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Reliability Analysis For Shear In Lightweight Reinforced Concrete Bridges Using Shear Beam Database

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RELIABILITY ANALYSIS FOR SHEAR IN LIGHTWEIGHT REINFORCED CONCRETE BRIDGES USING SHEAR BEAM DATABASE

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ABSTRACT

The objective of this report is to analyze and calibrate the reliability indices for shear in reinforced concrete bridge girders. Existing statistical models are based on limited experimental data from only a few research tests. These existing models show that our current procedures for analysis are about 10-15% less conservative for lightweight concrete compared to an analysis for normal weight concrete. Accurate load models are used to find shear and moment envelopes of loads applied to bridges. Analysis is based on different span lengths, span number and girder dimensions. Design calculations are performed using design values and loads calculated from load models. Different strength of concrete are also used to compare the reliabilities of various parameters. Results show that when using a professional factor of 1.0 and variability of 0.0 and a resistance factor of 0.8 can be applied to the AASHTO design equation for shear in reinforced concrete. After sorting approximately 100 previous lightweight beam tests a better understanding of lightweight concrete girders is known.

Keywords: Concrete, Shear, Reinforced, AASHTO

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1. INTRODUCTION

The objective of this paper is to find and calibrate reliability indices, bias factors and COV's, as well as Professional factors for shear design in reinforced concrete bridges. Using AASHTO 2012 general procedure, shear analysis reliability indices can be found and compared to an acceptable value or target value. Previous tests and analysis have been performed on lightweight concrete for simple span bridges, however this report looks at lightweight concrete in multi span bridges. Using structural reliability theory, resistance factors can be found using a five step system similar to that explained in Paczkowski and Nowak in 2010.

The first step is to select and model loads for dead load and live load that will be applied to the bridge girders. A design approach outlined in "Design of Highway Bridges by Barker and Puckett (2007) is similarly used for the design aspect of this analysis. Because this study looks at a single girder and not a bridge as a system, these loads will need to be factored for multiple girders present. The loads will also have to be factored for multiple trucks present as well as load and impact factor. Selection of an appropriate size of T-beam dimensions for a specific span of girder is done. Concrete and steel properties will also need to be selected. These values will allow for calculation of area of steel reinforcement which will increase the shear resistance to an accepted value.

The second step is to select typical and up to date statistical parameters for each of the variables used in analysis. This included finding distributions and parameters for the applied loads. Most variables are normally distributed, but with the large amount of variables and types of loads used in this analysis exact distributions of variables should be used. With a large amount of variables the analysis is not dependent on any one variable. This causes the output to change with minor input changes.

The third part is to develop the load resistance model. This is the comparison of the design equation to the actual loads applied to the system. This is equal to Resistance minus Load in a very simplified manner. AASHTO 2012 is used with the modified compressive theory of shear analysis. Explanation of the design procedure will be described further later in the paper. The next step is to perform a Reliability analysis of the system. The Reliability analysis will show the probability of failure that the system is designed too. In this case we are looking at shear in R/C T-beams designed with lightweight concrete so the reliability will be the safety of a single beam. Reliability will be calculated based on different span lengths and a resistance factor. Further analysis will be taken to look at a database of shear tests. This database will give us a better understanding of how lightweight concrete beams perform under different conditions due to difference in geometry as well as reinforcement steel used in the beam.

The final step in the calibration of the code will be to find an acceptable reliability for shear in concrete girders. A target reliability will be used for all cases of load and span, as well as different strengths of concrete and steel. The target reliability is set based on the consequences of failure and the marginal cost of safety. A target reliability is typically set to 3.5 in most cases for most bridge components. For this study a reliability of 3.5 will be used suggested by Nowak and Collins (2010) in Reliability of structures. Another step that could be applied is adjusting the factors that are applied to load. In this study, load factors will remain the same as previous studies and a factor applied to the resistance will be applied. The factor will have a range from 0.75 to 0.9 with increments of 0.05 which will show an increase in reliability with decrease in factor. The results of the analysis will be compared to the target reliability and an appropriate resistance factor will be chosen.

1.1 Lightweight concrete

A brief discussion on the differences between lightweight and normal weight concrete is needed due to the design differences. Many test have been conducted on normal weight concrete but very few test have been performed on lightweight girders. The term lightweight concrete describes a wide range of special aggregate bases. The specific gravities of these aggregates are significantly lower than that of normal weight concrete. Lightweight concrete can be used for insulative and nonstructural concrete composed of extremely light porous aggregate. It is also used for structural concrete usually being composed of expanded clays and shale's. Air is trapped in the aggregate which causes the reduced weight of the concrete mixture. This means the lighter the concrete the more air that is trapped in the aggregate and therefore causes the concrete to have less strength. The properties of lightweight concrete varies with the type of aggregate and the source and size of the aggregate (Ramirez 2000).

Based on the definition from AASHTO LRFD lightweight concrete is defined as concrete having an air dry weight not exceeding 120 pcf. For this study, lightweight concrete based on average tests will be used. Sand based concrete is another popular type of lightweight concrete, this concrete is common but not used for this study. A reduction in the strength of lightweight concrete contribution to shear resistance is expressed by a factor applied to $\sqrt{f_c'}$. The reduction factor will be a 25 percent decrease in $\sqrt{f_c'}$ this is reflected as $.75\sqrt{f_c'}$ (Russell 2009). The weight of the concrete will also be set to a maximum of 120 pcf with a wearing surface unit weight of 140 pcf.

2. LITERATURE REVIEW

2.1 Lightweight concrete in shear

AASHTO uses a modified compression theory analysis to design for shear in prestressed and RC bridge girders. This analysis procedure was produced from tests on normal weight concrete only. The code uses reduction factors for lightweight concrete to account for the loss of strength. One of those factors being a 25% decrease in $\sqrt{f_c'}$. There is no sufficient experimental data to show that the code effectively takes into account the differences in strength. (Ramirez 2000)

Applying this reduction in shear has been said to account for three different phenomenon. The first being a loss in the strength of the concrete in tension. A shear failure in a concrete beam is usually a 45^{o} angle to the top and bottom face. This causes the concrete to pull away from each other in tension and, in turn, results in reduced shear capacity.

