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ABSTRACT

Pesticides have value because they help farmers control pests that destroy crops and decrease yields; however, their use will lead to insecticide resistance. In addition, their use has unintended other effects. These stem in part from pesticide residues on food and application drift that exposes humans and other non-target biologic populations to toxic agents. We analyze these interactions in a stochastic general equilibrium optimal control model. The necessary conditions of an optimal path are examined to identify the roles that Integrated Pest Management, spraying and pest resistant crops, and biological research can play along the optimal time path. Because the insecticides affect more than the target population, there are common property or externality effects. We identify expressions that can be used to formulate policies to adjust for these effects. Our results are consistent with the findings in the literature, and add some significant insights. These insights stem from our decomposition of the shadow values which identifies the expected values that can be estimated to estimate the shadow values. We then identify how to use these shadow values to correct the market signals for the common property effects.
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Pesticides have value because they help farmers control pests that destroy crops and decrease yields; however, their use will lead to insecticide resistance. In addition, their use has unintended other effects. These stem in part from pesticide residues on food and application drift that exposes humans and other non-target biologic populations to toxic agents. We analyze these interactions in a stochastic general equilibrium optimal control model. The necessary conditions of an optimal path are examined to identify the roles that Integrated Pest Management, spraying and pest resistant crops, and biological research can play along the optimal time path. Because the insecticides affect more than the target population, there are common property or externality effects. We identify expressions that can be used to formulate policies to adjust for these effects. Our results are consistent with the findings in the literature, and add some significant insights. These insights stem from our decomposition of the shadow values which identifies the expected values that can be estimated to estimate the shadow values. We then identify how to use these shadow values to correct the market signals for the common property effects.
Introduction
At the middle of the 20th century many individuals felt that with biotechnology we could control crop pests, weeds and diseases. In addition, there was the feeling that the 'vectors', mosquitoes, that spread infectious diseases such as malaria and yellow fever could be eradicated. These new chemicals triggered the abandonment of previously important farming techniques of controlling pests - principally biological and cultural controls. The beneficial effects of these chemicals, mainly DDT, were enormous; however, they were short lived as resistance soon developed in the pests. It also became clear that sprays had costly side effects. Many were broad spectrum pesticides that also killed non-target organisms, and some, such as DDT, were also persistent in the environment, causing problems in the food chain for years. There are also costs in terms of human health of the workers that apply the pesticides and the public that consumes the produce.

With respect to the development of resistance, “By the 1960s, mosquitoes resistant to DDT effectively prevented the worldwide eradication of malaria, and by 1990, over 500 species had evolved resistance to at least one insecticide,” Palumbi [2001]. “…[R]esistance is a natural evolutionary response to environmental stresses. As such, resistance will remain an ongoing dilemma in pest management and we can only delay the onset of resistance to pesticides.” Hoy [1998]. In addition, the development of resistance is related to the intensity of pesticide use. “There is little doubt that pests that have developed resistance rapidly have been frequently treated with insecticides.” Tabashnik [1990]. An additional part of the problem is that the pesticides also kill the natural enemies of the pest; thus, when the resistance has progressed to the point where the spray cannot effectively control the pest it may rebound beyond its initial level. Of course if a new chemical is ready for implementation the rebound need not happen.

To lengthen the half-life of these chemicals integrated pest management (IPM) practices have been introduced. Many of these practices were the main line of defense against pests before the 1950’s. They include such things a cultural controls, sanitation, crop rotation, cultivation, trap crops, and refuges, Flint and Bosch [1981, pp149-53].

There are also transgenic or genetically modified plants that produce the soil bacterium Bacillus thuringiensis (or Bt) that is a deadly toxic to specific insects but
harmless to humans. In each crop the Bt is engineered to be lethal to the relevant pests. Bt
resistance is a concern and occupies much of the literature in this area. In our model we
make no special provision for transgenic plants, instead we assume they can be included
with the other pest control activities.

Many aspects of the economics of pesticide resistance have been reported in the
literature. Much of this literature deals with farm level optimal decision making, for
example Hall and Norgaard [1973], and Harper and Zilberman [1989]; and some of the
farm level literature includes decision making under uncertainty, for example Feder
[1979] and Deen et al. [1993]. Saphores [2000] analyses uncertainty in this problem
using options analysis. Some of the literature, including some of the above, deals with the
externalities generated by pesticide use and some with the related public policies, for
example Zilberman and Millock [1997]. A portion of this work is reviewed in Waibel,
Zadoks and Fleischer [2003].

We have not found anyone that has modeled the resistance problem as we have.
The closest is Goeschl and Swanson [2003], who include in their analysis an R&D sector
as we do. A basic difference is that theirs is a deterministic model while ours is
stochastic; hence the main thrusts of the two papers are very different.

Below we build a stochastic optimal control problem and analyze it to identify the
types of information and how to organize it to make good policy choices. In the objective
function of the model food, human health, and health of the environment are important.
The constraints of the model recognize that pesticide use has both desirable and
undesirable effects in that its use kills pests and non-pests, and affects pesticide resistance
of the next generation of pests. The rate at which resistance advances is affected by the
rate of application of the pesticide and the integrated pest management and resistance
pest management strategies implemented. We include an R&D sector because new
pesticides can be developed. Since there is uncertainty about both the rate of advance of
resistance and the rate of development of new pesticides we make them stochastic. The
model is analyzed in a stochastic discrete time optimal control setting.

We next build the model and analyze it. Then in the following section we identify
and discuss the decomposition of the shadow values, costate variables. The results of this
decomposition are our main contribution, because this allows decision makers, both
private and public, to identify the items that need to be estimated to make informed decisions. The decomposition identifies the expected values that can be estimated to estimate the shadow values. We also identify in the necessary conditions where this information is to be implemented to correct the market signals caused by the common property aspects of the problem.

The final section is a summary.

