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Optimal Discounting of Benefits From Cleanup at Waste Sites

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**OPTIMAL DISCOUNTING OF BENEFITS FROM CLEANUP AT WASTE
SITES**

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INTRODUCTION

The conventional thinking on discounting is that if production externalities are absent and if the future is certain, then all future benefits should be discounted at the market rate of interest. (See Weitzman 1994 and Weitzman 1998 for a discussion of the discount rate when production externalities and uncertainty are considered). Weitzman (1999) says "I don't see fundamental reasons why we should not keep on discounting [at] the rate of return to capital." Bradford (1999) and Montgomery (1999) support this statement. But this sentiment is far more widespread, of course, than the few citations given here. However, this paper demonstrates that discounting at the market rate of interest is not optimal for certain types of future benefits.

In environmental problems such as whether to cleanup high level toxic waste sites, the principal concern is the affect of waste on human health. This paper shows that in such problems conventional cost-benefit analysis (i.e., discounting all benefits at the market rate of interest) is inappropriate. Improved human health through waste removal enhances the productivity of workers and simultaneously increases utility directly since the quality of life can be expected to increase.

We show that it is suboptimal to discount utility from improved health at the market rate of interest. Instead, the optimal rate of discount for such benefits is the rate of time preference. If this rate is zero, then zero discounting of utility from improved health is optimal in waste removal programs regardless of the productivity of capital in the market place. This result is important because there is significant interest in applying zero time preference to environmental problems whose consequences extend many generations into the future (see Cline 1999, Schelling 1999, Rothenberg 1999, and Toman 1999).

Conversely, productivity gains from improved health are discounted at the market rate of interest on an optimal growth path. These diverse conclusions stem from the toxic waste cleanup, neutralization, yielding utility to individuals and simultaneously yielding improved productivity of workers. These joint benefits show up in the shadow value of waste cleanup as separate terms that have different discount rates.

In the following section a general equilibrium optimal control model with waste disposal and human health is presented and analyzed. The toxic waste can be removed,

cleaned up, in accordance with social cleanup activities. Waste removal generates good health which we include as an argument in the utility function. This improved health also increases the productivity of workers which is included in the production function. The solution time paths for the costate variables for the capital and waste stocks jointly characterize the role of optimal discounting in waste removal projects. We then present a numerical example to illustrate the complex relationships that exist within the model. The final section is the summary.

OPTIMAL GROWTH MODEL WITH WASTE DISPOSAL AND HUMAN HEALTH

Consider an aggregate, optimal growth economy. The aggregate nature of the model is viewed as a strength, for its assumptions are minimal, and exact functional forms are neither specified nor needed to demonstrate the intended conclusion. And, the economy is assumed to grow *optimally* because the goal here is to discuss the extent to which future benefits are endogenously discounted on an optimal growth path. This model is a modification of the growth model presented in Lyon (1996, pp. 258-261).

The model economy produces output, income, $y(t)$ according to the following strictly concave production function

$$y(t) = f(k(t), h(w(t)))$$

where $k(t)$ is the stock of capital, and $h(w(t))$ is the level of human health which is a function of the level of toxic waste at a disposal site, $w(t)$. (A common concern with waste disposal sites is the potential for contamination of the groundwater and the subsequent affect on health). The assumption here is that toxic waste affects human health, which in turn impairs the productivity of the workforce.

Society can spend income on three items: a composite consumption commodity, capital, and waste cleanup activity. The variable $c(t)$ is consumption, $z(t)$ is aggregate investment, $g(t)$ is waste removal, and $C(g(t), w(t))$ is the cost of the cleanup activity. This cost function is convex with $\partial C / \partial g > 0$ and $\partial C / \partial w \leq 0$. Thus,

$$y(t) = f(k(t), h(w(t))) = c(t) + z(t) + C(g(t), w(t))$$

Capital and toxic waste are the state variables in this control problem. Capital accumulates according to

$$\frac{dk(t)}{dt} = z(t) - \delta k(t)$$

which can be rewritten as

$$(1) \quad \frac{dk(t)}{dt} = f(k(t), h(w(t))) - c(t) - C(g(t), w(t)) - \delta k(t)$$

where δ is the rate of capital depreciation.

For simplicity the level of waste is reduced only according to cleanup activity (there is no biological depreciation of the waste of an appreciable amount over the relevant planning horizon)

$$(2) \quad \frac{dw(t)}{dt} = -g(t)$$

Imagine a toxic waste site that was once an area for active dumping, but that is now abandoned. Equations (1) and (2) serve as constraints in the control problem.

Society receives utility from the consumption good and from the level of health

$$U(c(t), h(w(t)))$$

This function is strictly concave. Consider the following control problem on the interval $[0, T]$, where T is given exogenously:

$$\max : V = \int_0^T e^{-\rho t} U(c(t), h(w(t))) dt + e^{-\rho T} B(k(T), w(T))$$

subject to Equations (1) and (2), where $B(k(T), w(T))$ is a bequest function that measures the value of endowing distant generations with $k(T)$ and $w(T)$. There are also non-negativity constraints, which in this section we ignore, so as to concentrate on internal solutions. In the numerical example, however, we include the non-negativity constraints.

