THREE ESSAYS ON STOCK MARKET VOLATILITY

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Economics

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2008

ABSTRACT

Three Essays on Stock Market Volatility

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Utah State University, 2008

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Volatility is inherently unobservable, and thus the selection of models and their definition is crucial in financial research. This dissertation attempts to check the role of investor sentiment and forecast Value-at-Risk (VaR) of the stock market using both parametric and nonparametric approaches. In the first essay, based on daily return data of three stock indices and four individual stocks from January 1988 to December 2006, the role of day-of-the-week, as well as investor sentiment, is examined using two approaches: linear regression to test investor sentiment effect on stock returns and Logit regression to test the investor sentiment effect on market direction. The results indicate that there is a significant positive role of investor sentiment in the market. However, the outcome also shows that the role of the day-of-the-week effect varies among stocks.

Based on the results presented in the first essay, in the second paper investor sentiment effect was included in both mean and conditional variance equations of GARCH models. By comparing augmented GARCH models considering investor sentiment effect with traditional GARCH models, the result demonstrated that augmented GARCH models are significantly better than traditional GARCH models where AIC, BIC, log-likelihood, and out-of-sample VaR forecasting were employed.

The research indicates that a significant role of investor sentiment in forecasting conditional mean and conditional volatility and the accuracy of GARCH models is improved by accounting for investor sentiment effect.

Compared with the first and second essays employing a parametric method to analyze the stock market, the third paper adopts a nonparametric approach to estimate the conditional probability distribution of asset returns. It is evident that the exact conditional mean and conditional variance is inherently unobservable for time series. In practice, conditional variance is often achieved from different parametric models, such as GARCH, EGARCH, IGARCH, etc., by assuming different distributions such as normal, student's t, or skewed t. Therefore, the accuracy of forecasting strongly depends on the distribution assumption. The nonparametric method avoids the need for a distribution assumption by using a neural network to estimate the potentially nonlinear relationship between VaR and returns. Our results show that the neural network approach outperforms traditional GARCH models.

(96 pages)

DEDICATION

This dissertation is dedicated to my father, Shunmin Li; my mother, Caiyun Zhang; and my husband, Christophe Tricaud.

ACKNOWLEDGMENTS

The completion of my thesis would never have happened if not for the help and support of many people. I sincerely appreciate the support and guidance I received from each member of my committee. Special recognition is extended to my advisor, Dr. Christopher Fawson, who has been a great supporter and mentor throughout the whole process. His words of encouragement and advice when times were challenging motivated me to accomplish this task. His understanding and inspiration throughout my course of study is greatly appreciated. I also owe a huge amount of gratitude to my co-chair, Basudeb Biswas, who provided numerous hours of critical input and support. I especially thank him for his patience and encouragement throughout my graduate school career.

I also owe much gratitude to my committee members: Dr. Dwight Israelsen, Dr. Kenneth Lyon, Dr. Austin Kwag, and Dr. Yangquan Chen, who have provided me with resources to accomplish this task. I appreciate their effort and time in this matter. The macroeconomic knowledge of Dr. Israelsen, the mathematical knowledge of Dr. Lyon, the financial knowledge of Dr. Kwag, and the quantitative knowledge of Dr. Chen were crucial throughout the entire process. I also want to express my sincere thanks to Dr. Tom Jones, who helped me tremendously with written English.

I was fortunate to receive ongoing inspiration from my family members. I appreciate their love and efforts to help me through the most difficult period. Thank you for all you have done. My husband, Christophe, has been a tremendous support during this time. To him, I give my sincere gratitude and love. My parents have also been a driving force in my achievements. Their work ethic and continuous efforts to improve have provided me with the foundation to succeed. I love and admire them with all my heart. Thank you for all you have done.

Qianru Li

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Chapter 1

INTRODUCTION

Over the last decade, research on conditional mean and conditional volatility of asset prices has been a topic of expanding interest in the field of finance. It is common knowledge that asset prices vary on a daily basis, i.e., they demonstrate dynamic behavior. For economists, the essential feature of asset prices is their obviously increased volatility during periods with greater amounts of news or information. Expertise in understanding the dynamics of asset prices and forecasting them should not be withheld from investors. Academic researchers have developed mathematical tools facilitating analysis and prediction of asset prices, and quantitative analysis is the backbone of this theory. Based on this statement, this dissertation incorporates additional variables, such as investor sentiment in the forecast of the stock market, and demonstrates how it enhances predictability of asset price behavior. Estimation performance is compared using both parametric and nonparametric methods. The first two papers employ a parametric method, whereas the third paper applies a nonparametric method. An overall description of the relationship between the three papers is given in Fig. 1.1.

The first two papers analyze the changes in stock prices by incorporating investor sentiment through an indicator based on trading volume.

In classical asset-pricing theory, the expected return of an asset depends upon the risk-free interest rate, β, and the expected return of the market, $E(R_m)$. This is reflected in equation 1.1 below:

$$E(R_i) = R_f + \beta_{im}(E(R_m) - R_f), \tag{1.1}$$

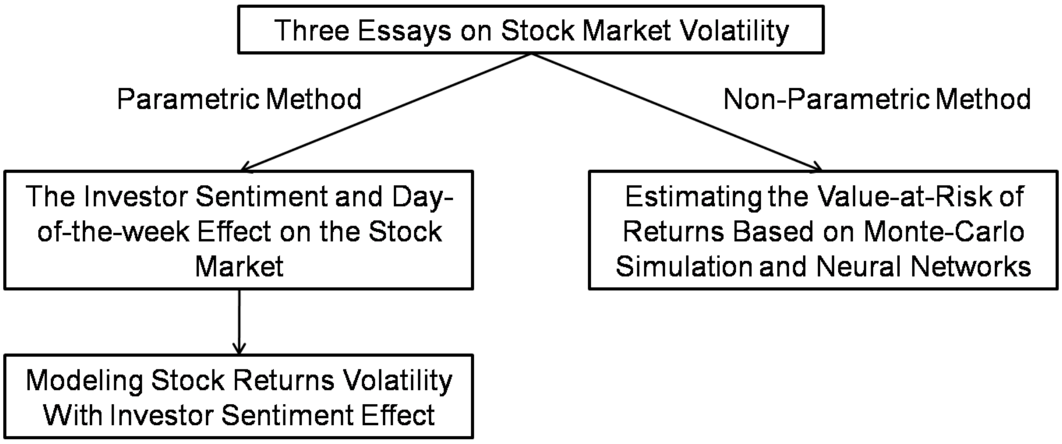

Fig. 1.1: Three essays on stock market volatilities.

where $E(R_i)$ stands for the expected return on the capital asset; R_f represents the risk-free rate of interest; β_{im} is the beta coefficient, which indicates the sensitivity of the asset returns to market returns; and $E(R_m)$ constitutes the expected return of the market, which is determined simultaneously with trading volume within a structural framework. Under this traditional asset-pricing approach, prices of traded assets, such as stocks, bonds, or property, already reflect all known information , therefore, are unbiased in the sense that they reflect the collective beliefs of all investors about future prospects. Therefore, at first glance, investor sentiment is not generally used to forecast the stock market. For example, in the work of Grossman (1976), prices bear all the information, while trading volume is not assigned any role in informing investors. The reason behind this approach is the belief that the equilibrium price alone reflects all the relevant information. For a long period in empirical research, the role of investor sentiment has not been recognized as a valid variable of interest. This traditional perspective underlies the notion that asset-pricing models rely on rational factors and complete information. In traditional models, rational investors make efficient use of information, while in empirical research investors are assumed to be rational yet imperfect. Investor heterogeneity suggests differences in investors' beliefs, risk aversion, and time preference. Such striking events as the 1987 crash and the internet bubble of 1990 have thrown doubt on the standard finance model in which stock prices equal the rational expectations of unemotional investors. After 20 years of discussion, the question is no longer whether stock prices are affected by investors' emotions, but rather how to measure and quantify these sentiments. However, it is obvious that measurement of information flow and investor sentiment is not straightforward, and proxies such as surveys, mood proxies, trading volumes, mutual fund flows, etc., have been used in the literature as an investor sentiment index (Baker and Wurgler, 2007).

Hong and Stein (2007) state that the importance of trading volume is pinned down by the unanticipated liquidity and portfolio rebalancing needs of investors in traditional pricing models. However, these drives seem to be too small to account for the huge amount of trading volume observed. This dissonance makes even the most ardent defenders of the rational asset-pricing models to admit that the bulk of volume must be from something else, such as the differences in investors beliefs.

In the research of Hong and Stein (2007), prices and returns are tightly correlated to movements in volume, and higher volume is more likely to accompany higher price levels. The authors contend that high-priced glamour stocks tend to be exchanged in higher volume than low-priced value stocks, and they calculate a high significant correlation of 0.49 between trading volume and prices of the $S\&P$ index from 1901 to 2005. Furthermore, an increasing number of empirical studies find a strong positive contemporary correlation between trading volume and return volatility [Karpoff, 1987, 1988; Gallant et al., 1992; Lamoureux and Lastrapes, 1994; Bollerslev, 2003]. Based on these empirical facts, the question is this: what is the economic sense behind this phenomenon? What drives volume and price change? Based on Kindleberger and Aliber (2005), the process is driven by speculation bubbles. In the work of Hong and Stein (2007), volume can stand for investor disagreement and it should be included in asset-pricing models. Baker and Wurgler (2007) argue the importance of investor sentiment, and volume can be a proxy for it.

However, in many early studies on stock returns and volatility of stock prices, the role of trading volume was set aside because of the endogeneity of trading volume. Trading volume and price are decided simultaneously by structural factors. While the research of Hong and Stein (2007) and Cassidy (2002) show that trading volume may be an indicator of investors' sentiment, in Baker and Wurgler (2007) only part of the trading volume may be seen as a proxy for sentiment, and the other part is used for the structural effect. They remove the economic fundamentals in volume data by regressing them on macroeconomic factors. The residual from this regression is used as the instrumental variable for investor sentiment.

Based on this method, Hong and Stein proposed "asset-pricing theories in which volume plays a central role." One key question they raised is this: what are the underlying mechanism to induce the disagreement in investors' belief to trading volume?

The primary reason for investors' disagreement regarding the importance of the trading volume effect is gradual information flow. Some investors have access to a certain kind of information earlier than others as a result of either the distribution of information technology, or investor segmentation and specialization. Different from the traditional rational-expectations theory, investors are not aware the fact that they may be at an informational disadvantage, and therefore do not draw the right inferences from others. Secondary reasons include limited attention and heterogeneous priors. Investors base their predictions from only a fraction of publicly available information. In addition, given the same information, investors rarely reach consensus about their decisions.

According to the research by Shefrin (2000), the market is not perfectly efficient. Investors have access to a heterogeneous pool of information and have different reactions to the same piece of information. Many investors are inexperienced and underinformed, rushing from one idea to another. Instead of following a random pattern, prices can sometimes be pushed up violently by mania while sometimes be pushed down by panics. For example, in the late stages of a bull market, the market is driven by buyers who take little notice of underlying risk. Towards the end of a crash, investors are hesitant to participate regardless of the unusually good value that their positions represent. Many investors are unaware of the irrationality of the market at its extremes. Many researchers believe that market participants are influenced by a complex set of factors that drive personal expectations about the future movement of prices. For example, Blanchard (1982) comments that public reaction to the same piece of information can arbitrarily be optimistic or pessimistic, depending on the individual's perception regarding the state of the economy.

Because of the challenge in quantifying the factors that influence information flow and investors' behavior, an efficient indicator of investor sentiment is necessary. Trading volume reflects the flow of transactions that underpin supply and demand factors operating in the market at a particular moment. Increasing volatility in the stock market manifests the presence of greater pressure or urgency by both buyers and sellers to engage in a transaction. Therefore, looking at trading volume as a signal of the strength of sentiment to trade seems to be a reasonable starting point.

Following Baker and Wurgler (2007), we regress trading volume on the three-month treasury bills interest rate to disassemble the macro structure imbedded in trading volume from investor sentiment. The residuals from this regression reflect the part standing for investor sentiment:

$$Volume_t = f(Interest\ rate) + \epsilon_t, \tag{1.2}$$

where ϵ_t is utilized as an indicator for the part remaining for investor sentiment in trading volume.

In the first paper, a high/low volume indicator variable in Donaldson and Kamstra (2004) is introduced to distinguish when the market has higher or lower sentiment. If the residual from regressing the investor sentiment indicator is greater than the previous week's average, the indicator is set to 1, and to 0 otherwise. This high/low investor sentiment variable is used to evaluate the relationship between investor sentiment, stock returns, and the change in direction of stock returns. Day-of-the-week dummy variables are also employed together with this investor sentiment indicator to check the effect on stock returns and changing directions.

Two models are set up in the first paper: a linear regression is introduced to demonstrate the investor sentiment effect on stock returns; and a Logit model is set up to visualize the effect of investor sentiment on the probability of an upward market.

The first paper aims to evaluate: (i) a positive effect of investor sentiment on stock return; and (ii) a positive effect of investor sentiment on market direction. Results show that investors' sentiments, as reflected in volume data, have a significant positive effect on the average of stock returns and market direction.

Based on conclusions from the first paper, in the second paper, the investors' sentiment variable is included in the mean and conditional variance equations of generalized autoregressive conditional heteroskedasticity models (GARCH), and their performance is compared to that of traditional GARCH models.

Traditional GARCH models forecast return and volatility based on lagged returns and innovations. It is important to emphasize that Engle (1982) and Bollerslev (1986) allow the inclusion of exogeneous variables in the conditional mean and variance equations. Considerable research has been done to include the day-of-the-week or holiday dummies into the return and conditional variance in GARCH models. (See Hsieh, 1989; Schwert, 1990; Tonchev and Kim, 2004; Malik and Hassan, 2004.) As suggested by Hong and Stein (2007), adding the investor sentiment into the mean and variance equations of the GARCH model may prove to be a more effective way to analyze the stock market.

Four of the most innovative GARCH models are compared: GARCH, FIGARCH (a fractional GARCH model considering long-range dependence), EGARCH (an exponential GARCH model accounting for leverage effect), and riskmetrics GARCH (an integrated GARCH model that was first introduced by the Morgan group and then widely adopted by banks and securities companies). The performance of these four GARCH models with and without investor sentiment is compared using in-sample AIC, BIC, log likelihhod and out-of-sample Value-at-Risk (VaR) forecasting.

VaR forecasting is commonly utilized as a stock-returns density prediction. Forecasting density has been a critical issue in the research of finance and economics. Its purpose is to model the potential uncertainty via parametric or nonparametric distribution functions. Historically, more attention has been given to evaluating point forecasts, while there has been less discussion given to interval forecasts, e.g., Chatfield (1993), Christoffersen (1998), and Clemen et al. (1995). In recent years, more and more interest has been paid to evaluating a density forecast. According to the research of de Diebold et al. (1998), Bao et al. (2004), and Raaij and Raunig (2005), the rapid development of density forecast originates from the booming demand for derivative products and financial risk management. Furthermore, the improvement of computer technology and simulation techniques has promoted more straightforward and precise density forecasts.

Compared with the first two papers, the third paper adopts a nonparametric method. As mentioned before, research on conditional volatility of asset prices has been the subject of intense investigation over the last few years. Most of the estimation has been parametric, e.g., GARCH family (Baillie and Bollerslev, 1989) and stochastic volatility models (Mahieu and Schotman, 1994). The nonparametric smoothing approach offers a flexible tool in analyzing unknown regression relationships between dependent and independent variables. Sometimes a preselected parametric model might be too restrictive or too low-dimensional to fit unexpected features. As an alternative, a nonparametric approach can be adopted without reference to a specific functional form.

In contrast with the parametric methods adopted in the first two papers, nonparametric methods can have certain advantages. According to Hardle and Sperlich (1998), the nonparametric approach to estimating a regression has the following benefits. First, it offers a versatile method of exploring a general relationship between the dependent and independent variables. Second, it provides predictions without respect to reference to a fixed parametric model. In the third paper, the use of a neural network is adopted, thanks to its ability to avoid the distribution assumption for the stock returns by forecasting VaR based on historical stock returns simulated from a Monte Carlo simulation.

In the third paper, a small Monte Carlo experiment, similar to the work by Park (2002), is adopted to generate stock returns. A series of standard normal distributions (heteroskedastic, but neither skewed nor leptokurtic), a series of standard student's-t distributions (heteroskedastic, not skewed but leptokurtic), and a series of skewed student's-t distributions (heteroskedastic, skewed, and leptokurtic) will be generated. For each of the series, a neural network is trained and simulated to forecast the VaR, and results are compared with traditional GARCH models.

As a nonparametric tool, neural networks have been used in domains of finance, such as portfolio selection, market distribution analysis, stock prediction, bond risk assessment, credit card fraud detection, and exchange rate forecast, etc. (Hamid, 2004). This method has proven to outperform linear models in a variety of circumstances (Hamid, 2004), especially in capturing complicated relationships in which traditional models fail to perform well (White, 1989; Kuan and White, 1994). White's (1988) research on IBM daily common-stock returns concluded that neural network method is capable of capturing some of the dynamic behavior of stock returns. In other words, a neural network approach has been proven to outperform traditional GARCH models. Based on these findings, the third paper forecasts VaR of stock returns using a neural network.

The results from the third paper illustrate that the performance of neural networks outperforms traditional GARCH models by both in-sample and out-of-sample mean squared error and VaR analysis. Thus, nonparametric methods provide a superior alternative to forecasting VaR.

Chapter 2

THE INVESTOR SENTIMENT AND DAY-OF-THE-WEEK EFFECT ON THE STOCK MARKET

2.1 Introduction

In our society, wherein the power of the media is continuously growing, the stock market's overall trend has a tendency to reflect the population's reaction to broadcast news: when information is positive, or when the public's reaction to news is favorable, the stock market typically reflects a general upward trend in prices. Conversely, a decline in the stock market's prices frequently originates from negative information. Comprehending what information is incorporated in asset prices through the trading process and the process by which it is incorporated is an essential research direction in finance. In this paper, we address these issues by empirical investigation of the role of trading volume on returns and market direction.