The Model

The objective function for the model is the present value of the utility stream, and utility is posited to depend upon consumption of both an agricultural commodity and a nonagricultural commodity and an index of human health and the health of the environment. We combine these two types of health to keep the model from becoming more complex. The discrete time utility function is

\[ \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, h_t) \]

where \( c_1 \) is the nonagricultural composite commodity, \( c_2 \) is the agricultural composite commodity, \( h \) an index of both human health and health of the environment, and \( \beta = 1/(1 + \rho) \) where \( \rho \) is the rate of time preference. Nonagricultural income is denoted \( y \), and is used as \( c_i \) and \( z \) which is the investment good,

\[ y = c_1 + z. \]

The production function for income is

\[ y_t = f_1(k_{1t}, l_{1t}, w_t) \]

where \( k_1 \), \( l_1 \) and, \( w_t \) are capital, labor and the flow of harmful or toxic waste, respectively. The last one, harmful waste is a by product of the production process. We expect the first partial derivative of all of these to be positive and \( f_1 \) to be concave. An important feature of this relationship is that if harmful waste, \( w_t \), is reduced while income, \( y \), is held constant then at least one of capital, \( k_1 \), and labor, \( l_1 \), must be increased.

The production function for the agricultural composite commodity is

\[ c_{2t} = f_2(k_{2t}, l_{2t}, s_p, m_t, s_t) \]
where the new symbols are \( s_p, m, \) and \( s \). The first, \( s_p \), is insect spray volume of a standardized intermediate good, where the standardization is with respect to effectiveness against the current pests\(^1\). The second, \( m \), is an index of pest management. This incorporates integrated pest management, IPM, and resistance pest management, RPM, strategies, and \( s \) is an index of susceptibility of the current pests to the current sprays and resistant crops. The partial derivatives of \( f^*_z \) with respect to all of its arguments except \( m \) are expected to be positive and with respect to \( m \) is expected to be negative. Some of the methods of IPM and RPM are crop rotations, cultivation and tillage practices, trap crops, habitat diversification and refuges\(^2\). See Flint and Bosch [1981, pp.149-158].

Insect spray is produced in the economy and has the production function

\[
(5) \quad s_{p_t} = f_z(k_{3t}, l_{3t}, A_t, s_t, x_t)
\]

with all partial derivatives positive and \( f_z \) strictly concave. The new variable \( A \) is a state variable and is the level of scientific knowledge about pest control and the interaction of pest control and the environment. We name this scientific bio-pest knowledge, so we can have a reasonably short name for \( A_t \). The other new variable, \( x \), is the level of toxicity of the spray. Other things equal, one spray would be less toxic than another if its impact on nontarget biota is smaller. We assume that toxic sprays are cheaper to produce with respect to capital and labor than are less toxic sprays holding effectiveness constant.

The laws of motion for the state variable capital is

\[
(6) \quad k_{t+1} = z_t + (1 - \delta_t)k_t,
\]

for pest control scientific knowledge and how to apply it is

\[
(7) \quad A_{t+1} = f_A(k_{4t}, l_{4t}) + (1 - \delta_A)A_t + \varepsilon_{A_t},
\]

\(^1\) The standardization could be with respect to LD\(_{50} \), which is the volume required to generate a lethal dose for 50 percent of the target population; thus, the physical volume of a unit of the spray will become larger through time as susceptibility in the target population decreases.

\(^2\) All items in this list except refuges were common practices which were severely decreased by the adoption of chemical controls. In addition, all items except cultivation and tillage practices cause a decrease in the value of current ag produce and thus a decrease in the composite ag commodity. Cultivation practices were modified because it was cheaper to use chemical control and the others because value increased and costs decreased. We use a negative partial derivative because \( m \) is a composite input even though the sign is not consistent for one of its components.
for the index of susceptibility
(8) \[ s_{t+1} = g_s(s_p, m_t) + s_t + \varepsilon_s. \]

The function \( f_s \) is for research and development of pest control and its effect on the environment, and the term \(-\delta_A A\) is for depreciation of the knowledge. The usefulness of this type of knowledge will depreciate because the pests will mutate and evolve. In addition, there is a stochastic element in this production process, which is evidenced by \( \varepsilon_A \), that is independently and identically distributed through time with mean zero and density function \( P_t^\delta(\varepsilon_A) \). The function \( g_s \) is for the effects of spraying and integrated pest management practices upon the drift of susceptibility. It also is expected to be a stochastic variable. Similar to above, \( \varepsilon_s \) is assumed to be independently and identically distributed through time with mean zero and density function \( P_t^\delta(\varepsilon_s) \).

There is one more state variable, the stock of harmful or toxic waste in the environment, \( W \), and its law of motion is given by
(9) \[ W_{t+1} = g_w(w_t, x_t) + (1 - \delta_w) W_t. \]
The function \( g_w \) is increasing in both arguments and is strictly concave. The flows of waste into the environment, \( w_t, x_t \), cause an increase in the stock of waste in the environment. Also, \(-\delta_w W\) is the natural breaking down of waste materials in the environment. We assume that the variable health in the utility function is a strictly decreasing function of the state variable, \( W \).
(10) \[ h_t = h(W_t) \]
We hold labor constant at \( l \) through time, and have the constraint
(11) \[ l_{1_t} + l_{2_t} + l_{3_t} + l_{4_t} - l = 0. \]
Capital evolves through time as given by Equation (6) and we have the constraint
(12) \[ k_{1_t} + k_{2_t} + k_{3_t} + k_{4_t} - k_t = 0. \]