The current value Hamiltonian function is

$$H(c(t), g(t), k(t), w(t), \lambda(t), \eta(t)) = U(c(t), h(w(t))) + \lambda(t)(f(k(t), h(w(t))) - c(t) - C(g(t), w(t)) - \delta k(t)) - \eta(t)g(t)$$

where $c(t)$ and $g(t)$ are the only control variables, $k(t)$ and $w(t)$ are the state variables, and $\lambda(t)$ and $\eta(t)$ are current value costate variables that correspond to $k(t)$ and $w(t)$, respectively. The costate variables or 'shadow values' represent the change in the solution value of the objective functional resulting from a unit increase in the respective

state variables at time t , valued at time t . See, e.g., Kamien and Schwartz (1991) or Hoy et al. (1996) for a discussion of these ideas.

The first order necessary conditions of this control problem are

$$(3a) \quad \frac{\partial U(c^*(t), h(w^*(t)))}{\partial c(t)} - \lambda^*(t) = 0$$

$$(3b) \quad -\lambda^*(t) \frac{\partial C(g^*(t), w^*(t))}{\partial g(t)} - \eta^*(t) = 0$$

$$(3c) \quad \frac{d\lambda^*(t)}{dt} = \left\{ \rho + \delta - \frac{\partial f(k^*(t), h(w^*(t)))}{\partial k(t)} \right\} \lambda^*(t)$$

$$(3d) \quad \begin{aligned} \frac{d\eta^*(t)}{dt} = & \rho \eta^*(t) - \frac{\partial U(c^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} \\ & - \lambda^*(t) \left(\frac{\partial f(k^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} - \frac{\partial C(g^*(t), w^*(t))}{\partial w(t)} \right) \end{aligned}$$

where optimal values are denoted by an asterisk. The boundary conditions – the initial conditions for the state variables and the transversality conditions for the costate variables – are omitted; they are required for a complete characterization of the optimum, but they are not central to the analysis that follows.

It will be assumed that the market rate of interest is given by

$$r(t) \equiv \frac{\partial f(k^*(t), h(w^*(t)))}{\partial k(t)} - \delta.$$

We use this definition because it is implied by the implicit user cost of capital, which can be expected to hold for this model.¹ Thus, we can write equation (3c) as

$$(3c') \quad \frac{d\lambda^*(t)}{dt} = (\rho - r(t)) \lambda^*(t)$$

Equations (3a) and (3b) can be written as

¹ Along the optimal path we have

$$p \frac{\partial f(k^*(t), h(w^*(t)))}{\partial k(t)} = (r(t) + \delta)q(t) - \frac{dq(t)}{dt}$$

where the right-hand side is the implicit user cost of capital, p is the price of a unit of output (income), and q is the price of capital. The statement of income and its uses defines both of these prices to be one, implying $dq(t)/dt = 0$. Our definition follows from this.

$$\frac{\partial U(c^*(t), h(w^*(t)))}{\partial c(t)} = \lambda^*(t) = \frac{-\eta^*(t)}{\frac{\partial C(g^*(t), w^*(t))}{\partial g(t)}}$$

This condition ensures that at the optimum, the marginal utility of consumption equals the value of an extra unit of capital as well as the value of one dollar of cleanup expenditure, measured in units of utility. Since $\eta^*(t)$ is the value of a unit of waste, $-\eta^*(t)$ is the value of a unit of waste removal. In addition we consider consumption, c , to be the numeraire, and we have one unit of cleanup costs equal to one unit of consumption implying that $\partial C / \partial g$ is measured in units of consumption. Thus, at the optimum, income is divided between consumption, investment, and waste cleanup expenditures in a manner consistent with the above equation.

Equation (3c') is a linear first order differential equation and has a particular solution (see the Appendix A for a derivation of Equation 4)

$$(4) \quad \lambda^*(t) = \lambda^*(0) \exp \left[\rho t - \int_0^t r(s) ds \right]$$

Since the objective of this paper is to discuss the degree to which future returns to waste cleanup are discounted, consider Equation (3d) which is a linear differential equation with constant coefficient and variable term. A particular solution is (see the Appendix A for a derivation of Equation 5).

$$(5) \quad \begin{aligned} \eta^*(0) = & \eta^*(T)e^{-\rho T} + \int_0^T \frac{\partial U(c^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} e^{-\rho t} dt \\ & + \int_0^T \lambda^*(t) \left(\frac{\partial f(k^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} - \frac{\partial C(g^*(t), w^*(t))}{\partial w(t)} \right) e^{-\rho t} dt \end{aligned}$$

Equation (5) gives the value of an additional unit of waste at time zero, which is negative. Additionally, the negative of Equation (5) can be interpreted as the value of a unit of waste removal, which is the interpretation that will be used in what follows. Equation (5) represents a decomposition of the value of a unit of waste removal into its four components.

