. \(N_F = 5.7 S^{0.43} (d/k_{25})^{32} f(\theta)\)

C. \(N_F = 5.5 S^{0.66} (d/k_{25})^{27} f(\theta)\)

D. \(N_F = 4.0 S^{0.45} (d/k_{25})^{28} f(\theta)\)

E. \(N_F = 3.5 S^{0.51} (d/k_{25})^{30} f(\theta)\)

The function \(f(\theta)\) associated with each experiment is difficult to express in analytic terms but has a minimum for values of \(\theta\) between 0.10 and 0.20, Fig. 15.

The average exponent of the slope \(S\) is 0.51 which shows that the channel conductance \(N_F\) varies with the slope as assumed in the Chezy equation. An examination of Fig. 12 gives a range of \(N_F\) from about 0.40 to 1.60. The plot for experiment A for the two steeper slopes seems to indicate that slope in excess of 4 percent does not affect \(N_F\) number; however, in experiment A extrapolation was necessary to obtain data at the proper levels and this may account for the deviation of A from the other experiments. The average exponent of the relative roughness is 0.30. This compares to values commonly observed.

Using average values

\(N_F = 4.0 S^{0.50} (d/k_{25})^{33} f_6(\theta)\) \hspace{1cm} (43)

or

\(C/(g)^{1/2} = 4.0 f_6(\theta) (d/k_{25})^{33}\) \hspace{1cm} (44)
Figure 12. $H_s$ versus slope—factorial analysis of Abdelsalam's data.
Figure 13. $H_{d/k}$ versus $d/k$—factorial analysis of Abdelsalam's data.
Figure 14. Ho versus θ—factorial analysis of Abdelsalam’s data.

Figure 15. Froude number versus slope for short bermuda grass—Ree and Palmer data.
Grass-lined channels

Ree and Palmer (1949) made a field study of the flow of water in channels protected with vegetative linings. Both green and dormant bermuda grass that had been cut was used for the study. The data are listed in Table 5. Because of the wide range of variables the following analysis was possible.

The spacing parameter could not be determined, however, the green and dormant beds are assumed to be similar and constant. Relative roughness for each discharge can be considered constant. Mannings n, for the tests considered, varied from 0.030 to 0.049 with seven of the values between 0.035 and 0.039. The variation with depth in this case was small. However, n is smaller for the larger discharge than for the smaller discharge in all cases. A similar comparison can be made using Chezy’s C.

Table 5. Hydraulic data from Ree and Palmer. Short Bermuda grass.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Exper. No.</th>
<th>Test No.</th>
<th>Discharge cfs</th>
<th>Mean Velocity ft./sec.</th>
<th>Hydraulic Radius ft.</th>
<th>Effective Slope</th>
<th>Center Depth ft.</th>
<th>Scour in./hr.</th>
<th>Froude v/gD</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1-6</td>
<td>1</td>
<td>5</td>
<td>14.3</td>
<td>7.98</td>
<td>.308</td>
<td>.2012</td>
<td>.49</td>
<td>.27</td>
<td>2.00</td>
</tr>
<tr>
<td>B1-6</td>
<td>1</td>
<td>8</td>
<td>29.3</td>
<td>10.08</td>
<td>.406</td>
<td>.1977</td>
<td>.66</td>
<td>.15</td>
<td>2.19</td>
</tr>
<tr>
<td>B1-5</td>
<td>2</td>
<td>5</td>
<td>14.7</td>
<td>6.56</td>
<td>.348</td>
<td>.0980</td>
<td>.57</td>
<td>.01</td>
<td>1.53</td>
</tr>
<tr>
<td>B1-5</td>
<td>2</td>
<td>8</td>
<td>29.8</td>
<td>8.74</td>
<td>.426</td>
<td>.1002</td>
<td>.73</td>
<td>.04</td>
<td>1.80</td>
</tr>
<tr>
<td>B2-8</td>
<td>2</td>
<td>6</td>
<td>15.2</td>
<td>5.18</td>
<td>.583</td>
<td>.0341</td>
<td>1.00</td>
<td>.16</td>
<td>0.91</td>
</tr>
<tr>
<td>B2-8</td>
<td>2</td>
<td>9</td>
<td>29.8</td>
<td>6.30</td>
<td>.752</td>
<td>.0348</td>
<td>1.39</td>
<td>.09</td>
<td>0.94</td>
</tr>
<tr>
<td>B2-17</td>
<td>1</td>
<td>5</td>
<td>14.6</td>
<td>2.46</td>
<td>.602</td>
<td>.0111</td>
<td>1.04</td>
<td>-.03</td>
<td>0.43</td>
</tr>
<tr>
<td>B2-17</td>
<td>1</td>
<td>8</td>
<td>30.0</td>
<td>3.40</td>
<td>.718</td>
<td>.0102</td>
<td>1.31</td>
<td>.02</td>
<td>0.53</td>
</tr>
<tr>
<td>Supply Canal</td>
<td>4</td>
<td>5</td>
<td>14.9</td>
<td>1.40</td>
<td>1.063</td>
<td>.0020</td>
<td>1.74</td>
<td>----</td>
<td>0.19</td>
</tr>
<tr>
<td>Supply Canal</td>
<td>4</td>
<td>8</td>
<td>30.4</td>
<td>1.71</td>
<td>1.394</td>
<td>.0013</td>
<td>2.44</td>
<td>----</td>
<td>0.19</td>
</tr>
</tbody>
</table>

B1-6 Veg. submerged, all tests water surface very rough for low flow. Extremely rough for high flows. Self aeration considerable. Mannings n 0.038 and 0.036 resp.

B1-5 Veg. submerged, water surface rough for low flows and extremely rough for high flows. Some aeration. Mannings n 0.035 and 0.030 resp.

B2-8 Veg. submerged, water surface slightly rough for low flows to moderately rough for high flows. Evidence no aeration. Mannings n 0.037 and 0.036 resp.

B2-17 Veg. submerged, water surface was fairly smooth. No aeration. A few waves and boils. Mannings n 0.046 and 0.035 resp.

Supply Canal Veg. submerged, water surface fairly smooth for all flow. Mannings n 0.049 and 0.039 resp.
The values of \( n \) are comparatively large (concrete is about 0.016) indicating that the channels are quite rough. This is also indicated by the description of the free surface in the remarks. This should mean that

\[
N_F = f_n(S) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (45)
\]

\( N_F \) is plotted as a function of \( S \) in Fig. 15. The equations for the two lines can be written as

\[
N_F = C_o S^{1/2} \quad \text{where } C_o \text{ varies with } Q, \text{ or, in effect, } d. \text{ For } Q = 30 \text{ cfs, } C_o = 5.0; \text{ for } Q = 14 \text{ cfs, } C_o = 4.4. \text{ This variation is as expected and confirms the importance of relative roughness.}
\]

For the smaller discharge (and also the smaller relative roughness)

\[
N_F/S^{1/2} = C/(g)^{1/2} = 4.4 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (46)
\]

Combining with Eq. 44 gives

\[
(d/k)^{1/3} f_6(\theta) = 1.1 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (47)
\]

**Effect of spacing parameter \( \theta \)**

The measurable variables in the simulated fixed natural channels are discharge, slope, flow depth, and bed characteristics. Referring to Eq. 38 and considering that \( N_F \) varies as \( S^{1/2} \) gives

\[
V/(g d S)^{1/2} = f(C_{sf}, \theta, d/k_n) \ldots \ldots \ldots \ldots \ldots \ldots \ldots (48)
\]

For any particular bed, \( \theta \) and \( k_n \) are constant. Assuming that the channel shape factor \( C_{sf} \) and relative roughness \( d/k_n \) combine as products in power form and that discharge per unit width is a measure of velocity, Eq. 48 for a particular channel becomes

\[
q/S^{1/2} = C_1 (d)^{h} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (49)
\]

\( C_1 \) is a constant for any particular channel but includes the effect of \( \theta \) and \( k_n \) and therefore is a function of the bed.

Kharrufa (1962) stated that \( h \) has a value of 3/2 for rapid flow and 11/6 for subcritical (tranquil and tumbling) flow. For the higher values of \( \theta \) it is difficult to distinguish between the values of \( h \) for rapid and for subcritical flow. However, at the lower values of \( \theta \) there is a marked difference. The datum for the depth measurements is the average of all grid points which compares to the level if all the elements were "melted" down. As has been pointed out by many authors the choice of the proper datum is critical and, as is the case in Abdelsalam's work, where the depths of flow are small, makes considerable difference in the results. The difference in the exponents \( h \) for rapid and subcritical flow are plotted relative to spacing parameter in Fig. 16. The greatest difference occurs near the value of \( \theta \) that gives maximum resistance, but the difference usually is not large.
Figure 16. Difference in the exponents \( h \) for rapid and sub-critical flow versus spacing parameter.

