

MULTIPLE SPACECRAFT FIXED FORMATION CONTROL FOR LARGE BASELINE INTERFEROMETRY

Jonathan Lawton Randal W. Beard
Electrical and Computer Engineering Department
Brigham Young University

Fred Y. Hadaegh
Jet Propulsion Laboratory
California Institute of Technology

Abstract

In this paper we present a spacecraft control for rigid fleet rotations. This is done by creating a fleet template which is slowly rotated to generate desired trajectories for each individual spacecraft. By rotating the template slow enough each spacecraft is able to track these trajectories to within a given tolerance in the presence of actuator saturation. Simulations for a three spacecraft fleet are given.

1 Introduction

Travel to neighboring galaxies would require space voyages lasting thousands of years. As a result further space exploration can only be practically achieved by indirect observation of astronomical objects by means of spectral analysis. Much can be determined about an astronomical object from the light that the object emits. To make such delicate observations a space based interferometer with baselines on the order of one to ten of kilometers have been proposed.

In [Decou, 1991a] and [Decou, 1991b] a free-flying multiple spacecraft interferometer is proposed. The simplest free-flying multiple spacecraft interferometer would consist of three spacecraft. These would be used to sample the light from an astronomical source striking the U-V plane which is the plane perpendicular to the direction of incoming light from an astronomical source. The

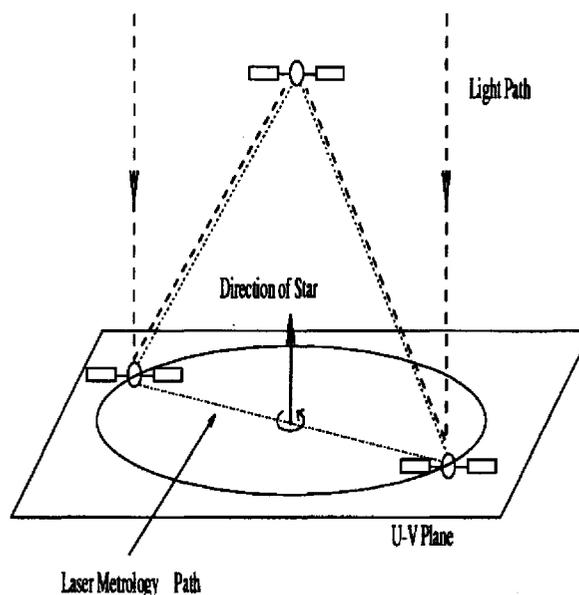


Figure 1: A Three Spacecraft Free-Flying Interferometer In The U-V plane

three spacecraft would be oriented in a rigid triangular formation. Two spacecraft, positioned within the U-V plane, would collect light from the distant astronomical source (see Figure 1). The light would then be reflected from each spacecraft to the third spacecraft where the interference pattern would be observed. The fleet must then be moved to another position and orientation in the U-V plane to make another measurement until a sufficient portion of the light striking the U-V plane in sampled.

Each fleet member will have its sensors locked

on a neighboring spacecraft while conducting each measurement. It is necessary for spectral interferometry to maintain the relative distance and relative alignment between spacecraft to very fine tolerances. If sensor lock is lost then the costly process of formation re-initialization must be done before proceeding with another measurement [Wang et al., 1997].

The main result of this paper is to rigidly rotate the fleet given actuator saturation constraints. We provide control laws for each spacecraft to move it from its initial position to the desired final position. Spacecraft attitude control will be considered in another paper.

Preliminary work on fleet formation control for free-flying multiple spacecraft interferometry was done in [Wang and Hadaegh, 1996], where the authors developed a fleet hierarchy which classified some spacecraft as leaders and others as followers. Given trajectories for the leaders, desired trajectories for the followers were derived. Furthermore, control laws were derived such that each follower would follow its desired trajectory. In [Beard and Hadaegh, 1998] a non-hierarchical approach is considered, this extending prior work by developing trajectories for every member of the fleet. The authors treated the fleet as if it were a rigid body. A pseudo-torque was applied to this body which generated trajectories for each spacecraft to follow. Then control laws were derived such that each spacecraft would track its desired trajectory. Our paper builds on the work done in the second paper, while taking actuator saturation constraints into account.

