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DETERMINATION OF HYDRAULIC CONDUCTIVITY-CAPILLARY PRESSURE RELATIONSHIP FROM SATURATION-CAPILLARY PRESSURE DATA FROM SOILS

A Description of a Computer Program for Numerically Evaluating the Burdine Integrals

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INTRODUCTION

Saturation (or moisture content) of soils has been studied and measured widely in the laboratory for many years. In more recent years such instruments as the newtron moisture meter or gamma probes have been developed for moisture content determinations. With these instruments field measurements as well as laboratory moisture contents can readily be obtained. A variety of tensiometers, osmotic devices, pressure transducers and psychrometric devices have also been developed for relatively rapid measurements of capillary pressure (a negative quantity), within partially saturated soil. With available instrumentation, therefore, saturation-pressure data can be obtained readily both in the field and the laboratory.

The hydraulic conductivity (or permeability) and capillary pressure relationship is vital in analyzing moisture movement through unsaturated soils. This latter relationship must be known to utilize a basic flow equation such as Darcy' s Law. Unfortunately, measurements of hydraulic conductivity versus capillary pressure are difficult and time consuming to obtain in the laboratory and virtually impossible to obtain from field measurements.

Fortunately, theory proposed by Burdine (1953) is available for obtaining hydraulic conductivity capillary pressure relationships from saturation-capillary pressure relationships. The approximations involved in this theory, which will be referred to hereafter as Burdine Theory or Burdine Integrals, as well as its uses, are discussed by Brooks and Corey (1964) and 1966) and Laliberte et al (1968). For further explanation of the Burdine Theory the reader should consult the references cited by these authors. In brief, the Burdine Theory gives reasonably accurate values of relative permeability from pressure saturation data. The result from the Burdine Theory is given by the following equation:

$$k_r = \frac{k}{k_o} = S_e^{\,2} \frac{\int_{S_r}^{S} \frac{dS}{p^2}}{\int_{S_r}^{1} \frac{dS}{p^2}} \qquad (1)$$

in which k_r is the relative intrinsic permeability with dimensions of length squared, k is the intrinsic permeability, k_o is the saturated intrinsic permeability, p is the capillary pressure (or pressure head), and S_e is the effective saturation defined by $S_e = (S - S_r)/(1 - S_r)$, in which S_r is the residual saturation. Physically S_r is that value of saturation at which moisture movement ceases. Its value is generally determined, however, by achieving as good a fit as possible of data to a functional relationship of S_e versus capillary pressure). Actually k_r in

Eq. 1 may be interpreted as the relative hydraulic conductivity $K_r = (\rho g/\mu)k_r$ with dimensions of velocity provided the density ρ nor viscosity μ of the fluid vary.

This paper describes a computer program (written in FORTRAN IV) which evaluates Equation 1 numerically using discrete data which define the saturation capillary pressure relationship for a given soil. Originally the program was developed as a SUBROUTINE in a program designed to solve problems of transient moisture movement from a circular infiltrometer during which solution values of relative hydraulic conductivity as well as its derivative with respect to the capillary pressure were required repeatedly (see Jeppson, 1970). Since a variety of needs exist for obtaining the permeability-pressure relationship from saturation-pressure data, the SUBROUTINE has been changed into a stand along program. The methology used in evaluating the Burdine Integrals, as well as a description of the input to and output from the program are given herein.

METHOD OF SOLUTION

The computer program is designed to read data defining the saturation-pressure relationship. This data should define this relationship in a reasonably smooth fashion so that derivatives will be continuous within ranges over several consecutive data points. Should experimental error cause the data to exhibit an irratic behavior, it should be smoothed by some appropriate technique prior to supplying it as input to the program. From this data the program will evaluate the following quantities at each of a specified number of increments of pressure: (1) the saturation, S, (2) the effective saturation, S_e, (3) the relative permeability, k_r, (4) $(1/k_r)(\partial S/\partial p)$ and (5) $(1/k_r)(\partial k_r/\partial p)$. The pressure increment between consecutively computed values is one-half as large over the first half of the total range as over the second half. These quantities are obtained by passing a second degree polynomial through each three consecutive data points supplied as input advancing one point at a time as necessary to evaluate Equation 1 over the entire range of the input data.