To see the intended result, insert Equation (4) into Equation (5)

$$\begin{aligned}
(6) \quad \eta^*(0) = & \eta^*(T)e^{-\rho T} + \int_0^T \frac{\partial U(c^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} e^{-\rho t} dt \\
& + \int_0^T \exp\left[-\int_0^t r(s)ds\right] \frac{\partial f(k^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} dt \\
& - \int_0^T \exp\left[-\int_0^t r(s)ds\right] \frac{\partial C(g^*(t), w^*(t))}{\partial w(t)} dt
\end{aligned}$$

where the innocuous normalization $\lambda^*(0) \equiv 1$ has been employed. This normalization is justifiable because utility is an ordinal concept and therefore can be normalized such that the marginal utility of consumption at time zero is one.

In Equation (6) $\eta^*(0)$ can be viewed as the present value of three return streams and a deferred item caused by an additional unit of waste reduction at time zero. We call

$$\eta^*(T)e^{-\rho T}$$

the undesirable abundance effect,

$$+ \int_0^T \frac{\partial U(c^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} e^{-\rho t} dt$$

the utility effect,

$$+ \int_0^T \exp\left[-\int_0^t r(s)ds\right] \frac{\partial f(k^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} dt$$

the productivity effect, and

$$- \int_0^T \exp\left[-\int_0^t r(s)ds\right] \frac{\partial C(g^*(t), w^*(t))}{\partial w(t)} dt$$

the cost effect.

The undesirable abundance effect is similar to the scarcity effect in the nonrenewable resource analysis (see Lyon (1999) and Lyon and Lee (2004)), and is the terminal shadow value of waste discounted to the present. The utility and productivity effects are the result of an altered path of health caused by this unit of waste reduction

$$dh(t) = \frac{\partial h(w^*(t))}{\partial w(t)} dw(t)$$

where $dw(0) = -1$. This time path of $dh(t)$ simultaneously generates these two streams of benefits. The utility effect is the direct effect of $dh(t)$, and the productivity effect is the indirect effect. It comes indirectly through added productivity of the workers. The cost

effect exists only if $\partial C(g(t), w(t)) / \partial w(t)$ is nonzero for some $t \in [0, T]$, and is the result of $dw(t)$ causing an altered path of cleanup costs. We posit that it becomes more costly to cleanup the toxic waste as its stock gets smaller. This could be due to the remaining waste being more dispersed or further in the ground. We note that the productivity and cost effects are discounted at the interest rate and the utility effect is discounted at the rate of time preference.

Part of why this result is of interest relates to the concern of many economists that the rate of time preference should be zero in long-term environmental problems, of which toxic waste removal is an important example. In addition, whenever the interest rate and the rate of time preference are different, discounting the utility effect at the rate of interest will give biased results. We illustrate this bias in our numerical examples below.

When speaking of time preference, it seems that the majority of economists agree that *individuals* have a positive rate of time preference (see Olson and Bailey 1981 for a convincing argument). Individuals would prefer benefits now as opposed to later. But when a proposal spans very long periods of time such as the impact of toxic waste sites, many feel that positive time preference is inappropriate.

Cline (1999) states that in social cost-benefit analysis of very long-term projects the rate of pure time preference should be zero. Cline (1999) states very strongly that “individually, [positive time preference] is a recipe for living like a prince in youth but a pauper in old age. For those who [make] decisions on behalf of society, it is irresponsible.” Schelling (1999) also states emphatically that “there can be no ‘time preference’ of the traditional sort [in projects that affect the distant future]. Time preference, often associated with impatience, relates to impatience about one’s own future consumption, [not to the consumption of distant future generations].” Similarly, Rothenberg (1999) states that there “...is no place for pure time preference in the usual sense of impatience: events occurring at different times beyond the lifetime of the present generation are irrelevant to the actual experiences of this generation.” Toman (1999) also supports this view. The basic idea here is that positive time preference may be an *intra*-generational phenomenon that perhaps should not be applied to *inter*-generational problems like waste disposal cleanup.

However, Arrow takes the position that zero time preference implies a potentially unreasonable amount of capital accumulation. The basic point of Arrow's argument is easily seen at the stationary state of the model in this paper. If $d\lambda(t)/dt = 0$, then zero time preference also implies that the market rate of interest is zero (see Equation 3c'). That is, ultimately the market rate of interest would converge to zero because extra capital accumulation would equalize the marginal product of capital and the rate of depreciation.

NUMERICAL EXAMPLE

INTRODUCTION

In this section we present two optimal numerical time paths to illustrate the characteristics of the model. The first we call the Scenario E (for exhaustion) because the optimal cleanup activity completely neutralizes all of the toxic waste, and the second we call the Scenario N (for non-exhaustion) because cleanup costs become sufficiently high that optimal cleanup activity becomes zero before all of the waste is neutralized.

We select functional forms for the utility, production and cost functions, specify parameter values, and solve for the optimal time paths using the discrete time form of the problem. We use discrete time because the techniques we tried for solving the continuous time problem were unsuccessful. We solve this problem point-wise for a time independent numerical policy function using value function iterations on a grid. We divide the state space, capital stock and waste stock, into a 1000 point even spaced grid and a 200 point even spaced grid, respectively. Then numerically solve the Bellman equation contraction at each of these points. The result of this is a discrete numerical policy function, $[k_{t+1}, w_{t+1}] = F(k_t, w_t)$. This solution technique has the advantage that as the step size in the grid approaches zero the solution on the grid approaches the true solution. The details of this procedure can be found in Bertsekas (1987).