Considering Kharrufa’s and Abdelsalam’s flume data, \( C_1 \) and \( h \) may be computed. Eq. 49 can be written

\[
V/(g d S)^{1/2} = C_1 (d/h) - 1.50 \sqrt{g} \quad (50)
\]

Assuming that the variables can be combined as a product of powers with the relative roughness to the \( 1/3 \) power

\[
V/(g d S)^{1/2} = C_1 k_{50}^{1/3} \frac{-h}{d} - 1.83 \frac{(d/k_{50})^{1/3}}{g} \quad (51)
\]

Values of \( \theta \) and \( h \) are plotted in Fig. 17 to give

\[
h = 1.83 = 2.4 (\theta - 0.14) \quad (52)
\]

While there is considerable scatter, the relationship is apparent. The factor \((d/w)^{2.4(\theta - 0.14)}\) where \( w \) is the channel width might be justified from the standpoint of nonuniform distribution of shear at the boundary; the relative importance of wall shear compared to bed shear i.e., at low values of \( \theta \) there is more smooth open area between the elements; or to account for the variation in hydraulic radius due to the roughness of the bed and for dimensional homogeneity. In other words

\[
R^{1/2} = \frac{-1/2}{d} (d/w)^{2.4(\theta - 0.14)} \quad (53)
\]
Figure 17. \((h - 1.83)\) versus \(\theta\) for Abdelsalam's and Kharrufa's data.
This would give, in the form of Eq. 44

$$C/(g)\frac{1}{2} = f(\theta) \left(\frac{d}{k_5}\right)^{1/3}$$  \hspace{1cm} (54)

and

$$V/(g d S)^{1/2} = k_1 f(\theta) \left(\frac{d}{w}\right)^{2.4(\theta - .14)} \left(\frac{d}{k_5}\right)^{1/3}$$  \hspace{1cm} (55)

Values of $f(\theta)$ and $\theta$ for each bed are plotted in Fig. 18; $k_1$ is near 1.0. Also plotted on Fig. 18 are the results of the Fenzel and Davis (1964) analysis of Charlu's and Mirajgaoker's work in a 3-foot flume with natural roughness. From the previous discussion of $\theta$, particularly with cubes as roughness elements, there must be a value of $\theta$ between 0.0 and 1.0 where the resistance is a maximum. From Fig. 18 it appears to be near $\theta = 0.10$.

For the simulated natural channels Eq. 55 is satisfactory. Values of $f(\theta)$ can be selected from Fig. 18.

---

![Figure 18](image-url)

Figure 18. $f(\theta)$ versus $\theta$ for simulated natural channels.
NATURAL CHANNELS

Most flume research on flow in large-bed-element channels has been conducted using geometrically shaped roughness elements usually arranged in regular patterns. In natural channels the elements are not geometrical in shape and may not, at first look, follow any regular pattern. Therefore, the study of the stream bed must be statistical and, at present, on a macroscopic basis. In this study the field channels were assumed to have "fixed beds" and in this respect are different from alluvial channels without large bed elements where the boundary under most considerations is mobile.

Beginning in the summer of 1962 field data were collected on the following streams—Utah: Logan River, Blacksmith Fork River, Providence Creek, High Creek, Smithfield Creek, American Fork Creek, and Ashley Creek; Colorado: Boulder Creek and Clear Creek; New Mexico: Red River. Judd (1963) reported on some Utah sites.

A "site" is a reach of stream in unconsolidated sediment selected to be as straight and uniform as possible. One or more sites were selected on each stream. At some sites only bed information was collected; at others both bed and flow data were obtained. The Appendix gives a detailed description of each site as well as pictures of the site at various stages of flow. Methods of obtaining field data are described in the following paragraphs.

Description of bed elements

Methods and equipment have been developed for sampling bed load and suspended matter in normal alluvial streams. Sampling a cobble- and boulder-strewn stream poses different problems. Because of the size of the elements, a discrete sample is possible although very large particles must be measured in situ. Kharrufa (1962) and others have noted that the larger elements have the greatest effect on flow resistance and therefore, measurement of these is important.

Three methods of sampling were used: (1) a grid method attributed to Wolman (1954), (2) an areal method, and (3) a "grid-highest point" method as described by Judd (1963).

**Grid method.** Wolman proposed a method in which the observer arbitrarily established a grid, either actual or imaginary. A sample element was obtained at each point and the axis or axes measured. The resulting size distribution represents the bed material. This method gives no measure of spacing of the bed elements.

At several sites studied by the writers, a float type grid was established. This consisted of nylon cords with painted wood floats tied at each foot distance, anchored at a particular cross section, and allowed to float downstream. (See Fig. 51.) A sample was obtained by reaching down at the chosen spacing, obtaining a sample element, and measuring it. Some of the elements had to be measured in situ. This method was satisfactory for streams with shallow depths and as the measurements were taken when discharges were low, most sites could be sampled using this method.

Fig. 19 shows typical size distribution curves obtained. As expected, the estimated average diameter of each element gave results which were as satisfactory as averaging the diameters of the three major axes.
Figure 19. Typical grain-size distribution curves for various sites.
**Areal method.** In an attempt to include spacing information a selected area of the stream bed was blocked off by driving steel pegs at each corner of a 3' x 3' area and placing timber around the perimeter to stop the flow of water. The top layer of bed elements in the area was then collected. Because of local scour and other difficulties this method was abandoned.

**Grid-High Point method.** The results from the Grid-High Point method provided the most complete description of the bed and its configuration in relation to the hydraulic properties of the stream. The method involves taking measurements on an established "grid" and locating and measuring "highest points" in the bed whether on the grid or not. Since the field study was concerned with hydraulic variables at a reach as well as bed element size distribution and configuration, two different types of equipment were necessary; one for measuring the bed and one for determining an integrated water surface elevation at a particular cross section.

At each site the limits of the study reach were decided upon and a base line laid out along the bank parallel to the axis of the stream. This base line provided a reference for roughness studies, aided in establishing location of the grid, and was used for describing the location of the piezometric installations for measuring the water surface where these were required. For a typical layout see Fig. 20.

Piezometers were installed at each of three cross sections at the sites selected for flow studies. Each piezometer consisted of a 10-inch concrete pipe stilling well set vertically in the stream bank and a 1 ½-inch galvanized steel pipe with regularly spaced static tubes laid across the stream bed. (See Fig. 21.) The installation was similar to the horizontal piezometers used by Kharrufa (1962) in his laboratory study. The static tubes (Fig. 22) were constructed according to U.S. Geological Survey stream gaging station specifications (Pierce, 1941). The horizontal pipe was extended at least halfway across the channel. The number of static tubes installed depended on the width of the stream.

Static tube effectiveness was tested at two installations. The transverse pipe was installed first without the static tubes, plugs were then placed in all of the tees except one in which a static tube was placed and the elevation of the water surface in the stilling well was measured. The process was repeated until individual readings had been determined for each tee. All of the static tubes were then installed, and an integrated reading taken. Table 6 shows the comparison between the integrated reading and the average of the individual readings.

As part of the analysis using the "grid-high" point method a mean plane was fitted to the bed. The validity of using a plane for reference may be questioned because of channel shape. However, examination of cross sections showed that basically the channels had flat bottoms with curvature near the banks, and that a plane would be satisfactory, provided the grid were established relatively near the center of the channel. A set of vertical measurements from some reference datum provided the bed description rather than the grain sizes of the bed particles, as has been customary. Two sets of measurements were taken: one set at the intersection points on a horizontal-coordinate grid, the other by measuring the elevation of the highest points in the bed area under investigation. The reference datum is the mean plane of the grid point data as determined by linear-regression. As the "highest points" were measured the location of each was recorded. The distance from the reference plane to the top of the highest points was used to characterize bed element size. Figs. 23 and 24 show actual measurement operations.
Figure 20. Typical site layout.
Table 6. Comparison between individual static tube elevations and an average elevation at two sites.