The paper is organized as follows. In Section 2 we develop spacecraft trajectories for each fleet member. In Section 3 we present an adaptive saturated control to track the desired trajectories. In Section 4 we apply our controller to the three spacecraft free-flying interferometer. Then in Section 5 we give our conclusions.

2 Trajectory Generation

To derive trajectories for each spacecraft such that the fleet will rotate as if it were a rigid body, we will treat the fleet as if it were one a giant composite mass. We will refer to this fictitious body as the fleet template. A virtual torque is applied to the fleet template and the resultant template trajectory generates the desired spacecraft trajectories. By designing control laws to track these trajectories within a fine tolerance we will cause

the entire fleet to rotate as if it were rigidly connected.

To set up the fleet rotation problem, we will choose the unit quaternion to measure the fleet orientation, which we represent as $q = \hat{q} + q_4$, where \hat{q} and q_4 are the vector and scalar components of the quaternion respectively. Without loss of generality, we may assign $q = 1$ to be the initial attitude of the fleet orientation. The desired final orientation of the fleet will be

$$q_d = \sin\left(\frac{\theta_d}{2}\right)\mathbf{v} + \cos\left(\frac{\theta_d}{2}\right).$$

To ensure that all spacecraft remain in the U-V plan, we wish to rotate the fleet about a fixed axis. Thus the attitude trajectory of the fleet will be given by

$$q_T = \sin\left(\frac{\theta_T}{2}\right)\mathbf{v} + \cos\left(\frac{\theta_T}{2}\right), \quad (1)$$

where $\theta_T(0) = 0$ and $\theta_T \rightarrow \theta_d$ as $t \rightarrow \infty$. We will refer to θ_T as the template angle. This kind of rotation about a fixed axis is called an eigenaxis rotation. An eigenaxis rotation has been studied for a single spacecraft [Wie et al., 1989]. When considering a fleet eigenaxis rotation the results simplify somewhat.

Since we have chosen to use the unit quaternion representation of attitude we must also give a kinematic relationship between the unit quaternion, q_T , and the angular velocity ω_T . This is given by

$$\begin{aligned} \dot{q}_T(t) &= \frac{1}{2}\omega_T(t) \times \hat{q}_T(t) + \frac{1}{2}q_{T4}\omega_T(t) \\ \dot{q}_{T4}(t) &= -\frac{1}{2}\hat{q}_T(t)^T\omega_T(t). \end{aligned}$$

In [Wie et al., 1989] it is shown that a necessary and sufficient condition for an eigenaxis rotation is that $\omega_T(t)$ and $\hat{q}_T(t)$ are parallel. This implies that $\omega_T = \Omega\mathbf{v}$. Substitution of $\omega_T = \Omega\mathbf{v}$ in to the kinematic relationship results in $\Omega = \dot{\theta}_T$. Therefore the kinematic relationship for an eigenaxis rotation is

$$\omega_T = \dot{\theta}_T\mathbf{v}. \quad (2)$$

To arrive at a desired trajectory for the system, suppose that the fleet template is connected to a torsional spring such that the equilibrium orientation is q_d . Assuming the fleet template starts at rest the fleet template dynamics may be modeled by

$$\begin{aligned} \ddot{\theta}_T + d_T\dot{\theta}_T + k_T(\theta_T - \theta_d) &= 0 \\ \dot{\theta}_T(0) &= 0 \\ \theta_T(0) &= 0, \end{aligned} \quad (3)$$

where d_T and k_T are the systems damping and spring constants respectively. Since equation (3) is a second order differential equation with positive coefficients the eigenvalues are stable and $\theta_T \rightarrow \theta_d$.