Blanchard (1982) found that public reaction to the same information can either be optimistic or pessimistic, depending on the health of the economy. Such a paradoxical result demonstrates the challenge residing in forecasting investors' sentiment. Another challenging task is quantifying the flow of information, as no reliable source reflects it. As a practical solution for traders and agents, a proxy could be employed. Thus, a promising candidate is trading volume (Kalotychou and Staikouras, 2006). Chordia and Swaminathan (2000) observe that daily and weekly returns on high trading volume portfolios lead returns on low trading volume portfolios. Kuo et al.'s (2004) research results regarding the relationship between trading volume and cross-autocorrelations of stock returns are different from the efficient market hypothesis that trading volume does not exhibit predictive power. Lamoureux and Lastrapes (1994) state the following regarding the imprecise role of the trading-volume effect in financial research: "Volume is likely to contain information about the disequilibrium dynamics of asset markets." Empirical surveys illustrate that time periods with high trading volume tend to correspond to periods of increasing volatility. For instance, trading volume and volatility are inclined to be greater during open and closed periods. Based on their study of intra-day volume and price movements, Brock and Kleidon (1992) point out that periods of increased trading volume tend to be periods of increased return variability. In particular, volume and return variability are higher at the open and closed periods of trading. Donaldson and Kamstra (2004) conclude that volume does, indeed, have predictive power for forecasting stock market returns despite previous studies reporting that trading volume can not forecast stock price and volatility directly.

Numerous empirical studies have discussed the strong positive contemporaneous correlation between trading volume and stock market returns [Karpoff, 1987, 1988; Gallant, et al., 1992; Lamoureux and Lastrapes, 1994; Bollerslev, 2003]. Hence it appears to be possible to analyze stock market returns based on knowledge of the trading volume.

While trading volume may signal a flow of information into the market, calendar effects can also constitute an important factor affecting the stock market return dynamics. And despite the fact that "efficient market theory states that anomalies may disappear once they are described by academics to the investment community because any profitable opportunities will be traded out of existence" (Taylor, 2005, p. 59), empirical studies have shown that anomalies from calendar effects exist throughout a long period in the history of stocks prior to the last thirty years (Taylor, 2005). Furthermore, calendar effects have been observed in stock market return data in very diverse institutional settings. For example, Jaffe and Westerfield (1985) introduce the presence of the day-of-the-week effect in four countries: Australia, Canada, Japan, and the UK. They additionally observe that the lowest mean returns occurred on Tuesday for Australia and Japan during different periods between 1950 and 1983.

Heckman and Rubinstein (2001) show that the Monday effect is the most potent of calendar anomalies. Their research indicates that, for all twelve nonoverlapping five-year periods from 1928 to 1987, not only have Monday returns been negative but also Monday has demonstrated the lowest returns of the week. However, this research has also indicated that, from 1989 to 1998, Monday returns appeared to be positive and Monday was the best day of the week. Taylor (2005) stated that this Monday effect anomaly is puzzling because, as long as the public knows the information, such a behavior should not be observed.

This chapter incorporates the day-of-the-week as another explanatory variable in the model for determining the effect of investor sentiment on stock price. The rationale is to have a model correctly specified such that a relevant variable would not be excluded even if, after calibration, it proves to be meaningless. The work reported here uses daily return data for three stock indices data, S&P 500, Dow Jones Industrial Average, and Nasdaq Composite, and four individual stocks Exxon (XOM), Walmart(WAL), General Electric (GE), and Texas Instruments (TXN). Based on daily returns from January 1988 to December 2006, this chapter examine the effect of investor sentiment as reflected in day-of-the-week and trading volume on stock returns and on the change in direction.

The chapter is organized as follows. Section 2.2 provides a review of existing literature. Section 2.3 introduces models to test investor sentiment effect on returns and market direction through linear regression and Logit regression approaches. Section 2.4 discusses investor sentiment effect on stock returns. Section 2.5 presents the analysis of investor sentiment effect on the changing direction of stock returns.

2.2 Literature Review

Previous studies report that the absolute price change in the stock market is positively correlated with trading volume. A summary of Karpoff's (1987) survey about the relationship between price changes and trading volume is given in Table 2.1. In this study, both equity and futures market are examined. By using different measurements of price changes and trading volume, almost all studies suggeste a positive correlation between absolute return and trading volume.

However, using data on 12 futures contracts from the Chicago Board of Trade (CBOT) during the period 1972-79, Karpoff (1988) finds little correlation between absolute daily return and volume. On the other hand, he finds positive and statistically significant correlation for equity markets, wherein there are restrictions on short sale contracts. In other words, when investors are restrained from acting on their information or beliefs by urgent decision-making, there is no correlation between absolute return and volume.

Table 2.1: Summary of the Research Regarding Volume (Karpoff, 1987).

Author	Year	Sample Data	Sample period	sample interval	Support positive | p |, V) Correlation?
Godfrey et al.	1964	Stock market aggregates, 3 common stocks	1959-62, 1951-53,63 daily transactions	Weekly	No
Cornell	1981	Future contracts for 17 commodities	1968-79	Daily	Yes
Harries	1983	16 common stocks	1981-83	Transactions, daily	Yes
Rutledge	1984	Future contracts for 13 commodities	1973-76	Daily	Yes
Wood et al.	1985	946 common stocks, 1138 common stocks	1971-72 1982	Minutes	Yes
Grammatikos and Saunders	1986	Futures contracts for 5 foreign currencies	1978-83	Daily	Yes
Harris	1986	479 common stocks	1976-77	Daily	Yes
Jain and Joh	1986	Stocks market aggregates	1979-83	Daily	Yes
Richardson et al.	1989	106 common stocks	106 common stocks	weekly	Yes
Pettengill and Jordan	1988	S&P 500 and Small firm index	1962-85	Daily	yes
Martikainen et al.	1994	HeSE daily index	1977-88	Daily	yes for 1983-88, no for 1977-82.
Karpoff	1988	12 commodities from CBOT	1972-79	Daily	yes for equity market, no for future market

Pettengill and Jordan (1988) set up a general linear model to assess the relative importance of the various seasonal influences on stock returns from 1962 to 1985. The stock returns regress on dummy variables of months, day-of-the-week and holidays:

$$R_t = F(TOM, TOY, JAN, EXP, TRD, DAY, PRE, PST), \tag{2.1}$$

where TOM is a dummy variable indicating turn of the month ($TOM = 1$ if it is the first trading day of a month), TOY accounts for any additional effect of the turn of the year, ($TOY = 1$ for the first trading day of a year), JAN stands for January, EXP and TRD are dummy variables examining a potential explanation for the lower returns in the third week; DAY measures the day-of-the-week effect, and PRE and PST are dummy variables associated with holiday effects.

The results demonstrate that for stock price returns for both large firms, represented by S&P 500 and small firms, the day-of-the-week effect is highly pronounced. However, the turn of the month appears to be less significant for stock price returns of small firms than for large firms.

2.3 The Model

It is common knowledge that information flow and investor sentiment are not straightforward to measure, and proxies, such as surveys, mood proxies, trading volume, mutual fund flows, etc., have been used in the literature as investor sentiment indices (Baker and Wurgler, 2007).

Hong and Stein (2007) state that the importance of trading volume is pinned down by the unanticipated liquidity and portfolio rebalancing needs of investors in traditional pricing models. However, these drives seem to be too small to account for the huge amount of trading volume observed. This dissonance makes even the most ardent defenders of the rational asset-pricing models to admit that the bulk of volume must be from something else, such as the differences in investors beliefs.

From these observations, Hong and Stein formulate a theoretical foundation for asset-pricing theories in which volume plays a central role. Based on this definition, it is first required to disaggregate the trading volume effect because it reflects both economic fundamentals and investor sentiment. The decomposition method introduced in Baker and Wurgler (2007) is selected in this paper. To remove the structural effect, volume is regressed on three-month treasury bill interest rates, which can be considered as a macroeconomic indicator, and the residuals of the regression are treated as the part remaining for investor sentiment.

$$Volume_t = f(Interest\ rate) + \epsilon_t, \tag{2.2}$$

where $Volume_t$ is the money value of trading volume traded per day, and ϵ_t can be viewed as an indicator for the part remaining for investor sentiment in trading volume.

We consider a similar high/low volume indicator variable V_t as in Donaldson and Kamstra (2004) to evaluate the relationship between investor sentiment, stock returns, and the change in direction of stock returns.

In the euqtion below, TV stands for the investor sentiment portion of trading volume, which is ϵ_t in equation 2.2. V_t is a dummy variable. Whenever the TV exceeds the previous week's average (average of last 5 days' investor sentiment), it is equal to 1 and to 0 otherwise, shown as follows:

$$V_t = \begin{cases} 1 & \text{if } TV_t > \frac{TV_{t-1}+TV_{t-2}+TV_{t-3}+TV_{t-4}+TV_{t-5}}{5} \\ 0 & \text{otherwise} \end{cases}$$

The day-of-the-week dummy variables (D_{Monday}, $D_{Tuesday}$, $D_{Thursday}$, and D_{Friday}) associated with Monday, Tuesday, Thursday, and Friday will be used together with V_t to validate our hypothesis regarding the effect on stock returns and changes in directions.

We define r_t as the continuously compounded return at time t:

$$r_t = 100Log\frac{P_t}{P_{t-1}}, \tag{2.3}$$

where P_t is the adjusted closing price at time t that considers dividend and stock split. *Log* refers to the natural logarithm.

Hong and Stein (2007) showed that a positive relationship may exist between trading volume and returns because trading volume may reflect the investor sentiment. The following models aim to prove that there is a positive effect of investor sentiment on stock returns and market direction.

Model 1: an ordinary linear regression is used to show the evidence of a positive effect of investor sentiment on stock returns. Stock return is regressed on investor sentiment and day-of-the-week dummy variables.

$$r_t = \alpha + \beta V_t + \gamma_1 D_{Monday} + \gamma_2 D_{Tuesday} + \gamma_3 D_{Thursday} + \gamma_4 D_{Friday} + \upsilon_t \tag{2.4}$$

Model 2: a Logit regression is used to show the evidence of a positive effect of investor sentiment on stock market direction.

There are two possibilities for stock market direction: upward market or downward. Suppose a positive return means an upward market, and the possibility of having an upward market is :

$$Pr(r_t > 0) = Pr(Y = 1) = p, \tag{2.5}$$

while the probability of a downward stock market is:

$$Pr(r_t \leq 0) = Pr(Y = 0) = 1 - p \tag{2.6}$$

A Logit model can be constructed defining $Pr(r_t > 0)$ as the probability of an upward market:

$$Pr(r_t > 0) = \frac{e^{\beta V_t + \gamma_1 D_{Monday} + \gamma_2 D_{Tuesday} + \gamma_3 D_{Thursday} + \gamma_4 D_{Friday}}}{1 + e^{\beta V_t + \gamma_1 D_{Monday} + \gamma_2 D_{Tuesday} + \gamma_3 D_{Thursday} + \gamma_4 D_{Friday}}} \tag{2.7}$$

These two models attempt to predict: (i) a positive effect of investor sentiment on stock return; and (ii) a positive effect of investor sentiment on market direction.

Tables 2.2 and 2.3 show the descriptive statistics for return sorted by the investor sentiment dummy variable and the day-of-the-week dummies. Obviously, if investor sentiment is higher than the previous week's average, the mean value and standard deviation are significantly larger for each stock. In addition, the mean of the return is often negative when investor sentiment is lower than the week's average. We can also remark that the day-of-the-week effect is not same for all the stocks.

Table 2.4 demonstrates the assorted statistics of stock returns by $V = 0$ and $V = 1$ for GM, TXN, Dow Jones, Nasdaq, and $S\&P$ 500 for the period from 1988 through 2006. Together with the results from Tables 2.2 and 2.3, it is easy to verify that the return and variance are much higher when $V = 1$. The most significant observation lies in the fact that two out of seven stocks show negative average returns when $V = 0$. This finding shows that when investor sentiment is high, stock return is more likely to be higher and vice versa.

Table 2.2: Descriptive Statistics for Returns of XOM.

XOM	N	Mean	Std. Dev	Minimum	Maximum
V=0	2113	-0.02889	1.166656	-4.32276	6.495001
V=1	1652	0.143469	1.703413	-8.83019	9.279536
MONDAY	715	0.106204	1.437522	-7.70241	5.259007
TUESDAY	771	0.066062	1.405638	-5.29968	9.242972
WEDNESDAY	769	0.041336	1.458194	-4.56493	9.279536
THURSDAY	757	-0.02014	1.410986	-8.83019	6.495001
FRIDAY	753	0.043233	1.436382	-7.00882	3.940708

Note: V=1 indicates the days when investor sentiment is higher than the previous week average, and V=0 otherwise. Monday, Tuesday, Wednesday, Thursday, and Friday indicate the stock returns on that specified day. Minimum and Maximum indicate the minimum and maximum value of stock return.

2.4 Impact of Investor Sentiment on Stock Returns

An ordinary linear regression is employed to demonstrate the evidence of positive

Table 2.3: Descriptive Statistics for Returns of WAL.

WAL	N	Mean	Std. Dev	Minimum	Maximum
V=0	2093	-0.02647	1.549424	-7.3052	7.008676
V=1	1591	0.215468	2.4535	-10.2389	8.706984
MONDAY	699	0.145034	2.029291	-10.2389	8.706984
TUESDAY	754	0.064955	1.964679	-7.01636	7.605729
WEDNESDAY	753	0.101378	2.044586	-9.95746	8.124767
THURSDAY	741	0.026133	1.999458	-8.64632	6.61398
FRIDAY	737	0.056117	1.935617	-8.80701	7.64399

Note: V=1 indicates the days when investor sentiment is higher than the previous week average, and V=0 otherwise.
Monday, Tuesday, Wednesday, Thursday, and Friday indicate the stock returns on that specified day. Minimum and Maximum indicate the minimum and maximum value of stock return.

Table 2.4: Descriptive Statistics for Returns Sorted by $V = 0$ and $V = 1$.

Stock		Mean	Std. Dev	Minimum	Maximum
GM	V=0	-0.12088	1.524594	-6.77529	7.531746
	V=1	0.200623	2.53518	-15.0282	16.6511
TXN	V=0	-0.06264	2.326772	-10.7654	14.22925
	V=1	0.205998	3.557287	-20.1265	21.563
Dow Jones	V=0	6.77E-05	0.810679	-3.76729	5.273169
	V=1	0.078565	1.128674	-7.4549	6.154722
Nasdaq	V=0	-0.05548	1.261729	-7.50649	7.637188
	V=1	0.134753	1.577885	-10.1684	13.25464
$S\&P$ 500	V=0	-0.01021	0.829008	-3.58671	5.266658
	V=1	0.085714	1.123277	-7.11274	5.574432

Note: V=1 indicates the days when investor sentiment is higher than the previous week average, and V=0 otherwise.
Minimum and Maximum indicate the minimum and maximum value of stock return.

investor sentiment effect on stock returns. Stock return is regressed on investor sentiment and day-of-the-week dummy variables. The purpose is to testify the investor sentiment effect and possible day-of-the-week effect. Monday, Tuesday, Thursday and Friday will be selected from the week and Wednesday will be left as the base.

$$r_t = \alpha + \beta V_t + \gamma_1 D_{Monday} + \gamma_2 D_{Tuesday} + \gamma_3 D_{Thursday} + \gamma_4 D_{Friday} + \epsilon_t \qquad (2.8)$$

The outcome from the ordinary linear regression as shown in Table 2.5 suggests that there is a significantly positive relationship between return and investor-sentiment dummies. The result is quite robust. All of the parameters regarding investor sentiment are positive and significant at 1%. This indicates that if investor sentiment in a given day day is larger than the previous week's average, there is a positive effect on stock returns. However, it is difficult to conclude whether a common day-of-the-week effect exists for all stocks. It is true that the Monday effect appears for some series, while for other stocks there is no significant effect.

Table 2.5: Parameter Estimated by Linear Regression.

STOCK	Linear Regression				
	Investor Sentiment Effect	MONDAY	TUESDAY	THURSDAY	FRIDAY
Dow Jones	0.19628	0.15163	0.04517	-0.04698	0.00050790
	(0.02994)*	(0.04737)*	(0.04497)	(0.04502)	(0.04530)
Nasdaq	0.17070	-0.09244	-0.10027	-0.03420	-0.04103
	(0.04470)*	(0.07112)	(0.06674)	(0.06696)	(0.06760)
SP	0.10364	0.06381	0.00409	-0.04753	-0.00373
	(0.03063)*	(0.04875)	(0.04590)	(0.04605)	(0.04632)
XOM	0.21804	0.13076	0.06993	-0.04155	0.02060
	(0.04662)*	(0.07413)***	(0.07159)	(0.07181)	(0.07196)
WAL	0.26928	0.14381	0.00427	-0.03055	-0.00203
	(0.06816)*	(0.10819)	(0.10449)	(0.10491)	(0.10508)
GM	0.35160	0.34118	0.01909	-0.08311	-0.05442
	(0.05997)*	(0.09546)*	(0.09245)	(0.09291)	(0.09305)
TXN	0.23840	-0.17322	-0.15572	-0.13113	0.37141
	(0.08932)*	(0.14228)*	(0.13727)	(0.13785)	(0.13838)*

Note: $*$ indicates significance at 1% level.
$**$ indicates significance at 5% level.
$***$ indicates significance at 10% level.
The parameters are estimated by a linear regression.

2.5 Impact of Investor Sentiment on the Direction of Stock Market

Numerous studies are based on the relationship between detrended trading volume and stock returns; refer to Hong and Stein (2007). However, none of these pieces of work provide an explanation for the relationship between investor sentiment and stock market direction. Suppose that a positive stock returns demonstrates an upward market, while a negative sign of stock returns demonstrates a downward market. Given this assumption, a Logit regression can be employed and compared to the linear regression above. Specifically, the major contribution of this study is to further prove that higher investor sentiment does have a positive effect on stock returns, and to demonstrate the relationship between market direction and investor sentiment.

Table 2.6: Parameter Estimated by Logit Regression.

