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A Quick Optimization of a Rocket Trajectory Using MCMC Method.

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ABSTRACT

In these years, small satellites have grown in performance greatly and it is important to develop the good launcher for small satellites. One of the promising launch systems for small satellites is air launch. In Japan there is no successful small launch vehicle as it is now and some researchers have proposed air launch rockets using existing solid motors. To achieve air launch, some new technologies are necessary. As candidate of the one of such new technologies, the quick optimization of the rocket trajectory with MCMC (Markov chain Monte Carlo) method is introduced in this paper.

BACKGROUND AROUND SMALL SATELLITE

Small satellites have so great potential that high performance will be achieved with a low cost and short time range of development. Businesses, governments, universities and other organizations around the world are starting their own small satellite plans.

In Japan, small satellites increasingly became popular in late years. METI and NEDO, with high expectation for small satellites to establish new market and promote commercialization in space, officially announced that they sponsor three-year development program of the low cost micro/small satellite from FY2008.

Institute of Space and Astronautical Science (ISAS) has started up small scientific satellite series project to achieve unique scientific missions frequently and rapidly. They are planning to make a standard satellite bus of 250 kg to 400 kg class. They define four phases of standardization. The first phase is standardization of design and development method. The second is standardization of electrical, thermal, and mechanical interface. The third is standardization of payloads. The final phase is standardization of configuration of satellite. They have stated that most scientific observation mission is cable to aim the final phase. It is interesting that they adopted a small CPU box called Spacecube for processing data on the second phase. (Fig. 1) Spacecube is based on Teacube which is not a special computer for space but a public product controlled by Toron OS. It also has three Space Wire interface. Therefore Spacecube can organize so-called network bus.

Figure 1: Spacecube (52mm*52mm*55mm; SHIMAFUJI)

The circumstance mentioned above prove that small satellite projects will be important more and more under commercial and scientific application. Thus launching system for small satellites is important too. Next section will explain the circumstance about past and future launching systems in Japan.

HISTORY AND FUTURE OF JAPANESE SOLID ROCKETS

The first rocket in Japan was launched on April 12, 1955. This tiny rocket called Pencil was launched horizontally. Rocket research was not continued effectively after the Second World War. because most scientists concerned with research into rockets destroyed their materials for fear that they would be accused of war crimes. Thus the launch of Pencil is usually considered as the start of rocket development in Japan. The Pencil group was led by Prof. Hideo Itokawa, University of Tokyo, who is popular for the Father of Japanese Rocketry. (Fig. 2)

Figure 3: Prof. Itokawa with Pencil

The group developed their rocket technology into a sounding rocket called K-series to join 3rd International Geophysical Year (IGY). They succeeded in reaching 60 km, a minimum altitude required to join IGY, by K-6 type rocket in June, 1958. K-6 was 25 cm in 1st stage diameter, 5.4 m in total length and 255 kg in weight. In 1964, the Institute of Aeronautics and the rocket group of the Institute of Industrial Science of University of Tokyo were merged into a new institute, the Institute of Space and Aeronautical Science (the former ISAS) attached to the University of Tokyo. ISAS developed several types of sounding rockets among S-210, S-310, S-520, K-9M and L-3H which contributed largely to the progress of Japanese space science. The L-4S project started as a precursor project to simulate the payload injection scheme for the MU project which is Japanese artificial satellite plan. The L-4S was equipped with a fourth kick stage on a three-stage L-3H sounding rocket to give a last push to the payload. In February, 1970, the fifth attempt succeeded in sending a 24 kg payload into an elliptical orbit around the earth. The first Japanese satellite was named OHSUMI.

Then the age of the Mu rocket. ISAS has proceeded step by step by improving the launch performance of the Mu rocket series. The first generation was the four-stage M-4S type. The M-4S adopted the gravity-turn method as a payload injection scheme, and stabilized the attitude by tail fins and spin. A technological test