The trajectory generated by equation (3) may be used to give trajectories for the desired coordinates of each spacecraft.

$$\begin{aligned} r_{id}(t) &= q_T^* r_{id}(0) q_T \\ \dot{r}_{id}(t) &= \dot{\theta}_T \mathbf{v} \times r_{id}(t) \\ \ddot{r}_{id}(t) &= \ddot{\theta}_T \mathbf{v} \times r_{id}(t) + \dot{\theta}_T \mathbf{v} \times (\dot{\theta}_T \mathbf{v} \times r_{id}(t)), \end{aligned} \quad (4)$$

where r_{id} is the desired position of the i th spacecraft with respect to an inertial reference frame with origin at the center of the fleet template. The first equation is simply a vector rotation of the initial desired position by the current attitude of the rigid body. The other two equations come from standard expressions for the velocity and acceleration of rotating vectors.

It is imperative to carefully choose our control gains k_T and d_T such that the resultant trajectories as given in equation (4) are tractable given spacecraft actuator constraints. Theorem 2.1 will give bounds on $\|\ddot{r}_{id}\|$ for the i th spacecraft in terms of the control gains k_T and d_T . These bounds will be useful to ensure that the fleet template is rotating slow enough that each spacecraft will be able to track their desired trajectories. Before presenting Theorem 2.1 two lemmas will be derived to help obtain the desired result.

Lemma 2.1 *We may place the following bounds on the template dynamics*

$$\begin{aligned} |\dot{\theta}_T| &\leq \sqrt{k_T} |\theta_d| \\ |\theta_T - \theta_d| &\leq |\theta_d| \end{aligned} \quad (5)$$

Proof:

First observe that if we define our state vector $x = [\theta_T, \dot{\theta}_T]^T$ then

$$V(x) = \frac{1}{2} \dot{\theta}_T^2 + k_T \frac{1}{2} (\theta_T - \theta_d)^2$$

is a Lyapunov function. To see this we may take the time derivative of $V(x)$

$$\begin{aligned} \dot{V}(x) &= \dot{\theta}_T (\ddot{\theta}_T + k_T (\theta_T - \theta_d)) \\ &= -d_T \dot{\theta}_T^2, \end{aligned}$$

where the last line follows from equation (3). Since $\dot{V}(x(t))$ is negative semi-definite then $V(x(t)) \leq$

$V(x(0))$. The first bound in equation (5) follows:

$$\begin{aligned} \dot{\theta}_T(t)^2 &\leq \dot{\theta}_T(t)^2 + k_T (\theta_T(t) - \theta_d)^2 \\ &= 2V(x(t)) \\ &\leq 2V(x(0)) \\ &= k_T \theta_d^2. \end{aligned}$$

The last line is true since $\dot{\theta}_T(0) = \theta_T(0) = 0$.

Similarly, the second bound in equation (5) is established since

$$\begin{aligned} (\theta_T - \theta_d)^2 &\leq \frac{1}{k_T} \dot{\theta}_T(t)^2 + (\theta_T(t) - \theta_d)^2 \\ &= \frac{2}{k_T} V(x(t)) \\ &\leq \frac{2}{k_T} V(x(0)) \\ &= \theta_d^2. \end{aligned}$$

■

Lemma 2.2 *Given equation (4), if r_{id} is decomposed into its components parallel and perpendicular to \mathbf{v}*

$$r_{id} = r_{id\parallel} \mathbf{v} + r_{id\perp} \mathbf{w}_1(t), \quad (6)$$

where $\mathbf{w}_1(t)$ is an appropriately chosen unit vector perpendicular to \mathbf{v} then

$$\|\ddot{r}_{id}\|_2 = r_{id\perp} \sqrt{\ddot{\theta}_T^2 + \dot{\theta}_T^4}.$$

Recall from equation (4) that

$$\ddot{r}_{id}(t) = \ddot{\theta}_T \mathbf{v} \times r_{id}(t) + \dot{\theta}_T(t) \mathbf{v} \times (\dot{\theta}_T(t) \mathbf{v} \times r_{id}(t)).$$