<table>
<thead>
<tr>
<th>Static tube number</th>
<th>Site 32</th>
<th>Site 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (nearest bank)</td>
<td>95.48</td>
<td>95.72</td>
</tr>
<tr>
<td>2</td>
<td>95.48</td>
<td>95.65</td>
</tr>
<tr>
<td>3</td>
<td>95.47</td>
<td>95.60</td>
</tr>
<tr>
<td>4</td>
<td>95.43</td>
<td>95.62</td>
</tr>
<tr>
<td>5</td>
<td>95.44</td>
<td>-</td>
</tr>
<tr>
<td>Numerical average of individual tubes</td>
<td>95.46</td>
<td>95.65</td>
</tr>
<tr>
<td>Integrated average elev. using all static tubes</td>
<td>95.44</td>
<td>95.67</td>
</tr>
</tbody>
</table>

Figure 21. Typical installation of one of the three piezometers at Site 32 on the Logan River.
Figure 22. Static tube details.

Four 13/32" holes per ring

ASSEMBLED PIPE UNIT
(no scale)

STATIC TUBE DETAILS
(no scale)
Figure 23. Stream bed measurements being taken. Note grid wires, 2 x 4 timber supports, and steel pegs.

Figure 24. Measurement of grid points. Tags on grid wires are visible.
Size distribution curves

Size distribution curves, similar to grain-size analysis curves, were obtained using the height above the mean plane as the representative size of each element. Typical curves plotted on normal probability paper are shown in Figs. 25 and 26. Examination of these figures shows that the curves are nearly straight lines indicating normal distribution.

These distribution curves can be used to obtain a $k_n$ value corresponding to any percentile size desired. The percentile of the set larger than $k_n$, rather than percent smaller, was used because of the emphasis on the larger bed elements.

Size distribution curves normally are expected to be log-normal; however, the method of sampling which considered only the highest points means that a good number of the smaller points would be overlooked. The grain-size sample indicates log-normal tendencies as expected and the highest point curve seems to follow a normal distribution.

Axis orientation

Sampling showed that in general the bed elements were oriented with the longest axis perpendicular to the direction of flow and the shortest axis perpendicular to the bed of the stream. The particles tend to roll along the stream bed about their longest axis and are most stable with the shortest axis perpendicular to the bed. If each individual element were free to move without interference from adjacent elements, all should be oriented this way. Detailed examination of in-place elements at Site 35 on the upper Logan River shows that between 50 and 75 percent of the elements were oriented as described. At Site 63 on American Fork Creek, where the elements are mostly angular, the orientation of axes is hard to distinguish.

Shape and orientation affect the value of $\theta$

Consider an oblate spheroid with the longest axis $a$, median axis $b$, and shortest axis $c$. If the elements were oriented as suggested, the correct value of $\theta$ would be calculated using $\pi ac$ as the projected area. However the average value is more readily obtained and $\theta$ would be calculated using $\pi b^2$. The ratio of $\pi b^2/\pi ac$ should equal 1.00 if the $b$ readings are to give correct values of $\theta$. The relation between $b^2$ and $ac$ for sites on High Creek and Smithfield Creek is shown in Figs. 27 and 28. For High Creek $b^2/ac$ is 1.37 and for Smithfield Creek, 1.22. Assuming that 50 percent of the elements are oriented systematically as described before, the values of $\theta$ obtained using $b^2$, rather than $ac$ could be 10 percent to 15 percent too high.

Spacing

Recalling Eq. 2

$$A^{1/2}/k_n = IN^u$$

Plots of $A^{1/2}/k_n$ and $N$, Figs. 29 through 36, give values of $l$ and $u$ for natural streams. The curves of Figs. 29 to 36 were obtained as follows. First, the entire area included in the grid was considered. Locations of the highest points, along with their heights above the mean plane, were plotted on a large plan drawing. A particular size level was selected and the total
Figure 25. Typical size distribution curves.
Figure 26. Typical size distribution curves.
Figure 27. Bed element axis comparison $b^2$ versus $ac$ for High Creek.
Figure 28. Bed element axis comparison, $b^2$ versus $ac$ for Smithfield Creek.
Figure 29. Spacing determination $A^{1/2}/k_n$ versus $N$ on Blacksmith Fork River, Site 11.

Figure 30. Spacing determination $A^{1/2}/k_n$ versus $N$ on High Creek.
Figure 31. Spacing determination $A^{1/2}/k_n$ versus $N$ on Logan River, Site 32.

Figure 32. Spacing determination $A^{1/2}/k_n$ versus $N$ on Logan River, Site 35.
Figure 33. Spacing determination $A^{1/2}/k_n$ versus $N$ on Ashley Creek.

Figure 34. Spacing determination $A^{1/2}/k_n$ versus $N$ on American Fork Creek, Site 62.
Figure 35. Spacing determination $A^{1/2}/k_n$ versus $N$ on Boulder Creek.

Figure 36. Spacing determination $A^{1/2}/k_n$ versus $N$ on Red River, Site 82.
number of bed elements of that size and larger was counted. A small increment (about 5 percent) of area was then eliminated from each end of the grid and the counting process repeated. Another increment was eliminated, and so on. Usually this process was continued until the remaining area was approximately square. After completion for one size level, another size level was selected and the procedure repeated. The resulting values of \( I \) (the y-intercept) for each size level was tabulated in Table 7 and the mean value for each site obtained. As the variation of the individual \( I \) values for different size levels at each site is generally small, the spacing parameter can be considered constant for any particular site regardless of size level chosen. Since the lines are approximately parallel the uniformity coefficient also appears to be independent of the particular value of \( k_n \) selected at any site. There is some variation for the 10-, 20-, and 30-percentile curves, but this is to be expected as there are fewer of the large size elements to be counted and the smaller sample would be less representative. When the roughness measurements were taken some of the smaller highest bed elements were unavoidably missed. This accounts for the 90 percent size level curves shifting to the left for each site. Because of these sampling variations the smallest value of \( I \) at each site may be the most representative. This value of spacing is defined as \( I_{\min} \).

The relationship between spacing and \( \theta \), Eq. 7 includes a constant \( B_o \) that depends on shape and size gradation.

\[
\theta = B_o I^{-2}
\]

(6)

\( B_o \) could vary from 0.784 to possibly 6.0.

The value of \( \theta \) for natural streams may be estimated by assuming that the bed elements are spheres bisected by the mean plane. The projected area of each element is \( \pi k^2/2 \) and

\[
\theta = \pi (\sum k^2)/2A
\]

(56)

Table 7. Spacing parameter \( I \) for natural streams.

<table>
<thead>
<tr>
<th>Percent Larger</th>
<th>11</th>
<th>23</th>
<th>32</th>
<th>35</th>
<th>61</th>
<th>62</th>
<th>71</th>
<th>82</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>27.5</td>
<td>6.2</td>
<td>14.2</td>
<td>4.7</td>
<td>12.0</td>
<td>10.8</td>
<td>9.4</td>
<td>15.6</td>
</tr>
<tr>
<td>50</td>
<td>22.1</td>
<td>5.0</td>
<td>12.5</td>
<td>4.6</td>
<td>11.8</td>
<td>13.1</td>
<td>9.5</td>
<td>10.0</td>
</tr>
<tr>
<td>60</td>
<td>14.5</td>
<td>5.4</td>
<td>13.5</td>
<td>4.5</td>
<td>11.0</td>
<td>13.1</td>
<td>10.7</td>
<td>7.8</td>
</tr>
<tr>
<td>70</td>
<td>10.0</td>
<td>5.9</td>
<td>10.0</td>
<td>4.8</td>
<td>11.3</td>
<td>12.2</td>
<td>11.9</td>
<td>6.0</td>
</tr>
<tr>
<td>80</td>
<td>8.8</td>
<td>6.9</td>
<td>9.6</td>
<td>5.1</td>
<td>12.5</td>
<td>12.8</td>
<td>13.0</td>
<td>10.0</td>
</tr>
<tr>
<td>90</td>
<td>9.1</td>
<td>8.5</td>
<td>14.0</td>
<td>6.4</td>
<td>14.0</td>
<td>13.0</td>
<td>14.0</td>
<td>15.6</td>
</tr>
<tr>
<td>Mean I</td>
<td>16.0</td>
<td>6.3</td>
<td>12.3</td>
<td>5.0</td>
<td>12.1</td>
<td>12.5</td>
<td>11.4</td>
<td>9.5</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>7.4</td>
<td>1.4</td>
<td>1.8</td>
<td>0.7</td>
<td>1.2</td>
<td>0.9</td>
<td>1.80</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>0.46</td>
<td>0.49</td>
<td>0.58</td>
<td>0.36</td>
<td>0.42</td>
<td>0.47</td>
<td>0.56</td>
</tr>
</tbody>
</table>
Table 8 gives values of $I$ calculated for most of the sites considered. Fig. 37 is a plot of $\theta$ and $I_{\text{min}}$ and substantiates

$$\theta = 5.0 I^{-2}$$

($57$)

$B_o$ equal to 5 implies 2.0 (shape) x 2.5 (gradation).