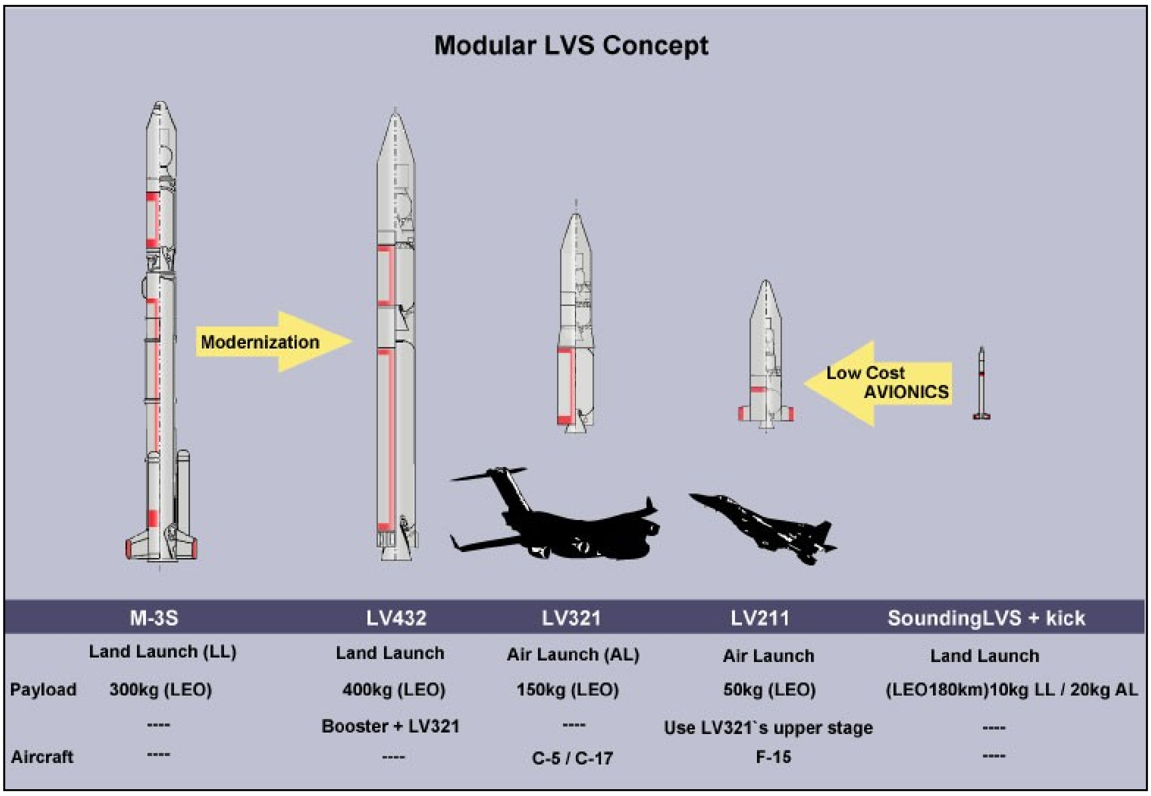


Figure 2: Modular LVS Concept (J-Solid NEXT Generation)

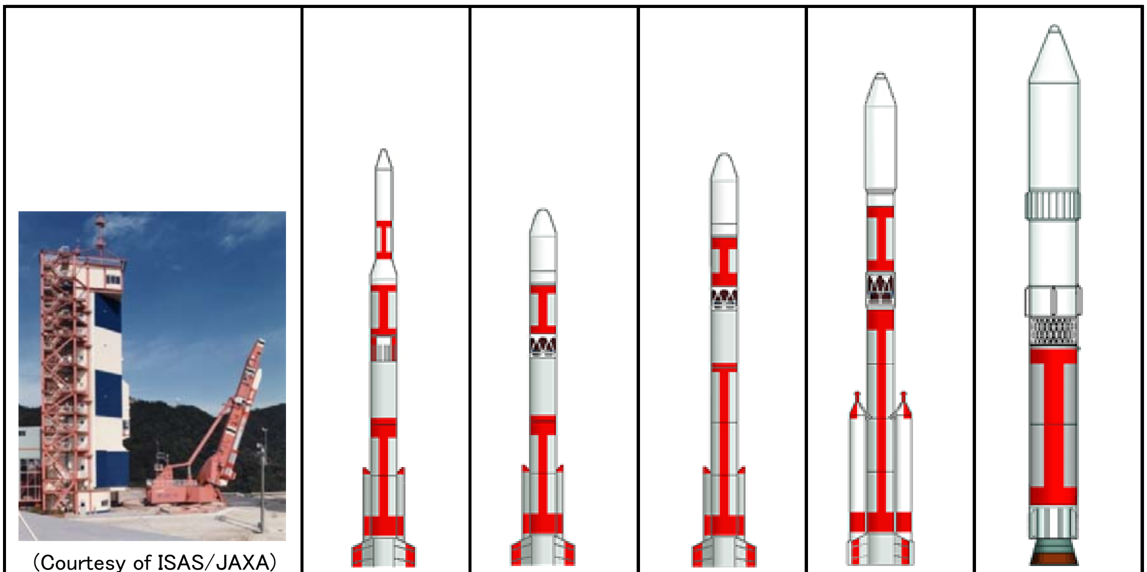
satellite TANSEI was launched by the first M-4S in February 1971. The second generation, M-3C, had three stages. Both 2nd and 3rd stages were newly developed, with the 2nd stage incorporating secondary injection thrust vector control (SITVC) and side jet (SJ) systems to improve orbit-injection accuracy. The M-3C sent three satellites into orbit. The third generation, M-3H, was developed by extending the length of the first stage motor with improved propellant to achieve a greater payload capability. The M-3H carried three satellites into orbit around earth. The fourth generation, M-3S, introduced a thrust vector control (TVC) system to the 1st stage, to improve the launch accuracy and ease restrictions on launch conditions. The M-3S successfully launched four satellites.