Lagrange's interpolation formula (see Kunz, 1957) has been used to define this second degree polynomial resulting in the equation

$$p = A_1 S^2 - A_2 S + A_3 \qquad (2)$$

in which $A_1 = C_1 + C_2 + C_3$, $A_2 = C_1 (S_1 + S_2) + C_2 (S_o + S_2) + C_3 (S_o + S_1)$ and $A_3 = C_1 S_1 S_2 + C_2 S_o S_2 + C_3 S_o S_1$. The subscripts of the S' s denote the values at the three consecutive data points; S_o is the first at the smallest value of pressure, S_1 is the second and S_2 is the third data point and the C' s are defined in turn by

$$C_1 = \frac{p_o}{(S_o - S_1)(S_o - S_2)}, \quad C_2 = \frac{p_1}{(S_1 - S_o)(S_1 - S_2)}$$

$$C_3 = \frac{p_2}{(S_2 - S_o)(S_2 - S_1)} \tag{3}$$

in which p_o, p_1, and p_2 are the values for pressures at the three consecutive data points corresponding to the saturation values S_o, S_1, and S_2 respectively.

Values for S are obtained by solving Equation 2 with the quadratic formula giving

$$S = \frac{A_2 - \sqrt{A_2^2 + 4A_1 (A_3 - p)}}{2 A_1} \tag{4}$$

(Note that the negative sign is used preceding the square root). Values for the partial derivative of S with respect to p are obtained by differentiating Equation 2, giving

$$\frac{\partial S}{\partial p} = \frac{dS}{dp} = \frac{1}{2 A_1 S - A_2} \tag{5}$$

Values for K_r corresponding to each of the specified equally incremented pressures are obtained by evaluating the Burdine integral, Equation 1.

Substituting from Equation 2 for the pressure gives the equation shown below

$$\int_{S_1}^{S_2} \frac{dS}{p^2} = \int_{S_1}^{S_2} \frac{dS}{\left[A_1 S^2 - A_2 S + A_3\right]^2} = \left[\frac{2 A_1 S - A_2}{8p} + \frac{4A_1}{q^{3/2}} \tan^{-1} \frac{2 A_1 S - A_2}{q^{1/2}}\right]_{S_1}^{S_2} \tag{6}$$

in which $q = 4A_1 A_3 - A_2^2$ and S_1 and S_2 are the values of saturation at the beginning and end of increment. S_1 is larger in value than S_2.

After carrying out this integration for each of the specified increments, it is necessary to evaluate the remaining increment of the integral from the smallest value of saturation given in the input data to the saturation equal to the residual saturation S_r. This incremental integration is accomplished by assuming that the effective saturation-pressure relation over this range of saturation plots as a straight line on log-log paper, i.e., $S_e = (p_b / p)^{\lambda}$ in which p_b is the bubbling pressure as defined in the references cited earlier, and λ is the slope of the line on log-log paper and has been referred to as the grain size distribution coefficient. This remaining increment is given by

$$\int_{S_r}^{S_s} \frac{dS}{p^2} = \frac{\lambda}{p_b^2 \,(1 - S_r)^{2/\lambda}} \; \frac{(S_s - S_r)^{2/\lambda+1}}{2+\lambda} \tag{7}$$

in which S_s is the smallest value of saturation given in the input data. Justification for the assumption that S_e plots against p as a straight line on log-log paper, particularly in this interval of pressure, is given by Brooks and Corey (1964) and is substantiated by White's (1970) more recent work.

The integrals accumulated at each integration step are stored temporarily in the computer's core memory. The final accumulated integral which is obtained by adding the value from Equation 7 to the previously accumulated integrals from Equation 6 equals the value of the integral in the denominator of Equation 1. Let this accumulated amount be denoted by T, that is

$$T = \int_{S_r}^{1} \frac{dS}{p^2} \tag{8}$$

and the temporarily stored values of the accumulated integral be denoted by t(I) in which $I = 1, 2 \ldots,$ number of specified increments such that t(1) corresponds to the largest values of the input saturation and pressure data, and t(N) corresponds to the smallest of these values. Values for K_r corresponding to the N equal increments of pressure are subsequently obtained by

$$K_r(I) = \left[\frac{S(I)-S_r}{1-S_r}\right]^2 \quad t\,(N+1-I)/T \tag{9}$$

Values for the partial derivative of the relative permeability with respect to the pressure are obtained by noting that

$$\frac{\partial K_r}{\partial p} = \frac{dK_r}{dp} = \frac{dK_r}{dS_e} \cdot \frac{dS_e}{dS} \cdot \frac{dS}{dp} \tag{10}$$

in which S_e as defined previously is the effective saturation, i.e. $S_e = (S-S_r)/(1-S_r)$ and $dS_e/dS = 1/(1-S_r)$. Values for dS/dp have already been obtained from Equation 5. Values for the derivative dK_r/dS_e are given by

$$\frac{dK_r}{dS_e}(I) = \frac{1}{T}\; S_e^2(I)\, \frac{1-S_r}{p^2\,(I)} + 2\, S_e(I)\, t\,(N+1-I) \tag{11}$$