**Width, depth, and velocity relationship**

Leopold and Maddock (1953) proposed equations relating width, depth, and velocity as power functions of discharge, as follows:

$$w = aQ^{b}$$

$$d = cQ^{f}$$

$$V = pQ^{m}$$

where $w$ is the width, $d$ the mean depth, and $V$ the mean velocity and $a$, $b$, $c$, $f$, $p$, and $m$ are numerical constants. These equations have been used both to relate $w$, $d$ and $V$ to $Q$ occurring at specific sites and to characterize channel geometry of a stream or class of streams as $Q$ changes along its length. From continuity

$$Q = w d V = a c p Q^{b} Q^{f} Q^{m}$$

($59$)

Therefore

$$a c p = 1$$

($60$)

and

$$b + f + m = 1$$

($61$)

**Table 8.** $I$ and $\theta$ values for natural streams.

<table>
<thead>
<tr>
<th>Site No.</th>
<th>11</th>
<th>22</th>
<th>23</th>
<th>23A</th>
<th>30</th>
<th>32</th>
<th>35</th>
<th>36</th>
<th>37</th>
<th>41</th>
<th>51</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.036</td>
<td>0.125</td>
<td>0.220</td>
<td>0.142</td>
<td>0.042</td>
<td>0.103</td>
<td>0.098</td>
<td>0.110</td>
<td>0.056</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>16.0</td>
<td>8.0</td>
<td>6.3</td>
<td>5.6</td>
<td>8.1</td>
<td>12.3</td>
<td>5.7</td>
<td>7.4</td>
<td>8.0</td>
<td>9.5</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>8.8</td>
<td>6.5</td>
<td>5.0</td>
<td>4.3</td>
<td>6.1</td>
<td>9.6</td>
<td>4.5</td>
<td>5.6</td>
<td>6.4</td>
<td>8.4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site No.</th>
<th>61</th>
<th>62</th>
<th>63</th>
<th>63A</th>
<th>70</th>
<th>71</th>
<th>81</th>
<th>82</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.107</td>
<td>0.087</td>
<td>0.111</td>
<td>0.160</td>
<td>0.103</td>
<td>0.054</td>
<td>0.046</td>
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<tr>
<td>$I$</td>
<td>12.1</td>
<td>12.5</td>
<td>8.5</td>
<td>3.1</td>
<td>11.4</td>
<td>12.2</td>
<td>9.5</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>11.0</td>
<td>10.8</td>
<td>5.9</td>
<td>2.5</td>
<td>9.4</td>
<td>10.6</td>
<td>6.0</td>
<td></td>
</tr>
</tbody>
</table>
Figure 37. $I_{\text{min}}$ versus $\theta$ for natural streams.
Wolman (1955) in his study of Brandywine Creek, Pennsylvania, determined average values of \( b = 0.04 \), \( m = 0.55 \), and \( f = 0.41 \). The mean depth \( d \) equals the cross-sectional area divided by the water surface width. The depth \( D \) is the "statistical" depth, defined as the depth of flow referenced to the mean plane through the bed elements. The resulting equation is

\[
D = c' Q^{f'}
\]  

Typical curves relating these parameters at the Utah State University sites are shown in Figs. 38 through 42 and values of \( b, m, f', a, c, c', p \) are tabulated in Table 9. The average values for the sites studied are \( b = 0.11 \), \( m = 0.48 \), \( f = 0.42 \), and \( f' = 0.46 \) which gives \( b + m + f = 1.01 \), and \( b + m + f' = 1.05 \). The average of the products \( acp \) and \( ac'p \) are respectively 0.99 and 1.10.

Also listed in Table 9 are values for three sites on Oak Park Canal near Vernal, Utah, a canal in alluvial material which has formed its own channel. Values of \( b, m, \) and \( f \) are respectively 0.14, 0.42, and 0.44.

### Froude number in natural channels

The appearance of gravity waves in open channel flow indicates that Froude number is important in accounting for energy dissipation. Observations indicate that for natural LBE channels Froude numbers for the entire stream rarely if ever exceed 1.0. Flume experiments

<table>
<thead>
<tr>
<th>Site</th>
<th>( b )</th>
<th>( m )</th>
<th>( f )</th>
<th>( f' )</th>
<th>( a )</th>
<th>( c )</th>
<th>( c' )</th>
<th>( p )</th>
<th>( acp )</th>
<th>( ac'p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.07</td>
<td>0.42</td>
<td>0.55</td>
<td>0.55</td>
<td>27.5</td>
<td>0.086</td>
<td>0.08</td>
<td>0.38</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>22</td>
<td>0.06</td>
<td>0.44</td>
<td>0.50</td>
<td>0.47</td>
<td>12.5</td>
<td>0.15</td>
<td>0.20</td>
<td>0.57</td>
<td>1.07</td>
<td>1.42</td>
</tr>
<tr>
<td>23</td>
<td>0.08</td>
<td>0.57</td>
<td>0.34</td>
<td>0.38</td>
<td>12.7</td>
<td>0.28</td>
<td>0.29</td>
<td>0.30</td>
<td>1.07</td>
<td>1.11</td>
</tr>
<tr>
<td>30</td>
<td>0.11</td>
<td>0.38</td>
<td>0.49</td>
<td>0.45</td>
<td>14.5</td>
<td>0.14</td>
<td>0.29</td>
<td>0.61</td>
<td>1.24</td>
<td>2.56</td>
</tr>
<tr>
<td>32</td>
<td>0.05</td>
<td>0.58</td>
<td>0.37</td>
<td>0.46</td>
<td>36.0</td>
<td>0.21</td>
<td>0.13</td>
<td>0.14</td>
<td>0.98</td>
<td>0.66</td>
</tr>
<tr>
<td>35</td>
<td>0.07</td>
<td>0.44</td>
<td>0.46</td>
<td>0.47</td>
<td>28.0</td>
<td>0.12</td>
<td>0.14</td>
<td>0.33</td>
<td>1.11</td>
<td>1.29</td>
</tr>
<tr>
<td>61</td>
<td>0.09</td>
<td>0.54</td>
<td>0.35</td>
<td>0.35</td>
<td>24.0</td>
<td>0.25</td>
<td>0.32</td>
<td>0.17</td>
<td>1.02</td>
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</tr>
<tr>
<td>62</td>
<td>0.13</td>
<td>0.48</td>
<td>0.45</td>
<td>0.47</td>
<td>9.8</td>
<td>0.16</td>
<td>0.19</td>
<td>0.46</td>
<td>0.72</td>
<td>0.85</td>
</tr>
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<td>0.14</td>
<td>0.51</td>
<td>0.38</td>
<td>0.38</td>
<td>11.8</td>
<td>0.26</td>
<td>0.27</td>
<td>0.30</td>
<td>0.92</td>
<td>0.95</td>
</tr>
<tr>
<td>63A</td>
<td>0.17</td>
<td>0.27</td>
<td>0.69</td>
<td>0.66</td>
<td>8.6</td>
<td>0.10</td>
<td>0.11</td>
<td>0.84</td>
<td>0.72</td>
<td>0.79</td>
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<tr>
<td>70</td>
<td>0.20</td>
<td>0.52</td>
<td>0.29</td>
<td>-</td>
<td>13.5</td>
<td>0.38</td>
<td>-</td>
<td>0.20</td>
<td>1.03</td>
<td>-</td>
</tr>
<tr>
<td>71</td>
<td>0.16</td>
<td>0.49</td>
<td>0.32</td>
<td>0.55</td>
<td>15.0</td>
<td>0.26</td>
<td>0.09</td>
<td>0.27</td>
<td>1.05</td>
<td>0.36</td>
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<td>81</td>
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<td>0.43</td>
<td>0.48</td>
<td>0.48</td>
<td>17.0</td>
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<td>0.10</td>
<td>0.67</td>
<td>1.02</td>
<td>1.14</td>
</tr>
<tr>
<td>82</td>
<td>0.12</td>
<td>0.58</td>
<td>0.29</td>
<td>0.31</td>
<td>10.5</td>
<td>0.26</td>
<td>0.26</td>
<td>0.38</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>Ave</td>
<td>0.11</td>
<td>0.48</td>
<td>0.42</td>
<td>0.46</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.99</td>
<td>1.10</td>
</tr>
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</table>