The second cause for a reduction is the characteristics of friction along the shear failure surface. A shear failure will push concrete sections together on the shear plane, this causes the aggregate to grind against each other and allow for some shear resistance even after the concrete has cracked. Lightweight concrete has smaller aggregate and the strength of that aggregate is usually less. Having these different characteristics hurt the strength of lightweight concrete. The smaller aggregate causes a smoother surface for the concrete to slide against and reduce in aggregate interlock that is present in normal concrete. The weaker aggregate also fractures easier and crumbles when the concrete slides against each other.

Another cause of reduction in shear capacity is the origin of the lightweight concrete. Because lightweight concrete is not as widely used. Universal types of aggregates are not used like in normal weight. This causes a wider range of aggregate types and strengths. A reliability analysis can take into account the variability of different parameters and allows for this type of phenomenon.

Ramirez (2000) conducted a comparison study looking at the differences between lightweight and ordinary concrete in bridge girders with a shear failure. 12 tests were performed on rectangular R/C girders with varying strengths of concrete. Studies concluded that tensile strengths and modulus of rupture on lightweight concrete are comparable to that of normal weight concrete. It also concluded that AASHTO LRFD provides for conservative estimates of shear resistance. From that same study a bias between the experimental and calculated resistance was found to be 1.1 with a standard deviation of 0.06. For normal weight concrete the bias is 1.29 with a standard deviation of 0.11. Bias and standard deviation will be recorded with this study as well.

Accuracy in any design method to predict test results is considered as the professional factor. Represented by a bias and a coefficient of variation. Bias is represented by equation (11). The bias is a value around 1.0 that will show how reliable the design procedure is. If the value is below one then the design procedure is under predicting the strength. If the value is above one then the design analysis is predicting the value of resistance above that of load (i.e. conservative).

$$\lambda = \frac{\mu_{tests}}{\mu_{nominal}} \tag{1}$$

$$COV = \frac{\sigma}{\mu} \tag{2}$$

A bias value should be determined by tests performed on sections where the structural materials as well as geometry of the section is known. This then can have a theoretical failure strength calculated by a design procedure and be compared to the actual breaking strength. When structural mechanics of the analysis are well understood and the design equation closely predicts how the material is behaving, there is a bias close to one. In the case of a flexure beam bias values are close to 1.0 and variations around 0.05. Then again, when material mechanics are not understood and design equations are based on empirical results and assumptions. Which is the case for normal reinforced concrete beams. The bias values are greater than one with greater variations on the mean.

3. AASHTO 2012

In this study most variable in used are considered random variables except in the case of a few unknown distributions. The nominal shear resistance is calculated based on AASHTO 2012 where the resistance is the lesser of:

$$V_n = V_c + V_s + V_p \tag{3}$$

AASHTO (5.8.3.3-1)

$$V_n = .25 f_c' b_v d_v + V_p \tag{4}$$

AASHTO(5.8.3.3-2)

In which

$$V_c = .0316\beta\sqrt{f_c'}\,b_v d_v \tag{5}$$

AASHTO(5.8.3.3-3)

$$V_s = \frac{A_v f_y d_v (cot\theta + cot\alpha) sin\alpha}{s} \tag{6}$$

AASHTO(5.8.3.3-4)

In the case of $\alpha = 90$

$$V_s = \frac{A_v f_y d_v (cot\theta)}{s} \tag{7}$$

AASHTO(C5.8.3.3-4)

According to equation (5) the shear strength of the steel is calculated assuming that the placement of the transverse shear steel is vertical ($\alpha = 90^0$) to the top and bottom of the concrete section. A minimum of transverse reinforcement is also included which is determined as:

$$A_v \geq .0316\sqrt{f_c'}\frac{b_v s}{f_y} \tag{8}$$

AASHTO(5.8.2.5-1)

The parameters for the shear resistance in both the concrete and the steel are:

$$\beta = \frac{4.8}{(1 + 750\varepsilon_s)} \quad (9)$$

AASHTO(5.8.3.4.2-1)

For this study, the section will always include the minimum amount of shear reinforcement so a revision to the previous equation is not needed. If the section did not include the minimum amount of reinforcement eq. (5.8.3.4.2-3) would be used.

$$\theta = 29 + 3500\varepsilon_s \quad (10)$$

AASHTO(5.8.3.4.2-3)

$$\varepsilon_s = \frac{\frac{|M_u|}{d_v} + .5N_u + |V_u - V_p| - A_{ps}f_{po}}{E_sA_s + E_pA_{ps}} \quad (11)$$

AASHTO(5.8.3.4.2-4)

$M_u$is not to be taken less than $|V_u - V_p|d_v$

An explanation for ε_s is the stress that is induced in the longitudinal reinforcement. M_u and V_u increase the stress that is felt in the steel. Large moments and shears cause our theta (θ) to increase and the beta (β) multiplier in the concrete shear to decrease. Therefore and increase in loads causes the concrete to carry a lower percentage of shear.

This study is only going to be looking at reinforced concrete so many factors in this equation will go away. The factor for a normal force to the face of the girder is also going to be set to zero for this study. Therefore this equation which is the longitudinal strain in the extreme tension strand of the section, reduces to using the moment and shear and longitudinal steel parameters in a location.

In all equations the definition for variables are defined as:

b_v = effective web width taken as the minimum web width within the depth d_v

d_v = effective shear depth determined by:

$$d_v = max\begin{Bmatrix} d_e - \frac{a}{2} \\ 0.9d_e \\ 0.75h \end{Bmatrix} \quad (12)$$

d_e = distance from extreme compression side to tension reinforcement

a = 0.85 times distance from compression side to neutral axis. Defined as Whitney Stress Block

s = spacing of transverse reinforcement measured in a direction parallel to the longitudinal reinforcement this will be taken as a deterministic 12 inches in this study

β = factor indicating ability of diagonally cracked concrete to transmit tension and shear

θ = angle of inclination of diagonal compressive stresses

α = angle of inclination of transverse reinforcement to longitudinal axis

A_v = area of shear reinforcement within a distance s or 12 inches

V_p = Component in the direction of the applied shear of the effective prestressing force. In this case of R/C girders a value of zero for all prestressing variables will be used.

A_c = area of concrete on the flexural tension side of the member as shown in Figure 1

A_s = area of nonprestressed steel on the flexural tension side of the member at the section under consideration. Shown in Figure 1.

N_u = factored axial force, taken as positive if tensile and negative if compressive

M_u = factored moment at design section

V_u = factored shear force at design section

Figure 5.8.3.4.2-1 from AASHTO 2012 shows an illustration of the shear parameters on a typical girder section. This diagram shows how an appropriate shear failure should occur.