The stored values from Equations 5, 9, and 11 permit values for

$(1/K_r)(\partial K_r/\partial p_h)$ and $(1/K_r)(\partial S/\partial p_h)$ to be computed for each of the N equal increments of capillary pressure referred to earlier.

PROGRAM INPUT AND OUTPUT

The names of the FORTRAN variables which are required as input to the computer program are listed in the first column of Table 1. The first card shown in this table contains all the control variables. The columns of the card into which the values of these variables are to be punched are shown in the third column of Table 1, and a description of the significance of each variable is contained in the last column of the table. Subsequent cards, as many in number as required, contain the saturation capillary pressure head data which is used in the evaluation of the Burdine Integrals.

A listing of the output from the program is given in Table 2. This output resulted from execution of the program using the data contained at the end of the listing of FORTRAN statements in Figure 1.

Table 1. Data required as input to the FORTRAN program

FORTRAN variable name	Card no.	Columns (right justif.)	Description of Variable
N	1	1-5	An integer variable which specifies the number of data points of saturation and pressure head data which are to be read in subsequently.
NFT	1	6-10	An integer variable which specifies whether the input pressure head data is in units of feet or centimeters. If in feet NFT = 0. If in centimeter NFT>0.
NINC	1	11-15	An integer variable which specifies the number of increments at which the various quantities are to be printed. The increment of pressure between consecutively printed values will be one-half as large for the first portion of the table as for the last portion of the table.
AMBDA	1	16-25	A real variable whose magnitude equals the pore size distribution exponent λ referred to in the literature. Its magnitude is determined as the slope of the capillary pressure effective saturation data for smaller values of saturation when plotted on log-log graph paper.
SR	1	26-35	A real variable whose magnitude equals the residual saturation. The residual saturation may be interpreted physically as the saturation at which moisture movement ceases, but its value is generally determined to get as

Table 1, Continued

FORTRAN variable name	Card no.	Columns (right justif.)	Description of Variable
			good a fit as possible of data for effective saturation versus pressure to a straight line on a log-log plot (see Brooks and Corey, 1964).
PB	1	36-45	A real variable whose magnitude equals the bubbling pressure head. The magnitude of the bubbling pressure, as defined by Brooks and Corey, 1964, is the pressure at unit saturation given by a straight line on log-log paper through data for effective saturation versus pressure.
PO	1	46-55	A real variable whose magnitude equals the actual pressure head at which the saturation first begins to decrease from unity. Its magnitude is generally less in absolute value than the bubbling pressure head.
PC(I) and S(I)	2	array across entire card with 8F10.5	The array PC respresents the capillary pressure head and is given in terms of positive values. S(I) is an array representing the saturation, with magnitudes equal to unity or less. This data is punched such that the saturation corresponding to each pressure head follows the pressure head data immediately. The first values are for the smaller pressure heads (largest values of saturation) and the final values are for the larger values of pressure (smaller values of saturation).

Figure 1. Listing of FORTRAN statement which constitutes the program for obtaining the relative permeability from saturation capillary pressure data.

```
@FOR,IS
      REAL RK(100),SAT(100),R(100),DSP(100),DKP(100),PC(60),S(60)
   10 READ(5,100) N,NFT,NINC,AMBDA,SR,PB,PO
      IF (N .GT. 60) STOP
  100 FORMAT(3I5,4F10.5)
C   PO IS THE PRESSURE AT UNIT SATURATION
      WRITE(6,102) AMBDA,SR,PB,PO
  102 FORMAT(' LAMBDA=',F8.3,' SR=',F8.4,' PB=',F8.3,' PO=',F8.4)
      UNITY=-1.0
      N1=N-1
      READ(5,101) (PC(I),S(I),I=1,N)
  101 FORMAT(8F10.5)
      IF(NFT .GT. 0) GO TO 12
      FTCM=1.0/30.4801
      DO 13 I=1,N
   13 PC(I)=FTCM*PC(I)
   12 I1=1
      WRITE(6,107) (I,PC(I),S(I),I=1,N)
  107 FORMAT(' CAPILLARY TENSION AND SATURATION INPUT DATA',10(/,1H , 3(
     $I3,2F10.5)))
      I2=2
      I3=3
      NC3=NINC/3
      NCB=2*NC3
      NCF=NINC-NCB
      DELT1=(PC(N)-PC(I1))/FLOAT(NCB+2*(NCF-1))
      DELT2=2.*DELT1
      NCBP=NCB+1
      SRM1=1.0-SR
      ALPHA=3.*AMBDA+2.
      P=PC(1)
      SUM1=0.0
      C1=PC(I1)/((S(I1)-S(I2))*(S(I1)-S(I3)))
      C2=PC(I2)/((S(I2)-S(I1))*(S(I2)-S(I3)))
      C3=PC(I3)/((S(I3)-S(I1))*(S(I3)-S(I2)))
      C=C1+C2+C3
```