Upon substituting equation (6) for r_{id} , $\ddot{r}_{id}(t)$ simplifies to

$$\ddot{r}_{id}(t) = r_{id\perp} (\ddot{\theta}_T (\mathbf{v} \times \mathbf{w}_1(t))) + \dot{\theta}_T(t)^2 \mathbf{v} \times (\mathbf{v} \times \mathbf{w}_1(t)).$$

since $(\mathbf{v} \times \mathbf{w}_1(t))$ and $\mathbf{v} \times (\mathbf{v} \times \mathbf{w}_1(t))$ are orthogonal

$$\|\ddot{r}_{id}(t)\|_2 = r_{id\perp} \sqrt{\ddot{\theta}_T(t)^2 + \dot{\theta}_T(t)^4}.$$

Theorem 2.1 (Acceleration Bounds) *Let*

$$r_{id\perp} = \|r_{id} \times \mathbf{v}\|_2$$

(i.e. the component of r_{id} perpendicular to \mathbf{v}), then we may establish the following bound on the desired acceleration of the i th spacecraft:

$$\|\ddot{r}_{id}\|_2 \leq r_{id\perp} \sqrt{(d_T \sqrt{k_T} + k_T)^2 \theta_d^2 + k_T^2 \theta_d^4} \quad (7)$$

Proof:

From Lemma 2.2

$$\begin{aligned}\|\ddot{r}_{id}\|_2 &= r_{id\perp} \sqrt{\dot{\theta}_T^2 + \dot{\theta}_t^4} \\ &= r_{id\perp} \sqrt{(d_T \dot{\theta}_T(t) + k_T(\theta_T - \theta_d))^2 + \dot{\theta}_T^4} \\ &\leq r_{id\perp} \sqrt{(d_T \sqrt{k_T} + k_T)^2 \theta_d^2 + k_T^2 \theta_d^4},\end{aligned}$$

where the last line follows from Lemma 2.1. ■

3 Spacecraft Control

In applications of spectral interferometry, it is necessary to maintain the fleet in a very rigid formation to within a small tolerance. This may be accomplished by requiring that each spacecraft remain within some tolerance ϵ of its desired spacecraft position as defined by the fleet template. This is especially challenging given a spacecraft actuator constraint

$$|u| \leq u_{max}.$$

To complicate the problem further, during space flight the exact mass of the spacecraft may become uncertain.

To solve the mass uncertainty problem, we implement an adaptive control law which continuously updates the estimated spacecraft mass while applying the controller. To solve the actuator constraint problem, we will place bounds on control gains to keep the spacecraft thrusters from going into saturation. The first problem will be addressed by Theorem 3.1 and the second problem will be addressed by Theorem 3.2. In both theorems we will assume that the spacecraft dynamics may be modeled by

$$\begin{aligned}m_i \ddot{r}_i &= u_i \\ |\{u_i\}_j| &\leq u_{max},\end{aligned}\quad (8)$$

where $\{u_i\}_j$ is the j th component of u_i .

Theorem 3.1 presents an adaptive controller with sufficient conditions on the spacecraft control gains such that the spacecraft will track the desired trajectories from Section 2 within a tolerance of $\|\ddot{r}_i(t)\|_2 < \epsilon$.

Theorem 3.1 [*Adaptive Spacecraft Control*]

Given the adaptive control law u_i defined by

$$\begin{aligned}u_i &= \hat{m}_i v_i, \\ v_i &= \ddot{r}_{id} - \gamma_p \dot{r}_i - \gamma_v \ddot{r}_i, \\ \dot{\hat{m}}_i &= -\gamma_m v_i^T \dot{r}_i \\ \hat{m}_i(0) &= \frac{m_a + m_b}{2}\end{aligned}\quad (9)$$

where

1. $0 < m_a \leq m_i \leq m_b$
2. \hat{m}_i is the approximate mass of the i th spacecraft
3. $\ddot{r}_i = r_i - r_{id}$
4. $\gamma_p, \gamma_v > 0$ are control gains
5. $\gamma_m > 0$ is an adaption gain,