Targeting Halley’s comet which would reappear in 1985 after 76 years, ISAS began research and development of the fifth generation MU vehicle, M-3SII, in April 1981. It utilized the first stage motor of its predecessor M-3S, while the 2nd and the 3rd stage motors and strap-on boosters were newly developed to enhance the payload capability. M-3SII carried the first Japanese interplanetary probes, SAKIGAKE and SUISEI in January and August 1985 toward Halley’s comet. It also sent five satellites into earth orbit.

M-V rockets are a substantially improved version based on technologies developed and nurtured on previous Mu rockets. M-V sent HALCA, NOZOMI, HAYABUSA, SUZAKU, AKARI and HINODE into orbit around the earth and interplanetary space. M-V rockets were highly praised as “the best solid propellant rockets in the world.” However, after the launch of M-V-7 in September 2006, production of the M-V series was discontinued.

That is why heavy lift is the only vehicle that is available in Japan now, while operational and responsive small launch vehicles such as Minotaur and VEGA have been successfully developed in US and Europe. It is grave situation for Japanese space activities to lose their own small launch vehicle. Then some researchers in Japan have studied new small satellite launch vehicles. They conducted tradeoff study of launch platforms of land, sea and air carried out in terms of performance, operation, ground systems and economics. And they concluded that air launch was valued highly as most ideal launch system for small satellites because of its launch capability, minimum ground support, higher flexibility and mobility advantage, and affordable cost.

Matsuda and his coauthors proposed the modular launch vehicle systems (LVS) concept. (Ref.1) They proposed three types of vehicles which are capable of delivering small satellites of 50kg, 100kg and 300kg respectively to circular orbit of 500km. (Fig. 3) Small



Rocket	M-4S	M-3C	M-3H/M-3S	M-3SII	M-5
Launch capacity	180 kg	195 kg	300 kg	770 kg	1800 kg
1st Stage Motor	M10	M10	M13	M13	M14
2nd Stage Motor	M20	M22	M22	M23	M24/M25
3rd Stage Motor	M30	M-3A	M-3A	M-3B	M34
4th Stage/Kick Motor	M40	–	KM-A/KM-B	–	KM-V1

Figure 4: Modular LVS Concept in Mu Project

LVS family is configured by the combination of various solid motors. LV321 uses M-3S upper stages derivative which incorporates technologies derived from M-V such as CFRP motor case, control system, structure to reduce weight and increase launch capability. LV211, derivative of upper stages of LV321, is capable of lifting 50kg-class payload. Conventional LV432 land launch system (Booster + LV321) is for launching 300kg-class operational small satellites. In addition, avionics and simplified launch control and operation systems are unified to those of sounding rockets. This modular concept is not new in Japan. It is glimpsed in progress of Mu rockets. (Fig. 4)

According to Matsuda, the primal technologies of air launch are considered as

1. Vehicle Loading and Deployment
2. Ignition Attitude Stabilization
3. Launch Sequence
4. Safety of Solid Motor Loading
5. Inertia Navigation System Initialization.