Figure 1, Continued

```
      C5=2.*C
      CCC=C1*(S(I2)+S(I3))+C2*(S(I1)+S(I3))+C3*(S(I1)+S(I2))
      A=C1*S(I2)*S(I3)+C2*S(I1)*S(I3)+C3*S(I1)*S(I2)
      B=-CCC
      SAT1=1.0
      DELTAP=DELT1
      DO 1 I=1,NINC
      IF(P .LT. PC(I2) .OR. I2 .EQ. N1) GO TO 3
    2 I1=I1+1
      I2=I2+1
      I3=I3+1
      IF(P .GT. PC(I2) .AND. I2 .LT. N1) GO TO 2
      C1=PC(I1)/((S(I1)-S(I2))*(S(I1)-S(I3)))
      C2=PC(I2)/((S(I2)-S(I1))*(S(I2)-S(I3)))
      C3=PC(I3)/((S(I3)-S(I1))*(S(I3)-S(I2)))
      C=C1+C2+C3
      C5=2.*C
      CCC=C1*(S(I2)+S(I3))+C2*(S(I1)+S(I3))+C3*(S(I1)+S(I2))
      A=C1*S(I2)*S(I3)+C2*S(I1)*S(I3)+C3*S(I1)*S(I2)
      B=-CCC
    3 IF(I .EQ. NCBP) DELTAP=DELT2
      P2=P+DELTAP
      BB4A=B*B-4.*(A-P)*C
      SBB4A=DSQRT(BB4A)
      SAT2=(CCC-SBB4A)/(2.*C)
      SAT22=(CCC+SBB4A)/(2.*C)
      SLT22=S(I2)+(S(I1)-S(I2))*(P-PC(I2))/(PC(I1)-PC(I2))
      IF(ABS(SAT22-SLT22) .LT. ABS(SAT2-SLT22)) SAT2=SAT22
      R(I)= (SAT2-SR)/SRM1
      Q=4.*A*C-B*B
      SM0=C5*SAT1+B
      SM=C5*SAT2+B
      SAT(I)=SAT2
      DSP(I)=UNITY/SM
      SQQ=SQRT(ABS(Q))
      IF(I .LT. NINC) GO TO 86
```

Figure 1, Continued

```
      SE2=(SAT2-SR)/SRM1
      AB2=(2.+AMBDA)/AMBDA
      SPBA=SRM1/(4.*AB2)
      SM1=SPBA*SE1**AB2
      SUM=SPBA*SE2**AB2
      SUM1=SUM1+SM1-SUM
      RK(I)=SUM1
      SUM1=SUM1+SUM
      GO TO 85
 86   SM2=SM/(Q*P)+4.*C/(Q*SQQ)*ATAN2(SM,SQQ)
      SM1=SMO/(Q*PO)+4.*C/(Q*SQQ)*ATAN2(SMO,SQQ)
      SUM1=SUM1+ABS(SM1-SM2)
      RK(I)=SUM1
      SAT1=SAT2
      PO=P
 85   DKP(I)=P
  1   P=P2
      WRITE(6,109)
109   FORMAT('0 NO.   PRESSURE   SATURATION   EFFECT. SAT.   RELATIVE K
     $DS/DP/KR    DKR/DP/KR')
      P=PC(1)
      DELTAP=DELT1
      DO 4 I=1,NINC
      RK(I)=SUM1-RK(I)
      RR=R(I)**2
      RRK=RR*RK(I)/SUM1
      DKSE=(RR*SRM1/DKP(I)**2+2.*RK(I)*R(I))/SUM1
      DKP(I)=DSP(I)*DKSE/(SRM1*RRK)
      RK(I)=RRK
      DSP(I)=DSP(I)/RRK
      WRITE(6,110) I,P,SAT(I),R(I),RK(I),DSP(I),DKP(I)
      IF(I .EQ. NCBP) DELTAP=DELT2
  4   P=P+DELTAP
110   FORMAT(1H ,I3,3F12.4,3E13.6)
      GO TO 10
      END
```