THE ALGEBRAIC EQUATIONS

The algebraic utility function

$$U(c, h) = a_c \frac{c^{\frac{1-\frac{1}{\tau}}{\tau}}}{1 - \frac{1}{\tau}} + a_h \frac{h^{\frac{1-\frac{1}{\tau}}{\tau}}}{1 - \frac{1}{\tau}} \quad \text{with } \tau, a_c, a_h > 0.$$

The health function

$$h(w) = a_1 - b_1 w \text{ with } a_1, b_1 > 0.$$

The production function

$$f(k, h) = Ak^\alpha h^\gamma \text{ with } A, \alpha, \gamma > 0 \text{ and } \alpha + \gamma < 1.$$

The cleanup cost function

$$C(g, w) = a_2 g + \frac{a_3 g^2}{2} - a_4 gw \text{ with } a_2, a_3 > 0, a_4 \geq 0.$$

DISCRETE TIME ANALYSIS

Bellman's Equation:

$$(D1) \quad V_t(k_t, w_t) = \underset{c, g}{\text{Max}} \{U(c_t, h(w_t)) + e^{-\rho} V_{t+1}(k_{t+1}, w_{t+1})\}$$

Subject to:

$$(D2) \quad k_{t+1} = f(k_t, h(w_t)) - c_t - C(g_t, w_t) + (1 - \delta)k_t$$

$$(D3) \quad w_{t+1} = w_t - g_t$$

$$(D4) \quad g_t, w_{t+1} \geq 0$$

Substitution of (D2) and (D3) into (D1) yields:

(D5)

$$V_t(k_t, w_t) = \underset{c, g}{\text{Max}} \{U(c_t, h(w_t)) + e^{-\rho} V_{t+1}(f(k_t, h(w_t)) - c_t - C(g_t, w_t) + (1 - \delta)k_t, w_t - g_t)\}$$

subject to (D4).

The Lagrangean function

(D6)

$$L(c_t, g_t, \mu_{1t}, \mu_{2t}; k_t, w_t) = U(c_t, h(w_t)) + e^{-\rho} V_{t+1}(f(k_t, h(w_t)) - c_t - C(g_t, w_t) + (1 - \delta)k_t, w_t - g_t) \\ + \mu_{1t} g_t + \mu_{2t} (w_t - g_t)$$

The necessary conditions:

$$(D7.1) \quad \frac{\partial U(c_t^*, h(w_t^*))}{\partial c_t} - e^{-\rho} \frac{\partial V_{t+1}(k_{t+1}^*, w_{t+1}^*)}{\partial k_{t+1}} = 0$$

$$(D7.2) \quad -e^{-\rho} \left\{ \frac{\partial V_{t+1}(k_{t+1}^*, w_{t+1}^*)}{\partial k_{t+1}} \frac{\partial C(g_t^*, w_t^*)}{\partial g_t} + \frac{\partial V_{t+1}(k_{t+1}^*, w_{t+1}^*)}{\partial w_{t+1}} \right\} + \mu_{1t}^* - \mu_{2t}^* = 0$$

$$(D7.3) \quad g_t^*, \mu_{1t}^* \geq 0, \quad \mu_{1t}^* g_t^* = 0$$

$$(D7.4) \quad (w_t^* - g_t^*), \mu_{2t}^* \geq 0, \quad (w_t^* - g_t^*) \mu_{2t}^* = 0$$

By the envelope theorem we have:

$$(D8.1) \quad \frac{\partial V_t(k_t^*, w_t^*)}{\partial k_t} = e^{-\rho} \frac{\partial V_{t+1}(k_{t+1}^*, w_{t+1}^*)}{\partial k_{t+1}} \left(\frac{\partial f(k_t^*, h(w_t^*))}{\partial k_t} + (1 - \delta) \right)$$

$$(D8.2) \quad \begin{aligned} \frac{\partial V_t(k_t^*, w_t^*)}{\partial w_t} &= \frac{\partial U(c_t^*, h(w_t^*))}{\partial h_t} h'(w_t^*) \\ &+ e^{-\rho} \left\{ \frac{\partial V_{t+1}(k_{t+1}^*, w_{t+1}^*)}{\partial k_{t+1}} \left(\frac{\partial f(k_t^*, h(w_t^*))}{\partial h_t} h'(w_t^*) - \frac{\partial C(g_t^*, w_t^*)}{\partial w_t} \right) + \frac{\partial V_{t+1}(k_{t+1}^*, w_{t+1}^*)}{\partial w_{t+1}} \right\} + \mu_{2t}^* \end{aligned}$$

$$\text{Define } \lambda_t^* = \frac{\partial V_t(k_t^*, w_t^*)}{\partial k_t} \quad \text{and} \quad \eta_t^* = \frac{\partial V_t(k_t^*, w_t^*)}{\partial w_t}$$