We now examine the maximization of
(13) \[ E_0[V] = E \sum_{t=0}^{\infty} \beta^t u(c_{1_t}, c_{2_t}, h_t) \]
subject to Equations (2)-(12). The choice, control, variables of the problem are 
\( v = \{ c_1, s_p, m, x, w, k_i, l_i \} \) for each \( t \). There are also nonnegativity constraints which we are ignoring because we are interested in the characteristics of an internal solution. The necessary conditions can be found using Bellman’s equation. We write for \( t \geq 0 \):

\[
V_t(k_t, A_t, s_t, W_t) = \max_{\nu_t} \left\{ \left[ u(c_1, \ldots, c_n, h_t) + \beta V_{t+1}(k_{t+1}, A_{t+1}, s_{t+1}, W_{t+1}) \right] \right\}
\]

subject to Equations (2)-(12) with \( k_t, A_t, s_t, W_t \) given. Substituting Equations (4) and (10) into the utility function and substituting Equations (2) and (3) into (6), and then substituting this result and Equations (7)-(9) into (14) yields:

\[
V_t(k_t, A_t, s_t, W_t) = \max_{\nu_t} \left[ u(c_1, f_2(k_{t-1}, l_{t-1}, s_{p_{t-1}}, m_{t-1}, s_{s_{t-1}}) h(W_t)) + \beta \int \left[ V_{t+1}(k_{t+1}, A_{t+1}, s_{t+1}, W_{t+1}) \right] \right]
\]

subject to Equations (5), (11), and (12). The Lagrangean function is:

\[
L = u(c_1, f_2(k_{t-1}, l_{t-1}, s_{p_{t-1}}, m_{t-1}, s_{s_{t-1}}) h(W_t)) + \beta \int \left[ V_{t+1}(k_{t+1}, A_{t+1}, s_{t+1}, W_{t+1}) \right] \]

The integrals are over the domains of \( \epsilon_A \) and \( \epsilon_s \), respectively. The relevant necessary conditions for \( k_t, A_t, s_t, W_t \) given are:

\[
\frac{\partial u}{\partial c_i} \left|_{c_1} \right. + \frac{\partial u}{\partial f_2} \left|_{k_i} \right. + \frac{\partial u}{\partial s_{p_i}} \left|_{l_i} \right. + \frac{\partial u}{\partial m_i} \left|_{m_i} \right. + \frac{\partial u}{\partial s_{s_i}} \left|_{s_{s_i}} \right. h(W_t) = 0
\]
\[
\frac{\partial u(c_2, f_2(k_2^*, l_2^*, s_{p_1}^*, m_1^*, s_1^*), h(W_t))}{\partial c_2} + \frac{\partial g_2(s_{p_1}^*, m_1^*)}{\partial s_{p_1}} = \psi_{t_1}
\]

(16b)

\[
\frac{\partial u(c_2, f_2(k_2^*, l_2^*, s_{p_1}^*, m_1^*, s_1^*), h(W_t))}{\partial c_2} + \frac{\partial g_2(s_{p_1}^*, m_1^*)}{\partial m_1} = 0
\]

(16c)

\[
\frac{\partial u(c_2, f_2(k_2^*, l_2^*, s_{p_1}^*, m_1^*, s_1^*), h(W_t))}{\partial c_2} + \frac{\partial g_2(s_{p_1}^*, m_1^*)}{\partial c_2} = \psi_{t_2}
\]

(16d)

\[
\frac{\partial u(c_2, f_2(k_2^*, l_2^*, s_{p_1}^*, m_1^*, s_1^*), h(W_t))}{\partial c_2} + \frac{\partial g_2(s_{p_1}^*, m_1^*)}{\partial l_2} = \psi_{t_2}
\]

(16e)

\[
\left(\frac{\partial u(c_2, f_2(k_2^*, l_2^*, s_{p_1}^*, m_1^*, s_1^*), h(W_t))}{\partial c_2} + \frac{\partial g_2(s_{p_1}^*, m_1^*)}{\partial s_{p_1}}\right) = \psi_{t_2}
\]

(16f)

\[
\beta \int \sum_{s_{p_1}^*, m_1^*} \frac{\partial f_1(k_1^*, l_1^*, w_{t_1}^*)}{\partial l_1} = \psi_{t_1}
\]

(16g)

\[
\beta \int \sum_{s_{p_1}^*, m_1^*} \frac{\partial f_1(k_1^*, l_1^*, w_{t_1}^*)}{\partial k_1} = \psi_{t_1}
\]

(16h)

\[
\frac{\partial u(c_2, f_2(k_2^*, l_2^*, s_{p_1}^*, m_1^*, s_1^*), h(W_t))}{\partial c_2} + \frac{\partial g_2(s_{p_1}^*, m_1^*)}{\partial s_{p_1}} = \psi_{t_2}
\]

(16i)

\[
\beta \int \sum_{s_{p_1}^*, m_1^*} \frac{\partial f_1(k_1^*, l_1^*, w_{t_1}^*)}{\partial k_1} = \psi_{t_1}
\]