The first, second, and third technologies need new research and development, while the fourth and fifth one will be achieved by extending conventional LVS technology and introducing commercial technology.

TRAJECTORY OPTIMIZATION

In this paper, the technique to simplify above-mentioned the second and third technologies is discussed. In air launch, the attitude of the separated rocket is unstable. So, the launch angle (the attitude at the ignition) is unpredictable and can stray too far from the nominal. When the launch angle is much different from the nominal, it is necessary to revise the trajectory program of the rocket quickly. There are different methods to optimize the rocket trajectory. For example, in the launch of M-V rockets, the trajectory optimization is conducted with nonlinear programming method (Ref.4). Such a method can deal with the trajectory optimization only when the difference from the nominal is enough small. In air launch, the difference can be large. Moreover in air launch, it is needed to optimize the trajectory as soon as possible after ignition. Because of these requirements the traditional method of trajectory optimization is not suitable for air launch. So the new technique is necessary. Then this research suggests applying MCMC method to the trajectory optimization of air launch rocket.

Markov chain Monte Carlo (MCMC) methods are a class of algorithms for sampling from probability distributions based on constructing a Markov chain that has the desired distribution as its equilibrium distribution(Ref.5) The state of the chain after a large number of steps is then used as a sample from the desired distribution. The quality of the sample improves as a function of the number of steps.

In this time, MCMC method is not used as a sampling algorithm but as random walk searching algorithm in the high-dimensional space. With the probability step that is the critical step of MCMC method it is able to search the point efficiently at which the objective function has the maximum value or the minimum value. By dividing the rocket trajectory into some nodes and treating attitude of the rocket on each node as variable, the optimization problem can be regarded as the search problem in high-dimensional space. Thus the MCMC method can be used for the optimization of the rocket trajectory

The critical step of MCMC method is very simple. It is shown in the following sequence

1. Calculating the value of f($\mathbf{x}$). Here $\mathbf{x}$ is the current position.
2. Generating $\Delta\mathbf{x}$ using random numbers and calculating the value of f($\mathbf{x}+\Delta\mathbf{x}$)
3. Calculating the transition probability P from the values of f($\mathbf{x}$) and f($\mathbf{x}+\Delta\mathbf{x}$).
4. Moving from $\mathbf{x}$ to $\mathbf{x}+\Delta\mathbf{x}$ at the probability P

By repeating above 1- 4 steps with memorizing the values of f($\mathbf{x}$), it is possible to search the point at which f($\mathbf{x}$) takes the maximum of minimum value. Moreover two artifices shown below are applied.

- Changing $\Delta\mathbf{x}$ as the searching step proceeds. $\Delta\mathbf{x}$ is large in the initial step and reduced gradually.
- Changing the calculation method of the transition probability as the searching step proceeds. When f($\mathbf{x}$) has to be maximized, in earlier stage, the transition probability can be large even if f($\mathbf{x}+\Delta\mathbf{x}$) is smaller than f($\mathbf{x}$). Gradually changing calculation method, in later stage, the transition probability approaches zero when f($\mathbf{x}+\Delta\mathbf{x}$) is smaller than f($\mathbf{x}$). In the case of minimization of f($\mathbf{x}$), it is inversion.

By above-mentioned artifices, it is able to avoid falling into the local extremum and to search wider space. And the accuracy of the optimization gets better.

If the transition probability is calculated as shown below, it meets the requirement.

$$P(x, x+\Delta x) = \begin{cases} \frac{1}{2} e^{\beta\left(\frac{f(x)}{f(x+\Delta x)}-1\right)} & (1) \\ 1-\frac{1}{2} e^{\beta\left(\frac{f(x)}{f(x+\Delta x)}-1\right)} \end{cases}$$

$$\beta = \frac{1}{T_0 T_r^{\frac{n}{N}}} \qquad (2)$$

T_0 and Tr are positive numbers of one or less. n and N are positive integers and n indicates the step number. When T0 = 1.0, Tr = 0.95 and N = 5 the transition probability is calculated as shown in Fig.5.