Using these, the necessary conditions become:

$$(D7.1') \quad \frac{\partial U(c_t^*, h(w_t^*))}{\partial c_t} - e^{-\rho} \lambda_{t+1}^* = 0$$

$$(D7.2') \quad -e^{-\rho} \left\{ \lambda_{t+1}^* \frac{\partial C(g_t^*, w_t^*)}{\partial g_t} + \eta_{t+1}^* \right\} + \mu_{1t}^* - \mu_{2t}^* = 0$$

$$(D7.3') \quad g_t^*, \mu_{1t}^* \geq 0, \quad \mu_{1t}^* g_t^* = 0$$

$$(D7.4') \quad (w_t^* - g_t^*) \mu_{2t}^* \geq 0, \quad (w_t^* - g_t^*) \mu_{2t}^* = 0$$

And

$$(D8.1') \quad \lambda_t^* = e^{-\rho} \lambda_{t+1}^* \left(\frac{\partial f(k_t^*, h(w_t^*))}{\partial k_t} + (1 - \delta) \right)$$

$$(D8.2') \quad \eta_t^* = \frac{\partial U(c_t^*, h(w_t^*))}{\partial h_t} h'(w_t^*) + e^{-\rho} \left\{ \lambda_{t+1}^* \left(\frac{\partial f(k_t^*, h(w_t^*))}{\partial h_t} h'(w_t^*) - \frac{\partial C(g_t^*, w_t^*)}{\partial w_t} \right) + \eta_{t+1}^* \right\} + \mu_{2t}^*$$

$$(D8.3') \quad k_{t+1} = f(k_t, h(w_t)) - c_t - C(g_t, w_t) + (1 - \delta)k_t$$

$$(D8.4') \quad w_{t+1} = w_t - g_t$$

The algebraic necessary conditions are the result of substituting the algebraic equations into Equations (D7.1')-(D8.4').

$$(DA7.1') \quad a_c c_t^{\frac{-1}{\tau}} - e^{-\rho} \lambda_{t+1} = 0$$

$$(DA7.2') \quad -e^{-\rho} \lambda_{t+1} (a_2 + a_3 g - a_4 w) - e^{-\rho} \eta_{t+1} + \mu_{1t} - \mu_{2t} = 0$$

$$(DA7.3') \quad g_t, \mu_{1t} \geq 0, \quad \mu_{1t} g_t = 0$$

$$(DA7.4') \quad (w_t - g_t), \mu_{2t} \geq 0, \quad (w_t - g_t) \mu_{2t} = 0$$

And

$$(DA8.1') \quad \lambda_t - e^{-\rho} \lambda_{t+1} (\alpha A k_t^{\alpha-1} h_t^\gamma + (1-\delta)) = 0$$

$$(DA8.2') \quad \eta_t + a_h h_t^{\frac{-1}{\tau}} b_1 - e^{-\rho} \lambda_{t+1} (-b_1 \gamma A k_t^\alpha h_t^{\gamma-1} + a_4 g_t) - e^{-\rho} \eta_{t+1} - \mu_{2t} = 0$$

$$(DA8.3') \quad k_{t+1} - A k_t^\alpha h_t^\gamma + c_t + \left(a_2 g_t + \frac{a_3 g_t^2}{2} - a_4 g_t w_t \right) - (1-\delta) k_t = 0$$

$$(DA8.4') \quad w_{t+1} - w_t + g_t = 0$$

$$(DA8.5') \quad h_t - a_1 + b_1 w_t = 0$$

The components of the shadow values are found by back substitution on Equations (D8.1') and (D8.2'). Defining $\beta = e^{-\rho}$, manipulation of (D8.1') yields

$$\begin{aligned} \lambda_0^* &= \beta \lambda_1^* \left(\frac{\partial f_0^*}{\partial k_0} + (1-\delta) \right) \\ &= \beta^2 \lambda_2^* \left(\frac{\partial f_1^*}{\partial k_1} + (1-\delta) \right) \left(\frac{\partial f_0^*}{\partial k_0} + (1-\delta) \right) \\ &= \beta^3 \lambda_3^* \left(\frac{\partial f_2^*}{\partial k_2} + (1-\delta) \right) \left(\frac{\partial f_1^*}{\partial k_1} + (1-\delta) \right) \left(\frac{\partial f_0^*}{\partial k_0} + (1-\delta) \right) \\ &= \beta^4 \lambda_4^* \left(\frac{\partial f_3^*}{\partial k_3} + (1-\delta) \right) \left(\frac{\partial f_2^*}{\partial k_2} + (1-\delta) \right) \left(\frac{\partial f_1^*}{\partial k_1} + (1-\delta) \right) \left(\frac{\partial f_0^*}{\partial k_0} + (1-\delta) \right) \\ &= \beta^{T+1} \lambda_{T+1}^* \prod_{j=0}^T \left(\frac{\partial f_j^*}{\partial k_j} + (1-\delta) \right) \end{aligned}$$