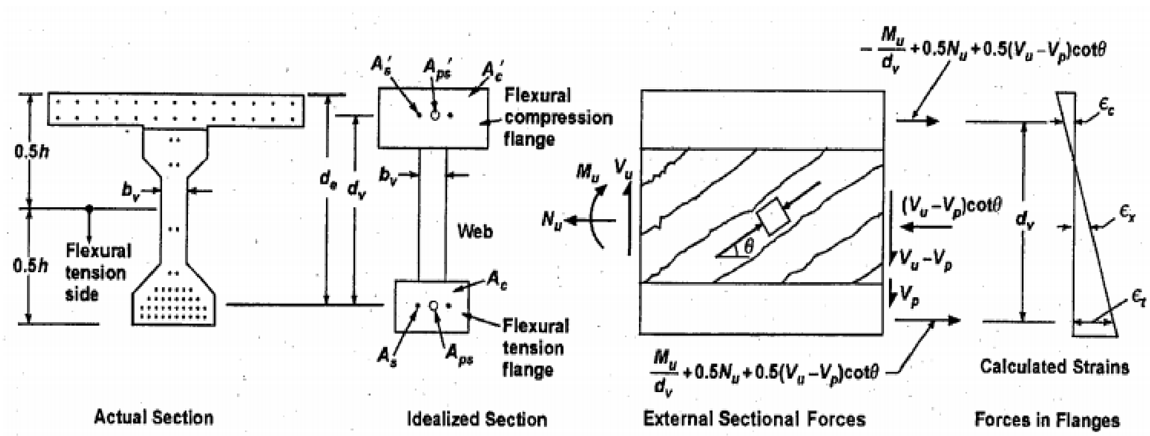

Figure 1- Illustration of Shear Parameters for Section Containing AASHTO 2012

More accurate procedures can be used to calculate the concrete strength of sections. However with lightweight concrete it is shown in this study that the ratio of V_c to V_s is very low in most cases due to the reduction in strength. Having the ratio so small will allow for minute changes in results if different methods are used in calculating the V_c differently.

3.1 ASSESEMENT OF AASHTO 2012 SHEAR PROCEDURES

Reliability analyses performed previous to this report used very limited results. Paczkowski and Nowak compiled test data from researches done by Hamadi and Regan, Walraven and Al-Zubi and Ramirez et. al. The experimental data was split into two different groups. The first consisted of 13 lightweight concrete beams. The second consisting of 13 normal weight control beams. Table 1 is from Paczkowkski and Nowak showing the properties of beams used in analysis.

Table 1- Experimental Database Paczkowski and Nowak 2010

Reference	Beam Id	Type of Aggregate		f_c	V_{Test}	Beam Id	Type of Aggregate		f_c	V_{Test}
		Coarse	Fine	[ksi]	[kip]		Corase	Fine	[ksi]	[kip]
Hamadi[9]	**LT-1**	Exp. Clay	Exp. Clay	3.2	19.1	**GT-1**	Gravel	Sand	3.2	27.0
Hamadi[9]	**LT-2**	Exp. Clay	Exp. Clay	4.1	26.7	**GT-2**	Gravel	Sand	4.1	33.4
Hamadi[9]	**LT-3**	Exp. Clay	Exp. Clay	2.9	27.0	**GT-3**	Gravel	Sand	2.9	37.1
Hamadi[9]	**LT-4**	Exp. Clay	Exp. Clay	5.1	33.8	**GT-4**	Gravel	Sand	5.1	51.1
Hamadi[9]	**LT-5**	Exp. Clay	Exp. Clay	3.2	33.8	**GT-5**	Gravel	Sand	3.2	50.1
Ramirez[11]	**5LWLA**	Haydite	Sand	6.3	73.8	**1NWLA**	Gravel	Sand	6.7	98.1
Ramirez[11]	**6LWLB**	Haydite	Sand	6.2	83.7	**2NWLB**	Gravel	Sand	6.7	109.1
Ramirez[11]	**7LWLC**	Haydite	Sand	6.3	80.3	**3NWLC**	Gravel	Sand	6.8	94.7
						4NWLD	Gravel	Sand	6.6	78.3
Ramirez[11]	**8LWLD**	Haydite	Sand	6.4	63.0	**9NWLD**	Limestone	Sand	6.4	69.8
						10NWHD	Limestone	Sand	5.8	80.8
Ramirez[11]	**11LWHD**	Haydite	Sand	10.5	89.1	**12NWHD**	Limestone	Sand	10.9	89.3
Walraven[10]	**Lg301**	Lytag	Sand	2.8	72.9					
Walraven[10]	**Lr301**	Liapor	Sand	4.0	74.3	**Gd301**	**Gravel**	**Sand**	**3.3**	**80.9**
Walraven[10]	**Ae301**	Aardelite	Sand	3.3	72.3					

Tests performed on lightweight concrete are presented on the left side of Table 1. Whereas the comparative normal weight concrete values are presented on right hand side. The two tests compared to each other are similar in properties but using different aggregate. A graph comparing the strengths of the two test is shown in Figure 2 from Paczkowski and Nowak 2010. Ratios of experimental results to calculated value from AASHTO LRFD 2007 is presented. Lightweight concrete ratio is presented on the vertical axis and the ratio for normal weight concrete on the horizontal axis. The line represents a ratio of normal weight to lightweight ratios equal to one. It can been seen that the ratio of lightweight concrete is less than that of normal weight concrete (Paczkowski and Nowak).

Cumulative distributions functions of the ratios of experimental to calculated shear results were plotted. From these graphs, statistical parameters of the test can be found. The mean values for ratio of normal weight concrete were equal to 1.2 with coefficient of variation 0.11. For lightweight concrete the mean value was 1.0 with a coefficient of variation of 0.1. Most research projects are done with hundreds of test data points. Which allows for variables in all aspects of the tests. Based on the severely limited data at the time it was concluded that shear resistance is about 15% lower for beams made of light weight concrete. It was concluded that a professional factor bias for shear of 0.91 and a coefficient of variation of 0.1should be used for analysis (Paczkowski and Nowak 2010).