Oak Park Canal

<table>
<thead>
<tr>
<th>Site</th>
<th>( b )</th>
<th>( m )</th>
<th>( f )</th>
<th>( f' )</th>
<th>( a )</th>
<th>( c )</th>
<th>( c' )</th>
<th>( p )</th>
<th>( acp )</th>
<th>( ac'p )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.10</td>
<td>0.41</td>
<td>0.48</td>
<td>-</td>
<td>7.8</td>
<td>0.17</td>
<td>-</td>
<td>0.77</td>
<td>1.02</td>
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</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>0.38</td>
<td>0.40</td>
<td>-</td>
<td>6.1</td>
<td>0.26</td>
<td>-</td>
<td>0.63</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
<td>0.46</td>
<td>0.43</td>
<td>-</td>
<td>6.9</td>
<td>0.24</td>
<td>-</td>
<td>0.56</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>Ave</td>
<td>0.14</td>
<td>0.42</td>
<td>0.44</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.98</td>
<td></td>
</tr>
</tbody>
</table>
Figure 38. Width, depth, and velocity related to discharge on Blacksmith Fork River, Site 11.
Figure 39. Width, depth, and velocity related to discharge on High Creek, Site 23.
Figure 40. Width, depth, and velocity related to discharge on Logan River, Site 35.
Figure 41. Width, depth, and velocity related to discharge on Ashley Creek.
Figure 42. Width, depth, and velocity related to discharge on Red River, Site 81.
by Hariri (1964) using model self-formed LBE beds, showed that when Froude number approached 1.0 as slope was increased with constant $Q$, the larger elements began to move, combination roughnesses were formed and Froude number did not continue to increase. Scour action would be increased greatly when the Froude number is near one because of the increased unbalanced forces associated with gravity waves. The resulting system of standing waves appears to strengthen the formation of combination roughnesses so rapidly that velocities in excess of critical are unlikely to occur in self-scoured LBE channels.

**Velocity formulas**

Eq. 55 gives a velocity formula for simulated natural channels. Beginning with Eq. 49 and using $D$ the statistical mean depth, rather than $d$ gives

$$v/S^{1/2} = A D^t$$

(63)

From this basic equation a velocity formula for natural LBE channels can be obtained. Assuming as for simulated natural channels that a function of spacing, relative roughness and channel shape combine in a power form, Eq. 63 becomes

$$v/(gD)^{1/2} = f(0) (D/w)^{t-0.83} (D/k_{50})^{1/3}$$

(64)

Fig. 43 shows $(t - 0.83)$ plotted as a function of $\theta$. The equation is

$$t - 0.83 = 7.0 (\theta - 0.08)$$

(65)

The constants in Eq. 65 are different than those in Eq. 52 but Eq. 52 is for rectangular channels while Eq. 65 is for natural channels. Accordingly, for natural channels the following equation similar to Eq. 55 is obtained

$$v/(gD)^{1/2} = f(0) (D/w)^{7.0(\theta - 0.08)} (D/k_{50})^{1/3}$$

(66)

Values of $f(\theta)$ are plotted in Fig. 44.

As explained in the section dealing with flume experiments and assuming $R \approx D$ the channel shape factor can be combined with the left hand side of Eq. 66 to obtain

$$C/(g)^{1/2} = f(\theta) (R/k_{50})^{1/3}$$

(67)

where $C$ is Chezy's coefficient. Relating this to Mannings equation gives

$$n/k_{50}^{1/6} = (1.49/g)^{1/2} (1/f(\theta)) (D^{-1/6}/k_{50})$$

(68)

or

$$n(D/k_{50})^{1/6} = 0.262 (k_{50})^{1/6}/f(\theta)$$

(69)

Chow (1959) gives an equation for $n/k_{50}^{1/6}$. Comparing Eq. 68 with his Eq. (8-27) one finds that for LBE channels the denominator of his equation, for rough channels,

$$21.9 \log 12.2 \frac{R}{k}$$
Figure 43. \((t - 0.83)\) versus \(\theta\) for natural streams.
Figure 44. $f(\theta)$ versus $\theta$ for natural streams.
should be replaced with \( 3.82 f(\theta) \left( \frac{R}{k} \right)^{1/3} \) to get

\[
n/k^{1/6} = \left( \frac{R}{k} \right)^{1/6}/3.82 f(\theta) \left( \frac{R}{k} \right)^{1/3}
\]

\[
(70)
\]

Bed forming process

For streams transporting silt and sand the formation of dunes and ripples complicates attempts to understand the channel forming processes. As velocity increases, the scouring power of the stream increases, therefore, for LBE streams the bed elements that are left in place become also increasingly larger. For the same bed forming discharge, it is logical to reason that the steeper the channel the greater the projected area of bed elements required to resist the flow, therefore,

\[
\theta = f(S)
\]

\[
(71)
\]

where \( \theta \) is the area of bed elements projected on a plane perpendicular to the direction of flow per unit area of bed and \( S \) is the slope of the stream.

Values of \( \theta \) and corresponding values of slope are plotted in Fig. 45. The trend is quite definite with surprisingly little scatter for field data even though the bed-forming discharges are different. The condition of the banks, the relative width of the channel, and the gradation of sediment particle sizes would also affect the results. In the figure the points are separated into three groups, 1) those for quite narrow channels with large elements protruding from the bank, 2) those that may have less bank resistance and are relatively wider, and 3) those that have still less bank resistance and are wider than those of group 2. With this grouping the resulting relation between \( \theta \) and \( S \) is

\[
S = B \theta
\]

\[
(72)
\]

where \( B \) is a constant that depends on the influence of the channel banks and other properties as mentioned previously.

Also plotted in Fig. 45 are data from Hariri's work using a laboratory flume. Eq. 72 still fits the data but the constant \( B \) must include other factors. There was no seepage in the flume bed thus only the bouyant weight of the particles was available to resist motion. For the same velocity larger particles would remain in the bed resulting in higher values of \( \theta \). Further, water was clear and discharge was constant so there was no receding flood hydrograph which would permit redeposition of smaller particles. A close examination of the molds of Hariri's channels shows that the bank resistance was zero as there were no banks. This would also tend to result in higher values of \( \theta \).

The propelling force \( F_p \) on a prism of water in a channel is \( WS \) where \( W \) is the weight of the prism. Assuming that the shear force is negligible, the resisting force is the drag force imposed by the bed elements, i.e.

\[
F_D = \sum_{i=1}^{n} F_i
\]

\[
(73)
\]

where \( F_i \) is the form drag on the (i)th element and \( n \) is the number of elements per unit area.
Figure 45. Slope versus $\theta$ for natural streams with different bank resistance.
F_i can be written as

\[ F_i = C_D A_i \rho V^2 / 2 \]  \( \text{(74)} \)

Writing \( W = \gamma D \) for a prism of unit width and length and assuming that \( C_D \) and \( V \) are constant for each element

\[ S = (C_D \frac{V^2}{2 \cdot g \cdot d}) \sum A_i \]  \( \text{(75)} \)

\( \sum A_i \) per unit area is \( \theta \), so

\[ S = C_D \theta N_F^2 / 2 \]  \( \text{(76)} \)

If the relationship shown by Eq. 72 is valid then at the time the bed-forming process occurs the product of the composite drag coefficient \( C_D \) and the Froude number \( N_F \) should be relatively constant. Field data are not available to investigate this but Hariri's laboratory results give the following comparison. The range of \( N_F \) during scour, was 0.76 to 1.05 with a mean of 0.88 and a standard deviation of 0.12. Using Eq. 76 and calculating \( C_D \) gives values that range from 0.093 to 0.358 with a mean of 0.18 and a standard deviation of 0.10.