STOCK	Linear Regression				
	Investor Sentiment Effect	MONDAY	TUESDAY	THURSDAY	FRIDAY
Dow Jones	0.3507	0.3063	-0.0285	-0.1022	0.0765
	(0.0614)*	(0.0973)*	(0.0919)	(0.0920)	(0.0927)**
Nasdaq	0.3071	-0.1577	-0.2508	-0.0829	-0.0213
	(0.0622)*	(0.0990)	(0.0931)	(0.0937)	(0.0946)
SP	0.2130	0.1109	-0.1326	-0.1438	0.0148
	(0.0610)*	(0.0965)	(0.0909)	(0.0911)	(0.0918)
XOM	0.3519	0.1409	-0.0542	-0.0931	0.0364
	(0.0680)*	(0.1080)	(0.1044)	(0.1048)	(0.1047)
WAL	0.3826	0.1411	-0.0322	0.0377	0.1225
	(0.0688)*	(0.1093)	(0.1057)	(0.1060)	(0.1060)
GM	0.4094	0.3978	0.0856	-0.0431	0.0269
	(0.0603)*	(0.0960)*	(0.0930)	(0.0937)	(0.0936)
TXN	0.2531	-0.0759	-0.0824	-0.1648	-0.3121
	(0.0602)*	(0.0957)	(0.0924)	(0.0929)***	(0.0935)*

Note: $*$ indicates significance at 1% level.
$**$ indicates significance at 5% level.
$***$ indicates significance at 10% level.
The parameters are estimated by a Logit model.

In this section, a similar approach is adopted; we assume there are two possibilities in the market: to earn, or a positive return, meaning an upward market or not to earn, meaning a downward market. An upward market can be seen as:

$$Pr(r_t > 0) = Pr(Y = 1) = p, \tag{2.9}$$

while a downward stock market can be seen as:

$$Pr(r_t \leq 0) = Pr(Y = 0) = 1 - p, \tag{2.10}$$

where Y follows a Bernoulli distribution. According to Greene (1997), a Logit model can be constructed defining $Pr(r_t > 0)$ as the probability of positive returns:

$$Pr(r_t > 0) = \frac{e^{\beta V_t + \gamma_1 D_{Monday} + \gamma_2 D_{Tuesday} + \gamma_3 D_{Thursday} + \gamma_4 D_{Friday}}}{1 + e^{\beta V_t + \gamma_1 D_{Monday} + \gamma_2 D_{Tuesday} + \gamma_3 D_{Thursday} + \gamma_4 D_{Friday}}} \tag{2.11}$$

Estimates from the Logit approach, as well as the associated t-statistics, are reported in Table 2.7. Similar to the outcome from linear regression, investor sentiment has a positive effect on the odds ratio at the 1% significance level, which can be seen as a positive effect on stock market direction. The day-of-the-week effect varies according to stocks.

To define the relationship between investor sentiment and stock-price movement, the Logit model is particularly suitable because the direction of movement is binary in nature, i.e., either up or down. In this section, we will adopt a Logit model and compare it with the linear regression method.

In recent years, there have been a growing number of studies using Logit model to analyze the relationship between stock returns and fundamental variables. By using a Logit model to check the relationship between the probability of a positive daily stock return and cloudiness in New York City, Hirshleifer and Shumway (2003) observed a statistically significant link between returns and cloudiness. Similar research employing a Logit model has been reported by Loughran and Schultz (2006). By examining 26 stock exchanges internationally from 1982 to 1997, using Log regression Hirshleifer and Shumway (2003) found that daily stock returns are significantly correlated with sunshine. This finding suggests

Table 2.7: Parameter Estimated by Logit Regression.

STOCK	Linear Regression				
	Investor Sentiment Effect	MONDAY	TUESDAY	THURSDAY	FRIDAY
Dow Jones	0.3507	0.3063	-0.0285	-0.1022	0.0765
	(0.0614)*	(0.0973)*	(0.0919)	(0.0920)	(0.0927)**
Nasdaq	0.3071	-0.1577	-0.2508	-0.0829	-0.0213
	(0.0622)*	(0.0990)	(0.0931)	(0.0937)	(0.0946)
SP	0.2130	0.1109	-0.1326	-0.1438	0.0148
	(0.0610)*	(0.0965)	(0.0909)	(0.0911)	(0.0918)
XOM	0.3519	0.1409	-0.0542	-0.0931	0.0364
	(0.0680)*	(0.1080)	(0.1044)	(0.1048)	(0.1047)
WAL	0.3826	0.1411	-0.0322	0.0377	0.1225
	(0.0688)*	(0.1093)	(0.1057)	(0.1060)	(0.1060)
GM	0.4094	0.3978	0.0856	-0.0431	0.0269
	(0.0603)*	(0.0960)*	(0.0930)	(0.0937)	(0.0936)
TXN	0.2531	-0.0759	-0.0824	-0.1648	-0.3121
	(0.0602)*	(0.0957)	(0.0924)	(0.0929)***	(0.0935)*

Note: $*$ indicates significance at 1% level.
$**$ indicates significance at 5% level.
$***$ indicates significance at 10% level.
The parameters are estimated by a Logit model.

that weather may be related to investor sentiment, which in turn affects stock returns and market direction. In their model, the probability of a positive return is positively correlated to total sky cover (SKC), which ranges from 0 (clear) to 8 (overcast):

$$Pr(r_{it} > 0) = \frac{e^{r_i SKC_{it}}}{1 + e^{r_i SKC_{it}}} \tag{2.12}$$

2.6 Concluding Comments

Based on daily return data of three stock indices and four individual stocks from January 1988 to December 2006, the role of day-of-the-week, as well as investor sentiment, is examined by two approaches: linear regression to test the investor sentiment effect on stock returns and Logit regression to test the investor sentiment effect on market direction. The results indicate that there is a significant positive role of investor sentiment in the market. However, the outcome also shows that the role of day-of-the-week effect varies according to stocks.

Chapter 3

MODELING STOCK RETURNS VOLATILITY WITH INVESTOR SENTIMENT EFFECT

3.1 Introduction and Literature Review

Traditional GARCH models estimate return and volatility depending on lagged returns and innovations. A short list of documented research includes ARCH process (Engle, 1982), GARCH process (Bollerslev, 1986; Bollerslev et al., 1992), exponential GARCH considering asymmetric volatility (Nelson, 1991; Duffee, 1995), long memory ARCH (Bollerslev and Mikkelsen, 1996, 1999) and Integrated GARCH (Nelson, 1990). All these publications estimate the conditional mean and variance based on the past information set. However, lagged returns are not the only candidate as a measure of market information. It is evident that the flow of information cannot be easily quantified, and a proxy could be the tool used by traders and agents. A promising candidate is trading volume (Kalotychou and Staikouras, 2006). Lamoureux and Lastrapes (1994) discuss the issue of disregarding the imprecise role of trading-volume effect in financial research: "Volume is likely to contain information about the disequilibrium dynamics of asset markets." Huffman (1992) demonstrates that asset prices increase with trading volume. Lamoureux and Lastrapes (1990) used daily trading volume as a proxy for information arrival time, showing that trading volume possesses significant explanatory power regarding daily stock return volatility. Empirical surveys have proven that periods of high trading volume tend to be periods of increasing volatility. For example, trading volume and volatility tend to be higher during the opening and closing periods. Although financial research has long recognized that trading volume may provide valuable information associated with stock returns, the efficient-market hypothesis holds that trading volume should have no predictive power (Kuo et al., 2004). In the work of Grossman (1976), trading volume plays no role in informing investors because the equilibrium price alone can provide all the relevant information.

Recently, an increasing number of studies have addressed the issue of the strong positive contemporaneous correlation between trading volume and volatility (Clark, 1973; Karpoff, 1987, 1988; Gallant et al., 1992; Lamoureux and Lastrapes, 1994; Bollerslev, 2003). Kalotychou and Staikouras (2006) include either contemporaneous or lagged value of trading volume into GARCH models, and either is found to have a significant coefficient. Therefore, it appears to be possible to estimate volatility based on knowledge of the trading volume.

In the second essay of this dissertation, based on daily return data for three stock index data and four individual stocks from January 1988 to December 2006, the role of day-of-the-week as well as investor sentiment is examined. Through linear regression and Logit regression approaches, we observe the presence of a significant positive role for investor sentiment in the market. The results demonstrate that not only does investor sentiment have a significant positive effect on stock returns, but also on change in stock market. The research gives a theoretical background for adding investor sentiment effect in GARCH models.

Moreover, it is important to emphasize that Engle (1982) and Bollerslev (1986) allow the inclusion of exogeneous variables in the conditional mean and variance. A vast amount of research has been done to include the day-of-the-week or holiday effect into GARCH models. Hsieh (1989) demonstrates a GARCH model with return and conditional variance regressed on day-of-the-week and holiday dummies to analyze five daily exchange rates from 1974 to 1983:

$$r_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i r_{t-i} + \alpha_M D_{Mt} + \alpha_T D_{Tt} + \alpha_W D_{Wt} + \alpha_F D_{Ft} + \alpha_H HOL_t + \varepsilon_t \qquad (3.1)$$

$$h_t = \omega + \sum_{i=1}^{q} \beta_i \varepsilon_{t-1}^2 + \beta_M D_{Mt} + \beta_T D_{Tt} + \beta_W D_{Wt} + \beta_F D_{Ft} + \beta_H HOL_t, \tag{3.2}$$

where D_{Mt}, D_{Tt}, D_{Wt}, D_{Ft}, and HOL_t are dummy variables standing for Monday, Tuesday, Wednesday, Friday, and the number of holidays (excluding weekends) in a year.

Similar research regarding adding the day-of-the-week effect into GARCH models has been performed by Schwert (1990) and Schwert (1999). Complementing the day-of-the-week effect, Tonchev and Kim (2004) included dummy variables modeling the January effect, the half-month effect, and the turn of the month effect. The research of Malik and Hassan (2004) adds dummy variables indicating each point of sudden changes of variance onward into the conditional variance equation in GARCH models.

Compared to the much-discussed calendar effect in GARCH modeling, the trading volume effect has been less widely discussed. Lamoureux and Lastrapes (1990) add trading volume into conditional variance:

$$r_t = \mu_{t-1} + \varepsilon_t h_t = \alpha_1 \varepsilon_{t-1} + \alpha_2 h_{t-1} + \alpha_3 V_t \tag{3.3}$$

$$h_t = \alpha_1 \varepsilon_{t-1} + \alpha_2 h_{t-1} + \alpha_3 V_t, \tag{3.4}$$

where μ_{t-1} is the mean r_t conditional on past information, and V_t is the trading volume. After comparing GARCH models with and without a volume effect on 20 individual daily stocks returns, Lamoureux and Lastrapes (1990) demonstrate that the persistence of variance as measured by the sum $\alpha_1 + \alpha_2$ was significantly lower for a GARCH model that included volume. In other words, the GARCH effect tends to fade when volume is included in the variance equation.

It has been well documented that both linear and nonlinear time trends exist in trading-volume series (Gallant et al., 1992). Therefore, it is important to determine a method to measure the volume variable. The research of Lamoureux and Lastrapes (1990) adds raw trading volume into the variance equation. Most research uses detrended trading volume to test the effect on the stock market. Chen et al. (2001) adopt a quadratic time trend term to detrend the trading volume series:

$$V_t = \alpha + \beta_1 t + \beta_2 t^2 + \epsilon_t, \tag{3.5}$$

where V_t is raw trading volume, and detrended trading volumes are the residuals from the equation. Other measures include turnover, log turnover adjusted by a moving average, and log of volume (Groenewold, 2006).

This paper adopts an approach similar to that described by Donaldson and Kamstra (2004) to measure investor sentiment effect. TV stands for investor sentiment portion of trading volume, which is ϵ_t in equation 2.2. V_t is a dummy variable that equals 1 whenever the investor sentiment exceeds the week's average (the previous five days investor sentiment average), otherwise it is equal to 0, such that:

$$V_t = \begin{cases} 1 & \text{if } TV_t > \frac{TV_{t-1}+TV_{t-2}+TV_{t-3}+TV_{t-4}+TV_{t-5}}{5} \\ 0 & \text{otherwise} \end{cases}$$

The paper is organized as follows. Section 3.2 provides a review of the traditional GARCH models, including GARCH, EGARCH, FIGARCH, FIEGARCH, and Riskmetrics. Section 3.3, the GARCH models considering investor sentiment effect are presented. In section 3.4, a comprehensive analysis of the distributional, statistical, and time series properties of the data is explained. Section 3.5 and 3.6, both in-sample and out-of-sample forecasts are compared and discussed.

3.2 Review of GARCH Models

In this section, four different traditional GARCH models (GARCH, FIGARCH, EGARCH, Riskmetrics) are introduced.

3.2.1 *Traditional GARCH* $(1, 1)$

Prior to discussing a GARCH $(1, 1)$ process, it is important to present the ARMA (p, q) process:

Let$\{x_t\}_{t=1}^{\infty}$ be a stationary process defined by:

$$\phi(B)x_t = \theta(B)\varepsilon_t, \tag{3.6}$$

where $\phi(B)$ is the autoregressive polynomial operator, $\phi(B) = 1 - \alpha_1 L - \alpha_2 L ... - \alpha_p L$; $\theta(B)$ is the moving average polynomial operator, and $\theta(B) = 1 - \beta_1 L - \beta_2 L ... - \beta_q L$. ϵ_t is a white noise process normally distributed with mean zero and finite variance σ^2.

In the ARMA(p, q) process, the variance of the disturbance term is assumed to be constant, namely homoskedastic. However, many time series data series exhibit volatility clustering. Engle (1982) defines a stochastic process whose conditional variance is a linear function of the square of the estimated residuals, called an autoregressive conditional heteroskedastic (ARCH) model. Bollerslev (1986) extends Engle's work by allowing the conditional variance to be an ARMA process, which is a generalized ARCH model, also known as a GARCH model. One important difference between GARCH and ARMA processes is that the former allows volatility shocks to persist over time. The key feature of GARCH models is that both autoregressive and moving average components are included in the heteroskedastic variance (Bollerslev, 1989, 1990; Bollerslev and Mikkelsen, 1996a; Bollerslev et al., 1994).

A GARCH (1, 1) model is the most popular model in empirical research and is defined as:

$$r_t = \mu + \varepsilon_t \tag{3.7}$$

$$\varepsilon_t = h_t^{\frac{1}{2}} z_t \tag{3.8}$$

$$h_t = \omega + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}, \tag{3.9}$$

where $\omega \geq 0$, $\beta_1 \geq 0$ and $\beta_2 \geq 0$. h_t is the conditional variance. ε_t stands for the residual of the process and z_t is defined as the standardized residuals by:

$$z_t = \frac{\epsilon_t}{\sqrt{h_t}}, \tag{3.10}$$

the distribution of z_t conditional on previous x is:

$$z_t | x_{t-1}, x_{t-2} \ldots \sim N(0,1) \tag{3.11}$$

Such a process is stationary if and only if $\beta_1 + \beta_2 < 1$. In Taylor (2005), when this condition is satisfied, the unconditional variance is finite, the unconditional kurtosis is always greater than 3 and can be infinite, the correlation is zero between any two variables in the time-series data, and the correlation is positive between any squared residuals.

3.2.2 *FIGARCH(1,d,1)*

Some empirical research demonstrates the existence of long memory in time series data. This long memory, or long run dependence, exists when the autocorrelation function displays a slow decay, while the process remains stationary. The research of this long-memory phenomenon has received much attention in recent years. Fractionally differenced time series models have been adopted widely in practice.

The processes in ARCH models developed by Engle and GARCH models by Bollerslev are short memory since the response of a shock on the conditional variance decreases at an exponential rate. For a weakly stationary time series wherein sample mean μ_i is independent of t and correlation $\rho(t+h, t)$ is independent of t for each h, the autocorrelation function (ACF) is defined as:

$$\rho(k) = \frac{E[(x_t - \mu)(x_{t+k} - \mu)]}{\sigma^2} \tag{3.12}$$

where μ is the mean and σ^2 is the variance.

If $\sum_{k=-\infty}^{\infty} |\rho(k)| < \infty$, the time series exhibits short memory or short range dependence, or weak dependence, while if $\sum_{k=-\infty}^{\infty} |\rho(k)| = \infty$ as $k \to \infty$, and the time series displays long memory.

The correlation function $\rho(k)$ satisfies $\rho(k) \sim C_p|k|^{-2(1-H)}$ as $k \to \infty$, where C_p is a constant, $C_p > 0$ and the symbol $\sim$ means "asymptotically equal to."

In Hosking (1981) and Brockwell and Davis (1987), the fractional ARMA (p, d, q) model is defined as:

$$
\begin{aligned}
\Phi(B)(1-B)^d x_t &= \theta(B)\epsilon_t \\
(1-B)^d &= \sum_{k=0}^{\infty}(dk)(-B)^k = \sum_{k=0}^{\infty}\pi_k B^k \\
\pi_k &= \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} = \prod_{1\le j\le k}\frac{j-1-d}{j} \\
\Gamma(x) &= \left\{ \begin{array}{lr} \int_0^{\infty} t^{x-1}e^{-t}dt \quad , & x > 0 \\ \infty \quad , & x = 0 \\ x^{-1}\Gamma(1+x) \quad , & x < 0, \end{array} \right|
\end{aligned}
$$

where $(1-B)^d$ is the fractional differentiating operator and $\Gamma(.)$ is the gamma function.

For $d \in (-0.5; 0.5)$ the process defined is stationary and invertible.

As mentioned previously in this chapter, the processes in ARCH models and GARCH models developed by Engle (1982) and Bollerslev (1986) are short memory. While more and more research has proven the existence of long run dependence in the conditional variance process, volatility tends to vary at a slow rate over time, and the autocorrelation is dominated by a hyperbolic rate of decay.

Different from GARCH models which focus on the short term volatility specification and forecast, FIGARCH models illustrate a finite persistence of volatility shocks. A FIGARCH model possesses a fractional ARMA structure on the variance. When $d = 1$, a

FIGARCH model is reduced to an integrated GARCH model; and when $d = 0$, it is reduced to a GARCH model.

A FIGARCH(1, d, 1) model is defined as:

$$r_t = \mu + \varepsilon_t \tag{3.13}$$

$$\varepsilon_t = h_t^{\frac{1}{2}} z_t \tag{3.14}$$

$$h_t = \omega + \beta_1 h_{t-1} + (1 - \beta_1 Ł - (1 - \varphi_1 Ł)(1 - L)^d)\varepsilon_{t-1}^2, \tag{3.15}$$

where z_t is a white noise with mean 0 and variance 1, following an AR(1) process, and the conditional variance h_t is represented by a FIGARCH process.

$$(1 - \beta_1 Ł)h_t = \omega + (1 - \beta_1 Ł - (1 - \varphi_1 Ł)(1 - L)^d)\varepsilon_{t-1}^2, \tag{3.16}$$

where all the roots of $\varphi_1 Ł$ and $(1 - \beta_1 Ł)$ lie outside of the unit circle, and $0 < d < 1$.