if

$$\begin{aligned}\dot{\ddot{r}}_i(0) &= 0 \\ \ddot{r}_i(0) &\leq \delta < \epsilon \\ \frac{1}{\gamma_m} &< [\epsilon^2 - \delta^2] \frac{4\gamma_p m_a}{(m_b - m_a)^2} \\ |\{\hat{m}_i v_i\}_j| &\leq u_{max}\end{aligned}$$

then $\forall t > 0$

$$\|\ddot{r}_i(t)\|_2 < \epsilon, \quad (10)$$

$$\|\dot{\ddot{r}}_i(t)\|_2 < \sqrt{\gamma_p} \epsilon \quad (11)$$

$$|\bar{m}_i(t)| \leq \Delta M_i(\gamma_p, \gamma_m), \quad (12)$$

$$\hat{m}_i(t) \leq m_b + \Delta M_i(\gamma_p, \gamma_m) \quad (13)$$

where $\bar{m}_i = \hat{m}_i - m_i$ and

$$\Delta M(\gamma_p, \gamma_m) = \sqrt{\gamma_m \gamma_p m_b \delta^2 + \frac{(m_b - m_a)^2}{4}}.$$

$$V_i(x) = \frac{1}{2} m_i \dot{\ddot{r}}_i^T \dot{\ddot{r}}_i + \frac{\gamma_p}{2} m_i \ddot{r}_i^T \ddot{r}_i + \frac{1}{2\gamma_m} \bar{m}_i^2, \quad (14)$$

where $\bar{m}_i = \hat{m}_i - m_i$ and the state vector is defined $x = [\ddot{r}_i^T, \dot{\ddot{r}}_i^T, \bar{m}_i]^T$. By taking the time derivative of $V_i(x)$ it can be verified that it is a Lyapunov function. Since $\dot{\bar{m}}_i = \dot{\hat{m}}_i = -\gamma_m \dot{\ddot{r}}_i^T v_i$,

$$\begin{aligned}\dot{V}_i(x) &= \dot{\ddot{r}}_i^T (m_i \ddot{r}_i + \gamma_p m_i \dot{r}_i - \bar{m}_i v_i) \\ &= \dot{\ddot{r}}_i^T (m_i \ddot{r}_i - m_i \ddot{r}_{id} + \gamma_p m_i \dot{r}_i - \bar{m}_i v_i) \\ &= \dot{\ddot{r}}_i^T (u_i - m_i v_i - \gamma_v m_i \dot{r}_i - \bar{m}_i v_i) \\ &= \dot{\ddot{r}}_i^T (u_i - \gamma_v m_i \dot{r}_i - \hat{m}_i v_i) \\ &= -\gamma_v m_i \dot{\ddot{r}}_i^T \dot{\ddot{r}}_i.\end{aligned}\quad (15)$$

To verify equation (10) note that since $\dot{V}_i(x(t))$ is negative semi-definite, $V_i(x(t)) \leq V_i(x(0))$. We know that

$$\begin{aligned} \|\tilde{r}_i(t)\|_2^2 &\leq \|\tilde{r}_i(t)\| + \frac{1}{\gamma_p} \|\dot{\tilde{r}}_i\|_2^2 + \frac{1}{\gamma_p \gamma_m m_i} \tilde{m}_i^2 \\ &= \frac{2}{m_i \gamma_p} V(t) \\ &\leq \frac{2}{m_i \gamma_p} V(0) \\ &\leq \frac{1}{\gamma_p} \|\dot{\tilde{r}}_i(0)\|_2^2 + \|\tilde{r}_i^T(0)\|_2^2 + \frac{1}{\gamma_m m_i \gamma_p} \tilde{m}_i^2(0). \end{aligned}$$