(16j)
\begin{align}
\tag{16k} & \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \frac{\partial f_t^* (k_{t+1}^*, l_{t+1}^*, w_{t+1}^*)}{\partial k_{t+1}} = \psi_{t+1}^* \\
\tag{16l} & \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \frac{\partial f_t^* (k_{t+1}^*, l_{t+1}^*, w_{t+1}^*)}{\partial w_{t+1}} + \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \frac{\partial g_{\alpha}^* (w_{t+1}^*, x_{t+1}^*)}{\partial w_{t+1}} = 0 \\
\tag{16m} & \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \frac{\partial g_{\alpha}^* (w_{t+1}^*, x_{t+1}^*)}{\partial x_{t+1}} \\
& \left( \frac{\partial u(c_{2}, f_2(k_{2}, l_{2}^*, s_{p_t}, m_t^*, s_t), h(W_t))}{\partial c_{2_t}} + \frac{\partial c_{2_t}}{\partial s_{p_t}} \right) + \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \frac{\partial g_{3}^* (s_{p_t}^*, m_t^*)}{\partial s_{p_t}} \\
& \left( \frac{\partial f_3(k_{3}, l_{3}^*, A_t^*, s_t^*, x_t^*)}{\partial x_t} \right) = 0 \\
\tag{16n} & \lambda_{\alpha t+1}^* = \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \left( 1 - \delta_{k} + \frac{\partial f_1^* (k_{1}^*, l_{1}^*, w_{1}^*)}{\partial k_{1}} \right) \\
\tag{16o} & \lambda_{\alpha t+1}^* = \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r (1 - \delta_{k}) \\
& \left( \frac{\partial u(c_{2}, f_2(k_{2}, l_{2}^*, s_{p_t}, m_t^*, s_t), h(W_t))}{\partial c_{2_t}} + \frac{\partial c_{2_t}}{\partial s_{p_t}} \right) + \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \frac{\partial g_{3}^* (s_{p_t}^*, m_t^*)}{\partial s_{p_t}} \\
& \left( \frac{\partial f_3(k_{3}, l_{3}^*, A_t^*, s_t^*, x_t^*)}{\partial x_t} \right) = 0 \\
\tag{16p} & \partial u(c_{2}, f_2(k_{2}, l_{2}^*, s_{p_t}, m_t^*, s_t), h(W_t)) + \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \\
& \left( \frac{\partial c_{2_t}}{\partial s_{p_t}} \right) + \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \frac{\partial g_{3}^* (s_{p_t}^*, m_t^*)}{\partial s_{p_t}} \\
& \left( \frac{\partial f_3(k_{3}, l_{3}^*, A_t^*, s_t^*, x_t^*)}{\partial x_t} \right) = 0 \\
\tag{16q} & \lambda_{\alpha t+1}^* = \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r (1 - \delta_{w}) \\
& \frac{\partial u(c_{2}, f_2(k_{2}, l_{2}^*, s_{p_t}, m_t^*, s_t), h(W_t))}{\partial h_t} + \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r (1 - \delta_{w}) \\
& \frac{\partial dh_t}{\partial W_t} = 0 \\
& \left( \frac{\partial c_{2_t}}{\partial h_t} + \frac{\partial c_{2_t}}{\partial h_t} \right) + \beta \int \lambda_{\alpha t+1}^* dP_t^A dP_t^r \frac{\partial g_{3}^* (s_{p_t}^*, m_t^*)}{\partial s_{p_t}} \\
& \left( \frac{\partial f_3(k_{3}, l_{3}^*, A_t^*, s_t^*, x_t^*)}{\partial x_t} \right) = 0 \\
\end{align}
We define the costate variables as \( \lambda_{kt}^*, \lambda_{kt}^*, \lambda_{st}^*, \lambda_{wt}^* \) and identify them using the envelope theorem. The transversality conditions are

\[
\lim_{t \to \infty} \beta^t \lambda_{it}^* = 0 \quad i = k, A, s, W
\]

These require that the costate variables be bounded as \( t \) increases without bound, and will be satisfied.

We next discuss the necessary conditions. We can write Equation (16a) in abbreviated form as (See the Appendix for the derivation of the expected value.)

\[
\frac{\partial u^*_i}{\partial c_{it}} - \beta \int \lambda_{kt+1}^* dP^t_i dP^s_i = \frac{\partial u^*_i}{\partial c_{it}} - E_{k_{t+1}, A_{t+1}, s_{t+1}, W_{t+1}} [\lambda_{kt+1}^* | k_t, A_t, s_t, W_t] = 0
\]

which states that the expected value of a unit of capital is equal to the marginal value of a unit of the composite non-ag consumption good. This is a usual relationship and is the result of Equation (2) which builds in a trade off of one unit of the main consumption good for one unit of the investment good, capital. The effect of this equation is to optimally ration production between consumption and investment.

Equation (16b) can be abbreviated as

\[
\frac{\partial u^*_i}{\partial c_{2t}} \frac{\partial f^*_2}{\partial s_{p_t}} + \beta \int \lambda_{st+1}^* dP^t_i dP^s_i \frac{\partial g^*_t}{\partial s_{p_t}} = \psi_{1t}^*,
\]

\[
\frac{\partial u^*_i}{\partial c_{2t}} \frac{\partial f^*_2}{\partial s_{p_t}} + \beta E_{k_{t+1}, A_{t+1}, s_{t+1}, W_{t+1}} [\lambda_{st+1}^* | k_t, A_t, s_t, W_t] \frac{\partial g^*_t}{\partial s_{p_t}} = \psi_{1t}^*.
\]

This states that the marginal value of a unit of spray, \( \psi_{1t}^* \), is its value of the marginal product in producing the composite ag consumption good plus the expected shadow value of a unit of susceptibility, \( \beta \int \lambda_{st+1}^* dP^t_i dP^s_i \), times the marginal effect of spraying on future susceptibility, which is negative. This has long been recognized in the literature. Spraying has a positive initial effect but has a negative future effect as it leads to additional resistance in the future. The first term is the private effect of spraying and is captured by the individual doing the spraying. The second is the social or common property effect and is only partially experienced by the individual doing the spraying.

Equation (16c), which can be written as,
states the marginal condition for the optimal level of integrated pest management and resistance pest management strategies. The first term is the current year value of the marginal product of these management practices, and is negative because the current year effect is to decrease production, $\frac{\partial f^*_2}{\partial m} < 0$. The second term is the present value of future effects of the current pest management practices. It is the expected shadow value of susceptibility times the marginal effect of these practices on the growth of susceptibility.

Equations (16d)-(16g) all have an expression equal to the shadow value of labor, $\psi_{2t}^*$, they say that labor has the same marginal value in all four of its uses. The first two

$$\frac{\partial u^*_i}{\partial c_{2t}} \frac{\partial f^*_2}{\partial m} = \frac{\partial u^*_i}{\partial c_{2t}} \frac{\partial f^*_2}{\partial l_{2t}} = \psi_{2t}^*$$

state that the value of the marginal product of labor in the production of the main product and the ag product are equal. The third one

$$\left(\frac{\partial u^*_i}{\partial c_{2t}} \frac{\partial f^*_2}{\partial s_{p_1}} + \beta \left[ \frac{\partial g^*_si}{\partial s_{p_1}} \right] \frac{\partial f^*_3}{\partial s_{p_1}} \right) \frac{\partial f^*_3}{\partial l_{3t}} = \psi_{2t}^*$$

is the value of the marginal product of labor in the production of standardized spray. The value term for this is