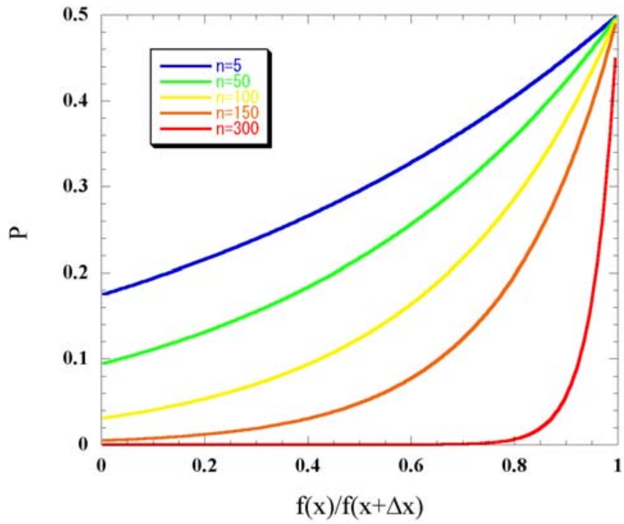


Figure 5: The Transition Probability

SIMULATION

The simulation of the trajectory optimization of an air launch rocket was conducted. The assumed rocket is a 9-ton class air launch rocket proposed by Matsuda and his coauthor in their paper (Ref. 1). They studied and propose d the flight sequence and vehicle configuration shown Fig. 6 and Fig. 7. Fig. 6 shows the flight path (-□indicates the flight path of the air launch rocket). Fig. 7 shows the flight sequence in case of Boeing-747 captive on bottom air launch. In the simulation, the flight sequence like shown in Fig. 7 is assumed.

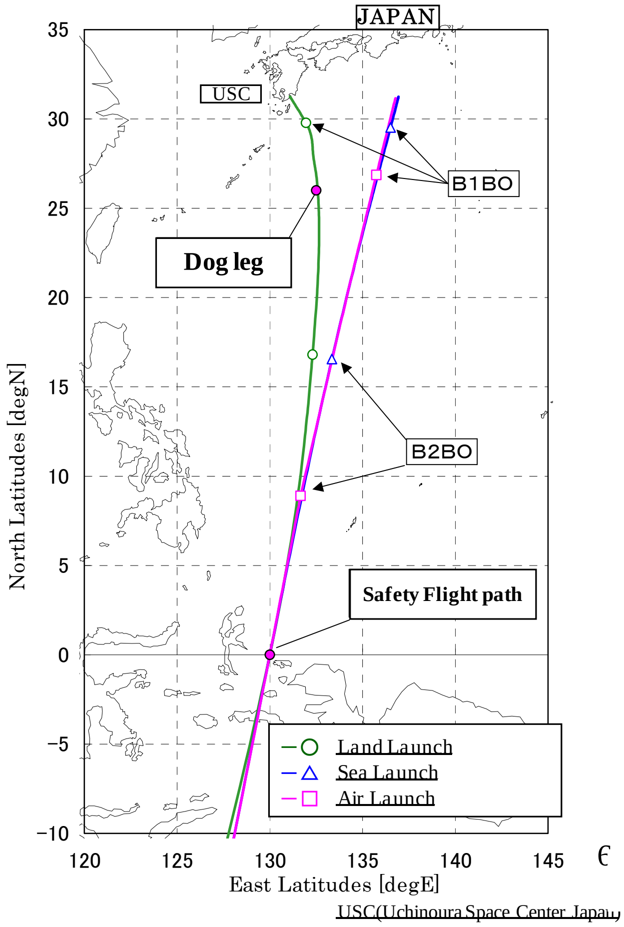


Figure 6: The Flight Path

In this simulation, the trajectory after 1st stage ignition till 1st stage separation is optimized using the MCMC method. The initial state and the objective state are shown in table 1.

Table 1: The initial/objective state

	The initial state	The objective state
Altitude	12km	40km
Mach	0.8	8.4
Gamma	9°	27°
time	0[s]	50[sec]

As mentioned in the previous section, the rocket trajectory is divided into some nodes and the attitude of the rocket on each node is treated as variable **x**. In this time, the trajectory is divided ten nodes and the motion of the rocket is calculated in the point mass model.