Since $\dot{\tilde{r}}_i(0) = 0$, $\|\tilde{r}_i(0)\| < \delta$ and $|\tilde{m}_i| \leq (m_b - m_a)/2$

$$\begin{aligned} \|\tilde{r}_i(t)\|_2^2 &\leq \delta^2 + \frac{(m_b - m_a)^2}{4\gamma_m m_a \gamma_p} \\ &< \epsilon^2, \end{aligned}$$

where the last line follows from the hypothesis. This shows equation (10) is true. By an analogous argument

$$\begin{aligned} \|\dot{\tilde{r}}_i\| &\leq \frac{2}{m_i} V(x(0)) \\ &= \gamma_p \left(\frac{2}{m_i \gamma_p} V(x(0)) \right) \\ &\leq \gamma_p \epsilon^2, \end{aligned}$$

which establishes equation (11).

Now to verify equation (12). Again since $V_i(x(t)) \leq V_i(x(0))$.

$$\begin{aligned} \tilde{m}_i^2(t) &\leq \gamma_m m_i (\|\dot{\tilde{r}}_i(t)\|_2^2 + \gamma_p \|\tilde{r}_i(t)^T\|_2^2) + \tilde{m}_i(t)^2 \\ &= 2\gamma_m V_i(x(t)) \\ &\leq 2\gamma_m V_i(x(0)) \\ &= m_i \gamma_m \gamma_p \|\tilde{r}_i(0)\|_2^2 + \tilde{m}_i^2(0) \\ &\leq m_b \gamma_m \gamma_p \delta^2 + \frac{(m_b - m_a)^2}{4}. \end{aligned}$$

It follows directly that

$$|\tilde{m}_i(t)| \leq \Delta M(\gamma_p, \gamma_m).$$

Now to proof equation (13) we apply the definition of \tilde{m} to show that

$$|\hat{m}_i - m_i| \leq \Delta M(\gamma_p, \gamma_m).$$

Now if we add m_i to both sides and apply the triangle inequality we derive the desired result

$$\begin{aligned} \hat{m}_i &\leq m_i + \Delta M(\gamma_p, \gamma_m) \\ &\leq m_b + \Delta M(\gamma_p, \gamma_m). \end{aligned}$$

This verifies equation (13).

Theorem 3.1 requires that the spacecraft thruster are not saturated. Theorem 3.2 will place sufficient conditions on the spacecraft and fleet template gains to keep the spacecraft thrusters from saturating.

Theorem 3.2 *Given control law (9), if γ_p , γ_v , k_T and d_T are chosen such that*

$$B_i \leq u_{max},$$

where

$$\begin{aligned} B_i &= (\Delta M(\gamma_p, \gamma_m) + m_b) \\ &\quad (r_{id\perp} \sqrt{(d_T \sqrt{k_T} + k_T)^2 \theta_d^2 + k_T^2 \theta_d^4} \\ &\quad + \epsilon(\gamma_v \sqrt{\gamma_p} + \gamma_p)). \end{aligned}$$

then $|\{\hat{m}_i v_i\}_j| \leq u_{max}$.

Proof:

From Theorem 3.1

$$\begin{aligned} |\{\hat{m}_i v_i\}_j| &= |\hat{m}_i| |\{v_i\}_j| \\ &\leq (\Delta M(\gamma_p, \gamma_m) + m_b) \|v_i\|_2 \\ &\leq (\Delta M(\gamma_p, \gamma_m) + m_b) \\ &\quad \times (\|\tilde{r}_{id}\|_2 + \gamma_v \|\dot{\tilde{r}}_i\|_2 + \gamma_p \|\tilde{r}_i\|_2) \\ &\leq (\Delta M(\gamma_p, \gamma_m) + m_b) \\ &\quad \times (\|\tilde{r}_{id}\|_2 + \gamma_v \sqrt{\gamma_p} \epsilon + \gamma_p \epsilon) \\ &= (\Delta M(\gamma_p, \gamma_m) + m_b) \\ &\quad \times (\|\tilde{r}_{id}\|_2 + \epsilon(\gamma_v \sqrt{\gamma_p} + \gamma_p)). \end{aligned}$$