$$\frac{\partial u^*_i}{\partial c_{2t}} \frac{\partial f^*_2}{\partial s_{p_1}} + \beta \left[ \frac{\partial g^*_si}{\partial s_{p_1}} \right] \frac{\partial g^*_si}{\partial s_{p_1}}$$

where the first term is value of the marginal product of spraying on the current ag product, and the second is present expected value of the future effects of the current spraying. Current spraying causes a
decrease in future susceptibility by lowering its growth rate, $\dfrac{\partial g_s^*}{\partial s_p} < 0$, and

$$\beta \int \lambda_{s_{t+1}} dP^s_t dP^s_t$$

is the expected shadow value of susceptibility. Equation (16g)

$$\beta \int \lambda_{t+1} dP^s_t dP^s_t \dfrac{\partial f^s_{t+1}}{\partial l_i} = \psi_{2t}$$

$$\beta E_{\lambda_{t+1}} \left[ \lambda_{t+1} \mid k_t, A_t, s_t, W_t \right] \dfrac{\partial f^s_{t+1}}{\partial l_i} = \psi_{2t}$$

gives the expected value of the marginal product of labor in the production of scientific bio-pest knowledge. The value term, $\beta \int \lambda_{t+1} dP^s_t dP^s_t$, is the expected shadow value of this knowledge.

Equations (16h)-(16k) can be described as the previous four equations with the appropriate change of capital for labor. Equation (16l) can be written

$$\dfrac{\partial u_i^*}{\partial c_{t_i}} \dfrac{\partial f^s_{t+1}}{\partial w_{t_i}} + \beta \int \lambda_{w_{t+1}} dP^s_t dP^s_t \dfrac{\partial g_{w_{t_i}}^*}{\partial w_{t_i}} = 0$$

$$\dfrac{\partial u_i^*}{\partial c_{t_i}} \dfrac{\partial f^s_{t+1}}{\partial w_{t_i}} + \beta E_{\lambda_{t+1}} \left[ \lambda_{w_{t+1}} \mid k_t, A_t, s_t, W_t \right] \dfrac{\partial g_{w_{t_i}}^*}{\partial w_{t_i}} = 0$$

and gives the optimal marginal condition for the discharge of harmful waste by the producers of the main commodity. The first term is the value of the marginal product of harmful waste in the production of this commodity, and the second is the expected shadow value of the stock of harmful waste in the environment, $\beta \int \lambda_{w_{t+1}} dP^s_t dP^s_t < 0$, times the marginal responsiveness of harmful waste on the growth of the stock of harmful waste, $\dfrac{\partial g_{w_{t_i}}^*}{\partial w_{t_i}}$. The first term is felt by the private firms producing the main commodity, and the second is a social or common property term. Thus, some type of social input is necessary to achieve this equality.

Equation (16m)
The first term is the expected shadow value the stock of harmful waste in the environment, 

\[
\beta \left[ \int \lambda_{t+1} \rho^u dP^s \frac{\partial g_{st}}{\partial x_i} \right] + \left( \frac{\partial u^*_t}{\partial c_{t+1}} \frac{\partial f_{st}}{\partial s_{p,t}} + \beta \left[ \int \lambda_{t+1} \rho^u dP^s \frac{\partial g_{st}}{\partial s_{p,t}} \right] \right) \frac{\partial f_{st}}{\partial x_i} = 0
\]

gives the optimal marginal condition for the level of toxicity of the spray. The first term is the expected shadow value the stock of harmful waste in the environment, 

\[
\beta \left[ \int \lambda_{t+1} \rho^u dP^s \right] < 0
\]
times the marginal responsiveness of the toxicity of the spray on the growth of the stock of harmful waste, \( \frac{\partial g_{st}}{\partial x_i} \). Toxic sprays cause deterioration of environment, and at the margin the value of this effect is balanced against the other terms in this equation. The second term is the value of the marginal product of toxicity in the production of the spray. It is the product of a value term and the marginal product of toxicity of the spray in the production of the spray. We posit that this marginal product is positive. The value term was discussed above in the discussion of Equation (16f) and is the sum of a positive current effect and a negative future effect. A more toxic spray will kill more insects today but will have a detrimental effect on future susceptibility and on the environment.

Discussion of the Costate Variables

To examine the information in the costate variables we recursively substitute the original equation into itself starting at \( t = 0 \) and moving forward in time. Starting with the shadow value of capital in Equation (16n) we have

\[
\lambda_{k_0} = \beta \left[ \int \lambda_{k_1} dP^s dP^t \left( 1 - \frac{\partial f_1}{\partial k_{10}} \frac{k_{10}^*}{w_{0}^*} \right) \right]
\]

\[
= \beta \left[ \int \beta \left[ \int \lambda_{k_2} dP^s dP^t \left( 1 - \frac{\partial f_1}{\partial k_{10}} \frac{k_{10}^*}{w_{0}^*} \right) \right] dP^t dP^t \left( 1 - \frac{\partial f_1}{\partial k_{10}} \frac{k_{10}^*}{w_{0}^*} \right) \right]
\]
To understand this last step note that

$$ \int \int \int \int a_{k_3}^x dP_2^t dP_2^t \left( 1 - \delta_k + \frac{\partial f_{1,2}^*}{\partial k_{12}} \right) dP_1^t dP_0^t \left( 1 - \delta_k + \frac{\partial f_{1,0}^*}{\partial k_{10}} \right)$$

$$= \beta \int \int \int \int a_{k_3}^x dP_2^t dP_2^t \left( 1 - \delta_k + \frac{\partial f_{1,2}^*}{\partial k_{12}} \right) dP_1^t dP_0^t \left( 1 - \delta_k + \frac{\partial f_{1,0}^*}{\partial k_{10}} \right)$$

$$= \beta^3 \int \int \int \int \int a_{k_3}^x \left( 1 - \delta_k + \frac{\partial f_{1,2}^*}{\partial k_{12}} \right) \left( 1 - \delta_k + \frac{\partial f_{1,1}^*}{\partial k_{11}} \right) \left( 1 - \delta_k + \frac{\partial f_{1,0}^*}{\partial k_{10}} \right) dP_2^t dP_2^t dP_1^t dP_0^t dP_0^t$$

$$= \beta^3 E_{\{k_0, A_0, s_0, W_0\}} \left[ a_{k_3}^x \left( 1 - \delta_k + \frac{\partial f_{1,2}^*}{\partial k_{12}} \right) \left( 1 - \delta_k + \frac{\partial f_{1,1}^*}{\partial k_{11}} \right) \left( 1 - \delta_k + \frac{\partial f_{1,0}^*}{\partial k_{10}} \right) \right]$$