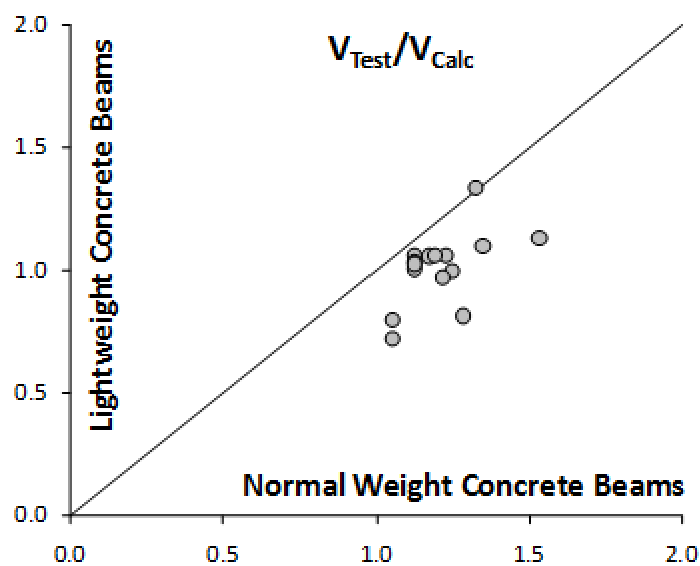

Figure 2-Ratio of Experimental to Calculated Shear resistance. Paczkoski and Nowak 2010

In the current study a database of hundreds of tests have been compiled and will be used to find trends in tested data. These trends can then be used to see how beams act under different situations. An example of database values is presented in Figure 3. It shows the statistical distribution of professional bias factors plotted on a log-normal scale. The average value of bias of these tests are on the range of 1-1.25. The compiled database are tests on reinforced and prestressed concrete as well as normal weight and lightweight concrete. The database will be sorted through and tests credibility will be chosen based on parameters used. Tests done on lightweight concrete will be singled out and bias factors and coefficient of variations will be found based on credible tests of lightweight concrete failing in shear. Using a larger database of tests we will make the bias and coefficient of variation very reliable and accurate. Resulting in a more informal, uniform and safe bridge design code.

The tests that were performed and compiled are from:

- Moody (1954)
- Taylor (1963)
- Bresler (1963)
- Mattock (1969)
- Krefeld (1966)
- Mphonde (1984)
- Bhal (1968)
- Hansen (1958-1963)
- Gaston (1952)
- Placas (1971)
- Diaz de Cosiso (1960)
- Van der Berg (1962)
- Mathey (1963)
- Taylor (1960)
- Ivey & Buth (1967)
- Ahmad (1986)
- Ahmad et al (1994
- Ahmad (2011)
- Kani (1967)
- Hamadi (1980)
- Rajagopalan (1968)
- Taylor (1972)
- Chana (1981)
- Walraven (1978)
- Walraven (1995)
- Murayama (1986)
- Salandra (1989)
- Collins (1999)
- Mattock (1986)
- Clark (1951)
- Johnston (1939)
- Mattock (1984)
- Elazanty (1986)
- Leonhardt (1962)

- Moretto (1945)
- Palakas (1980)
- Haddadin (1971)
- Malone (1999)
- Scott (2010)
- Heiser (2010)
- Malone (1999)
- Rakoczy (2013)
- Paczkowski (2010)

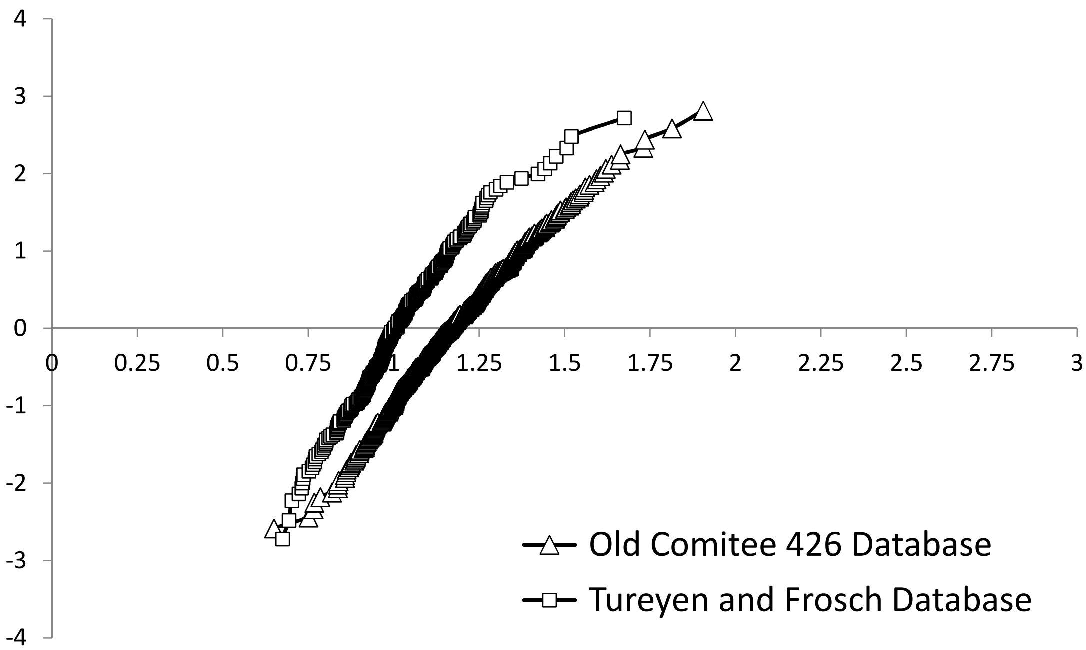

Figure 3-Compiled Database Values from Tureyen and Froshch & Old Committee 426

4. SHEAR DATABASE ANALYSIS

From the hundreds of shear tests that have been performed in previous studies, few are tests that have been performed on lightweight concrete. For this analysis two different compiled databases were actually investigated. An older database of concrete girder tests which used many of the papers mentioned previous were of normal weight concrete. However these older papers did also have tests on lightweight girders that weren't recorded in the database. These values were looked at and chosen based on the accuracy of the tests and failure mode. Another current large database compiled by Reineck, Bentz, Kuchma, and Bayrak is also available to the research committee. This database was a joint effort between European and American researchers. After looking at the ACI published database titled "ACI-DAfStb" most if not all tests were also shear tests on normal weight concrete. This cannot be confirmed because the authors would not respond to communications. Due to these setbacks a database of 95 lightweight concrete bridge girder tests were compiled separately. Table 2 shows all compiled test data for lightweight concrete failing in shear only.

