A Quick Optimization of a Rocket Trajectory Using MCMC Method.

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Introduction

Next Generation Space is **Convenient**.

**Low Cost, Quick, Adaptable**

- **Micro Space**
  - Small Satellites
  - Responsive Launchers
- **Standardization**
  - Standard Bus
  - Modular LV
  - Scalable Architecture

Japan is getting down to above things.

Today’s presentation
- Introducing Japanese micro space.
- Introducing our research concerning technical problems of air launch.
Japanese Micro Space – Satellites

4 phases of Standardization of Satellite

- Availability like USB
- Scalable Architecture
- Spacewire Network

Network-type Satellite Architecture

SpaceCube 2 (JAXA, NEC/TOSHIBA)

Useful middleware for handling deference on CPU and I/O of hardware.
Japanese Micro Space – Satellites

Advanced Small Satellite Bus

- Remote Sensing Optical Sensor
- Remote Sensing SAR Sensor
- Engineering Test
- Astronomical Observation

NeXT (New exploration X-Ray Telescope)

- Launched in 2013
- Weight / 2400 kg
- To study the high-energy non-thermal Universe

- Launched in 2011
- Size / W950D950H950
- Weight / >300kg
- Payload Power / MAX 300 W
- Development Term / >2 year
  (after ICD fixed)

Spacewire will be adapted
<table>
<thead>
<tr>
<th></th>
<th>M-3S</th>
<th>LV432</th>
<th>LV321</th>
<th>LV211</th>
<th>SoundingLVs + kick</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Launch</strong></td>
<td>Land Launch</td>
<td>Air Launch (AL)</td>
<td>Air Launch</td>
<td>Land Launch</td>
<td></td>
</tr>
<tr>
<td><strong>Payload</strong></td>
<td>300kg (LEO)</td>
<td>400kg (LEO)</td>
<td>150kg (LEO)</td>
<td>50kg (LEO)</td>
<td></td>
</tr>
<tr>
<td><strong>Aircraft</strong></td>
<td>----</td>
<td>Booster + LV321</td>
<td>----</td>
<td>C-5 / C-17</td>
<td>F-15</td>
</tr>
<tr>
<td><strong>Total Length</strong></td>
<td>16.5m</td>
<td>23.6m</td>
<td>23.8m</td>
<td>23.8m</td>
<td>30.7m</td>
</tr>
<tr>
<td><strong>Diameter</strong></td>
<td>0.735m</td>
<td>1.41m</td>
<td>1.41m</td>
<td>1.41m</td>
<td>2.5m</td>
</tr>
<tr>
<td><strong>Total Weight</strong></td>
<td>9.4t</td>
<td>43.6t</td>
<td>41.6t</td>
<td>48.7t</td>
<td>139t</td>
</tr>
<tr>
<td><strong>Payload to LEO</strong></td>
<td>26kg</td>
<td>180kg</td>
<td>195kg</td>
<td>300kg</td>
<td>770kg</td>
</tr>
<tr>
<td><strong>Rate (s/1)</strong></td>
<td>1/5</td>
<td>3/4</td>
<td>3/4</td>
<td>3/3</td>
<td>7/8</td>
</tr>
</tbody>
</table>

Seiji Matsuda, AIAA-RS6-2008-5004
Technical problems of Air Launch

Technical problems of air launch are

- Vehicle Loading and Deployment
- Ignition Attitude Stabilization
- Launch Sequence
- Inertia Navigation System Initialization
- ••• etc

- Initial errors
- Errors of system parameters
- Errors by uncertainty of parachute
- Errors by wind

The ignition Attitude may be different from the nominal.
Solution to the problem.

**Hardware Solution**
- Separation device.
- Control device.
- Advanced design.
- Many times experiment.
- ... etc

**Software Solution**
- Trajectory optimization quickly, widely, exactly (Trajectory adjustment) with **MCMC method.**
- Real time system identification with **Particle Filter.**

New method realizes smarter systems.

More complicated Systems

Classical way
- ex. linearization
- low calculation cost
- high

Mathematical statistical way
- ex. MCMC

Improvement of calculation device.
Optimization Algorithm with MCMC.

MCMC Step.

1. Calculating the value of $f(x)$. Here $x$ is the current position.
2. Generating $\Delta x$ using random numbers and calculating the value of $f(x + \Delta x)$.
3. Calculating the transition probability $P$ from the values of $f(x)$ and $f(x + \Delta x)$.
4. Moving from $x$ to $x + \Delta x$ at the probability $P$.

$$P ( x, x + \Delta x ) = \begin{cases} \frac{1}{2} e^{-\beta \frac{f(x)}{f(x + \Delta x)} - 1} & [ f(x) > f(x + \Delta x) ] \\ 1 - \frac{1}{2} e^{-\beta \frac{f(x)}{f(x + \Delta x)} - 1} & [ f(x) < f(x + \Delta x) ] \end{cases}$$

$$\beta = \frac{1}{T_0 T_r \frac{N}{n}}$$

- Searching algorithm in high dimensional space.
- Useful for multivariate optimization.
- Able to apply to nonlinear systems.
Trajectory Optimization with MCMC.

\[ f(x) = f(\theta_1, \theta_2, \ldots, \theta_n; \theta_0) \]

The objective function

\[ f(x) = \sqrt{\left( \frac{40000}{1000} - h_f \right)^2 + (27 - \gamma_f)^2} \]

\( h_f \): final altitude
\( \gamma_f \): final attitude angle

Result after the 127\textsuperscript{th} step.