To understand this last step note that

$$\int \int \int \int a_{t+1} dP_t^A dP_t^e = \int \int \int \int a_{t+1} P_t^A (\epsilon_t) d\epsilon_t P_t^e (\epsilon_t) d\epsilon_t$$

however,

$$P_t^A (\epsilon_t) \equiv P_t^A (A_{t+1} | k_t, A_t, s_t, W_t),$$

and

$$P_t^e (\epsilon_t) \equiv P_t^e (s_{t+1} | k_t, A_t, s_t, W_t).$$

Thus

$$\int \int \int \int a_{t+1} P_t^A (\epsilon_t) d\epsilon_t P_t^e (\epsilon_t) d\epsilon_t = \int \int \int \int a_{t+1} P_t^A (A_{t+1} | k_t, A_t, s_t, W_t) dA_{t+1} P_t^e (s_{t+1} | k_t, A_t, s_t, W_t) ds_{t+1}$$

$$= \int \int \int \int a_{t+1} P_t^A (A_{t+1}, s_{t+1} | k_t, A_t, s_t, W_t) dA_{t+1} ds_{t+1}$$

$$= E_{\{k_0, A_0, s_0, W_0\}} [a_{t+1} | k_t, A_t, s_t, W_t]$$

Repetition of this logic completes the explanation of this last step. Additional recursion yields:

$$\lambda_{t+1}^* = \beta E_{\{k_0, A_0, s_0, W_0\}} \left[ \lambda_t^* \left( 1 - \delta_k + \frac{\partial f_{1,2}^*}{\partial k_{12}} \right) \cdots \left( 1 - \delta_k + \frac{\partial f_{1,0}^*}{\partial k_{10}} \right) | k_0, A_0, s_0, W_0 \right]$$

The shadow value $\lambda_{t+1}^*$ is the per unit present value of an additional epsilon unit of capital.

We will call it an additional unit. If we insert a $dk$ unit of capital at time zero it will have a time path given by $dk_{t+1} = (1 - \delta_k) dk$, because it will depreciate through time. In
addition, \( \frac{\partial f^*_t}{\partial k_{1t}} \) is the gross return to a unit of capital at time \( t \), and \( \frac{\partial f^*_t}{\partial k_{1t}} - \delta_k \) is the net return. Thus, \( dk_{t+1} = \left( 1 - \delta_k + \frac{\partial f^*_t}{\partial k_{1t}} \right) dk \), gives the time path with reinvestment of the gross return at each \( t \). If we define

\[
\xi_{kt} = \left( 1 - \delta_k + \frac{\partial f^*_{t-1}}{\partial k_{1t-1}} \right) \cdots \left( 1 - \delta_k + \frac{\partial f^*_{0}}{\partial k_{10}} \right)
\]

to be the level of this additional unit of capital with reinvestment at time \( t \), then we can write

\[
\lambda^*_k = \beta E_{\{k_{t+1}, \ldots, k_{1t} \}} [\lambda^*_{k_{t+1}} \xi_{kt} | k_0, A_0, s_0, W_0] .
\]

In words \( \lambda^*_k \) is the present value of the expected future value of an additional unit of capital.

The shadow value for susceptibility can be decomposed in a similar way.

Equation (16p) can be written

\[
\lambda^*_s = \beta \int \lambda^*_{s_{t+1}} dP_t \left( 1 + \frac{\partial g^*_s}{\partial s_t} \right) + \frac{\partial u^*_t}{\partial c_{2t}} \left( \frac{\partial f^*_t}{\partial s_t} + \frac{\partial f^*_t}{\partial \alpha_t} \right).
\]

Using recursion and the logic above we get:

\[
\lambda^*_s = \frac{\partial u^*_0}{\partial c_{20}} \left( \frac{\partial f^*_{20}}{\partial s_{p0}} + \frac{\partial f^*_{0}}{\partial s_{s0}} \right) + \beta E_{\{k_{t+1}, \ldots, k_{1t} \}} \left[ \frac{\partial u^*_t}{\partial c_{21}} \left( \frac{\partial f^*_{21}}{\partial s_{p1}} + \frac{\partial f^*_{1}}{\partial s_{s1}} \right) \left( 1 + \frac{\partial g^*_s}{\partial s_{p0}} \right) \right] + \beta^2 E_{\{k_{t+1}, \ldots, k_{1t} \}} \left[ \frac{\partial u^*_t}{\partial c_{22}} \left( \frac{\partial f^*_{22}}{\partial s_{p2}} + \frac{\partial f^*_{21}}{\partial s_{s2}} \right) \left( 1 + \frac{\partial g^*_s}{\partial s_{p1}} \right) \left( 1 + \frac{\partial g^*_s}{\partial s_{p0}} \right) \right] + \ldots.
\]

\[
+ \beta^p E_{\{k_{t+1}, \ldots, k_{1t} \}} \left[ \frac{\partial u^*_t}{\partial c_{2p}} \left( \frac{\partial f^*_{2p}}{\partial s_{p_{p0}}} + \frac{\partial f^*_{2p-1}}{\partial s_{s_{p0}}} \right) \left( 1 + \frac{\partial g^*_s}{\partial s_{p_{p0}}} \right) \right] + \nu^*_{s_{t+1}} \left( 1 + \frac{\partial g^*_s}{\partial s_{p_{t+1}}} \right) \cdots \left( 1 + \frac{\partial g^*_s}{\partial s_{p_{t}}} \right) | k_0, A_0, s_0, W_0.
\]
To interpret this result, examine an additional unit of susceptibility inserted at time zero.