Each beam test has a recorded actual failure shear and then a calculated shear based on AASHTO 2012 shear design procedures 5.8.3.3 general method. The simplified method using approximations for β and θ is also being calculated. The results from the simplified procedure will be analyzed to the general method. The ratio of measured values to predicted values are presented, if the values are above one, the design procedures predicted a V_n smaller than the breaking strength, which is optimal. None of these tests are being compared to normal strength concrete beams but according to Paczkowski (2010) and Rakoczy (2013) the calculated difference between normal weight and lightweight concrete is approximately 10 to 15%.

The ratio between measured and predicted will be plotted against the compressive strength of the concrete (f'_c), the area of longitudinal reinforcement steel (A_s) as well as the reinforcement ratio (ρw), (a/d) being the shear ratio and then the width of each beam (b_w). These plots will show correlations between how the beam is built and how well the lightweight beam performs in shear.

All the shear beams range from a compressive strength of 2780 psi to 12,948 psi. The extremely high compressive strength are due to a specific study on the differences between normal strength and extreme high strength lightweight concrete by Ahmad et al (1994). The area of reinforcement ranges from no longitudinal reinforcement to the maximum reinforcement allowed by AASHTO 2012. A specific study was also dedicated to the ranges of reinforcement done by Heiser (2010). Shear ratio (a/d) and reinforcement ratio were variable throughout all tests. The size of beams ranged from 4.25” × 9” to 12” × 18” so a wide range of sizes were analyzed.

Table 2 Shear Database for Lightweight Concrete

			measured vs predicted shear							
Author	Specimen	f'_c (psi)	General method	Simplified Method	A_s (in^2)	d (in)	d_v (in)	a/d	pw	bw
Heiser	S-120-0-3	4150	1.14	1.39	0	18	14.4	3	0	12
Heiser	S-120-0-3-R	4150	1.4	1.71	6	18	14.4	3	2.77	12
Heiser	S-120-.25-3	4150	1.26	2.27	0	18	14.4	3	0	12
Heiser	S-120-.25-3-R	4150	1.12	2.03	6	18	14.4	3	2.77	12
Heiser	S-120-.5-2	4150	1.16	2.22	0	18	14.4	3	0	12
Heiser	S-120-.5-3-R	4150	1.14	2.18	6	18	14.4	3	2.77	12
Heiser	S-130-0-3	6750	1.1	1.28	0	18	14.4	3	0	12
Heiser	S-130-0-3-R	6750	1.02	1.19	6	18	14.4	3	2.77	12
Heiser	S-130-.25-3	6750	1.19	2.03	0	18	14.4	3	0	12
Heiser	S-130-.25-3-R	6750	1.27	2.10	6	18	14.4	3	2.77	12
Heiser	S-130-.5-3	6750	1.1	2.04	0	18	14.4	3	0	12
Heiser	S-130-.5-3-R	6750	1.12	2.07	6	18	14.4	3	2.77	12
Ramirez	5	6294	1.03	1.29	3.95	12.12	10.9	2.14	2.33	14

						5				
Ramirez	6	6236	1.17	1.46	3.95	12.125	10.9	2.14	2.33	14
Ramirez	7	6294	1.12	1.41	3.99	12	10.85	2.16	2.37	14
Ramirez	8	6425	1.08	1.42	3.99	12	10.85	2.16	2.37	14
Ramirez	11	10486	1.41	1.81	3.99	12	10.85	2.16	2.37	14
Ivey & Buth	(1) 1	4490	2.99	3.57	0.8	12	10.5	2	1.27	6
Ivey & Buth	(1) 2	4500	1.16	1.38	0.8	12	10.5	3.33	1.27	6
Ivey & Buth	(1) 3	4690	0.97	1.15	0.8	12	10.5	4.95	1.27	6
Ivey & Buth	1	4040	1.2	1.35	0.6	12	10.5	3.33	0.95	6
Ivey & Buth	2	4170	1.14	1.38	0.8	12	10.5	3.33	1.27	6
Ivey & Buth	3	4160	1.21	1.51	0.93	12	10.5	3.33	1.48	6
Ivey & Buth	1S	3730	1.07	1.33	0.6	12	10.5	3.33	0.95	6
Ivey & Buth	2S	3870	1.13	1.51	0.8	12	10.5	3.33	1.27	6
Ivey & Buth	3S	4060	1.07	1.48	0.93	12	10.5	3.33	1.48	6
Ivey & Buth	27-1	3360	2.42	2.99	0.8	12	10.5	2	1.27	6
Ivey & Buth	27-2	3710	1.25	1.53	0.8	12	10.5	3.33	1.27	6
Ivey & Buth	27-3	3420	0.91	1.12	0.8	12	10.5	4.95	1.27	6
Ivey & Buth	4	3560	1.07	1.35	0.4	9	7.42	3.33	1.27	4.3
Ivey & Buth	5	4290	1.1	1.32	0.8	12	10.5	3.33	1.27	6
Ivey & Buth	6	3820	1.22	1.45	1.24	15	13.1	3.34	1.27	7.5
Ivey & Buth	7	3760	1.15	1.34	1.76	18	15.55	3.34	1.27	8.9
Ivey & Buth	8	3030	1.84	2.15	0.6	12	10.5	2	0.95	6
Ivey & Buth	9	2960	2.18	2.74	0.8	12	10.5	2	1.27	6
Ivey & Buth	10	3250	2.02	2.59	0.93	12	10.5	2	1.48	6
Ivey & Buth	11	3010	2.57	3.63	1.32	12	10.5	2	2.1	6

Ivey & Buth	12	3270	1.61	2.26	1.32	12	10.5	3	2.1	6
Ivey & Buth	13	3200	1.15	1.61	1.32	12	10.5	4	2.1	6
Ivey & Buth	14	2780	1.26	1.79	1.32	12	10.5	4.95	2.1	6
Ivey & Buth	15	3130	1.23	1.43	0.6	12	10.5	3.33	0.95	6
Ivey & Buth	16	2780	1.4	1.77	0.8	12	10.5	3.33	1.27	6
Ivey & Buth	17	3860	1.21	1.53	0.93	12	10.5	3.33	1.48	6
Hansen 57-61	2A1	3680	1.15	1.47	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	3A1	3310	1.24	1.60	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	4A1	2980	1.2	1.56	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	5A1	3490	1.33	1.70	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	6A1	3670	1.19	1.51	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	7A1X	3210	1.61	2.08	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	7A1	4240	1.57	1.97	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	2B1	5350	0.93	1.14	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	3B1	4090	1.14	1.44	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	4B1	4890	1.16	1.43	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	5B1	4790	1.18	1.43	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	6B1	4870	1.05	4.46	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	7B1X	4680	1.55	4.29	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	7B1	5200	1.47	1.90	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	10B1	4860	1.07	1.80	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	13B1	4940	0.96	1.18	1.6	12	10.5	2.5	2.5	6
Hansen 57-61	9C1	6910	1.06	1.44	1.6	12	10.5	2.5	5	6
Hansen	4C1	7000	0.99	1.35	1.6	12	10.5	2.5	5	6