For the natural streams, choosing the limited case of no banks and estimating the best possible line gives

\[ S = 0.2 \theta \]  \( \text{(77)} \)

Eq. 76 gives for natural channels

\[ 0.2 = C_D N_F^2 / 2 \]  \( \text{(78)} \)

or

\[ C_D = 0.4 / N_F^2 \]  \( \text{(79)} \)

Assuming \( N_F \) during scour of natural LBE channels is in the same range as for Hariri's flume, the composite drag coefficient \( C_D \) would be near 0.4; if a value of \( C_D = 0.2 \) as indicated in the flume, applies, \( N_F \) at scour would be about 0.7.

**Step formation**

A bed element configuration designated as steps was observed at sites with slopes greater than 1 percent. The steeper the site the more prominent and regularly spaced were the steps. For channels with slopes greater than about 3 percent, the steps were fully developed and generally extended across the full width of the channel. For slopes between about 2 and 3 percent, there was a tendency for the steps to extend only part way across the channel. For slopes less than about 2 percent, steps were sometimes discernible and sometimes not. At high discharges and steep slopes, flow tended to skim across the steps.

Steps may not be visible until the discharge decreases. At the lower discharges, the steps tend to act as overfall weirs and average channel slope would appear to be important in
the velocity equation only as it influences bed characteristics. Flow over steps is similar to flow over the artificial bar roughness studied by Mohanty (1959) and Al-Khafaji (1961) in their flume studies.

The writers believe steps begin forming at high discharges. At some point along the channel there exists a large rock that cannot be moved by the stream. The smaller elements are moved along the bottom of the channel and become anchored against the larger rock. The bar grows as more elements become interlocked with it until a small dam results. This action triggers formation of a series of additional steps downstream, probably accelerated by a series of standing waves. A pool is scoured immediately downstream from the bar with deposition at the toe of the pool creating another bar and causing the sequence to repeat at relatively constant spacing. In the narrower streams irregularities in the conformation of banks may also help to initiate step formation. Steps may be similar or analogous to the formation of dunes or meanders in streams carrying finer sediments.

Field observations indicate that channel slope and size of bed elements available are primary factors in the spacing. Fig. 46 shows a relationship between spacing $L$, slope $S$, and a representative bed element height $k_{16}$. A possible equation is

$$L = \frac{k_{16}}{C S^z}$$

as shown in Fig. 46. For $z$ equal to 1.0, $C$ values of 1.5 and 3.6 give envelope curves that enclose all of the data. Using $C = 2.0$ gives

$$L = \frac{k_{16}}{2S}$$

Equation 81 shows that step spacing decreases with slope. Considering $k_{16}$ constant and letting $S$ increase, at some slope spacing and bed element height would become equal; however, the value of $k_{16}$ left in the bed would increase with slope.

Eq. 81 is based on limited information and should be interpreted only as indicative of the phenomenon. Field determination of spacing is difficult because of channel variability. Table 10 shows the mean spacing and standard deviation at seven sites. At the steeper sites, where the steps are more pronounced, the standard deviation is much less. An average of the individual spacings in the reach of stream at each site was used in plotting Fig. 46. Statistical mean bed slopes and $k_{16}$ values are from the highest point distribution curves. Figs. 47 and 48 as well as the photographs in the Appendix show step formation.

Tractive force

The unit tractive force $\gamma_o$ of a stream on a unit area of the bed of a wide open channel is

$$\gamma_o = \gamma d S$$

The value of the tractive force at which erosion begins is the critical tractive force used in the design of canals. Values of critical tractive force recommended by the U.S. Bureau of Reclamation in lbs./sq. ft. for coarse noncohesive material are given by

$$\gamma_o = 4.8 k_{25}$$

where $k_{25}$ is the particle size in feet.
Figure 46. Bed slope related to step spacing for natural streams.
At three sites during the writers' study, discharges large enough to move the large bed elements occurred. Also, at the sites on the Oak Park Canal, data were available to obtain the bed forming discharge. Critical tractive force values for these cases, values from Harp (1963), Lane and Carlson (1954) and Hariri (1964), are plotted in Fig. 49. Shown on the figure is Eq. 83. Also shown is a limiting envelope curve that provides safe design values for stable channels.

Table 10. Step spacing at various sites.

<table>
<thead>
<tr>
<th>Site</th>
<th>Mean Step Spacing</th>
<th>Standard Deviation</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>14</td>
<td>4.3</td>
<td>4.5</td>
</tr>
<tr>
<td>23</td>
<td>15</td>
<td>5.5</td>
<td>3.7</td>
</tr>
<tr>
<td>34</td>
<td>21</td>
<td>13.0</td>
<td>2.3</td>
</tr>
<tr>
<td>35</td>
<td>23</td>
<td>13.8</td>
<td>1.7</td>
</tr>
<tr>
<td>36</td>
<td>13</td>
<td>3.4</td>
<td>3.2</td>
</tr>
<tr>
<td>37</td>
<td>16</td>
<td>8.6</td>
<td>2.4</td>
</tr>
<tr>
<td>41</td>
<td>5</td>
<td>0.9</td>
<td>5.6</td>
</tr>
<tr>
<td>61</td>
<td>29</td>
<td>8.6</td>
<td>1.7</td>
</tr>
<tr>
<td>62</td>
<td>23</td>
<td>8.6</td>
<td>1.7</td>
</tr>
<tr>
<td>63</td>
<td>23</td>
<td>9.0</td>
<td>5.7</td>
</tr>
<tr>
<td>71</td>
<td>25</td>
<td>7.9</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Figure 47. Steps on Logan River, Site 36, and High Creek, Site 23.
Figure 48. Steps on Providence Creek, Site 41.
Figure 49. Tractive force related to particle size for noncohesive beds.
POSSIBLE APPLICATIONS

There are many cases in which an improved understanding of large-bed element hydraulics would be helpful. One of these is in estimating channel discharge. While not replacing current meter rating curves, it would be helpful for reconnaissance, for sections where rating curves are not available, and for extrapolating to large flows. It could provide some indication of peak discharges, especially for ephemeral streams. LBE hydraulics could provide the basis for stable channel design and could likely be used to design flow measurement structures. Use of LBE channels for aeration and mixing has application to water pollution control and for kinetic energy dissipation.

Estimation of discharge

For discharge estimates the procedure similar to that outlined in the previous section could be used and the mean velocity determined. A rating curve could be constructed for various water levels including high watermarks. Following is a summary of the steps to be taken.

1. Select a reach as uniform and straight as possible preferably with a length at least 10 times the width.

2. Lay out a base line, as shown in Fig. 20 (page 40), and select three cross sections.

3. Measure cross sections using the same reference elevation for all. Determine the water surface elevation at each cross section, including high watermark elevations.

4. Use the elevations at each cross section to determine the mean plane. This reference plane is used in obtaining the average depth, the size distribution of elements and the spacing parameter $\theta$.

5. Determine the location and the elevation of each bed element in a representative grid area in the reach of stream. Provided the reach of stream is uniform a grid area covering about two-thirds the width and a length of twice the width is satisfactory. The location and elevation of the bed elements can be obtained by establishing a grid as explained in the previous section or by using a level and alidade and actually mapping the position and elevation of each element.

6. Calculate $\theta$ using the formula

$$\theta = \pi \frac{\sum k^2}{2A}$$

where $k$ is the height of each element above the mean plane. Construct a size distribution curve using the values of $k$.

7. Use Fig. 44 to estimate $f(\theta)$ and then use Eqs. 66, 67 (modified), or 70 (modified to obtain the average velocity and compute the discharge using $Q = AV$. The values of $D$, $W$, $R$ and $A$ are the averages at each cross section, and $k_{50}$ is obtained from
the size distribution curve. The slope is the slope of the water surface which should be equal to the slope of the mean plane.

The following example is given for the site on Clear Creek near Golden, Colorado. The information was determined from field measurements taken previous to and in June, 1965. See Fig. 50.

The averages of the three cross sections are:

<table>
<thead>
<tr>
<th>Area</th>
<th>163 sq. ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>49.3 ft.</td>
</tr>
<tr>
<td>Depth, D</td>
<td>4.3 ft.</td>
</tr>
<tr>
<td>Hydraulic radius, R</td>
<td>3.8 ft.</td>
</tr>
</tbody>
</table>

The water surface slope was 2.3 percent.

The value of k for bed element A, Fig. 50, is determined as follows: The station of the element is 0 + 75 and its elevation is 97.67. Using the equation of the plane \( y = 0.03 \) (station) + 94.59 and letting the station = 75 feet the elevation of the mean plane at A is 96.84 so

\[
k = 97.67 - 96.84 = 0.83
\]

The values for each element are found this way and the resulting k values are used to determine \( \theta \) and the size distribution curve.