Application of Theorem 2.1 results in

$$\begin{aligned} \|\{\hat{m}_i v_i\}_j\| &\leq (\Delta M(\gamma_p, \gamma_m) + m_b) \\ &\quad (r_{id\perp} \sqrt{(d_T \sqrt{k_T} + k_T)^2 \theta_d^2 + k_T^2 \theta_d^4} \\ &\quad + \epsilon(\gamma_v \sqrt{\gamma_p} + \gamma_p)) \\ &= B_i. \end{aligned}$$

The required stability condition follows directly from the hypothesis of the Theorem 3.2. ■

To summarize the results of this section, Theorem 3.1 establishes sufficient conditions to allow each spacecraft to track the fleet template within a tolerance of ϵ as long as the spacecraft thrusters are not saturated. Theorem 3.2 establishes sufficient conditions to ensure that the spacecraft thrusters do not saturate.

4 Example

We will apply Theorem 3.2 to a three spacecraft free-flying interferometer as shown in Figure 1. Consider three identical spacecraft with

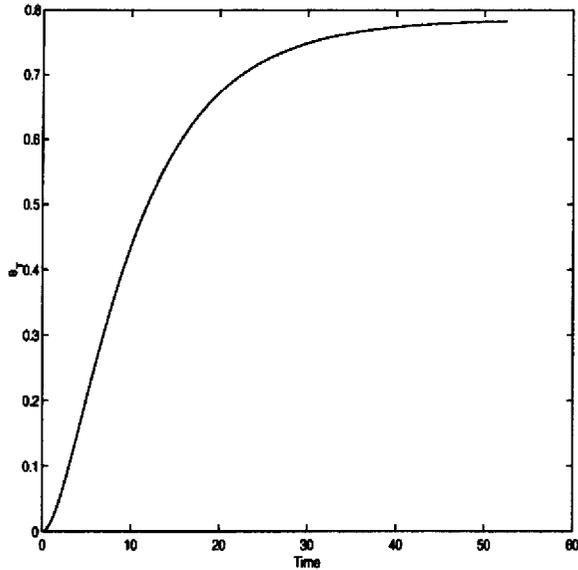


Figure 2: Trajectory of Template Angle θ_T

$m_a = 5$, $m_b = 10$ and $u_{max} = 25$. Let us further assume that the initial conditions on each spacecraft satisfy $\tilde{r}_i(0) \leq \delta \leq 0.1$ and $\max_i r_{id\perp} \leq 5$. We wish to rotate the fleet about the eigenaxis from $\theta_T = 0$ to a final desired angle $\theta_d = \frac{\pi}{4}$. Furthermore, we would like to impose an error tolerance of $\tilde{r}_i(t) \leq \epsilon \leq 0.2$ on each spacecraft.

We will first apply Theorem 3.1. This will make certain that the spacecraft control gains are large enough to track the desired trajectory. Choosing $\gamma_m = \gamma_p = 10$ will satisfy the conditions of Theorem 3.1, given $\epsilon = 0.1$ and $\delta = 0.2$.

Now we will apply Theorem 3.2. This will ensure that the fleet template and spacecraft gains are small enough to avoid saturation. By choosing $\gamma_v = 1$, $k_T = 0.44$ and $d_T = 0.5$, direct substitution of the control gains into the conditions of Theorem 3.2 ensures that the theorem holds.

The results of the simulation are given below. Figure 2 plots the trajectory of the fleet angle, θ_T , versus time. Figure 3, Figure 4 and Figure 5 give plots of the time history of the mean square spacecraft tracking error for each of the three spacecrafts.

5 Conclusion

In this paper we have develop spacecraft controls for rigid formation flying in the presence of actuator constraints. Previous attempts at fleet formation control did not take into account spacecraft actuator saturation.