57-61										
Hansen 57-61	4D1	8160	1.08	1.44	1.6	12	10.5	2.5	5	6
Hansen 57-61	2B2	4880	0.8	0.99	1.6	12	10.5	5	2.5	6
Hansen 57-61	3B2	4540	0.88	1.10	1.6	12	10.5	5	2.5	6
Hansen 57-61	4B2	5100	0.84	1.03	1.6	12	10.5	5	2.5	6
Hansen 57-61	5B2	4930	1.06	1.30	1.6	12	10.5	5	2.5	6
Hansen 57-61	6B2	5000	0.9	1.11	1.6	12	10.5	5	2.5	6
Hansen 57-61	7B2	4960	1.17	1.44	1.6	12	10.5	5	2.5	6
Hansen 57-61	10B2	4430	0.93	1.15	1.6	12	10.5	5	2.5	6
Hansen 57-61	13B2	5060	0.79	0.97	1.6	12	10.5	5	2.5	6
Hansen 57-61	2B3	4780	0.95	0.99	0.8	12	10.5	2.5	1.25	6
Hansen 57-61	3B3	4170	1.07	1.15	0.8	12	10.5	2.5	1.25	6
Hansen 57-61	4B3	5020	0.87	0.90	0.8	12	10.5	2.5	1.25	6
Hansen 57-61	5B3	4900	1.26	1.32	0.8	12	10.5	2.5	1.25	6
Hansen 57-61	6B3	4820	1.07	1.12	0.8	12	10.5	2.5	1.25	6
Hansen 57-61	7B3	4870	1.38	1.45	0.8	12	10.5	2.5	1.25	6
Hansen 57-61	10B3	5020	1.06	1.11	0.8	12	10.5	2.5	1.25	6
Hansen 57-61	13B3	5020	1.02	1.07	0.8	12	10.5	2.5	1.25	6
Hansen 57-61	3A4	3850	0.83	0.89	0.8	12	10.5	5	1.25	6
Hansen 57-61	4A4	3100	0.88	0.94	0.8	12	10.5	5	1.25	6
Hansen 57-61	10A4	3270	0.95	1.04	0.8	12	10.5	5	1.25	6
Hansen 57-61	2B4	4940	0.68	0.71	0.8	12	10.5	5	1.25	6
Hansen 57-61	3B4	4400	0.76	0.85	0.8	12	10.5	5	1.25	6
Hansen 57-61	4B4	5120	0.74	0.77	0.8	12	10.5	5	1.25	6

Hansen 57-61	6B4	4850	0.82	0.86	0.8	12	10.5	5	1.25	6
Hansen 57-61	10B4	4850	0.78	0.82	0.8	12	10.5	5	1.25	6
Hansen 57-61	13B4	5150	0.82	0.85	0.8	12	10.5	5	1.25	6
Hansen 57-61	10BW4	4530	0.75	0.79	0.8	12	10.5	5	1.25	6
Ahmad et al	LNW-2	5656	1.24	1.68	0.88	10	8.50	2	2.07	5
Ahmad et al	LNW-3	6467	0.91	1.25	0.88	10	8.50	3	2.07	5
Ahmad et al	LHW-2	12441	1.82	2.73	1.76	10	7.795	2	4.54	5
Ahmad et al	LHW-3	12948	1.23	1.85	1.76	10	7.795	3	4.54	5
Ahmad et al	LHW-3a	12789	1.24	1.74	1.76	10	7.795	3	4.54	5
Ahmad et al	LHW-3b	12615	1.25	1.66	1.76	10	7.795	3	4.54	5
Ahmad et al	LHW-4	12035	1.28	1.91	1.76	10	7.795	4	4.54	5

4.1 General Method shear database analysis

Figure 4 shows the cumulative distribution of the ratio between tested and calculated values. The mean value of tested value to calculated value is equal to 1.14 with a standard deviation of 0.373. These are found by fitting a line to the left half of the data in order to capture the lower tail of the distribution. Where the line crosses the zero mark is considered the mean value. A standard deviation can just be found by the data given. This is lower than the 20 values obtained from Rakoczy in 2013 but higher than the 13 values from Paczkowski in 2010. The values are actually close to the average of the two papers. This shows that with a larger data pool, better approximations can be acquired.

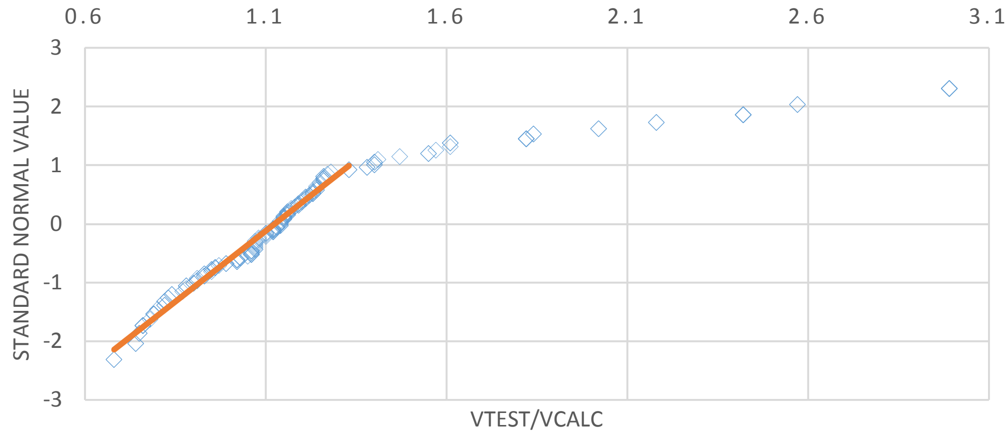

Figure 4 Cumulative Distribution vs tested value to calculated value

Figure 5 shows a comparison between the compressive strength of concrete and the ratio of experimental to calculated values. As the strength of concrete increases the ratio seems to have less variability and increase a small amount. There is no clear trend on the effect of f'c on the bias.