The law of motion of this unit is given by $dS_{t+1} = \left(1 + \frac{\partial g_{s1}^*}{\partial S_{p1}} \frac{\partial f_{s1}^*}{\partial S_t} \right) dS_t$. Note that this is a partial differential since this is the only deviation from the optimal path that is being analyzed. It is the partial differential of Equation (8) with Equation (5) inserted. Define

$$\xi_{s1} = \left(1 + \frac{\partial g_{s1}^*}{\partial S_{p1}} \frac{\partial f_{s1}^*}{\partial S_{t-1}} \right) \cdots \left(1 + \frac{\partial g_{s0}^*}{\partial S_{p0}} \frac{\partial f_{s0}^*}{\partial S_0} \right)$$

to be the level of susceptibility at time $t$ of this additional unit inserted at $t = 0$. Using this we can write

$$\lambda'_{s0} = \frac{\partial u'_0}{\partial c_{20}} \left( \frac{\partial f_{s0}^*}{\partial S_{p0}} \frac{\partial f_{s0}^*}{\partial S_0} + \frac{\partial f_{s0}^*}{\partial S_0} \right) + \beta E_{\{s_1, k_0, a_0, s_0, W_0\}} \left[ \frac{\partial u'_1}{\partial c_{21}} \left( \frac{\partial f_{s1}^*}{\partial S_{p1}} \frac{\partial f_{s1}^*}{\partial S_1} + \frac{\partial f_{s1}^*}{\partial S_1} \right) \xi_{s1} \right] k_0, A_0, s_0, W_0$$

$$+ \beta^2 E_{\{s_2, s_3, s_4, s_5\}} \left[ \frac{\partial u'_2}{\partial c_{22}} \left( \frac{\partial f_{s2}^*}{\partial S_{p2}} \frac{\partial f_{s2}^*}{\partial S_2} + \frac{\partial f_{s2}^*}{\partial S_2} \right) \xi_{s2} \right] k_0, A_0, s_0, W_0$$

$$+ \cdots + \beta' E_{\{s_{i-1}, s_i, \ldots, s_{n-1}\}} \left[ \lambda'_{s_i} \xi_{s_i} \right] k_0, A_0, s_0, W_0$$

This equation gives the present value of a stream of benefits. Each year has an entry where the benefit of that year is weighted by the remaining level of the additional unit of susceptibility, $\xi_{s_i}$. The annual benefit can be interpreted as the value of the marginal product of susceptibility. The marginal product of $s_i$ is $\frac{\partial f_{s1}^*}{\partial S_{p1}} \frac{\partial f_{s1}^*}{\partial S_i} + \frac{\partial f_{s1}^*}{\partial S_i}$. The first term is the indirect effect through the spray and the second is the direct effect on the production of the ag commodity through diminished crop damage. The value term is the marginal utility of the ag product, $\frac{\partial u'_i}{\partial c_{2i}}$.

The law of motion for the shadow value of scientific bio-pest knowledge is given in Equation (160), and can be written:
Examine a deviation from the optimal path for a 'small' deviation in the initial condition of stock of scientific bio-pest knowledge equal to \(dA_0\). The altered path of \(A_t\) is given by \(dA_t = (1 - \delta_A) dA_t\); thus for a given \(dA_0\) we have \(dA_{t+1} = (1 - \delta_A) dA_t\). This altered path of \(A_t\) yields two productivity effects each year in the future. The first is through

\[
\lambda^*_t = \beta \left( \lambda^*_t + dP_t \right) + \left( \frac{\partial u^*_t}{\partial c_{2t}} \frac{\partial f^*_2}{\partial s_{pt}} + \beta \left( \lambda^*_t + dP_t \right) \frac{\partial g^*_s}{\partial s_{pt}} \frac{\partial f^*_s}{\partial A_t} \right) \frac{\partial s^*_s}{\partial A_t}
\]

Recursion as used above yields:

\[
\lambda^*_{A0} = \frac{\partial u^*_0}{\partial c_{20}} \frac{\partial f^*_0}{\partial s_{p0}} \frac{\partial f^*_s}{\partial A_t}
\]

\[
+ \beta E_{\{A_t, s_t, A_t+1\}} \left[ \frac{\partial u^*_t}{\partial c_{21}} \frac{\partial f^*_s}{\partial s_{p1}} \frac{\partial f^*_s}{\partial A_t} \bigg| k_0, A_0, s_0, W_0 \right]
\]

\[
+ \beta^2 E_{\{A_t, s_t, A_t+1\}} \left[ \frac{\partial u^*_t}{\partial c_{22}} \frac{\partial f^*_s}{\partial s_{p2}} \frac{\partial f^*_s}{\partial A_t} \bigg| k_0, A_0, s_0, W_0 \right]
\]

\[
+ \beta E_{\{A_t, s_t, A_t+1\}} \left[ \lambda^*_{A1} \bigg| k_0, A_0, s_0, W_0 \right]
\]

\[
+ \beta^2 E_{\{A_t, s_t, A_t+1\}} \left[ \lambda^*_{A1} \bigg| k_0, A_0, s_0, W_0 \right]
\]

Examine a deviation from the optimal path for a 'small' deviation in the initial condition of stock of scientific bio-pest knowledge equal to \(dA_0\). The altered path of \(A_t\) is given by \(dA_t = (1 - \delta_A) dA_t\); thus for a given \(dA_0\) we have \(dA_{t+1} = (1 - \delta_A) dA_t\). This altered path of \(A_t\) yields two productivity effects each year in the future. The first is through

\[
\frac{\partial u^*_t}{\partial c_{2t}} \frac{\partial f^*_2}{\partial s_{pt}} \frac{\partial f^*_s}{\partial A_t}
\]

which is the value of the marginal product of scientific bio-pest knowledge. This value comes through the ag commodity and the production of the spray. The insertion of new knowledge in time zero has effects for several years in the future; however, the present value of these effects get smaller the further the effects are in the future. This is because of discounting, \(\beta\), and depreciation of the knowledge, \((1 - \delta_A)\). The second effect exists because \(dA_0\) causes an altered path of susceptibility. This altered path is caused

by \(\frac{\partial g^*_s}{\partial s_{pt}} \frac{\partial f^*_s}{\partial A_t}\) which is the marginal product of bio-pest knowledge on the growth of susceptibility. This comes through the effect of \(A_t\) on the production of spray.
marginal product is made value of the marginal product by its product with $\lambda^*_t$, the shadow value of a unit of susceptibility.