The conditional variance of the FIGARCH process is written as:

$$h_t = \frac{\omega}{1 - \beta_1 Ł} + \lambda(L)\varepsilon_{t-1}^2, \tag{3.17}$$

where

$$\lambda(L) = (1 - (1 - \varphi_1 Ł)(1 - L)^d))(1 - \beta_1 Ł)^{-1} \tag{3.18}$$

3.2.3 *EGARCH*

Traditional linear GARCH models place a nonnegativity condition on all parameters. However, Nelson and Cao (1992) argue that it is overly restrictive to confine all coefficients to be nonnegative because stock return and volatility can be negatively correlated from some empirical research. The work of Black (1976) illustrates that there is an inverse relationship between current return and future volatility. This fact was also proven by Christie (1982) and French et al. (1987). The EGARCH model proposed by Nelson (1991) thus concludes that the nonnegative constrains are too restrictive. Nelson (1991) introduces asymmetry, also known as leverage effect, into conditional variances. Whenever there is a negative return, the market responds more vigorously than whenever there is a positive return. In other words, the market is particularly sensitive to negative returns. In summary, price changes and volatility are more significantly negatively related by leverage effect.

In an EGARCH model, h_t, the conditional variance, is the exponential of an AR process. Furthermore, h_t is defined as the asymmetric function of lagged disturbances ε_{t-i} while there is no nonnegative restriction:

$$Ln(h_t) = \omega + \sum_{i=1}^{q} \alpha_i g(z_{t-i}) + \sum_{j=1}^{p} \gamma_j Ln(h_{t-j}) \tag{3.19}$$

$$g(z_t) = \nu_1 z_t + \nu_2(|z_t| - E|z_t|) \tag{3.20}$$

$$\varepsilon_t = h_t^{\frac{1}{2}} z_t \tag{3.21}$$

The three variance parameters are: γ_j, the autoregressive parameter for process, $Ln(h_t)$, the parameter accounting for leverage effect, and ν_1, a parameter appearing in $g(z_t)$. As long as the absolute value of γ_j is lower than 1, the $Ln(h_t)$ process is stationary. Nelson (1991) argues that the returns process is strictly stationary if and only if the AR process of $Ln(h_t)$ is strictly stationary.

3.2.4 *Riskmetrics (IGARCH)*

Riskmetrics is a type of IGARCH model. The process is stationary if and only if $\beta_1 + \beta_2 < 1$ in the GARCH process. However, the empirical research shows strong volatility persistence, when the sum of the parameters β_1 and β_2 is approaching one.

Under this condition, an IGARCH model is recommended as the underlying model. RiskMetrics, a kind of IGARCH model developed by J. P. Morgan, has been popular in empirical application by banks and security companies.

By assuming $\omega = 0$, Riskmetrics adopts a so called exponentially weighted moving average (EWMA) method:

$$r_t = \mu + \varepsilon_t \tag{3.22}$$

$$\varepsilon_t = h_t^{\frac{1}{2}} z_t \tag{3.23}$$

$$h_t = (1 - \lambda)\varepsilon_{t-1}^2 + \lambda h_{t-1} \tag{3.24}$$

The RiskMetrics classic concludes that "on average $\lambda = 0.94$ produces a very good forecast of one-day volatility, and $\lambda = 0.97$ results in good estimates for one-month volatility" (RiskMetrics Group, 1996).

One of the advantages of EWMA is that by discarding old information on asset returns, the effective number of days used in the volatility calculation is decided by the scale of the decay factor. In their example, if $\lambda = 0.94$, the effective number of days is 112 incorporating 99.9% of the information. For $\lambda = 0.97$, the last 227 days incorporate 99.9% of the information.

3.3 Augmented GARCH Models with Investor Sentiment Effect

3.3.1 *Augmented GARCH(1,1)*

In this paper, an innovative form of GARCH (1,1) with investor sentiment effect included in both return and variance equation is introduced and defined as:

$$r_t = \mu + \zeta V_t + \varepsilon_t \tag{3.25}$$

$$\varepsilon_t = h_t^{\frac{1}{2}} z_t \tag{3.26}$$

$$h_t = \omega + \delta V_t + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}, \tag{3.27}$$

where $\omega \geq 0$, $\beta_1 \geq 0$ and $\beta_2 \geq 0$. h_t is the conditional variance, ε_t is the residual of the process, and z_t is defined as the standardized residuals:

$$z_t = \frac{\epsilon_t}{\sqrt{h_t}} \tag{3.28}$$

The distribution of z_t conditional on previous x is:

$$z_t | x_{t-1}, x_{t-2} \ldots \sim N(0, 1) \tag{3.29}$$

3.3.2 *Augmented FIGARCH(1,d,1)*

A FIGARCH(1, d, 1) process incorporating the effect of investor sentiment is defined as:

$$r_t = \mu + \zeta V_t + \varepsilon_t \tag{3.30}$$

$$\varepsilon_t = h_t^{\frac{1}{2}} z_t \tag{3.31}$$

$$h_t = \omega + \delta V_t + \beta_1 h_{t-1} + (1 - \beta_1 \text{Ł} - (1 - \varphi_1 \text{Ł})(1 - L)^d) \varepsilon_{t-1}^2 \tag{3.32}$$

3.3.3 *Augmented EGARCH (1, 1)*

Adding investor sentiment effect into the mean and variance equations of an EGARCH model, we get the following definition:

$$r_t = \mu + \zeta V_t + \varepsilon_t \tag{3.33}$$

$$Ln(h_t) = \omega + \delta V_t + \sum_{i=1}^{q} \alpha_i g(z_{t-i}) + \sum_{j=1}^{p} \gamma_j Ln(h_{t-j}) \tag{3.34}$$

$$g(z_t) = \nu_1 z_t + \nu_2(|z_t| - E|z_t|) \tag{3.35}$$

$$\varepsilon_t = h_t^{\frac{1}{2}} z_t \tag{3.36}$$

3.3.4 *Augmented Riskmetrics*

Similar to the previous augmented GARCH models, investor sentiment effect is incorporated into the mean and variance equations of the Riskmetrics IGARCH model:

$$r_t = \mu + \zeta V_t + \varepsilon_t \tag{3.37}$$

$$\varepsilon_t = h_t^{\frac{1}{2}} z_t \tag{3.38}$$

$$h_t = (1-\lambda)\varepsilon_{t-1}^2 + \delta V_t + \lambda h_{t-1} \tag{3.39}$$

3.4 Analysis of the Stock Market

In this section, daily returns for Dow Jones from 1/1/1988 through 12/31/2006 is employed to test the efficiency of traditional GARCH models and augmented GARCH models in taking into account investor sentiment effect. We define r_t as the continuously compounded return at time t:

$$r_t = 100 Log \frac{P_t}{P_{t-1}} \tag{3.40}$$

where $logP_t$ refers to the natural logarithm of the adjusted closing price at time t.

3.4.1 *Background Information*

In this section, a comprehensive analysis of the distributional, statistical, and time series properties of the data are discussed. Our purpose is to determine which properties should to be included in the models.

The data consist of daily observations of closing price and trading volume from 1988 to 2006, totalling 4792 observations. The whole sample is divided into two periods, with the

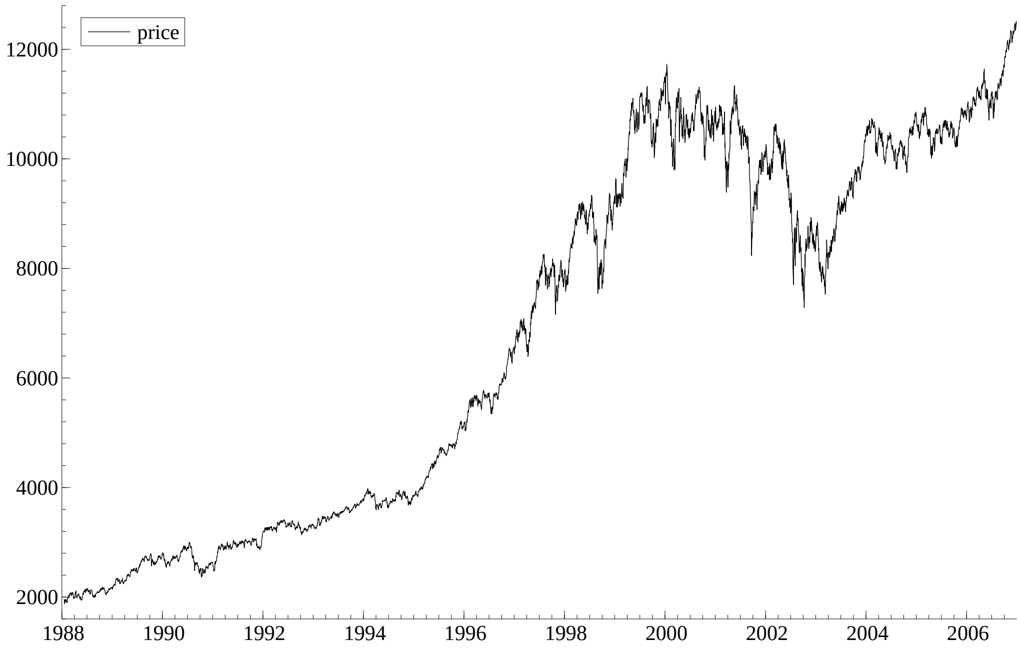

Fig. 3.1: The closing price for the Dow Jones from 1988-2006.

first 3700 observations as the in-sample estimation period, and the last 1000 observations as the out-of-sample forecasting period.

Figures 3.1 and 3.2 demonstrate the closing prices and returns for the Dow Jones index from 1988 to 2006. Figure 3.2 summarizes changes over time in the signs of variance clusters: high returns tend to be followed by high returns, and vice-versa. Furthermore, by studying 3.5, as well as the ARCH LM test and the Ljung-Box Q statistics Q test, we observe a significant serial correlation among residuals, which shows that there is conditional heteroscedasticity in the time series, thus GARCH modeling is recommended.

Table 3.1 gives the descriptive statistics for closing price, return, and trading volume of the Dow Jones.

From Table 3.1 as well as Figures 3.3, it is obvious that the closing price and return are left-skewed while the trading volume is right-skewed. The return series exhibits a leptokurtic distribution, that is to say they exhibit has a more acute "peak" around the mean and fat

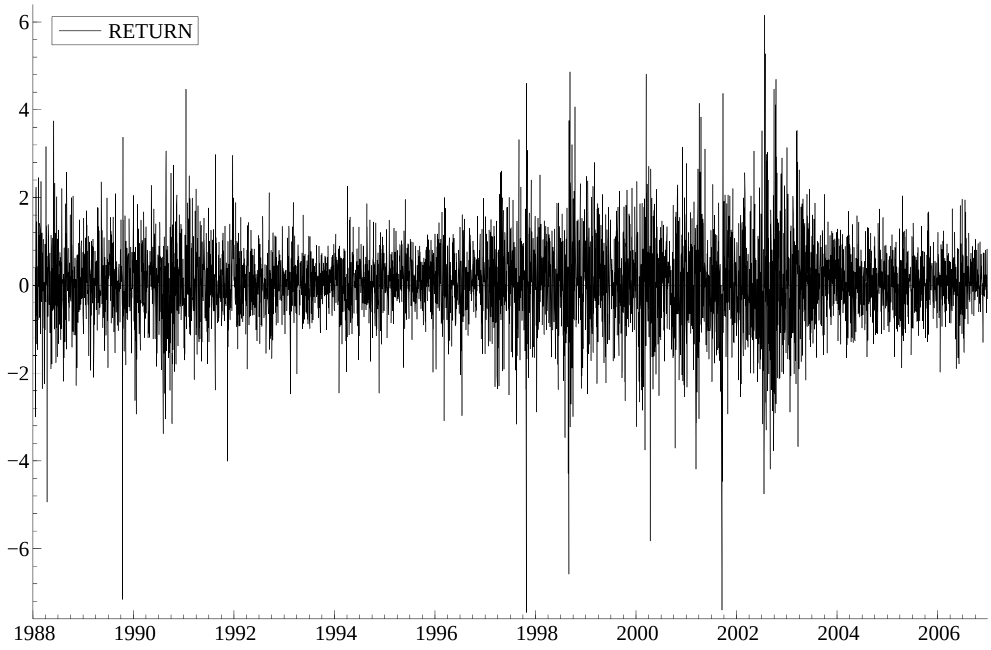

Fig. 3.2: The Dow Jones returns from 1988-2006.

Table 3.1: Descriptive Statistics Dow Jones.

	obser.	mean	Std. dev	Skewness	Excess Kurtosis	Jaque-Bera test
Closing price	4792	6781	3382.9	-0.034156 (10.261)*	-1.6620 (76.084)*	552.46 (.NaN)*
Return	4791	0.03803	0.9863	-0.36301	5.3822	5888.0
Trading Volume	4792	7.8616 e+008	6.7653 e+008	1.0775 (30.459)*	0.40981 (5.7938)*	960.71 (.NaN)*

1. The column regarding kurtosis describes the excess kurtosis, namely, the kurtosis in excess of 3. For a normal distribution, the excess kurtosis would be 0.
2. The numbers in parentheses under the parameter estimates are standard deviation.
3. $*$ indicates significance at 1% level.

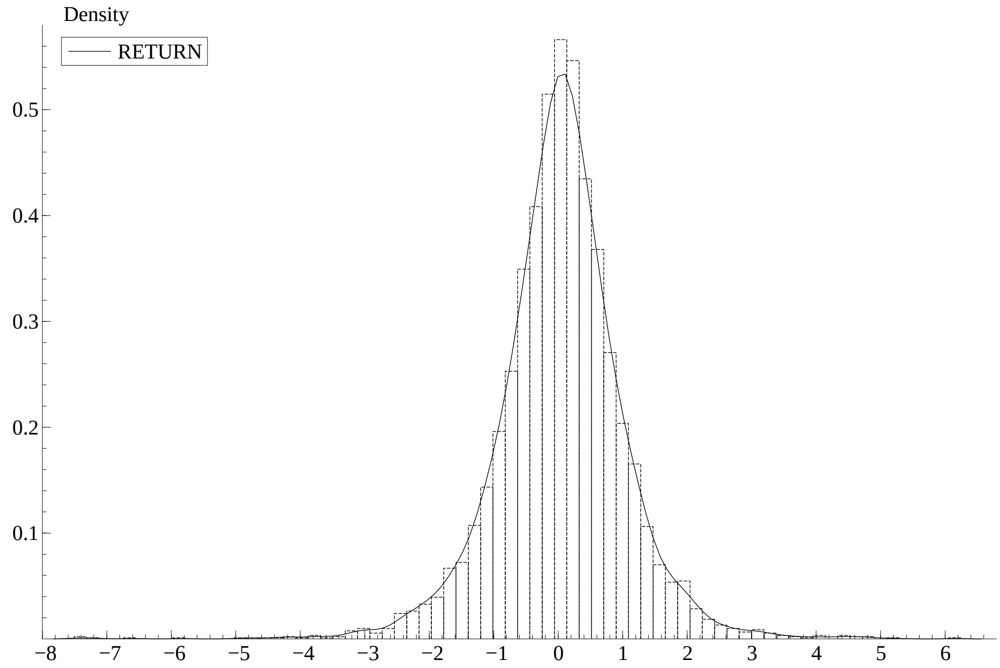

Fig. 3.3: The histogram of returns.

tails. Overall, none of them have been shown to possess a normal distribution.

3.4.2 *Stationarity*

Before performing any time-series analysis, the stationarity of the series must be determined. If a time series is stationary, "The mean, variance and autocorrelations can usually be well approximated by sufficiently long time averages based on the single set of realizations" (Enders, 1995, p. 124). For a stationary process, the effect of shocks is temporary, and the series reverts to its original trend. Under stationarity, the long-run forecasts of a time series converges to its unconditional mean. However, for a non-stationary process, time-dependence exists, no long-run mean exists, and the variance diverges to infinity as time progresses. Standard ARMA analysis requires a condition of stationarity of the time series, which can be verified by its autoCorrelation function (ACF) and its partial autocorrelation function (PACF). The ACF measures the correlation between a time series value and its lag. The PACF quantifies the additional correlation between a time-series value and a specific lag value, removing the influence of the other lag values. If the ACF of a time series dies out very slowly, the time series is possibly nonstationary.

However, judging whether the series is stationary directly from ACF can prove to be difficult according to different standards. This is due to the fact that a near-unit root process will have a very similar shape as a real-unit root process. Consider the case:

$$y_t = a_1 y_{t-1} + \epsilon_t, \tag{3.41}$$

where ϵ is an evenly distributed white-noise process with mean 0 and variance σ^2.

For the case wherein $a_1 = 0.99$, the ACF analysis exhibits a similar gradual decay as a nonstationary process. Therefore, ACF or PACF analysis does not suffice to assert the existence of a unit root, and the Dickey-Fuller and Phillips-Perron Tests should be performed. The Phillips-Perron Test incorporates an automatic correction to the Dickey-Fuller test procedure to allow autocorrelated residuals.

As shown in Figure 3.4, the ACF of closing prices do not demonstrate trailling off while the PACF of closing prices exhibits a significant first spike, indicating that the series may not be stationary. Further results from the Dickey-Fuller and Phillips-Perron tests point out that the closing price series is nonstationary. However, the results from ACF, PACF, as indicated by Figure 3.5, as well as the results from the Dickey-Fuller and Phillips-Perron Tests, prove that the return series is stationary.