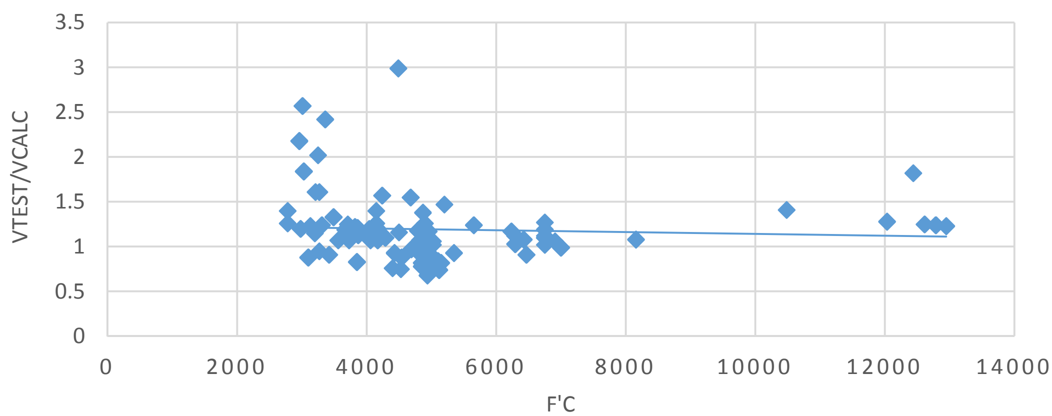

Figure 5 Experimental to Calculated value vs. Compressive Strength of Concrete

Figure 6 shows the ratio of experimental to calculated values to the shear ratio a/d. This also shows more of a variability with lower shear ratios. It also shows a decrease in experimental to calculated values with increase of shear ratio. As the a/d ratio decreases, higher shears are placed near the support, a disturbed region, where beam action is no longer valid. This might explain the additional variability associated with the prediction method for lower a/d.

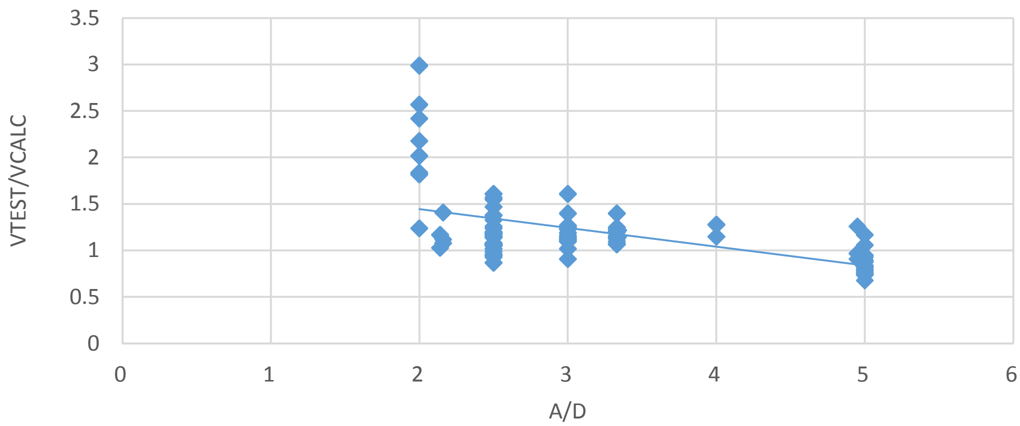

Figure 6 Experimental to Calculated ratio vs. Shear Span

Figure 7 shows the reinforcement ratio compared to the experimental to calculated ratio. Not much can be seen from this plot except the larger variability with lower reinforcement ratios. It can also be seen that with no reinforcement the ratio has very low variability, however there were only 4 tests with zero reinforcing.

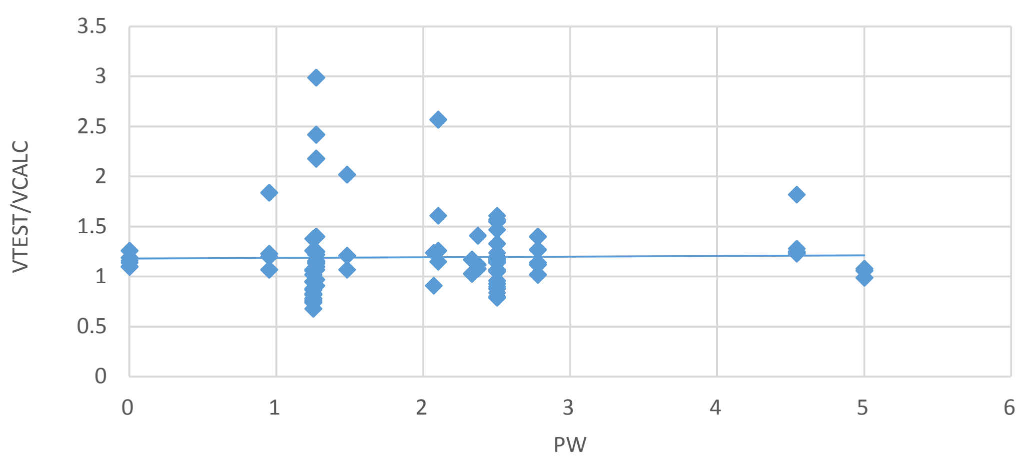

Figure 7 Experimental to Calculated ratio vs. Reinforcement Ratio

Figure 8 shows the experimental to calculated ratio vs the width of the beam tested. Because of the small amounts of data for larger beam sizes, it can't be concluded that these beams have large variability. However It can be seen that the 68 beam tested by Hanson in 61 and Ivey in 67, the widths of six inches has very large variability because of the mass amounts of beams tested at this size. A trend line applied to the plot shows that the ratio between experimental and calculated values to remain constant throughout different beam sizes.