Substitution of the definition of the interest rate, $r_t \equiv \frac{\partial f_t^*}{\partial k_t} - \delta$, yields

$$\lambda_0^* = \beta^{T+1} \lambda_{T+1}^* \prod_{j=0}^T (1 + r_j)$$

Also

$$\lambda_0^* = \beta^{t+1} \lambda_{t+1}^* \prod_{j=0}^t (1 + r_j)$$

(D8.1'')

$$\beta^{t+1} \lambda_{t+1}^* = \lambda_0^* \left[\prod_{j=0}^t (1 + r_j) \right]^{-1}$$

Back substitution on (D8.2') yields

$$\begin{aligned} \eta_t^* &= \frac{\partial U(c_t^*, h(w_t^*))}{\partial h_t} h'(w_t^*) + e^{-\rho} \left\{ \lambda_{t+1}^* \left(\frac{\partial f(k_t^*, h(w_t^*))}{\partial h_t} h'(w_t^*) - \frac{\partial C(g_t^*, w_t^*)}{\partial w_t} \right) + \eta_{t+1}^* \right\} + \mu_{2t}^* \\ \eta_0^* &= \frac{\partial U_0^*}{\partial h_0} h_0'^* + \beta \left\{ \lambda_1^* \left(\frac{\partial f_0^*}{\partial h_0} h_0'^* - \frac{\partial C_0^*}{\partial w_0} \right) + \eta_1^* \right\} + \mu_{20}^* \\ \eta_0^* &= \frac{\partial U_0^*}{\partial h_0} h_0'^* + \beta \lambda_1^* \left(\frac{\partial f_0^*}{\partial h_0} h_0'^* - \frac{\partial C_0^*}{\partial w_0} \right) + \beta \eta_1^* + \mu_{20}^* \\ &= \frac{\partial U_0^*}{\partial h_0} h_0'^* + \beta \lambda_1^* \left(\frac{\partial f_0^*}{\partial h_0} h_0'^* - \frac{\partial C_0^*}{\partial w_0} \right) + \beta \left\{ \frac{\partial U_1^*}{\partial h_1} h_1'^* + \beta \lambda_2^* \left(\frac{\partial f_1^*}{\partial h_1} h_1'^* - \frac{\partial C_1^*}{\partial w_1} \right) + \beta \eta_2^* + \mu_{21}^* \right\} + \mu_{20}^* \\ &= \frac{\partial U_0^*}{\partial h_0} h_0'^* + \beta \frac{\partial U_1^*}{\partial h_1} h_1'^* + \beta \lambda_1^* \left(\frac{\partial f_0^*}{\partial h_0} h_0'^* - \frac{\partial C_0^*}{\partial w_0} \right) + \beta^2 \lambda_2^* \left(\frac{\partial f_1^*}{\partial h_1} h_1'^* - \frac{\partial C_1^*}{\partial w_1} \right) + \beta^2 \eta_2^* + \beta \mu_{21}^* + \mu_{20}^* \\ &= \frac{\partial U_0^*}{\partial h_0} h_0'^* + \beta \frac{\partial U_1^*}{\partial h_1} h_1'^* + \beta \lambda_1^* \left(\frac{\partial f_0^*}{\partial h_0} h_0'^* - \frac{\partial C_0^*}{\partial w_0} \right) + \beta^2 \lambda_2^* \left(\frac{\partial f_1^*}{\partial h_1} h_1'^* - \frac{\partial C_1^*}{\partial w_1} \right) \\ &\quad + \beta^2 \left\{ \frac{\partial U_2^*}{\partial h_2} h_2'^* + \beta \lambda_3^* \left(\frac{\partial f_2^*}{\partial h_2} h_2'^* - \frac{\partial C_2^*}{\partial w_2} \right) + \beta \eta_3^* + \mu_{22}^* \right\} + \beta \mu_{21}^* + \mu_{20}^* \\ &= \frac{\partial U_0^*}{\partial h_0} h_0'^* + \beta \frac{\partial U_1^*}{\partial h_1} h_1'^* + \beta^2 \frac{\partial U_2^*}{\partial h_2} h_2'^* + \beta \lambda_1^* \left(\frac{\partial f_0^*}{\partial h_0} h_0'^* - \frac{\partial C_0^*}{\partial w_0} \right) + \beta^2 \lambda_2^* \left(\frac{\partial f_1^*}{\partial h_1} h_1'^* - \frac{\partial C_1^*}{\partial w_1} \right) \\ &\quad + \beta^3 \lambda_3^* \left(\frac{\partial f_2^*}{\partial h_2} h_2'^* - \frac{\partial C_2^*}{\partial w_2} \right) + \beta^3 \eta_3^* + \beta^2 \mu_{22}^* + \beta \mu_{21}^* + \mu_{20}^* \\ &= \sum_{j=0}^T \beta^j \frac{\partial U_j^*}{\partial h_j} h_j'^* + \sum_{j=0}^T \beta^{j+1} \lambda_{j+1}^* \left(\frac{\partial f_j^*}{\partial h_j} h_j'^* - \frac{\partial C_j^*}{\partial w_j} \right) + \beta^{T+1} \eta_{T+1}^* + \sum_{j=0}^T \beta^j \mu_{2j}^* \end{aligned}$$