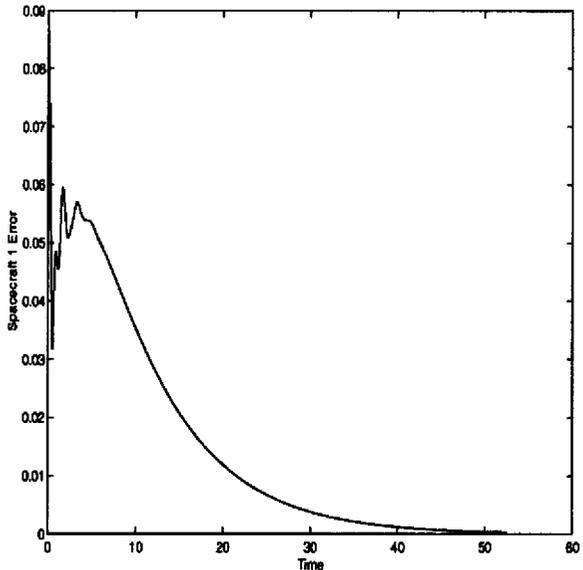


Figure 3: Evolution of Spacecraft 1 Position Error

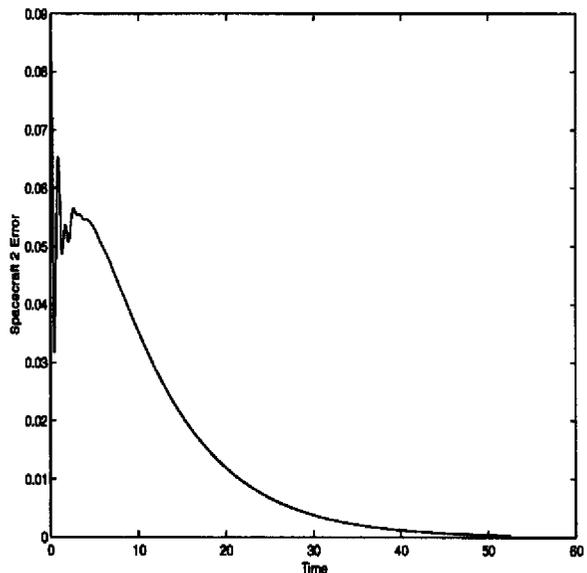


Figure 4: Evolution of Spacecraft 2 Position Error

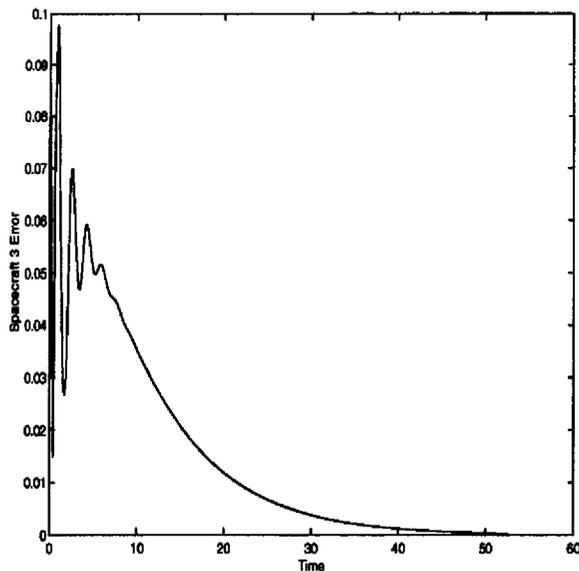


Figure 5: Evolution of Spacecraft 3 Position Error

To take saturation into account it is necessary to move the fleet slow enough to allow each spacecraft to track their desired fleet positions with in a fine tolerance. This was done by creating a fleet template attached to a fictitious torsional spring. The desired spacecraft trajectories for each spacecraft are generated from the fleet template trajectory. By adjusting the spring constants, we guarantee that the template is moving slow enough to allow each spacecraft to track its desired trajectory in the presence of actuator constraints. We derived several condition on the template and spacecraft gains to ensure spacecraft tracking.

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