For Clear Creek \( k_{50} = 0.74 \) ft. and \( \theta = 0.17 \).

Using Fig. 44, Eq. 67, and the above data, the computed average velocity was 11.2 ft. per second and the discharge was 1826 cfs. The discharge as determined at a gaging station 3 miles downstream was 1673 cfs.

**Design of stable channels**

Sometimes a design for a canal on a steeper slope than normal is desirable. When coarse material is available this can be achieved using the method previously presented. Two examples, the Oak Park Canal near Vernal, Utah, and the Boulder return ditch of the GarKane hydroelectric plant near Boulder, Utah, are discussed.

**Oak Park Canal.** The Oak Park Canal is a small canal conveying water stored on Brush Creek in Uintah County, Utah, to Ashley Creek for irrigation. The maximum discharge during the past 15 years was 52 cfs and the annual maximum was probably about 50 cfs. The canal was constructed through a variety of materials with a wide range of slopes (up to 20 percent in some places). Where slopes were less than 1 percent the channel has changed little. (See Fig. 51.) On the steeper slopes where a good supply of cobbles and boulders was available, a stable bed even with slopes of up to 6 percent has been formed. As can be seen from Figs. 52 through 53 the channel is extremely rough. In areas where sufficient large material was not present, considerable detrimental erosion has occurred and still is occurring (Figs. 54 and 55). With proper design and construction this condition could have been prevented. The tractive-force and grain size data from Oak Park Canal are shown on Fig. 49.
Figure 50. Clear Creek site for determination of discharge.
Figure 51. Oak Park Canal, near Vernal, Utah, slope 0.001.

Figure 52. Oak Park Canal, near Vernal, Utah, slope 0.012.
Figure 53. Oak Park Canal, near Vernal, Utah, slope 0.018.

Figure 54. Oak Park Canal, near Vernal, Utah, slope 0.040.
Figure 55. Oak Park Canal, near Vernal, Utah, slope 0.040.

Figure 56. Oak Park Canal, near Vernal, Utah, slope 0.165.
Boulder Return Ditch. The Boulder Return Ditch takes water from the GarKane hydroelectric plant near Boulder, Utah, and conveys it for several miles toward Boulder Creek. The water is then released down a steep hillside into the creek. In a length of 900 feet the drop is approximately 300 feet.

This uncontrolled water has caused considerable erosion damage. In order to correct this difficulty a new waste canal was proposed along the canyon wall at a slope of about 2 percent. However, a preliminary survey revealed a large outcrop of gypsum along the line of the canal, making this location not feasible. Accordingly, three alternate proposals, Fig. 58, A, B, and C, were investigated. Proposal A consisted of a 2100-foot canal at 1 percent slope along the canyon wall discharging into a small ravine connecting to Boulder Creek. Proposal B involved a 4700-foot canal on a slope of approximately 6 percent. Proposal C visualized a 2300-foot canal on a slope of 12 percent. For proposal A the stability of the ravine may be a problem. Fig. 59 shows a profile of the ravine compared with that of the present drop.

A canal on the 6 percent slope proposed for Alternate B would require some means of stabilization and energy dissipation. A 2-foot thick rip-rap lining of native rock as shown in Fig. 60 would be required. The rip-rap should be well graded and must meet the following size requirements:

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>2&quot; - 6&quot;</td>
</tr>
<tr>
<td>50%</td>
<td>6&quot; - 12&quot;</td>
</tr>
<tr>
<td>25%</td>
<td>12&quot; - 12&quot;</td>
</tr>
</tbody>
</table>
The difference in elevation between points 1 and 2 is 265 feet.

Scale: 1" = 1320'

Length of canals

A - 2100'
B - 4700'
C - 2300'

Figure 58. Layout of Boulder Return Ditch, Boulder Creek, Utah, with proposed canals.
Figure 59. Ravine profile near Boulder Return Ditch.

Excavation 53,000 yds
Graded rip-rap 8,000 yds

Rip-rap size
2.5% 2"-6"
50% 6"-12"
25% 12"-18"

Figure 60. Cross section of canal for proposal B on Boulder Return Ditch.
No rocks larger than 24" should be used. A 1:25 scale model of the rip-rap lined canal was built and tested at the College of Southern Utah at Cedar City. A 24-hour test at a prototype discharge of 50 cfs on a 6 percent slope showed no visible signs of scour at any time during the test (Fig. 61).

Other applications

A short masonry channel has attractive possibilities as a measurement weir. Large elements interspaced with smaller elements would be used to provide partial-depth bed elements at both large and small discharges. By proper selection of bed elements, the head loss through the weir would be comparable to that from an equal length of channel, thus avoiding upstream and downstream channel disturbances. Both suspended and bed load should pass readily through the device, which would be particularly advantageous on steep channel measurements for watersheds.

Where material is available, use of properly designed, LBE channels in place of concrete drops and chutes could be economical. Both natural and artificial channels with large-bed-elements could be used to increase aeration as a treatment for water carrying degradable solids or to increase oxygen content to desired levels.

Figure 61. Model of canal proposal B, Boulder Return Ditch, prototype discharge of 50 cfs.
SUMMARY

This paper discusses theoretical means of characterizing size, size distribution, spacing, and shape of bed elements in hydraulically significant terms for large bed element streams; these are related to flume experiments using simulated stream beds. Large bed element channels are defined as those channels in which the bed elements extend through a major portion of the flow depth.

Observed bed and flow characteristics are reported for a number of natural stream sites located in Utah, Colorado, and New Mexico. Detailed measurements made at the different sites are analyzed to determine the effects of channel roughness, depth, and slope of flow characteristics of natural streams.

In nature, large bed element streams usually become paved with large boulders and such channels may approach a fixed bed condition insofar as the large bed elements are concerned and only at very infrequent extreme discharge will the dominant elements be moved. Primarily this paper is concerned with the hydraulics of these streams at less than channel forming discharge.

Possibilities for use of large bed elements in open channel design are suggested.
APPENDIX A

General Description of Sites

Introduction

Appendix A gives a general description and several pictures of each site, its location, and its bed properties.

Blacksmith Fork River, Utah, Site 11

Site 11 is located 100 feet downstream from the U.S. Geological Survey gaging station. The left bank is covered with dogwood and at higher discharges this vegetation affects the flow. The right bank is relatively free of vegetation. The distribution of the bed elements is uniform and the absence of any extremely large elements is evident. The bed elements are angular and have the general appearance of talus debris.

\[
\begin{align*}
\text{bed slope} & \quad 0.0096 \\
\theta & \quad 0.36 \\
I_{\text{min}} & \quad 8.8 \\
(k_{50}, \text{high point}) & \quad 0.52 \text{ ft.}
\end{align*}
\]
Blacksmith Fork River, Utah, Site 12

This site, located about three miles downstream from Site 11, is straight and relatively free of vegetation. Gravel and alluvial material are exposed in both banks. Like those at Site 11, the bed elements are uniform but have a more rounded appearance. The picture shows bed measurements being taken.
High Creek, Utah, Site 22

This site is located in the lower section of the High Creek drainage basin. Bed elements are semirounded and both banks are lined with trees, which have helped to maintain the straight direction of the channel. Step formation was pronounced at this site.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>bed slope</td>
<td>0.0449</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.125</td>
</tr>
<tr>
<td>$l_{min}$</td>
<td>6.5</td>
</tr>
<tr>
<td>k_{50}</td>
<td>0.75 ft.</td>
</tr>
</tbody>
</table>
High Creek, Utah, Site 23

This site is located about one mile upstream from Site 22. Bed elements are angular to semirounded and about the same size as those at Site 22. The left bank is free of vegetation, there are trees several feet back from the right bank of the stream, and there is an overhanging ledge at the upstream end of the reach. The step formation is pronounced at Site 23.

| bed slope  | 0.0370 |
| 0         | 0.220  |
| $l_{min}$ | 5.0    |
| (high point) | $k_{50}$ | 0.72 |
High Creek, Utah, Site 23A

This site is the same location as Site 23 but measured two years later, after a major channel change. The description is the same.
Logan River, Utah, Site 30

This site is located near the mouth of Logan Canyon just below the Hyde Park diversion dam. The bed elements are large and angular. This reach is adjacent to highway U.S. 89 and was influenced by construction about two years before measurements were taken.