The law of motion for the shadow value of the stock of harmful or toxic waste can be similarly decomposed. Equation (16q) can be written:

$$
\lambda^*_w = \frac{\partial u^*_t}{\partial h_t} \frac{dh^*_t}{dW_t} + \beta \int \lambda^*_{w_{t+1}} dP^*_t dP^*_t (1-\delta_w),
$$

and recursion as applied above yields:

$$
\lambda^*_{w_0} = \frac{\partial u^*_0}{\partial h_0} \frac{dh^*_0}{dW_0} + \beta E_{\{A_{t-1}, \ldots, A_1\}} \left[ \frac{\partial u^*_{t-1}}{\partial h_{t-1}} \frac{dh^*_{t-1}}{dW_{t-1}} | k_0, A_0, s_0, W_0 \right] (1-\delta_w)
$$

$$
+ \beta^2 E_{\{A_{t-2}, A_{t-1}, A_t, \ldots, A_1\}} \left[ \frac{\partial u^*_{t-2}}{\partial h_{t-2}} \frac{dh^*_{t-2}}{dW_{t-2}} | k_0, A_0, s_0, W_0 \right] (1-\delta_w)^2
$$

$$
+ \cdots
$$

$$
+ \beta^{t-1} E_{\{A_{t-1}, \ldots, A_1\}} \left[ \frac{\partial u^*_{t-1}}{\partial h_{t-1}} \frac{dh^*_{t-1}}{dW_{t-1}} | k_0, A_0, s_0, W_0 \right] (1-\delta_w)^{t-1}
$$

$$
+ \beta^t E_{\{A_t, \ldots, A_1\}} \left[ \lambda^*_{w_0} | k_0, A_0, s_0, W_0 \right] (1-\delta_w)^t.
$$

Now examine an altered path of harmful waste caused by a ‘small’ deviation in its initial stock of $dW_0$. The altered path of $W_t$ is given by $dW_{t+1} = (1-\delta_w) dW_t$; thus for a given $dW_0$ we have $dW_{t+1} = (1-\delta_w)^t dW_0$. This altered path of $W_t$ yields a stream of disbenefits given that $dW_0 > 0$. This follows because $W$ harms the health of both humans and the environment. This stream of disbenefits has an entry for each year in the future, and each is discounted to year zero and weighted by $(1-\delta_w)^t$. Both of these cause the magnitude of the effects to become smaller as $t$ increases. The disbenefit is given by

$$
\frac{\partial u^*_t}{\partial h_t} \frac{dh^*_t}{dW_t},
$$

which gives the marginal utility of health times the marginal effect of harmful waste on health.

**SUMMARY**

We have analyzed the pesticide resistance problem in the context of a stochastic optimal control model. In the objective function of the model food, human health, and
health of the environment are important. The constraints of the model recognize that pesticide use has both desirable and undesirable effects in that its use kills pests and non-pests, and affects pesticide resistance of the next generation of pests. The rate at which resistance advances is affected by the rate of application of the pesticide and the integrated pest management and resistance pest management strategies implemented. We include an R&D sector because new pesticides can be developed. Since there is uncertainty about both the rate of advance of resistance and the rate of development of new pesticides we make them stochastic.

We identified and discussed the necessary conditions for an optimal path. There are terms in these conditions that will be met only if there is public input. All of these terms are related to the shadow values of susceptibility, scientific bio-pest knowledge, or of the stock of harmful waste. These all have a public or common property attribute; hence, we would not expect the private sector to correctly take the effects of these into account. This point, of course, has been previously stated in the literature. We, however, decompose the shadow values in a way that identifies the expected values that can be estimated to estimate the shadow values, and in the necessary conditions we identify where this information is to be implemented to correct the market signals.

REFERENCES


G. Feder [1979], Pesticides, Information, and Pest Management under Uncertainty, American Journal of Agricultural Economics, 61, 97-103.


APPENDIX

The derivation of the expected value of $\lambda_{k_{t+1}}$ is as follows:

$$\int \lambda_{k_{t+1}} dP^A dP^P = \int \lambda_{k_{t+1}} P^A_A(\varepsilon_A) d\varepsilon_A P^P_p(\varepsilon_s) d\varepsilon_s;$$

however,

$$P^A_A(\varepsilon_A) \equiv P^A_A(A_{t+1} | k_t, A_t, s_t, W_t),$$

and

$$P^P_p(\varepsilon_s) \equiv P^P_p(s_{t+1} | k_t, A_t, s_t, W_t).$$

Thus

$$\int \lambda_{k_{t+1}} P^A_A(\varepsilon_A) d\varepsilon_A P^P_p(\varepsilon_s) d\varepsilon_s = \int \lambda_{k_{t+1}} P^A_A(A_{t+1} | k_t, A_t, s_t, W_t) dA_{t+1} P^P_p(s_{t+1} | k_t, A_t, s_t, W_t) ds_{t+1}$$

$$= \int \lambda_{k_{t+1}} P(A_{t+1}, s_{t+1} | k_t, A_t, s_t, W_t) dA_{t+1} ds_{t+1}$$

$$= E_{(A_{t+1}, s_{t+1})} [\lambda_{k_{t+1}} | k_t, A_t, s_t, W_t]$$