Furthermore, in the time series considered here, ϵ is believed to be an identically distributed white-noise process with mean 0 and variance σ^2. However, the literature has documented that variance may vary. One possibility is to model the conditional variance employing an AR process defined as:

$$\sigma_t^2 = a_0 + \sigma_{t-1}^2 + \sigma_{t-2}^2 + \ldots + \sigma_{t-q}^2 + \nu_t, \tag{3.42}$$

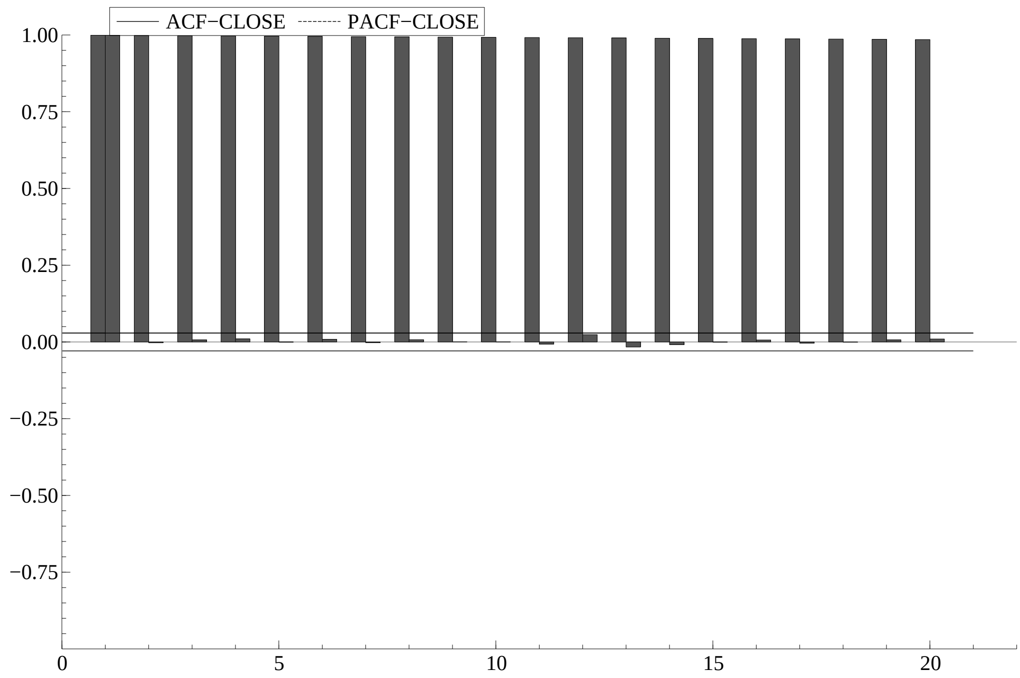

Fig. 3.4: ACF and PACF of closing prices.

where ν_t is a white-noise process. Equation 3.42 describes an autoRegressive conditional heteroskedastic (ARCH) model. In order to examine whether a time-series' variance is correlated, ARCH LM and Ljung-box Q tests must be performed. The test results indicate that there is significant autocorrelation in the variance of the return series.

3.4.3 *Long Memory and Hurst Parameter*

In this section, the intriguing feature of the data, particularly the property of the long memory are examined by mathematical tools. H is the Hurst parameter, whose name comes from the hydrologist who pioneered the topic (Hurst, 1951). The Hurst parameter is an indicator of the degree of long range dependence (LRD). If the process is turned back to its original state after a shock, then $H < 0.5$; this property is defined as antipersistance. Under this situation, the market is frequently "swinging" up and down. If the process tends to move away, then $H > 0.5$; and under this situation investors can observe a strong trend

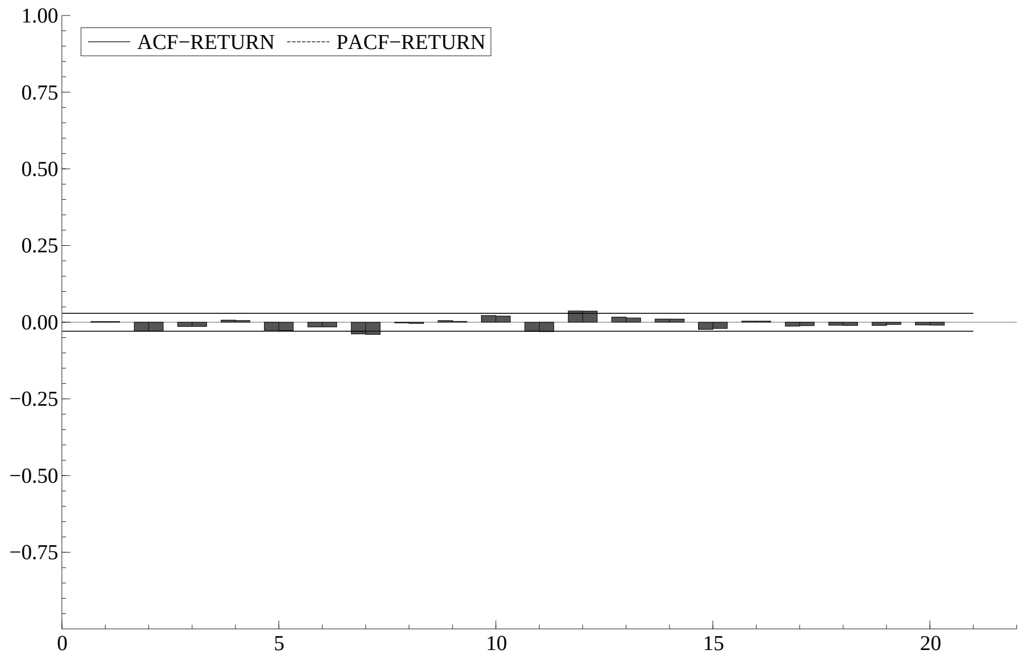

Fig. 3.5: ACF and PACF of returns.

(no matter up or down). If $H = 0.5$, the process has no memory; it is a regular Brownian motion. A Hurst parameter $0.5 < H < 1$ means that the process has long range dependence.

One of the most popular techniques to measure the Hurst parameter is the R/S parameter, as indicated in Mandelbrot and Wallis (2006) as well as Beran (1994): Let $R(n)$ be the range of the data aggregated over blocks of length n and $S(n)^2$ be the sample variance of the data aggregated at the same scale. For long-range dependent time series, the ratio $R/S(n)$ follows:

$$E[R/S(n)] \sim C_H n^H \tag{3.43}$$

where C_H is a positive, finite constant independent of n. Based on the data from ten financial market series, including Morgan Stanley Capital Index (MSCI) world, MSCI Europe, MSCI North America (NA), $S\&P500$, MSCI Emerging Markets Free (EMF), MSCI EMF

Asia, etc. , Kitt (2003) measured Hurst exponents and found the presence of long-term memory in this category of time series. Furthermore, the result from 50-year $S\&P500$ corresponds to $H = 0.64$, which confirms the results of previous studies (Kitt, 2003).

The Hurst parameter estimated by the R/S technique using the sample size from 1998 to 2006 is 0.546, demonstrating that there is long-run dependency in the data set. Therefore, models considering long memory should be recommended.

3.5 In-sample Results

The sample is separated into two parts: calibration and forecasting. Model parameters are estimated from the first $T - 1000$ observations, where T is the sample size, and 1000 forecasts are generated and compared with true observations to test model accuracy.

Among all the possibilities suggested by ACF and PACF, we choose $GARCH(1, 1)$ because it is the simplest specification and the most widely used in the literature.

The parameters estimated by GARCH, FIGARCH, EGARCH, and Riskmetrics are presented in Tables 3.2 to 3.9. All the parameters related to the investor sentiment effect are positive and significant at the 1% level, which is a robust proof that investor sentiment affects the conditional mean and variance of the returns.

For GARCH and augmented GARCH models, almost all parameters related to the ARCH effect, GARCH effect, tail, and asymmetry are significant at the 1% level. The Ljung-box test of significance for autocorrelations of 5 and 10 lags for returns residuals and squared returns residuals, respectively, rejecting the null hypothesis of autocorrelation. Most important of all, the results of AIC, BIC and log-likelihhod show that the in-sample performance of augmented GARCH models are significantly better than traditional GARCH models. Furthermore, these results show that models with a student's-t distribution perform the best in all three distributions.

For EGARCH models with and without investor sentiment effect, most of the parameters are significant at 1% except that the ARCH effect breaks down for both models with a student's-t distribution. Regarding the FIGARCH models with and without investor sentiment effect, d parameters in the models with three distributions are between 0 and 0.5 for FIGARCH, indicating the process is stable. Other results from ARCH and GARCH effect are less robust in models with investor sentiment effect. For Riskmetrics models with and without the investor sentiment effect, all parameters related to the ARCH effect, GARCH effect, tail, and asymmetry are at the 1% significant level. The overall performance of these models with investor sentiment effect proves to be better than traditional models according to the results of AIC, BIC, and log-likelihhod.

Conclusion 1

Augmented GARCH models with investor sentiment effect demonstrate significantly lower AIC, BIC, and higher log-likelihood compared with traditional GARCH models.

For all of the GARCH models previously discussed, if we compare augmented GARCH (1, 1) to traditional GARCH (1, 1) with the same distribution, the results clearly show that augmented GARCH models with investor sentiment effect possess significantly lower AIC and BIC and higher log likelihood. The result is robust.

Conclusion 2

Models with a skewed t-distribution have the lowest AIC, BIC and the highest log-likelihood compared with the t-distribution and the normal distribution.

Comparing the same GARCH models with different distributions, the skewed t-distribution shows the best result. Furthermore, the normal distribution has the highest AIC, BIC and the lowest log likelihood. These findings are consistent with the literature that the skewed t-distribution produces better results than the normal and student-t distributions. This is due to the fact that financial data are always leptokurtic and skew to one side.

Table 3.2: Augmented GARCH (1,1) Considering Investor Sentiment Effect.

	Normal	Student-t	Skewed t
Cst(M)	-0.032537	-0.014897	-0.039923
	(0.019009)***	(0.015338)	(0.01697)**
V (M)	0.184477	0.176965	0.195885
	(0.032056)*	(0.025749)*	(0.026533)*
Cst(V)	-0.002360	-0.007799	-0.007386
	(0.0087483)*	(0.0046922)*	(0.0047670)*
V (V)	0.044537	0.028262	0.028003
	(0.029339)*	(0.0091611)*	(0.0091823)*
ARCH	0.075331	0.048595	0.049861
	(0.030335)**	(0.011230)*	(0.010949)*
GARCH	0.908147	0.946884	0.945041
	(0.041962)*	(0.012743)*	(0.012471)*
STUDENT(DF)		5.912263	
		(0.59861)*	
Asymmetry			-0.085549
			(0.022511)*
Tail			6.136794
			(0.62944)*
Log-likelihood	-5086.890	-4938.937	-4932.051
AIC	2.699650	2.621753	2.618633
BIC	2.709566	2.633322	2.631854
Q(5)	10.7849 [0.0558163]	10.3914 [0.0648747]	10.3235 [0.0665716]
Q(10)	15.5832 [0.1122003]	15.3229 [0.1207245]	15.2514 [0.1231616]
$Q^2(5)$	1.79410 [0.6162192]	2.24081 [0.5239540]	2.06116 [0.5598101]
$Q^2(10)$	3.85688 [0.8697996]	3.67034 [0.8855755]	3.60006 [0.8912869]

*, **, and *** denote significant level at 1%, 5%, and 10%, respectively. Q(L) and $Q^2(L)$ denote the Ljung-Box test of significance of autocorrelations of L lags for returns residuals and squared-returns residuals, respectively. Autocorrelations are computed for standard residuals.

3.6 Out-of-Sample Results

The predictability of a model is assessed not only based on the in-sample fit, but also on the out-of-sample fit obtained from a sequence of rolling regressions. In this section, the conditional VaR framework is used to measure the performance of the one-step ahead VaR predicted by traditional GARCH models and augmented GARCH models considering investor sentiment effect with normal distribution. VaR adopts a “nearly worst case” scenario approach to measure the expected losses (Gordon and Tse, 2003). It gives a sense of maximum expected losses that one can expect over a given time horizon under a specific

Table 3.3: Traditional GARCH (1,1).

	Normal	Student-t	Skewed t
Cst(M)	0.055089 (.014534)*	0.064599 (0.012471)*	0.055670 (0.013222)*
Cst(V)	0.011574 (0.0064)***	0.05938 (0.0027)**	0.061147 (0.0027)**
ARCH	0.055092 (0.018268)*	0.047488 (0.010316)*	0.048044 (0.010212)*
GARCH	0.935024 (0.021652)*	0.947857 (0.011582)*	0.946927 (0.011507)*
STUDENT(DF)		5.997856 (0.62125)*	
Asymmetry			-0.045631 (0.021782)**
Tail			6.130201 (0.63871)*
Log-likelihood	-5115.104	-4967.507	-4965.468
AIC	2.713546	2.635837	2.635287
BIC	2.720157	2.644101	2.645203
Q(5)	10.9659 [0.0520597]	11.1592 [0.0483126]	11.1180 [0.0490895]
Q(10)	15.0316 [0.1309150]	15.2848 [0.1220191]	15.2111 [0.1245528]
$Q^2(5)$	2.18896 [0.5341261]	2.55278 [0.4658286]	2.42836 [0.4883778]
$Q^2(10)$	3.01048 [0.9336982]	3.28520 [0.9152065]	3.17713 [0.9227546]

*, **, and *** denote significant level at 1%, 5%, and 10%, respectively. Q(L) and $Q^2(L)$ denote the Ljung-Box test of significance of autocorrelations of L lags for returns residuals and squared-returns residuals, respectively. Autocorrelations are computed for standard residuals.

distribution at a given confidence level, e.g., 95%. In practice, VaR always focuses on the left-hand (negative) tail of the distribution of possible returns because a key aspect of risk management is to minimize the loss of negative events, supposing the investors have a long position. Throughout this section, both long and short positions in the financial market are analyzed. In the situation of a short position when prices of the assets tend to vary upward, the investors' concern is the portfolio's appreciation amount. Five confidence levels (99.75%, 99.5%, 99%, 95%, and 90%) are used to check the robustness of models for expected stock returns. The time horizon is one day.

3.6.1 *Value at Risk (VaR)*

VaR is a quantitative tool developed to measure market risk. The term VaR did not appear in financial market research until the last ten years, while the origins of the concept of VaR measures can be traced back to about 1922 where the NYSE ruled that

Table 3.4: Augmented EGARCH(1,1) Considering Investor Sentiment Effect.

	Normal	Student-t	Skewed t
Cst(M)	-0.048286 (0.017225)*	-0.038213 (0.017720)**	-0.045307 (0.015560)*
V (M)	0.173519 (0.025109)*	0.171413 (0.027588)*	0.171370 (0.025680)*
Cst(V)	-0.288862 (0.12491)**	12.935454 (11.275)*	-1.530730 (0.32764)*
V (V)	0.595761 (0.068905)*	0.551558 (0.054477)*	0.546550 (0.053331)*
ARCH	-0.426827 (0.13276)*	0.215554 (0.76283)	-0.353507 (0.12707)*
GARCH	0.982910 (0.0051966)*	0.958986 (0.067986)*	0.986844 (0.0037114)*
ν_1	-0.132654 (0.028562))*	-0.087015 (0.023958)**	-0.115221 (0.023716)*
ν_2	0.163168 (0.025005)*	0.111105 (0.030197)**	0.151485 (0.022793)*
Asymmetry			-0.097499 (0.022067)*
Tail			7.767901 (0.99056)*
STUDENT(DF)		6.855176 (0.73583)*	
Log-likelihood	-4951.006	-4879.484	-4852.932
AIC	2.628681	2.591298	2.577753
BIC	2.641902	2.606173	2.594280
Q(5)	10.2725 [0.0678724]	9.28175 [0.0983402]	10.3374 [0.0662196]
Q(10)	16.3709 [0.0894968]	14.7094 [0.1430208]	16.4475 [0.0875194]
$Q^2(5)$	1.90845 [0.5916233]	1.02046 [0.7963005]	0.954191 [0.8123344]
$Q^2(10)$	7.52434 [0.4812545]	5.88276 [0.6603632]	5.85877 [0.6630477]

*, **, and *** denote significant level at 1%, 5%, and 10%, respectively. Q(L) and $Q^2(L)$ denote the Ljung-Box test of significance of autocorrelations of L lags for returns residuals and squared-returns residuals, respectively. Autocorrelations are computed for standard residuals.

required firms to hold specific amounts of capital to cover their exposure to market risk. The development of VaRwas motivated by the volatility in US interest rates around 1980. VaR literature has grown remarkably in the last ten years due to the growth of the Riskmetrics approach developed by the J. P. Morgan group and the risk-adjusted measures of capital adequacy enforced by the Basel Committee (Giot, 2005a). Over time, VaR measures have been increasingly used by banks and securities firms as a way to estimate the potential loss of financial assets.

VaR can be seen as the left α quantile conditional probability distribution of asset returns:

Table 3.5: Traditional EGARCH(1,1).

	Normal	Student-t	Skewed t
Cst(M)	0.033954 (0.0075367)*	0.039114 (0.015345)***	0.039637 (0.010868)*
Cst(V)	0.099557 (0.12573)	6.483132 (1.0251)*	-0.779609 (0.27908)*
ARCH	-0.452028 (0.14738)*	0.597335 (0.43179)	-0.345849 (0.13888)**
GARCH	0.984518 (0.0049030)*	0.885548 (0.027106)*	0.988355 (0.0035115)*
ν_1	-0.130818 (0.031041)*	-0.090595 (0.030907)*	-0.100493 (0.022097)*
ν_2	0.147578 (0.024219)*	0.154042 (0.039570)*	0.139761 (0.023561)*
STUDENT(DF)		4.762409 (0.28894)*	
Asymmetry			-0.051012 (0.021570)*
Tail			6.825930 (0.79430)*
Log-likelihood	-5055.529	-4999.420	-4933.578
AIC	2.683026	2.653814	2.619442
BIC	2.692943	2.665383	2.632664
Q(5)	10.3932 [0.0648304]	10.0681 [0.0733291]	10.6282 [0.0592720]
Q(10)	16.0976 [0.0968721]	16.3563 [0.0898777]	16.1942 [0.0942074]
$Q^2(5)$	1.04402 [0.7906019]	2.25538 [0.5211223]	0.830746 [0.8420997]
$Q^2(10)$	2.34611 [0.9685212]	6.52878 [0.5882171]	1.93834 [0.9828443]

*, **, and *** denote significant level at 1%, 5%, and 10%, respectively. Q(L) and $Q^2(L)$ denote the Ljung-Box test of significance of autocorrelations of L lags for returns residuals and squared-returns residuals, respectively. Autocorrelations are computed for standard residuals.