<table>
<thead>
<tr>
<th>Bed slope</th>
<th>0.044</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{\text{min}}$</td>
<td>6.1</td>
</tr>
<tr>
<td>(high point) $k_{50}$</td>
<td>0.53 ft.</td>
</tr>
<tr>
<td>(grain size) $k_{90}$</td>
<td>1.53 ft.</td>
</tr>
</tbody>
</table>
Logan River, Utah, Site 32

This site is located near the mouth of Logan Canyon. The step formation is barely visible. Bed elements are almost all angular, even the smaller ones. There are few rounded elements similar to the ones found about twenty miles upstream at the other four sites on the Logan River. Clumps of willows grow on the left bank and dogwood on the right bank. This growth would have some affect on the flow at higher discharges.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>bed slope</td>
<td>0.0082</td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td>l_{min}</td>
<td>9.6</td>
<td></td>
</tr>
<tr>
<td>(high point) k_{50}</td>
<td>0.65 ft.</td>
<td></td>
</tr>
<tr>
<td>(grain size) k_{50}</td>
<td>0.55 ft.</td>
<td></td>
</tr>
</tbody>
</table>
Logan River, Utah, Site 34

This site is located on the Upper Logan near Tony Grove Ranger Station. Bed elements are mostly semirounded to rounded. The deeper holes and stagnation areas behind the large elements are partly filled with a loose gravel deposit graded from about 1/2-inch to 1/16-inch in size. These deposits occur throughout the main part of the river.
Logan River, Utah, Site 35

This site is on the Upper Logan River near the Utah State University Forestry Training Laboratory. The description is the same as Site 34.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>bed slope</td>
<td>0.0174</td>
</tr>
<tr>
<td>θ</td>
<td>0.103</td>
</tr>
<tr>
<td>l_{min}</td>
<td>4.5</td>
</tr>
<tr>
<td>(high point) k_{50}</td>
<td>0.70 ft</td>
</tr>
<tr>
<td>(grain size) k_{50}</td>
<td>1.11 ft</td>
</tr>
</tbody>
</table>
Logan River, Utah, Site 36

This site is located upstream about three miles from Sites 34 and 35 and has the same general description.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>bed slope</td>
<td>0.012</td>
</tr>
<tr>
<td>0</td>
<td>0.098</td>
</tr>
<tr>
<td>$l_{\text{min}}$</td>
<td>5.6</td>
</tr>
</tbody>
</table>
Logan River, Utah, Site 37

This site is located about three miles upstream from Site 36 and has the same general description.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>bed slope</td>
<td>0.0242</td>
</tr>
<tr>
<td>θ</td>
<td>0.110</td>
</tr>
<tr>
<td>L_min</td>
<td>6.4</td>
</tr>
</tbody>
</table>
Smithfield Creek, Utah, Site 51

This site is located toward the lower end of the drainage basin, below an irrigation diversion. The bed elements are small, and large trees are along both banks.

bed slope  0.0095
0  0.056
$1_{\text{min}}$  8.4
Site 52 is located about three miles upstream from Site 51 and has much larger roughness elements than Site 51. Roughness elements are angular to semirounded. The banks of this site are relatively free of vegetation. The step formation is visible at this site, but the spacing is quite irregular.
Ashley Fork Creek, Utah, Site 61

This site is located in the Ashley Creek Gorge near Vernal, Utah. Bed elements are rounded sandstone and the channel is straight.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Bed slope</td>
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</tr>
<tr>
<td>$\theta$</td>
<td>0.107</td>
</tr>
<tr>
<td>$l_{min}$</td>
<td>11.0</td>
</tr>
<tr>
<td>(high point) $k_{60}$</td>
<td>0.61 ft.</td>
</tr>
</tbody>
</table>
American Fork Creek, Utah, Site 62

Located above Timpanogos Cave National Monument, the reach is relatively free of vegetation. The large bed elements are angular and the smaller ones are rounded to semirounded.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>bed slope</td>
<td>0.0167</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.087</td>
</tr>
<tr>
<td>$l_{\min}$</td>
<td>10.8</td>
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<tr>
<td>(high point) $k_{50}$</td>
<td>0.55 ft.</td>
</tr>
</tbody>
</table>
American Fork Creek, Utah, Site 63A

Site 63A is located below Site 62. Bed elements are angular and about 2 to 3 feet in diameter. In the downstream wakes of the large elements are deposits of graded gravel up to 8 inches in diameter. This was the roughest reach measured.

<table>
<thead>
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<th>Value</th>
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<tbody>
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<tr>
<td>θ</td>
<td>0.160</td>
</tr>
<tr>
<td>( l_{\text{min}} )</td>
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</tr>
<tr>
<td>(high point) ( k_{50} )</td>
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</tr>
<tr>
<td>(grain size) ( k_{50} )</td>
<td>1.67 ft.</td>
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</table>
Boulder Creek, Colorado, Site 71

This site is located three miles west of the city of Boulder, Colorado, near the stream gaging station. The bed elements are semirounded with deposits of coarse sand behind the large ones.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</tr>
<tr>
<td>$l_{\text{min}}$</td>
<td>9.4</td>
</tr>
<tr>
<td>(high point)</td>
<td>$k_{50}$</td>
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</tbody>
</table>
Clear Creek, Colorado, Site 72

Site 72 is located about six miles west of the town of Golden, Colorado. The bed elements are angular. This site covers a portion of a test reach check by the U.S. Geological Survey in Denver.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>bed slope</td>
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<tr>
<td>0</td>
<td>0.167</td>
</tr>
<tr>
<td>(high point)</td>
<td>k₅₀</td>
</tr>
<tr>
<td></td>
<td>0.74 ft.</td>
</tr>
</tbody>
</table>
Red River, New Mexico, Site 81

This site is located on the Red River east of Questo, New Mexico. The bed elements are uniform and have a semiround shape. Both banks are covered with underbrush that may affect the discharge at high flows.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.0148</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.054</td>
</tr>
<tr>
<td>$l_{\text{min}}$</td>
<td>11.5</td>
</tr>
<tr>
<td>(high point) $k_{50}$</td>
<td>0.44 ft.</td>
</tr>
<tr>
<td>(grain size) $k_{50}$</td>
<td>0.37 ft.</td>
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</tbody>
</table>
Red River, New Mexico, Site 82

Site 82 is located about three miles upstream from Site 81. The bed elements are larger than those of Site 81, mean size of 12 to 18 inches, but of the same shape. There are deposits of graded gravel in front of and behind the large elements.

\[
\begin{array}{lcl}
\text{bed slope} & 0.0225 \\
\theta & 0.46 \\
I_{\text{min}} & 6.0 \\
\text{(high point)} & \kappa_{50} & 0.38 \text{ ft.}
\end{array}
\]
Providence Creek, Utah, Site 41

Located on Providence Creek, both banks are free of vegetation and bed elements are mostly semiangular. The steps at this site are pronounced and most regular.

bed slope 0.0547
## APPENDIX B

### Hydraulic Data

<table>
<thead>
<tr>
<th>Date</th>
<th>Discharge (Q, cfs)</th>
<th>Average cross section in square ft.</th>
<th>Hydraulic Radius, R, feet</th>
<th>Statistical Depth, D, feet</th>
<th>Average width, feet</th>
<th>Water surface slope</th>
</tr>
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<tbody>
<tr>
<td><strong>Site 11, Blacksmith Fork River, Utah</strong></td>
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</tr>
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<td>14.08</td>
<td>0.04111</td>
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<td></td>
</tr>
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<td><strong>Site 30, Logan River, Utah</strong></td>
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</table>
# APPENDIX B (continued)

## Hydraulic Data

<table>
<thead>
<tr>
<th>Date</th>
<th>Discharge Q (cfs)</th>
<th>Average cross section in square ft.</th>
<th>Hydraulic Radius, R, feet</th>
<th>Statistical Depth, D, feet</th>
<th>Average width surface slope</th>
</tr>
</thead>
<tbody>
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## APPENDIX B (continued)

### Hydraulic Data

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<th>Average width $W$, feet</th>
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**Site 62, American Fork Creek, Utah**

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**Site 63A, American Fork Creek, Utah**

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**Site 71, Boulder Creek, Colorado**

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**Site 72, Clear Creek, Colorado**

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APPENDIX B (continued)

Hydraulic Data

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<th>Statistical Depth, D, feet</th>
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Site 72, Clear Creek, Colorado (continued)

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Site 81, Red River, New Mexico

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<th>Statistical Depth, D, feet</th>
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