- \( f(x_{127}) = 0.09 \)
- \( h_f = 39964 \text{ [m]} \)
- \( \gamma_f = 26.92 \text{ [deg]} \)
Motion estimation with Particle Filter

To identify system parameters and predict the motion of the rocket exactly at real time with Particle Filter

Estimated motion at 3sec after separation.
Conclusion

Problems specific to Air Launch
- Attitude unstableness after separation.
- Ignition attitude error.

• Trajectory optimization with MCMC Method.
• Motion estimation with Particle Filter.

Software Approaches can smarten systems.

☆ New and powerful methods like MCMC or Particle Filter had not been able to be applied to space technologies until calculation devices improved in efficiency, cost and downsizing. These methods should be tools for rockets in next generation.

Classical way
- low calculation cost
- low capability

Mathematical statistical way
- high calculation cost
- high capability

Improvement of calculation device.

no problem
The objective function
\[
  f(x) = \sqrt{\left(\frac{40000}{1000} - h_f\right)^2 + (27 - \gamma_f)^2}
\]

- \(h_f\): final altitude
- \(\gamma_f\): final attitude angle

\[
  h_f = h(\theta_1, \theta_2, \ldots, \theta_n; \theta_0)
\]

\[
  \gamma_f = \gamma(\theta_1, \theta_2, \ldots, \theta_n; \theta_0)
\]

- \(\theta_i\): Attitude target at each node.
- \(\theta_0\): Attitude angle at ignition.
Assumed Air Launch Rocket.

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Rocket separation
Time: 0 sec
Altitude: 12 km (39370 feet)
Mach: 0.8
Q = 8.6 kPa
Alpha = 0°
Gamma = 0°

1st stage ignition
Time: 5 sec
Altitude: 12 km
Mach: 0.8
Q = 8.8 kPa
Alpha = 9°
Gamma = -9°

1st Stage separation/2nd Stage ignition (FITH)
Time: 55 sec
Altitude: 40 km
M = 8.4
V = 2640 m/s
Q = 14 kPa
Alpha = 0°
Gamma = 29°

MAX Q
Time: 28 sec
Altitude: 17 km
Mach: 3.7
\(V = 1090 \text{ m/s}\)
Q = 90 kPa / Alpha = 4°
Gamma = 29°

1st Stage separation
FAIRING SEPARATION
When 2nd stage coasting

2nd Stage Burn out
Time: 108 sec
Altitude: 126 km
V = 5340 m/s
Q = 0 kPa
Alpha = 3°
Gamma = 22°

MAX Pull-up acceleration
Time: 14 sec
Altitude: 12 km
Mach: 1.8
Q = 41 kPa
Alpha = 14°
Gamma = 11°

3rd Stage Ignition
Time: 451 sec
Altitude: 497 km
V = 4680 m/s
Alpha = -2°
Gamma = 2°

3rd Stage Burn out
Time: 513 sec
Altitude: 500 km
V = 7610 m/s
Alpha = -1°
Gamma = 0°

\(V\) : Inertial Velocity
\(Q\) : Dynamic Pressure
Alpha : Elevation
Gamma : Local Path Angle
Particle Filter.

A new powerful technique developed in mathematical statistics which provides quick and accurate estimation of high dimensional state vector. The system state is approximated by the distribution of particles.

1. Particles are updated by the governing equation of the system.
2. For each particle, the likelihood weight is calculated by the sensor data.
3. Particles are resampled at the probability.

Advantages

- can estimate high dimensional state vector exactly and quickly.
- can estimate system parameters.
- can be applied to nonlinear systems.
- easy to design the algorithm.
Analysis of internally carried type.

The motion of the rocket is different from the nominal for the reason of errors of parameters. It is necessary to estimate parameters quickly and exactly after separation.

Wind Velocity $+10[m] \sim -10[m]$
Calculation Model.

Entire weight : $9.0 \times 10^3$ [kg]

Initial condition
altitude : 10[km]
horizontal velocity : 243[m/s]
initial attitude angle : 5.0[deg]
initial pitch rate : 10[deg/s]

\[
M \frac{d^2}{dt^2} \begin{bmatrix} x \\ z \end{bmatrix} = G + L + D - T
\]

\[
I \frac{d^2\theta}{dt^2} = l_1 \times L \cdot n + l_1 \times D \cdot n + l_2 \times T \cdot n
\]

\[
L = \frac{1}{2} \rho W r^2 AC_{L\alpha}
\]

\[
D = \frac{1}{2} \rho W r^2 AC_{D\alpha}
\]

\[
G = g_0 \left( \frac{R}{R + h} \right)^2
\]

\[
T = \frac{1}{2} \rho W r^2 S_0 C_p
\]