Under the normal distribution:

$$VaR_{Long,t,\alpha} = \mu_t - \Phi_t \sigma_t and \tag{3.44}$$

$$VaR_{Short,t,\alpha} = \mu_t + \Phi_t \sigma_t, \tag{3.45}$$

where μ_t is the conditional mean, and σ_t^2 is the conditional variance at time t. Φ_t is the left α quantile for error distribution, $Pr(r < \Phi_t) = \alpha$. In this section, a normal distribution is assumed. Clearly, VaR measures depend on the mean, standard deviation, skewness, kurtosis, and higher moments of the distribution.

For example, for a standard normal distribution, if α =5%, the left α quantile Φ_t is -1.645. Conditional mean and conditional variance can be derived from GARCH models

Table 3.6: Augmented FIGARCH.

	Normal	Student-t	Skewed t
Cst(M)	-0.027468 (0.016485)***	-0.012745 (0.015247)	-0.026544 (0.015972)
V (M)	0.184759 (0.028470)*	0.178127 (0.026052)*	0.180371 (0.026123)*
Cst (V)	-0.005693 (0.027073)	-0.011528 (0.023887)	-0.021855 (0.023144)
V (V)	0.364833 (0.074984)*	0.258677 (0.096222)*	0.256523 (0.10181)*
d-Figarch	0.223348 (0.026331)*	0.251354 (0.047430)*	0.249739 (0.050192)*
ARCH	-0.044677 (0.12648)	0.087953 (0.19541)	0.078987 (0.21606)
GARCH	0.128324 (0.13859)	0.298514 (0.24718)	0.285450 (0.27215)
STUDENT(DF)		6.802575 (0.70534)*	
Asymmetry			-0.079236 (0.021490)*
Tail			7.079730 (0.75004)*
Log-likelihood	-5009.072	-4907.522	-4901.253
AIC	2.658930	2.605631	2.602837
BIC	2.670499	2.618852	2.617712
Q(5)	11.2464 [0.0467079]	11.5466 [0.0415573]	11.4690 [0.0428338]
Q(10)	18.0782 [0.0536591]	18.0536 [0.0540654]	17.9527 [0.0557661]
$Q^2(5)$	2.00766 [0.5708192]	3.53274 [0.3165405]	3.16362 [0.3670791]
$Q^2(10)$	10.1876 [0.2521022]	8.53901 [0.3826621]	8.23149 [0.4111908]

*, **, and *** denote significant level at 1%, 5%, and 10%, respectively. Q(L) and $Q^2(L)$ denote the Ljung-Box test of significance of autocorrelations of L lags for returns residuals and squared-returns residuals, respectively. Autocorrelations are computed for standard residuals.

or other time-series models, and thus VaR can be calculated at time t. When $VaR_{t,0.05} = -1.88$, the left 5% quantile of return distribution is -1.88, which can also be explained as the possibility of asset returns falling behind -1.88 equals 5%.

Consider the problem of comparing two different models by VaR. The model with the closest failure rate will outperform the other. For banks and securities companies, if the failure rate is too low, the model is too loose because it would underpredict the potential risk. On the other hand, if the failure rate is too high, the model is too conservative because it would unnecessarily jeopardize the profit opportunities.

3.6.2 *Kupiec LR Test*

The Kupiec LR test (Kupiec, 1995) is often applied to test the effectiveness of VaR

Table 3.7: Traditional FIGARCH.

	Normal	Student-t	Skewed t
Cst(M)	0.055989 (0.014488)*	0.064981 (0.012421)*	0.056916 (0.013133)*
Cst(V)	0.056474 (0.027986)*	0.031443 (0.010568)*	0.030976 (0.010490)*
d-Figarch	0.328519 (0.052978)*	0.389868 (0.047806)*	0.389079 (0.048249)*
ARCH	0.285117 (0.10582)*	0.296716 (0.045979)*	0.298346 (0.046285)*
GARCH	0.550555 (0.12141)*	0.638119 (0.056214)*	0.638005 (0.056803)*
STUDENT(DF)		6.288170 (0.63500)*	
Asymmetry		-0.043252 (0.021659)**	
Tail			6.408949 (0.65187)*
Log-	-5099.722	-4960.415	-4958.559
AIC	2.705922	2.632608	2.632154
BIC	2.714185	2.642524	2.643723
Q(5)	11.7832 [0.0378822]	11.5129 [0.0421067]	11.4946 [0.0424086]
Q(10)	16.6105 [0.0834382]	16.0690 [0.0976728]	16.0285 [0.0988198]
$Q^2(5)$	2.78183 [0.4264995]	4.20746 [0.2399157]	4.04531 [0.2566129]
$Q^2(10)$	3.37360 [0.9087754]	4.80339 [0.7783681]	4.64888 [0.7943599]

*, **, and *** denote significant level at 1%, 5%, and 10%, respectively. Q(L) and $Q^2(L)$ denote the Ljung-Box test of significance of autocorrelations of L lags for returns residuals and squared-returns residuals, respectively. Autocorrelations are computed for standard residuals.

models. It attempts to prove whether the observed frequency of exceptions conforms to the frequency of true exceptions according to the model and chosen confidence interval.

Let N be the number of times the asset returns are lower than VaR in a sample of size T. The number of exceptions, N, follows a binomial distribution. The probability of experiencing N or more exceptions is:

$$Pr(N|m,p) = \frac{m}{n} p^x (1-p)^{n-x}, \tag{3.46}$$

where p is the probability of an exception for a given confidence level, and m is the number of trials. The failure rate is defined as f, where:

$$f = \frac{N}{T}. \tag{3.47}$$

Table 3.8: Augmented Riskmetrics Considering Investor Sentiment Effect.

	Normal	Student-t	Skewed t
Cst(M)	-0.032831 (0.019178)	-0.014396 (0.015044)	-0.042041 (0.016985)**
V (M)	0.197477 (0.026482)*	0.177116 (0.025538)*	0.159917 (0.026465)*
V (V)	0.011214 (0.0025286)*	0.011666 (0.0026052)*	0.011433 (0.0025827)*
ARCH	0.060000	0.060000	0.060000
GARCH	0.940000	0.940000	0.940000
STUDENT(DF)		5.482625 (0.49926)*	
Asymmetry			-0.086889 (0.023059)*
Tail			5.656292 (0.52793)*
Log-likelihood	-5095.899	-4942.166	-4935.385
AIC	2.702836	2.621874	2.618810
BIC	2.707794	2.628485	2.627074
Q(5)	10.7512 [0.0565431]	10.9524 [0.0523324]	10.8390 [0.0546673]
Q(10)	15.3590 [0.1195108]	15.6442 [0.1102807]	115.5464 [0.1133738]
$Q^2(5)$	1.92932 [0.5872055]	2.01202 [0.5699167]	27.8929 [0.1119584]
$Q^2(10)$	3.28225 [0.9154174]	3.31897 [0.9127763]	54.6053 [0.3038577]

*, **, and *** denote significant level at 1%, 5%, and 10%, respectively. Q(L) and $Q^2(L)$ denote the Ljung-Box test of significance of autocorrelations of L lags for returns residuals and squared-returns residuals, respectively. Autocorrelations are computed for standard residuals.

Ideally, the failure rate should be equal to the prespecified VaR level α. Therefore, the null and alternative hypotheses are:

$$H_0 : f = \alpha \ and \tag{3.48}$$

$$H_1 : f \neq \alpha. \tag{3.49}$$

The appropriate likelihood ratio statistic is:

$$LR = 2Log[f^x(1-f)^{T-x}] - Log[\alpha^x(1-\alpha)^{T-x}] \sim \chi^2(1). \tag{3.50}$$

Note that the Kupiec test is a two-sided statistical test. Both high and low failure

Table 3.9: Riskmetrics.

	Normal	Student-t	Skewed t
Cst(M)	0.048206 (0.020623)**	0.065586 (0.013768)*	0.059551 (0.014146)*
STUDENT (DF)		6.708077 (0.57045)*	
Asymmetry			-0.044821 0.020267)**
Tail			6.821400 (0.58879)*
ARCH	0.060000	0.060000	0.060000
GARCH	0.940000	0.940000	0.940000
Log-likelihood	-5157.235	-4983.476	-4981.211
AIC	2.734288	2.642712	2.642041
BIC	2.735941	2.646018	2.646999
Q(5)	12.6314 [0.0270885]	12.7909 [0.0254193]	12.7378 [0.0259639]
Q(10)	16.7197 [0.0808022]	16.9596 [0.0752632]	16.8794 [0.0770756]
$Q^2(5)$	1.58252 [0.6633606]	1.62971 [0.6526712]	1.61272 [0.6565094]
$Q^2(10)$	2.74773 [0.9491781]	2.75748 [0.9486430]	2.75382 [0.9488442]

*, **, and *** denote significant level at 1%, 5%, and 10%, respectively. Q(L) and $Q^2(L)$ denote the Ljung-Box test of significance of autocorrelations of L lags for returns residuals and squareds-return residuals, respectively. Autocorrelations are computed for standard residuals.

rates tend to reject the null hypothesis. As mentioned before, both too-high and too-low failure rates will bring some problems: being either too conservative and jeopardizing the profit opportunity or being too liberal and underpredicting the potential risk. Therefore, for bank and securities firms, the two-sided test is appropriate to evaluate the performance of the model.

3.6.3 *Out-of-Sample Results of the Kupiec LR Test*

In this part, the forecasting capability of GARCH models are compared. The out-of-sample VaR is a one-step-ahead forecast based on the available information. 1000 out-of-sample VaRs are calculated for each model, and the performance of the model will be evaluated by the Kupiec's LR test given in Tables 3.10- 3.16. If the value of the Kupiec LR test appears to be NaN, the model captures perfectly all the characteristics of the series. Conclusion 1

Table 3.10: Out-of-Sample Forecast—GARCH—Short Position.

	Traditional GARCH(1,1)			GARCH(1,1) with investor sentiment effect		
Quantile	Failure rate	Kupiec LRT	P-value	Failure rate	Kupiec LRT	P-value
0.95	0.959	1.812	0.17827	0.965	5.2684	0.021716
0.975	0.982	2.224	0.13588	0.979	0.69355	0.40496
0.99	0.995	3.0937	0.078594	0.992	0.43374	0.51016
0.995	0.998	2.3439	0.12578	0.997	0.93906	0.33252
0.9975	0.998	0.10768	0.74281	0.999	1.1697	0.27947

Note: Quantile indicates ideal failure rate. Failure rate indicates the actual failure rate estimated by the model.
Lupiec LRT is to test whether actual failure rate equals the ideal failure rate.
Short indicates short position, which means investors buy and sell later.

Table 3.11: Out-of-Sample Forecast—GARCH—Long Position.

	Traditional GARCH(1,1)			GARCH(1,1) with investor sentiment effect		
Quantile	Failure rate	Kupiec LRT	P-value	Failure rate	Kupiec LRT	P-value
0.05	0.036	4.553	0.032861	0.041	1.812	0.17827
0.025	0.019	1.6082	0.20474	0.022	0.38455	0.53518
0.01	0.009	0.10452	0.74647	0.011	0.097834	0.75444
0.005	0.005	.NaN	1	0.004	0.21586	0.64222
0.0025	0.002	0.10768	0.74281	0.003	0.09418	0.75893

Note: Quantile indicates ideal failure rate. Failure rate indicates the actual failure rate estimated by the model.
Lupiec LRT is to test whether actual failure rate equals the ideal failure rate.
Long indicates long position, which means investors sell first and buy later.

The overall performance of Augmented GARCH models with investor sentiment effect demonstrate a lower rejection rate compared with traditional GARCH models.

By the Kupiec LR test, the null hypothesis is that the actual failure rate should equal the ideal failure rate, as indicated as the quantile in the tables. The overall performance of augmented GARCH models considering investor sentiment effect demonstrates a lower rejection rate compared with traditional GARCH models.

Conclusion 2

Generally, models with more conservative confidence levels perform better than in other cases.

Table 3.12: Out-of-Sample Forecast—FIGARCH—Short Position.

	Traditional FIGARCH(1,d, 1)			FIGARCH(1,d, 1) with investor sentiment Effect		
Quantile	Failure rate	Kupiec LRT	P-value	Failure rate	Kupiec LRT	P-value
0.95	0.964	4.553	0.032861	0.967	6.8784	0.008724
0.975	0.982	2.224	0.13588	0.983	2.9529	0.085722
0.99	0.994	1.8862	0.16963	0.994	1.8862	0.16963
0.995	0.998	2.3439	0.12578	0.996	0.21586	0.64222
0.9975	0.999	1.1697	0.27947	1	.NaN	1

Note: Quantile indicates ideal failure rate. Failure rate indicates the actual failure rate estimated by the model.
Lupiec LRT is to test whether actual failure rate equals the ideal failure rate.
Short indicates short position, which means investors buy and sell later.

Table 3.13: Out-of-Sample Forecast—FIGARCH—Long Position.

	Traditional FIGARCH(1,d, 1)			FIGARCH(1,d, 1) with investor sentiment Effect		
Quantile	Failure rate	Kupiec LRT	P-value	Failure rate	Kupiec LRT	P-value
0.05	0.033	6.8784	0.008724	0.032	7.7765	0.005293
0.025	0.016	3.8016	0.051203	0.014	5.8887	0.015238
0.01	0.007	1.0156	0.31356	0.004	4.706	0.030058
0.005	0.004	0.21586	0.64222	0.002	2.3439	0.12578
0.0025	0.002	0.10768	0.74281	0.001	1.1697	0.27947

Note: Quantile indicates ideal failure rate. Failure rate indicates the actual failure rate estimated by the model.
Lupiec LRT is to test whether actual failure rate equals the ideal failure rate.
Long indicates long position, which means investors sell first and buy later.

Obviously, when the confidence level is more conservative, such as 99.75% or 99.5%, the performance of both traditional GARCH models and augmented GARCH models are better than in other situations shown by a lower failure rate or higher accept rate. Although traditional Riskmetrics for short position at 0.0025 break down, generally speaking, the augmented GARCH models considering investor sentiment effect outperform traditional GARCH models.

Table 3.14: Out-of-Sample Forecast—EGARCH—Short Position.

	Traditional EGARCH			EGARCH with investor sentiment Effect		
Quantile	Failure rate	Kupiec LRT	P-value	Failure rate	Kupiec LRT	P-value
0.95	0.964	4.553	0.032861	0.961	2.7469	0.097444
0.975	0.987	7.145	0.007517	0.987	7.145	0.007517
0.99	0.995	3.0937	0.078594	0.995	3.0937	0.078594
0.995	0.999	4.7972	0.028506	0.998	2.3439	0.12578
0.9975	1	.NaN	1	1	.NaN	1

Note: Quantile indicates ideal failure rate. Failure rate indicates the actual failure rate estimated by the model.
Lupiec LRT is to test whether actual failure rate equals the ideal failure rate.
Short indicates short position, which means investors buy and sell later.

Table 3.15: Out-of-Sample Forecast—EGARCH—Long Position.

	Traditional EGARCH			EGARCH with investor sentiment Effect		
Quantile	Failure rate	Kupiec LRT	P-value	Failure rate	Kupiec LRT	P-value
0.05	0.027	13.278	0.000268	0.039	2.7469	0.097444
0.025	0.016	3.8016	0.051203	0.014	5.8887	0.015238
0.01	0.007	1.0156	0.31356	0.008	0.43374	0.51016
0.005	0.003	0.93906	0.33252	0.005	.NaN	1
0.0025	0.003	0.09418	0.75893	0.002	0.10768	0.74281

Note: Quantile indicates ideal failure rate. Failure rate indicates the actual failure rate estimated by the model.
Lupiec LRT is to test whether actual failure rate equals the ideal failure rate.
Long indicates long position, which means investors sell first and buy later.

Table 3.16: Out-of-Sample Forecast—Riskmetrics—Long Position.

	Traditional Riskmetrics			Riskmetrics with investor sentiment Effect		
Quantile	Failure rate	Kupiec LRT	P-value	Failure rate	Kupiec LRT	P-value
0.05	0.057	0.98893	0.32	0.041	1.812	0.17827
0.025	0.027	0.16	0.68916	0.018	2.224	0.13588
0.01	0.011	0.097834	0.75444	0.007	1.0156	0.31356
0.005	0.007	0.71463	0.39791	0.005	.NaN	1

Note: Quantile indicates ideal failure rate. Failure rate indicates the actual failure rate estimated by the model.
Lupiec LRT is to test whether actual failure rate equals the ideal failure rate.
Long indicates long position, which means investors sell first and buy later.

Table 3.17: Out-of-Sample Forecast—Riskmetrics—Short Position.

	Traditional Riskmetrics			Riskmetrics with Volume Effect		
Quantile	Failure rate	Kupiec LRT	P-value	Failure rate	Kupiec LRT	P-value
0.95	0.951	0.021187	0.88427	0.963	3.8953	0.048421
0.975	0.968	1.8494	0.17385	0.982	2.224	0.13588
0.99	0.988	0.37976	0.53773	0.997	0.68255	0.8986
0.995	0.998	2.3439	0.12578	0.998	2.3439	0.12578
0.9975	0.999	1.1697	0.27947	0.999	1.1697	0.27947

Note: Quantile indicates ideal failure rate. Failure rate indicates the actual failure rate estimated by the model.
Lupiec LRT is to test whether actual failure rate equals the ideal failure rate.
Short indicates short position, which means investors buy and sell later.

3.7 Concluding Comments

In this paper, investor sentiment effect is included in both mean and conditional variance equations of different GARCH models, such as FIGARCH, EGARCH, and Riskmetrics. By comparing augmented GARCH models considering investor sentiment effect with traditional GARCH models, the result proves that augmented GARCH models work significantly better than traditional GARCH models by AIC, BIC, log-likelihood, and out-of-sample VaR forecasting. The research indicates a significant role of investor sentiment in forecasting conditional mean and conditional volatility and the accuracy of the GARCH model is improved by accounting for investor sentiment effect.

Chapter 4

ESTIMATING THE VALUE-AT-RISK OF RETURNS BASED ON MONTE CARLO SIMULATION AND NEURAL NETWORK

4.1 Introduction

Forecasting density of asset prices has been a crucial issue in the research of finance and economics. Its purpose is to model the potential uncertainty via parametric or non-parametric distribution functions. Historically, more attention has been given to evaluating point forecasts, while less emphasis has been placed on interval forecasts (Chatfield, 1993; Christoffersen, 1998) and probability forecasts (Clemen et al., 1995). In recent years, more and more interest has centered on evaluating density forecasts. According to the research done by Diebold et al. (1998), Raaij and Raunig (2005), and Bao et al. (2004), the rapid development of density forecasting is based on the following reasons:

First, traditionally, density forecast is parially due to a lack of computer technology and simulation techniques. The rapid advance in computer technology has made straightforward and precise density forecasts possible.