Figure 1, Continued

```
@XQT
   35     0    75   1.56          .340         .38         .3275
  10.0        0.9996    20.0          .998      46.0         .995     74.0          .993
  88.0         .990     90.0          .988      92.0         .983     94.0          .981
  96.0         .979     98.0          .976     100.0         .962    101.8          .943
 103.6         .879    105.8          .834     107.8         .789    109.6          .767
 111.8         .727    113.7          .710     115.9         .684    117.9          .665
 119.5         .658    121.5          .648     131.0         .613    141.7          .563
 151.5         .549    161.5          .527     171.4         .511    181.2          .486
 201.5         .468    221.3          .458     243.2         .437    266.6          .425
 299.0         .406    354.8          .387     401.6         .380
   99
@FIN
```

Table 2. Example of output obtained from the computer program in Figure 1, using the input data contained at the end of this listing.

```
LAMBDA=   1.560 SR=   .3400 PB=    .380 PO=   .3275
CAPILLARY TENSION AND SATURATION INPUT DATA
  1     .32808      .99960  2     .65617      .99800  3    1.50918      .99500
  4    2.42781      .99300  5    2.88713      .99000  6    2.95275      .98800
  7    3.01836      .98300  8    3.08398      .98100  9    3.14960      .97900
 10    3.21521      .97600 11    3.28083      .96200 12    3.33988      .94300
 13    3.39894      .87900 14    3.47112      .83400 15    3.53673      .78900
 16    3.59579      .76700 17    3.66797      .72700 18    3.73030      .71000
 19    3.80248      .68400 20    3.86810      .66500 21    3.92059      .65800
 22    3.98621      .64800 23    4.29789      .61300 24    4.64893      .56300
 25    4.97046      .54900 26    5.29854      .52700 27    5.62334      .51100
 28    5.94486      .48600 29    6.61087      .46800 30    7.26047      .45800
 31    7.97898      .43700 32    8.74669      .42500 33    9.80968      .40600
 34   11.64038      .38700 35   13.17581      .38000

 NO.    PRESSURE  SATURATION   EFFECT. SAT.   RELATIVE K     DS/DP/KR      DKR/DP/KR
  1        .3281       .9996          .9994  .864263+00  .652713-02   .111914+01
  2        .4592       .9989          .9984  .727955+00  .683041-02   .602588+00
  3        .5903       .9983          .9974  .657479+00  .683702-02   .368847+00
  4        .7214       .9977          .9965  .632795+00  .780827-02   .286116+00
  5        .8525       .9971          .9956  .614285+00  .668342-02   .178356+00
  6        .9836       .9966          .9948  .602502+00  .595445-02   .121761+00
  7       1.1147       .9961          .9941  .594403+00  .542979-02   .884232-01
  8       1.2458       .9957          .9935  .588407+00  .502539-02   .671769-01
  9       1.3769       .9954          .9930  .583813+00  .470144-02   .528669-01
 10       1.5080       .9950          .9924  .580173+00  .443408-02   .427995-01
 11       1.6391       .9948          .9921  .578694+00  .312100-02   .263207-01
 12       1.7702       .9945          .9917  .577391+00  .330608-02   .247131-01
 13       1.9013       .9943          .9913  .576203+00  .352570-02   .236498-01
 14       2.0324       .9940          .9909  .576100+00  .379286-02   .230756-01
 15       2.1635       .9937          .9904  .574056+00  .412837-02   .229942-01
 16       2.2946       .9934          .9900  .573047+00  .456829-02   .234851-01
 17       2.4257       .9930          .9894  .572050+00  .517927-02   .247530-01
 18       2.5568       .9924          .9885  .570528+00  .890752-02   .398052-01
 19       2.6879       .9917          .9874  .568866+00  .108786-01   .457034-01
```