Inserting Equation (D8.1'')

$$(D8.2'') \eta_0^* = \sum_{j=0}^T \beta^j \frac{\partial U_j^*}{\partial h_j} h_j'^* + \sum_{j=0}^T \lambda_0^* \left[\prod_{i=0}^j (1 + r_i) \right]^{-1} \left(\frac{\partial f_j^*}{\partial h_j} h_j'^* - \frac{\partial C_j^*}{\partial w_j} \right) + \beta^{T+1} \eta_{T+1}^* + \sum_{j=0}^T \beta^j \mu_{2j}^*$$

THE SCENARIOS

In this section we present the numerical results for the two scenarios. These will present a comparative dynamic analysis of two economies with different costs associated with cleanup activities. We first present the parameter values that identify the scenarios, and then we discuss the resulting numerical time paths.

Both scenarios will have the following parameter values

$\rho = 0.05, \tau = 2, a_c = 7.8, a_h = 4.7, a_1 = 20, b_1 = 0.2, A = 2, \alpha = 0.5, \gamma = 0.4$, and $\delta = 0.1$. The values of a_c and a_h were originally 5 and 3, respectively, but were scaled to make $\lambda_0 = 1$ as discussed above. Scenario E will also have $a_2 = 9, a_3 = 10$, and $a_4 = 0$. These values make marginal cleanup costs an increasing function of cleanup activity, but independent of stock of waste; hence, cleanup activity will continue until all of the waste is neutralized. Scenario N will have $a_2 = 20, a_3 = 10$, and $a_4 = 6$ which make marginal cleanup costs an increasing function of cleanup activity and very responsive to waste stock size. As the remaining stock of waste decreases marginal cleanup costs increase at each level of cleanup activity. As a result of these increasing marginal costs the cleanup activity ceases before all of the waste is neutralized. These parameter values were selected to illustrate among other things the effect of the cost function parameters on the time path of the shadow value of toxic waste.

The model is of course a modification of a simple growth model; hence, in the implementation of the model we have the economies experience growth of capital,

income and consumption. Capital accumulation for both scenarios is depicted in Figure 1.

Figure 1 Capital Stock for Both Scenarios

At the level of resolution of the graph the initial growth of capital stock is nearly the same in the two scenarios; however, after the first few years Scenario N has the faster capital accumulation until about $t = 20$. Also capital stock is clearly higher in the stationary state for Scenario E. During the early years Scenario E is channeling more of its income into cleanup activities and less into net investment than does Scenario N. In the stationary state Scenario E has higher income because it has both higher capital and

health (lower toxic waste). These similar relationships are viewed in Figure 2

Figure 2 Consumption for Both Scenarios

where we observe that in Scenario N consumption is initially higher, but in the stationary state is lower than in Scenario E. We know from Equation (3a) that consumption and the shadow value of capital, λ_t , are functionally and inversely related. This is shown In

Figure 3.

Figure 3 Shadow Values of Capital and the Interest Rate for Both Scenarios

The interest rate for both scenarios is also plotted, and at this resolution is the same for the two scenarios. During the growth portion of the time horizon the interest rate is falling and in the stationary state is equal to the rate of time preference.

Figure 4 shows the time path of cleanup activity.

Figure 4 Cleanup Activity for Both Scenarios

Cleanup activity is for the first few years higher in Scenario N, because its cost function yields a very low marginal cost of clean up when the stock of waste is large. This relationship soon reverses and cleanup activity in Scenario E rapidly moves the toxic

waste stock to zero. Figure 5 shows the time paths for the stock of toxic waste,

Figure 5 Toxic Waste for Both Scenarios

and as already stated, for Scenario E the stock of waste goes to zero rapidly. For Scenario N the marginal cost of cleanup activity increases to a high enough level to stop cleanup activity before all of the waste is cleaned up.

Associated with these time paths are the time paths for the shadow value of waste, which is shown in Figure 6.

Figure 6 Shadow Value of Waste for Both Scenarios

The absolute value of this shadow value is the marginal value of a unit of cleanup activity, and for Scenario E this absolute value starts high when the stock of waste is high. It then gradually decreases until it reaches its stationary state value. This is as anticipated; however, for Scenario N the time path of the absolute value of η_t starts low and rapidly increases till it reaches its stationary state value. This path is less intuitive, but just as consistent with the model. The reason for this reversal is that the cost effect dominates the shape of the time path for Scenario N, but is absent in Scenario E.

These differences can be seen in Figure 7, which shows the decomposition of η_t for Scenario N.

Figure 7 Components of η_t for Scenario N

We graph only 99 years, time periods, since that is enough to illustrate the relationships. The line labeled 'Sum' is sum of the four effects, and it coincides very closely with the time path for η_t . This is reassuring since the opposite would indicate an error. Note that the utility and productivity effects are negative, as expected, and that they gradually go to zero. The cost effect is positive as expected, and is a measure of the impact of the marginal unit upon future cleanup costs. Suppose we were to cleanup an additional unit of waste at time t . This will affect the marginal unit all along the optimal path, starting at time t . In doing so it affects the cleanup costs all along the path from time t on. The cost effect is the present value of these additional costs, as of time t .