Furthermore, the booming area of derivative products and financial risk management increases the demand for density forecasts. Point or interval forecast is not adequate for loss analysis. More emphasis has been put on "tails" analysis in many classical finance theories, such as asset pricing, portfolio optimization, and option valuation. To handle risks in this global financial market, an efficient management of potential risk is required. One popular risk measurement tool is VaR, which is used to model the tails of portfolio return distributions and is also developed from the perspective of density forecasting. VaR is the left-quantile conditional probability distribution of asset returns, and it is broadly used to forecast the worst portfolio loss in practice. Another example is the Basle Accord, which is concerned with the need to hold a certain amount of capital that is commensurate with risk.

Supposing a random number Y is described by a distribution function $F(y)$ and supposing $F(y)$ is differentiable, the probability density function is defined as:

$$f(y) = \frac{dF}{dy}. \tag{4.1}$$

The $q - quantile(0 < q < 1)$ of Y is any value y such that $Pr(Y \leq (y) = q = F(Y)$.

sectionBackground The most popular generalized autoregressive conditional heteroskedastic model (GARCH (1,1)) in empirical research is described as:

$$r_t = E(r_t \mid \Omega_{t-1}) + \varepsilon_t \tag{4.2}$$

$$\varepsilon_t = h_t^{\frac{1}{2}} z_t \tag{4.3}$$

$$h_t = \omega + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}, \tag{4.4}$$

where Ω_{t-1} stands for the information set at time t-1, $E(r_t \mid \Omega_{t-1})$ is the conditional mean, r_t represents the asset return, ε_t is the error, h_t is the conditional variance, and z_t follows a white-noise process. The GARCH model described by equations 4.2, 4.3 and 4.4 displays two essential elements: the variance is an autoregressive moving average (ARMA) process and the standardized residuals are identically distributed.

It is known that volatility is inherently unobservable, and many conclusions drawn about volatility are either derived using models such as GARCH or from option-pricing models such as Black-Scholes. Absolute or squared returns are also used to represent volatility. However, none of them are completely correct (Andersen et al., 2003). For example, squared returns are always affected by noise, and Andersen and Bollerslev (1998) demonstrate that the variance of the noise is often larger than the amplitude of the signal.

The main feature of GARCH models is their ability to forecast unobservable conditional mean and conditional variance processes, and VaR is a popular measure of the efficiency of GARCH models in practice (Giot, 2005). VaR can be seen as the left α-quantile conditional probability distribution of asset returns. In practice, when VaR is mentioned, it usually stands for the 1-day-ahead forecast of the worst portfolio loss. For example, 1-day VaR equalling \$1 million at a 99% confidence level implies that under normal trading circumstances, tomorrow, one can expect a worst portfolio loss of no more than \$1 million with 99% probability.

One way of calculating VaR is by conditional mean, and conditional variance:

$$VaR_{t,\alpha} = \mu_t - \Phi_t \sigma_t, \tag{4.5}$$

where α is the confidence level, μ_t is the conditional mean and σ_t^2 is the conditional variance at time t. Φ_t is the left α-quantile for error distribution, which could follow a standardized normal, Student's-t, skewed-Student-t or any other distribution. Conditional mean and conditional variance can be derived from GARCH models or other time-series models and VaR can thus be calculated at time t.

The failure rate, namely violation rate, is the probability that the real return value is lower than the computed VaR.

Let N be the number of times the asset returns are lower than the VaR in a sample of size T. We define the failure rate as f, such that:

$$f = \frac{N}{T}. \tag{4.6}$$

Ideally, the failure rate should be equal to the prespecified α-level associated with VaR.

Using the conventional method to get a conditional probability distribution of returns, the conditional mean and conditional variance must be derived from different time-series models. Only after a specific distributional assumption has been imposed does it become possible to achieve a quantile forecast. During that process, it is important for the distribution assumption to be capable of approximating the "true" distribution of the series. In this paper, an innovative way to obtain the conditional probability distribution of returns is presented without a distribution assumption.

The paper is organized as follows. Section 4.2 provides a literature review, Section 4.3 presents a Monte Carlo experiment, and Section 4.4 gives a comprehensive introduction of neural networks. In Section 4.5, we assess the performance of the models by the failure rate as well as the mean squared error (MSE).

4.2 Literature Review

Using daily exchange rates from July 1988 to July 1996, Taylor (2001) discusses a new approach to estimate the conditional probability distribution of returns by using neural network algorithms. The benefit of this approach is that the use of a neural network does not require a distributional assumption. The result of Taylor (2001) shows that the neural network method gives a useful alternative for estimating VaR.

Taylor's nonparametric approach analyzes historical returns from a range of different holding periods: 1, 3, 5, 7, 10, 12, and 15 days, wherein conditional means for returns are assumed to be 0. The quantile distribution of returns is a function of k, the length of the holding period, and $\hat{\sigma}_{t+1}$, the 1-step-ahead conditional variance forecast. Supposing that the k-period volatility forecast equals $\hat{\sigma}_{t+1}$, inflated by $k^{\frac{1}{2}}$, i.e.,

$$Q_{t,k}(\theta) = \Phi_\theta \hat{\sigma}_{t,k} = \Phi_\theta k^{\frac{1}{2}} \hat{\sigma}_{t+1}, \tag{4.7}$$

where $Q_{t,k}(\theta)$ can be viewed as VaR at θ confidence level and k holding period. Notice that Equation 4.7 is a special case of Equation 4.5, assuming that the conditional mean is 0 and $k = 1$. The conventional approach to obtain the k-period forecast of $Q_{t,k}(\theta)$, the θth quantile distribution of returns, requires the assumption of a distribution. For different distributions, such as Gaussian, student's-t, or skewed student's-t, the value of Φ_θ differs. To avoid the assumption of distribution, Taylor uses a linear quantile regression model:

$$Q_{t,k}(\theta) = a + bk + ck\hat{\sigma}_{t+1} + dk^{\frac{1}{2}}\hat{\sigma}_{t+1}, \tag{4.8}$$

where $Q_{t,k}(\theta)$ is the θth quantile of return distribution, for which $Pr(r_{t,k} \leq (Q_{t,k}) = \theta$, and a , b, c, and d are constant parameters.

To estimate the relationship between $Q_{t,k}(\theta)$ and $\hat{\sigma}_{t+1}$, Taylor uses a more efficient approach called neural networks instead of the traditional approach. Taylor points out that neural networks are capable of avoiding the laborious and potentially inefficient regression procedure. In Taylor (2001), neural networks are used to analyze the model described by Equation 4.8 and to obtain the forecasts on future $Q_{t,k}(\theta)$ and $\hat{\sigma}_{t+1}$, which are calculated using Gaussian GARCH(1, 1) to analyze the exchange rate. It is acknowledged by Taylor (2001) that if the normal GARCH model is misspecified, the efficiency of the quantile regression neural network approach is affected, and a better alternatives might be available. However, according to Taylor (2001), this normal GARCH(1,1) may be the simplest and the least controversial choice.

The accuracy of Taylor's method depends in part on whether Gaussian GARCH (1, 1) is the most suitable model to analyze the exchange-rate return data. As mentioned in Taylor (2001), the Gaussian assumption is often inappropriate because the distribution is always skewed and leptokurtic, and alternatives such as the t-distribution and nonparametric methods may be more appropriate. If the Gaussian GARCH (1, 1) is not the best choice for the return data, it appears that the forecast $\hat{\sigma}_{t+1}$ is affected as well as the relationship between $Q_{t,k}(\theta)$ and $\hat{\sigma}_{t+1}$.

An example is used to illustrate the analysis of Dow Jones Industrial Index daily returns data back Chapter 3 of this dissertation. Table 4.1 demonstrates the descriptive statistics for the daily return data from 1998 to 2006. It is obvious that returns are left skewed and leptokurtic, which means that they have a smaller "peak" around the mean and thinner tails than the normal distribution. Figure 4.1 displays the histogram of returns compared with normal distribution. It is obvious that return distribution has a more acute "peak" around the mean and fat tails, which indicates that the empirical Dow Jones daily return data do not display a standard normal distribution.

Table 4.1: Descriptive Statistics for Dow Jones.

	obser.	mean	Std. dev	Skewness	Excess Kurtosis	Jaque-Bera test
Return	4791	0.03803	0.9863	-0.36301	5.3822	5888.0

Note: the column reporting kurtosis describes the excess kurtosis, that is, the kurtosis in excess of 3, where the excess kurtosis would be 0 for the normal distribution.
Skewness is to test whether the distribution is symmetric. If Skewness is 0, the it is symmetric. If Skewness is negative, it skews to the left.
Jaque-Bera test is test the normality.

To analyze the stock market using GARCH models, a distribution assumption is required. Although the real distribution of empirical data is not normal, the standardized student's t or skewed student t distribution assumption often work better. In Table 4.2, GARCH (1,1) models with normal, student's-t, and skewed student's-t distribution are compared by failure rate as explained in Chapter 3 of this dissertation. A popular measure of GARCH models' accuracy is whether the failure rate equals the prespecified level of significance; refer to Taylor (2001). The failure rates of 5%, 2.5%, and 1% left-quantile are presented respectively in Table 4.2. It is obvious that for different distributions, the failure rates are different. For 1% and 5% left-quantiles, skewed t outperforms the other two distributions, which confirms the results from the literature, that financial series is always skewed and leptokurtic. From this, it seems difficult to judge whether normal GARCH(1,1) is the best candidate in Taylor's method.

The accuracy of estimated conditional mean and variance clearly depends on models and distribution assumptions. Taylor mentions that if the results from other models are found to be notably different from those of the normal GARCH(1,1) model, it is difficult to determine whether the difference is caused mainly by the choice of models. In view of this

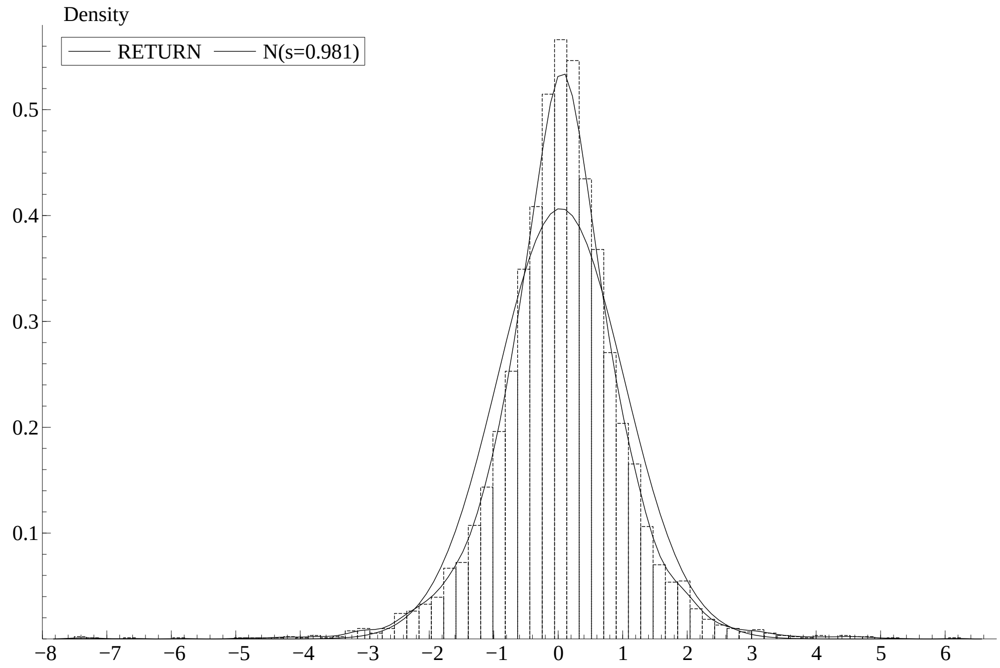

Fig. 4.1: Estimated density and histogram of returns.

problem, this paper attempts to use a Monte Carlo approach to generate a perfect normal distribution, a perfect standardized student's-t distribution, and a perfect skewed student's-t distribution. On the basis of this generated data, we compare the performance of the GARCH and neural network approaches. To make the method simple and straightforward, this paper uses 1-day return series and 1-step-ahead forecasting instead of the multi-period return series and the multi-step-ahead forecasting method of Taylor's (2001) paper. In other words, k=1 is assumed as in Taylor's model.

4.3 Monte Carlo Evidence

A small Monte Carlo experiment designed to generate the simulated conditional variance is adopted in this paper. The design of the experiment is similar to the work done by Park (2002). However, error distributions, parameter values in GARCH models, and sample sizes are constructed differently.

Table 4.2: Violation Rates for GARCH Models with Normal, Student's-t, and Skewed Student-t Distributions for Dow Jones Daily Returns.

	Failure Rate		
Quantile	Normal	Student's-t	Skewed t
0.05	4.28%	3.66%	4.77%
0.025	2.81%	1.94%	1.85%
0.01	1.36%	0.892%	0.0979%

Note: Quantile indicates the ideal failure rate.
Failure rate indicates actual failure rate by different distribution assumptions.

Following Taylor's (2001) method, the conditional mean for return series is supposed to be 0. We acknowledge that this assumption may have some problems because the mean of empirical data is not shown to be 0. However, this assumption of Taylor's (2001) will simplifies the analysis and calculation of VaR in the following sections. The Monte Carlo experiment is based on the following GARCH (1, 1) data-generating process:

$$r_t = h_t^{\frac{1}{2}} z_t \tag{4.9}$$

$$h_t = \omega + \beta_1 \varepsilon_t^2 + \beta_2 h_{t-1} \tag{4.10}$$

$$z_t | r_{t-1}, r_{t-1} ... \sim iid(0, 1), \tag{4.11}$$

where the values of parameters, ω, β_1, and β_2 are obtained from estimating the model using Dow Jones daily return data from 1988 to 2006. The innovations z_t are drawn as *i.i.d.* from N(0,1), standard student's-t distribution with 7 degrees of freedom, and skewed-t-distribution with 7 degrees of freedom and -0.069272 asymmetry as indicated by the historical data.

GARCH(1, 1) models with three different distributions are presented as:

1. Normal Distribution

A standard GARCH model with a conditional normal distribution (heteroskedastic, but neither skewed nor leptokurtic):

$$h_t = 0.005929 + 0.046914\varepsilon_t^2 + 0.948515h_{t-1} \tag{4.12}$$

$$VaR_{Long,t,\alpha} = \mu_t - \Phi_\alpha \sigma_t \tag{4.13}$$

$$VaR_{Short,t,\alpha} = \mu_t + \Phi_\alpha \sigma_t, \tag{4.14}$$

where Φ_α is the left α-quantile for a standardized normal distribution. σ_t is a conditional variance at time t, which is indicated as h_t in the GARCH model. μ_t is a conditional mean, which is assumed to be 0 following Taylor (2001). Long indicates a long position, which involves the purchase of an asset; short indicates short position when traders sell shares they do not own. For investors in long position, their concern is about prices falling, or left tail analysis on return distribution. On the other hand, the concern of those who are in short position is related to price increasing, or right tail analysis on return distribution.

2. Student's-t Distribution

Model GARCH (1,1) with standard student's-t distribution (heteroskedastic, not skewed but leptokurtic)

$$h_t = 0.005280 + 0.043433\varepsilon_t^2 + 0.950937h_{t-1} \tag{4.15}$$

$$VaR_{Long,t,\alpha} = \mu_t - St_{\alpha,\nu} \sigma_t \tag{4.16}$$

$$VaR_{Short,t,\alpha} = \mu_t + St_{\alpha,\nu} \sigma_t, \tag{4.17}$$

where $St_{\alpha,\nu}$ is the left α-quantile for student-t distribution. ν is the degree of freedom.

3. Skewed student's-t Distribution

Model GARCH (1,1) with skewed student-t distribution (heteroskedastic, skewed and leptokurtic):

$$h_t = 0.005529 + 0.044554\varepsilon_t^2 + 0.94996h_{t-1} \tag{4.18}$$

$$VaR_{Long,t,\alpha} = \mu_t - Skst_{\alpha,\nu,\zeta}\sigma_t \tag{4.19}$$

$$VaR_{Short,t,\alpha} = \mu_t + Skst_{\alpha,\nu,\zeta}\sigma_t, \tag{4.20}$$

where $Skst_{\alpha,\nu,\zeta}$ is the left α-quantile for student-t distribution, ν is the degree of freedom, and ζ measures the skewness.

Because 4700 Dow Jones data points are used to estimate the parameters of the model, 4700 hypothetical trials of daily returns for each model are generated. From each data generating process, h_t, the simulated conditional variance, and r_t, the simulated return, are achieved, and VaR in each step is calculated using $VaR_{t,\alpha} = \Phi_\alpha\sqrt{h_t}$ by assuming that the conditional mean is 0. The 1-step-ahead forecast of VaR for time t is:

$$VaR_{t+1,\alpha} = \Phi_\alpha\sqrt{\hat{h}_{t+1}}, \tag{4.21}$$

where $\hat{h}_{t+1}$ is the 1-step-ahead conditional variance forecast. $VaR_{t+1,\alpha}$ can be seen as a function of $\hat{h}_{t+1}$:

$$VaR_{t+1,\alpha} = g(\hat{h}_{t+1}). \tag{4.22}$$

Similar to Equation 4.9 and Equation 4.10, $\hat{h}_{t+1}$ can be expressed as the following equation:

$$\hat{h}_{t+1} = \omega + \beta_1\varepsilon_t^2 + \beta_2 h_{t-1} = \omega + \beta_1 r_t^2 + \beta_2(\frac{r_t}{z_t})^2. \tag{4.23}$$

Therefore, $\hat{h}_{t+1}$ can be seen as a function of r_t:

$$\hat{h}_{t+1} = f(r_t). \tag{4.24}$$

$VaR_{t+1,\alpha}$, the 1-step-ahead forecast of VaR for time t, can then be seen as a function of r_t:

$$VaR_{t+1,\alpha} = \phi(r_t). \tag{4.25}$$

Equation 4.25 demonstrates a certain relationship between VaR and returns. From the Monte Carlo simulation, conditional variance h_t and return r_t can be achieved at time t. $VaR_{t,\alpha}$ can be calculated through equation 4.5. Therefore, we have two series available, r_t and $VaR_{t,\alpha}$ at each time. A neural network is adopted to analyze the relationship between the two series, and $VaR_{t+1,\alpha}$ is forecasted based on previous values of VaR and returns.