Table 2, Continued

20	2.8190	.9907	.9860	.566938+00	.151787-01	.602339-01
21	2.9501	.9883	.9823	.562288+00	.611264-01	.229349+00
22	3.0812	.9811	.9714	.542240+00	.562365-01	.196720+00
23	3.2123	.9761	.9638	.533121+00	.981727-01	.325358+00
24	3.3434	.9369	.9043	.416986+00	.400025+01	.109146+02
25	3.4745	.8293	.7414	.189928+00	.653975+01	.104951+02
26	3.6056	.7557	.6298	.954946-01	.845147+01	.857495+01
27	3.7367	.7074	.5566	.546926-01	.742186+01	.520653+01
28	3.8678	.6651	.4925	.299378-01	.522998+01	.250648+01
29	3.9989	.6467	.4647	.252382-01	.411756+01	.168934+01
30	4.1300	.6326	.4433	.219833-01	.505514+01	.181952+01
31	4.2611	.6175	.4204	.189730-01	.637907+01	.198657+01
32	4.3922	.5798	.3633	.925946-02	.971882+01	.196059+01
33	4.5233	.5701	.3486	.744531-02	.842537+01	.145595+01
34	4.6544	.5628	.3376	.682471-02	.563467+01	.884495+00
35	4.7855	.5575	.3296	.639909-02	.661004+01	.959343+00
36	4.9166	.5516	.3207	.597099-02	.796943+01	.106652+01
37	5.0477	.5429	.3074	.489187-02	.151159+02	.174909+01
38	5.1788	.5340	.2940	.393958-02	.158266+02	.157068+01
39	5.3099	.5265	.2826	.356087-02	.121966+02	.109428+01
40	5.4410	.5205	.2735	.328061-02	.146149+02	.120326+01
41	5.5721	.5139	.2634	.299754-02	.180961+02	.135991+01
42	5.7032	.4980	.2394	.195470-02	.380563+02	.216211+01
43	5.8343	.4905	.2281	.156279-02	.293596+02	.142603+01
44	5.9653	.4850	.2197	.130993-02	.362056+02	.154745+01
45	6.0964	.4799	.2120	.110758-02	.290157+02	.109795+01
46	6.2275	.4762	.2063	.972100-03	.266332+02	.912236+00
47	6.3586	.4730	.2016	.869148-03	.256279+02	.803503+00
48	6.4897	.4703	.1974	.786262-03	.252399+02	.729590+00
49	6.6208	.4679	.1937	.747264-03	.179352+02	.489055+00
50	6.7519	.4661	.1910	.718736-03	.195844+02	.508492+00
51	6.8830	.4642	.1881	.690420-03	.215269+02	.532027+00
52	7.1452	.4600	.1818	.533704-03	.267239+02	.597155+00
53	7.4074	.4511	.1684	.458105-03	.789803+02	.139380+01
54	7.6696	.4435	.1569	.337531-03	.716504+02	.101262+01

Table 2, Continued

55	7.9318	.4379	.1483	.264527-03	.733577+02	.863222+00
56	8.1940	.4338	.1420	.230940-03	.662083+02	.688214+00
57	8.4562	.4297	.1359	.201625-03	.783638+02	.720500+00
58	8.7184	.4255	.1295	.175000-03	.935124+02	.758294+00
59	8.9806	.4194	.1203	.130642-03	.160559+03	.105253+01
60	9.2428	.4144	.1128	.101027-03	.170054+03	.922309+00
61	9.5050	.4103	.1064	.801614-04	.185897+03	.848423+00
62	9.7672	.4066	.1009	.645969-04	.206529+03	.801504+00
63	10.0294	.3998	.0907	.418996-04	.368285+03	.106332+01
64	10.2916	.3966	.0857	.332117-04	.314584+03	.766380+00
65	10.5538	.3941	.0820	.276478-04	.304359+03	.645304+00
66	10.8160	.3921	.0789	.235756-04	.307048+03	.575366+00
67	11.0782	.3903	.0762	.204064-04	.316024+03	.528521+00
68	11.3404	.3887	.0738	.178492-04	.328953+03	.494435+00
69	11.6026	.3872	.0715	.157349-04	.344827+03	.468294+00
70	11.8648	.3858	.0694	.139561-04	.363159+03	.447508+00
71	12.1270	.3845	.0675	.124396-04	.383715+03	.430547+00
72	12.3892	.3833	.0656	.111328-04	.406400+03	.416440+00
73	12.6514	.3822	.0639	.999731-05	.431194+03	.404936+00
74	12.9136	.3811	.0622	.900351-05	.458135+03	.394382+00
75	13.1758	.3800	.0606	.804988-05	.492060+03	.387484+00