Figure 8 illustrates for Scenario N the bias that results from discounting the utility effect using the interest rate instead of the rate of time preference.

Figure 8 Biased Utility Effect for Scenario E

The result is that the utility effect is under estimated, since the interest rate is high and decreasing during the growth portion of the time horizon. Of course, the two coincide during the stationary state since the interest rate and the rate of time preference are equal during this part of the time horizon.

SUMMARY

We have discussed optimal discounting of the benefits of the cleanup of toxic waste sites. Some have suggested that these benefits be discounted at a zero rate of time preference and others have suggested discounting at the market rate of interest. We have shown that it is optimal to discount the benefits that accrue to individuals in the form of increased utility at the rate of time preference, whether it is zero or positive, and that it is optimal to discount the benefits that accrue through the added productivity of workers at the rate of interest.

To achieve this we identified components of the shadow value of toxic waste cleanup, $-\eta_t$. These are the utility, productivity, cost and abundance effects. The utility effect is the present value of the stream of marginal utilities initiated by the removal of one unit of waste. This stream is discounted at the rate of time preference. The productivity effect is the present value of the stream of value-of-the-marginal-productivities initiated by this removal, and is discounted at the rate of interest. The cost effect is a measure of the impact of the marginal unit upon future cleanup costs, and is discounted at the rate of interest. The abundance effect is the present value of the terminal value of the shadow value, $-\eta_T$, discounted at the rate of time preference.

The relationships within this model were illustrated in two scenarios using a numerical discrete time implementation of the model. The cleanup cost function parameters were the only difference between the two scenarios. Scenario E's parameters resulted in complete cleanup of the toxic waste, while Scenario N's caused cleanup activities to cease before all of the waste was removed. These differences yielded strikingly different optimal time paths for the shadow value of toxic waste and its components.

APPENDIX A

DERIVATION OF EQUATION 4

Equation (3c') is a linear first order differential equation and has a particular solution

$$(A1) \quad \lambda^*(t) = C \exp \left[\rho t - \int r(t) dt \right]$$

where C is an arbitrary constant. To obtain a particular solution, let $R(t) \equiv \int r(t) dt$; thus,

$$\lambda^*(t) = C \exp[\rho t - R(t)]$$

and note that

$$\lambda^*(0) = C \exp[-R(0)]$$

or

$$\lambda^*(0) \exp[R(0)] = C$$

Insert this value of C into Equation (A1) and use the definite integral to obtain

$$\lambda^*(t) = \lambda^*(0) \exp \left[\rho t - \int_0^t r(s) ds \right]$$

DERIVATION OF EQUATION (5)

The general solution to Equation (3d) is (see, e.g., Simon and Blume 1994)

(A2)

$$\eta^*(t) = \left\{ D - \int \left[\frac{\partial U(c^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} + \lambda^*(t) \left(\frac{\partial f(k^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} - \frac{\partial C(g^*(t), w^*(t))}{\partial w(t)} \right) \right] \exp[-\rho t] dt \right\} \exp[\rho t]$$

where D is an arbitrary constant. To obtain a particular solution, let

$$F(t) \equiv \int \left[\frac{\partial U(c^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} + \lambda^*(t) \left(\frac{\partial f(k^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} - \frac{\partial C(g^*(t), w^*(t))}{\partial w(t)} \right) \right] \exp[-\rho t] dt$$

Equation (A2) can be rewritten as

$$\eta^*(t) = \{D - F(t)\} \exp[\rho t]$$

which implies

$$\eta^*(t) \exp[-\rho t] + F(t) = D$$

or evaluated at time T

$$\eta^*(T) \exp[-\rho T] + F(T) = D$$

Thus,

$$\eta^*(t) = \{\eta^*(T) \exp[-\rho T] + F(T) - F(t)\} \exp[\rho t]$$

Use the definition of $F(t)$ and the definite integral to obtain

$$\begin{aligned} \eta^*(t) = & \eta^*(T) \exp[-\rho(T-t)] + \int_t^T \left[\frac{\partial U(c^*(s), h(w^*(s)))}{\partial h(w(s))} \frac{\partial h(w^*(s))}{\partial w(s)} \right. \\ & \left. + \lambda^*(s) \left(\frac{\partial f(k^*(s), h(w^*(s)))}{\partial h(w(s))} \frac{\partial h(w^*(s))}{\partial w(s)} - \frac{\partial C(g^*(s), w^*(s))}{\partial w(s)} \right) \right] \exp[-\rho(s-t)] ds \end{aligned}$$

Evaluate at time zero

$$\eta^*(0) = \eta^*(T) \exp[-\rho T] + \int_0^T \left[\frac{\partial U(c^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} + \lambda^*(t) \left(\frac{\partial f(k^*(t), h(w^*(t)))}{\partial h(w(t))} \frac{\partial h(w^*(t))}{\partial w(t)} - \frac{\partial C(g^*(t), w^*(t))}{\partial w(t)} \right) \right] \exp[-\rho t] dt$$

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