4.4 Neural Networks

Inspired by Taylor (2001), Hamid (2004), Kulczycki and Scholer (1999), we use neural networks to fit the nonlinear relationship between the 1-step-ahead forecast of VaR and returns. The neural network is more flexible in modeling the relationship and most importantly, it can avoid making distribution assumptions. Taylor (2001) points out that returns do not always exhibit a normal or t-distribution. Thus, a nonparametric approach to quantile estimation is attractive. The main advantage of this method is that it allows a complete analysis of the nonlinear relationship between quantiles of asset returns and returns by avoiding any specific distribution assumption.

An artificial neural network (ANN) is a computational technique developed to mimic the ability of human brains to process data and to comprehend patterns. It can be viewed as a type of multiple regression accepting inputs and processing them to predict the output. ANNs are one of the most effective tools in learning and interpreting complicated real-world data (Mitchell, 1997).

The business world is becoming increasingly dependent on neural networks to estimate problems and to forecast the future. The networks have been used in domains such as portfolio selection, market distribution analysis, accounting, auditing, human resources evaluation, stock prediction, bond risk assessment, credit card fraud detection, exchange rate forecasting, options valuation, financial distress detection, commodity trading, mortgage risk assessment, and business cycling, etc. (Hamid, 2004). Wong et al. (1997) demonstrate that the most frequent application domains of ANN are productions and operations (53.5%)

and finance (25.4%).

Neural networks have proven to outperform linear models in a variety of circumstances (Hamid, 2004), especially in capturing complicated relationships in which traditional models fail to perform well (White, 1989; Kuan and White, 1994). White's (1988) research on IBM daily common-stock returns by the neural network conclude that ANNs are capable of capturing some of the complex dynamic behavior of stock returns. Shachmurove and Witkowska (2000) compare the performance of ordinary least squares, general linear regression, an artificial neural network model and multi-layer perception models to examine the dynamic interactions of some world stock markets. They conclude that the neural network outperformed other conventional techniques.

Which type of neural network performs best relies on the ability to forecast the data. There is no way to decide which kind of neural network is best before applying it to the data. The best strategy is to estimate different types of neural networks to find which fits the data best. In this paper, after comparing different neural networks, a multi-layer perception or MLP network is chosen with twenty inputs and two hidden layers. The twenty input nodes are ten lagged values of returns and ten lagged values of VaRs from time t-1 to t-10, and VaR at time t is the forecasted output.

$$Min\Sigma_{t=0}^{T}(y_t - \hat{y_t})^2 \tag{4.26}$$

$$n_{k,t} = \omega_{k,0} + \Sigma_{i=1}^{i^*}\omega_{k,i}x_{i,t} \tag{4.27}$$

$$N_{k,t} = \frac{1}{1 + e^{-n_{k,t}}} \tag{4.28}$$

$$p_{l,t} = p_{l,0} + \Sigma_{k=1}^{k^*}p_{l,k}N_{k,t} \tag{4.29}$$

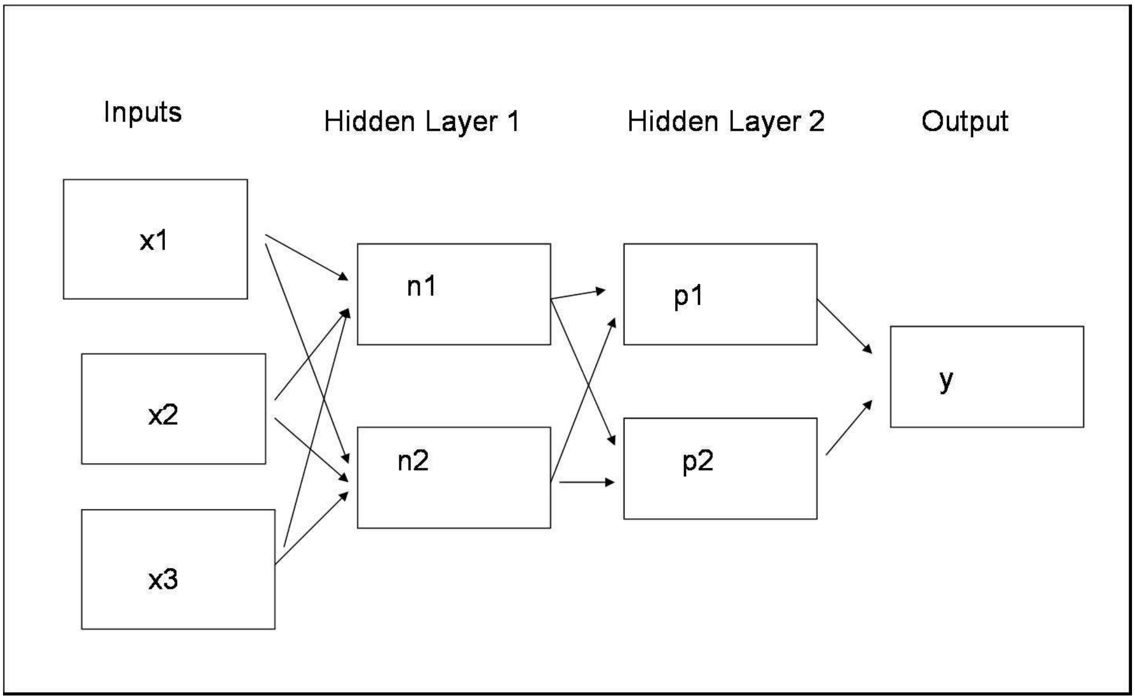

Fig. 4.2: MLP network.

$$P_{l,t} = \frac{1}{1 + e^{-p_{l,t}}} \tag{4.30}$$

$$\hat{y}_t = \gamma_0 + \Sigma_{l=1}^{l^*} \gamma_l P_{l,t}, \tag{4.31}$$

where y is the output, x is the input, and n and p are two hidden layers, N and P represent the logsigmoid activation function with the form $\frac{1}{1+e^{-n_{k,t}}}$ and $\frac{1}{1+e^{-p_{l,t}}}$. The reason why logsignoid activation function is adopted in MLP is that it can characterize many types of economic responses to change and reflects a form of learning behavior, as suggested by McNelis (2004). The set of k^* neurons or inputs are combined in a linear way with the coefficient vector $\{\gamma_l\}, l = 1, ...l^*$, and with a constant term, γ_0 to form the forecast $\hat{y}_t$ at time t. In this system, there are i^* input variables x, input weights $\omega_{k,i}$, k^* neurons in the first hidden layer, l^* neurons in the second hidden layer, and constants $\omega_{k,0}$, $p_{l,0}$ and γ_0.

Before using the neural network, scaling the data between 0 and 1 is a necessary step. The reason is that underflow or overflow problems can be caused by very high or low numbers in the series. Judd (1998) finds that the computer assigns a value of 0 to the values being minimized. Furthermore, for the logsigmoid approach used above, if the data are not scaled to a reasonable interval, such as $[0, 1]$ or $[-1, 1]$, the neural network will simply set the reasonably large values at 1, and the reasonably low values at 0 or -1. Without scaling, it is likely that a great deal of information will be lost (McNelis, 2004).

In this paper the linear and standardization methods from McNelis (2004) are combined:

$$x^*_{k,t} = \frac{z_{k,t} - min(z_k)}{max(z_k) - min(z_k)} \tag{4.32}$$

$$z_k = \frac{x_k - \overline{x}}{\sigma_x} \tag{4.33}$$

In the following section, the results from neural networks are compared with those of GARCH (1,1) models using the 4700 data generated from the Monte Carlo experiment under three data generation process: a normal distribution, a student-t distribution, and a skewed-t distribution.

Under each situation, the 4700 data is divided into two parts: estimation and forecasting. The first 3700 data are set to estimate the GARCH (1,1) model and to train the neural network, one-day is set as the order of the lagged term, and the last 1000 data are used as an out-of-sample validation data set to evaluate the forecasts. Simulated VaR and returns from the Monte Carlo experiment are compared with the VaR forecasts to test the forecasting ability of neural networks and GARCH models.

The predictability is judged from the in-sample fit of both GARCH and neural networks and out-of-sample fit obtained from a sequence of rolling regressions under each distribution condition.

4.5 Results

The performance of the models is evaluated by the failure rate, as well as the mean squared error (MSE) method, which are presented in Tables 4.3 - 4.14.

Table 4.3: In-sample Failure Rate by GARCH and NN-Normal Distribution.

	Simulated VaR	GARCH	Neural Networks
5%	4.73%	4.59%	4.73%
2.5%	2.3%	2.16%	2.27%
1%	0.92%	0.86%	0.92%

Note: The first column indicates the ideal failure rate.
Simulated VaR indicates the actual failure rate of simulated data.
GARCH indicates the actual failure rate of the simulated data estimated by a GARCH model.
Neural network indicates the actual failure rate of the simulated data estimated by a neural network approach.

Table 4.4: Out-of-Sample Failure Rate by GARCH and NN-Normal Distribution.

	Simulated VaR	GARCH	Neural Networks
5%	4.1%	3.8%	4.00%
2.5%	1.7%	1.7%	1.6%
1%	0.5%	0.5%	0.5%

Note: The first column indicates the ideal failure rate.
Simulated VaR indicates the actual failure rate of simulated data.
GARCH indicates the actual failure rate of the simulated data estimated by a GARCH model.
Neural network indicates the actual failure rate of the simulated data estimated by a neural network approach.

Ideally, the failure rate should be equal to the prespecified VaR level α. In practice, if the failure rate is too low, the model is too loose because it would underpredict potential risk. On the other hand, if the failure rate is too high, the model is too conservative because it would unnecessarily jeopardize profit opportunity. MSE can be used with the failure rate to measure the specific distance between the ideal value and estimated value:

Table 4.5: In-sample MSE by GARCH and NN-Normal Distribution.

	GARCH	Neural Networks
5%	0.001394	6.33432E-08
2.5%	0.001979	4.46477E-07
1%	0.002787	1.25818E-07

Note: The first column indicates the ideal failure rate.
GARCH indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a GARCH model.
Neural network indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a neural network approach.

Table 4.6: Out-of-sample MSE by GARCH and NN-Normal Distribution.

	GARCH	Neural Networks
5%	0.00277363	7.685E-08
2.5%	0.00394239	4.6927E-07
1%	0.00556993	1.5355E-07

Note: The first column indicates the ideal failure rate.
GARCH indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a GARCH model.
Neural network indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a neural network approach.

Table 4.7: In-sample Failure Rate by GARCH and NN-student's-t Distribution.

	Simulated	GARCH	Neural Networks
5%	5.07%	5.6911%	5.04%
2.5%	2.66%	2.5474%	2.60%
1%	1.14%	0.81301%	1.14%

Note: The first column indicates the ideal failure rate.
Simulated VaR indicates the actual failure rate of simulated data.
GARCH indicates the actual failure rate of the simulated data estimated by a GARCH model.
Neural network indicates the actual failure rate of the simulated data estimated by a neural network approach.

$$MSE = \frac{1}{N} \sum_{1}^{N} (V\hat{a}R_{t,\alpha} - VaR_{t,\alpha})^2 \qquad (4.34)$$

Table 4.8: Out-of-Sample Failure Rate by GARCH and NN-student's-t Distribution.

	Simulated VaR	GARCH	Neural Networks
5%	4.6%	5.6%	4.7%
2.5%	2.3%	2.6%	2.4%
1%	1.1%	0.8%	1.2%

Note: The first column indicates the ideal failure rate.
Simulated VaR indicates the actual failure rate of simulated data.
GARCH indicates the actual failure rate of the simulated data estimated by a GARCH model.
Neural network indicates the actual failure rate of the simulated data estimated by a neural network approach.

Table 4.9: In-sample MSE by GARCH and NN-student's-t Distribution.

	GARCH	Neural Networks
5%	0.006213749	2.8861396E-8
2.5%	0.001672545	4.4608918E-8
1%	0.041402473	7.1597047E-8

Note: The first column indicates the ideal failure rate.
GARCH indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a GARCH model.
Neural network indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a neural network approach.

Table 4.10: Out-of-sample MSE by GARCH and NN-student-t Distribution.

	GARCH	Neural Networks
5%	0.0048803	1.1982538E-6
2.5%	0.0016062	1.8667803E-6
1%	0.0321145	2.9955287E-6

Note: The first column indicates the ideal failure rate.
GARCH indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a GARCH model.
Neural network indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a neural network approach.

for $\alpha = 0.05, 0.25$, and 0.01

Table 4.11: In-sample Failure Rate by GARCH and NN-skewed student's-t Distribution.

	Simulated VaR	GARCH	Neural Networks
5%	4.65%	5.2162 %	4.65%
2.5%	2.73 %	2.7027 %	2.73 %
1%	1.38%	1.1351 %	1.38 %

Note: The first column indicates the ideal failure rate.
Simulated VaR indicates the actual failure rate of simulated data.
GARCH indicates the actual failure rate of the simulated data estimated by a GARCH model.
Neural network indicates the actual failure rate of the simulated data estimated by a neural network approach.

Table 4.12: Out-of-Sample Failure Rate by GARCH and NN-skewed student-t Distribution.

	Simulated VaR	GARCH	Neural Networks
5%	5.6%	6.4 %	5.7%
2.5%	3.6%	3.5%	3.6 %
1%	1.5%	1.1%	1.5 %

Note: The first column indicates the ideal failure rate.
Simulated VaR indicates the actual failure rate of simulated data.
GARCH indicates the actual failure rate of the simulated data estimated by a GARCH model.
Neural network indicates the actual failure rate of the simulated data estimated by a neural network approach.

Tables 4.3 - 4.6 summarize both the in-sample and out-of-sample performance of GARCH and neural networks under normal data-generating processes by comparison with the simulated VaRs obtained from the Monte Carlo simulation. Simulated VaR is calculated by simulated conditional variance, assuming different distributions, such as normal, student's-t or skewed t. Clearly, the in-sample fit of the neural network method outperforms GARCH, as indicated by the fact that the failure ratio for 5%, 2.5%, and 1% left-quantile is exactly the same as those of the simulated VaR and neural networks. The out-of-sample forecasting performance of both GARCH and neural networks are very close to the simu-

Table 4.13: In-sample MSE by GARCH and NN-skewed student's-t Distribution.

	GARCH	Neural Networks
5%	0.008943602	1.0960299E-7
2.5%	0.003109564	1.7014316E-7
1%	0.025626077	2.7340442E-7

Note: The first column indicates the ideal failure rate.
Simulated VaR indicates the actual failure rate of simulated data.
GARCH indicates the actual failure rate of the simulated data estimated by a GARCH model.
Neural network indicates the actual failure rate of the simulated data estimated by a neural network approach.

Table 4.14: Out-of-sample MSE by GARCH and NN-skewed student's-t Distribution.

	GARCH	Neural Networks
5%	0.0147448	0.0016686
2.5%	0.0045320	0.0025990
1%	0.1264481	0.0041769

Note: The first column indicates the ideal failure rate.
GARCH indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a GARCH model.
Neural network indicates the MSE estimated by the distance of ideal failure rate and actual failure rate derived from a neural network approach.

lated VaR. However, based on the MSE between the actual and fitted values, it is obvious that the neural networks method shows better performance than the GARCH method; the advantages lie in the tiny MSE for both in-sample and out-of-sample fit from neural networks.

Under the student's-t distribution data-generation process of the Monte Carlo experiment, Tables 4.7 - 4.10 summarize both in-sample and out-of-sample performance of GARCH and neural networks by comparism with the simulated VaRs from the Monte Carlo simulation. Similar to the case of normal distribution, both the in-sample fit and out-of-sample forecast of the neural networks method outperforms GARCH.

Tables 4.11 - 4.14 summarize both in-sample and out-of-sample performance of GARCH and neural networks under a skewed student's-t distribution data-generation process of the Monte Carlo experiment. It is obvious that both the in-sample fit and out-of-sample forecast of the neural networks method outperforms GARCH.

4.6 Concluding Comments

This paper adopts a nonparametric approach to estimate the conditional probability distribution of asset returns. It is evident that the exact conditional mean or conditional variance is inherently unobservable for time series. In practice, conditional variance is often achieved from different parametric models, such as GARCH, EGARCH, IGARCH, etc., by assuming different distributions such as normal, student's t, or skewed t. Therefore, the accuracy of forecast strongly depends on the assumption of distribution. The introduced method avoids the need to assume distribution by using a neural network to estimate the potentially nonlinear relationship between VaR (value at risk) and returns. Our results show that the neural network approach outperforms traditional GARCH models.

Chapter 5

CONCLUSIONS

In this three-essay dissertation, we examined the role of trading volume in the stock market and forecast Value-at-Risk using both parametric and nonparametric methods. It is known that volatility is inherently unobservable, thus the selection of models and how to define them is crucial for financial research. This research attempts to analyze and forecast the stock market by both parametric and nonparametric approaches. The first two essays use the parametric method. In the first essay, the role of the day-of-the-week as well as investor sentiment is examined on stock returns and market direction. Through linear regression and Logit regression approaches, robust results are achieved to show that there is a significant positive role for investor sentiment on returns and market direction. The day-of-the-week effect is dubious varying with individual stocks. Based on evidence from the first essay, an investor sentiment effect derived from trading volume is added to both the mean and conditional variance of Generalized Autoregressive Conditional Heteroskedastic (GARCH) models. Four GARCH models are examined including GARCH, FIGARCH, EGARCH, and Riskmetrics. By both in-sample and out-of-sample value-at-risk forecasts, GARCH models are significantly improved by accounting for the investor sentiment.

In contrast to the first two essays that use parametric methods to forecast stock market returns, the third essay uses a nonparametric approach to forecast value at risk of returns. A Monte Carlo experiment is used to generate stock-return data, including a series with a standardized normal distribution, a series with a standardized student's-t distribution and a series with a skewed student's-t distribution. A neural network approach is used to forecast Value-at-Risk and the result is compared with the traditional GARCH approach. These results suggest that nonparametric neural network methods can be a good alternative to forecasting Value-at-Risk in the market.

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