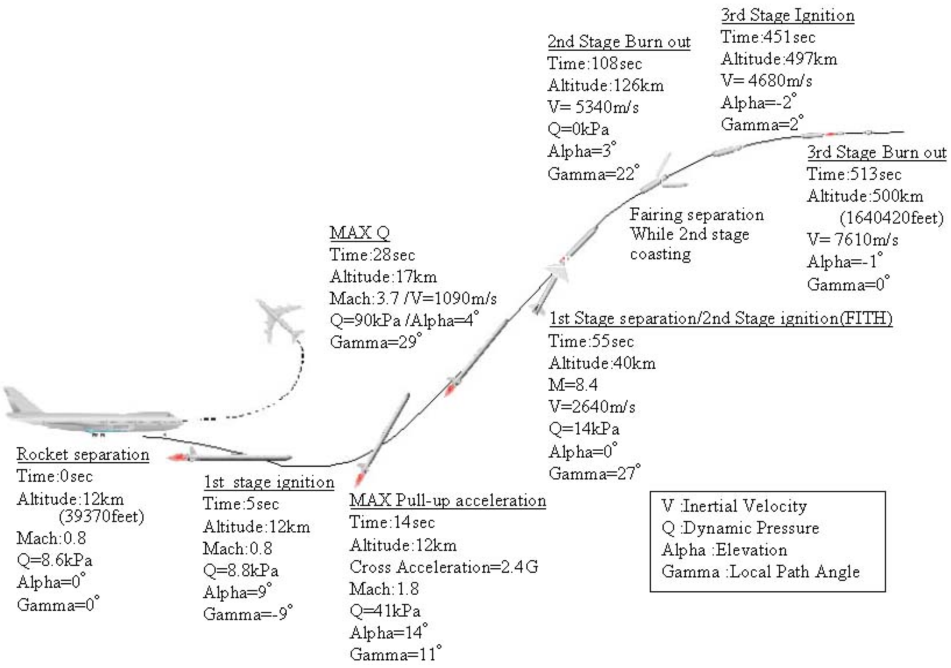


Figure 7: The Flight Sequence

The aerodynamic characteristics of the rocket are calculated with MISSILE DOTCOM that is a library-mathematics for aerodynamics developed by USAF (Ref. 6).

The objective function is provided as shown (3) so that near the objective state or more, the more the objective function becomes small. And **x** that minimize the objective function

$$f(\mathbf{x}) = \sqrt{\left(\frac{40000 - h_f}{1000}\right)^2 + \left(27 - \gamma_f\right)^2} \quad (3)$$

h_f [m] is the altitude in the last state(t=50[s]) and γ_f [°] is the attitude angle in the last state.

Under this situation, the optimization using the MCMC method was conducted and the result is shown in Fig. 8.

From the graph it is understood the value of f(**x**) decreased as the searching step proceeds. At last, after the 127th step, the value f(x) became lower than 0.1. So it succeeded in the trajectory optimization with considerable accuracy. In the simple search method it needs 10^{14} steps to achieve same accuracy.

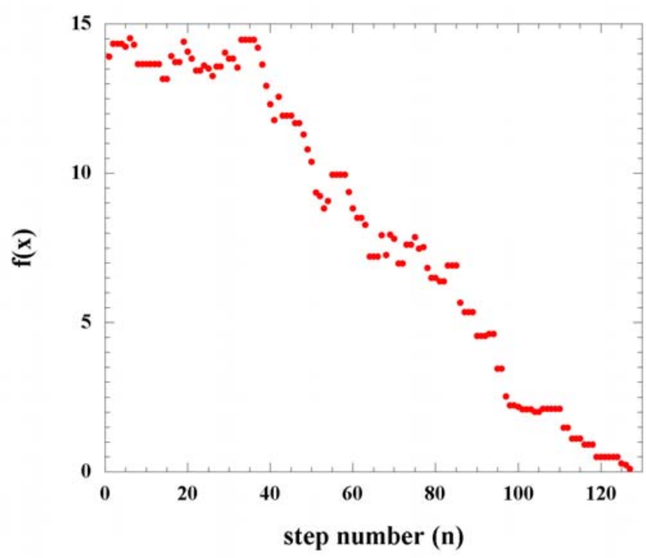


Figure 8: Change of f(x) as the step proceeds

CONCLUSION

From the simulation result, it is concluded that the optimization of the rocket trajectory with MCMC method is useful. With this method, it is possible to revise and optimize the trajectory program of air launch rocket quickly and exactly. Thus a part of technical problems of air launch can be simplified and it will make a contribution to the development of air launch systems.

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