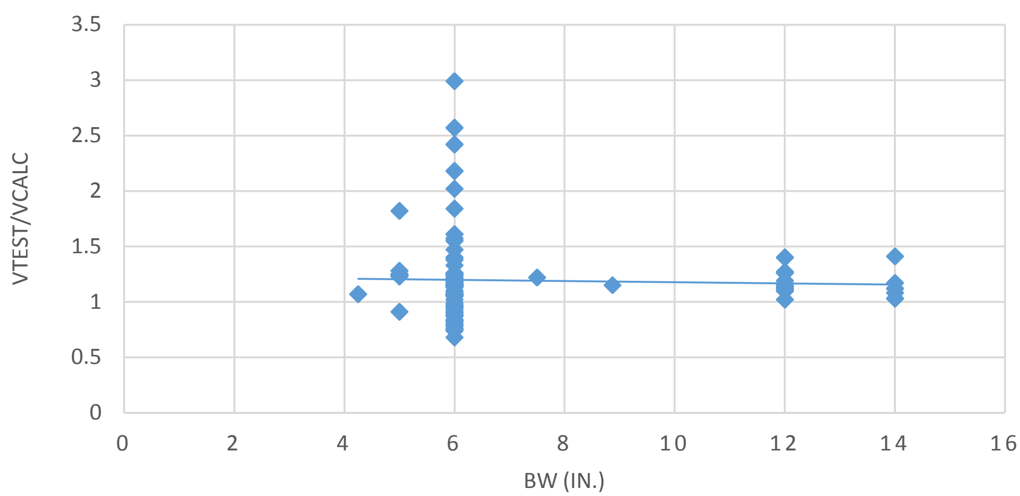

Figure 8 Experimental to Calculated ratio vs. Beam Width

Figure 9 also shows similar values to the previous four figures. The experimental to calculated values compared to the area of steel used shows higher variability in lower values of steel. This can also be due to the lack of data using higher amounts of steel. It does show as well that the experimental to calculated values due remain constant with changes in the amount of steel used.

Simple conclusion can be taken from each of these graphs but because larger amounts of data could not be obtained no definite conclusion can be made. These values can be used to form a hypothesis about future research in the lightweight concrete beam research community.

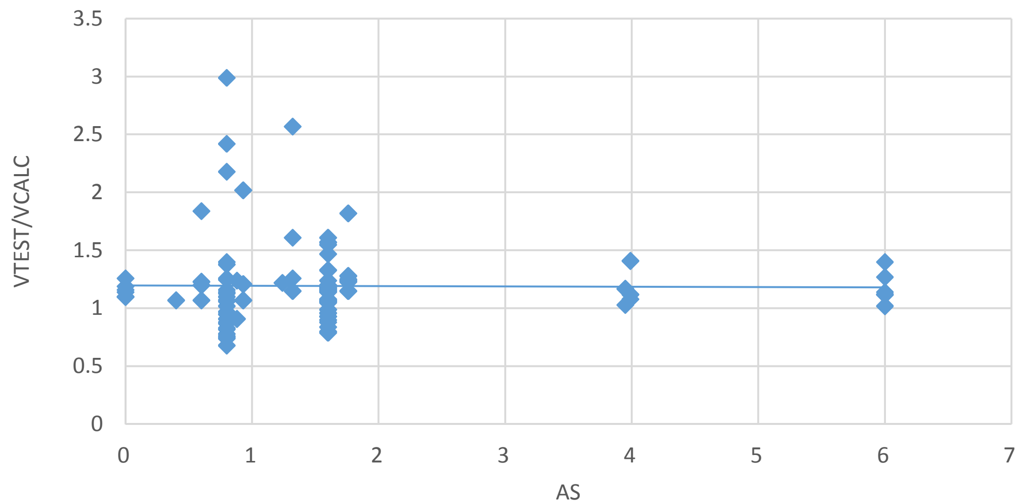

Figure 9 Experimental to Calculated ratio vs. Area of Steel

4.2 Simplified procedure shear database analysis

Typically using the simplified method causes the ratio of experimental to calculated values to be higher and more conservative. Only thirteen out of the ninety five tests have predicted higher shear values than the actual measured breaking shear. The few tests that have failed the ratio are tests that have a large shear span and then a low amount of reinforcing steel. Figure 10 shows the cumulative distribution of the bias of tested value to calculated value.

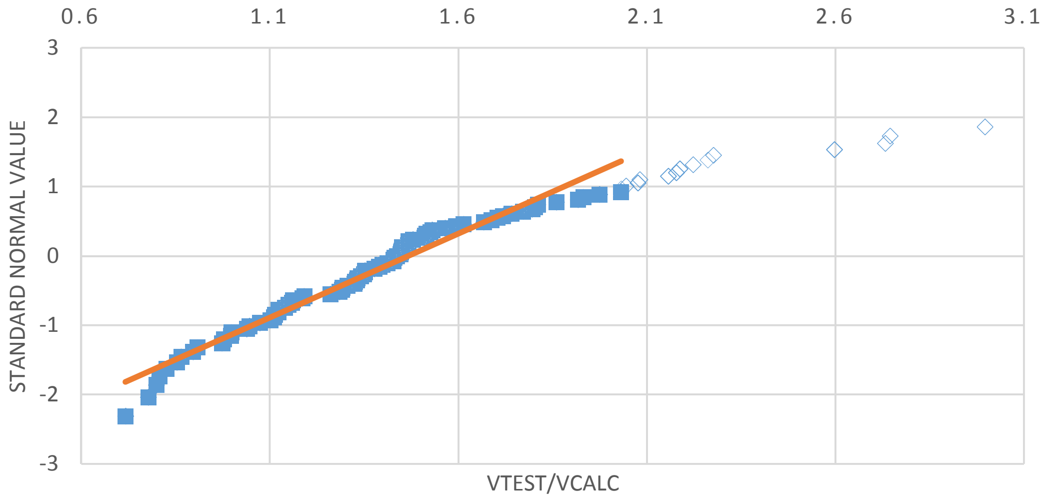

Figure 10 Cumulative Distribution vs calculated to tested value for simplified procedure

It can be seen that the cumulative distribution has increased to an average of 1.43 with a standard deviation of 0.55. The larger standard deviation can also be seen by the plotted points being farther spaced from each other around the mean.

Figure 11 through Figure 15 show the same distributions as shown for the general method. In each case the simplified method shows a larger variation in data at all levels. As stated earlier it can be seen that with a higher shear span or lower amounts of reinforcing steel less conservative beams are built. A single report done by Hansen (1961) showed that with lower amounts of reinforcing steel and larger shear spans contribute to the lower amounts of conservatism. Figure 12 shows only slight dependence on prediction performance of the simplified procedure with respect to shear span. Figure 12 actual indicates that there is little trend at all, but higher variability for lower shear spans. As the shear span decreases it is generally advisable to use a strut-and-tie model, which may account for some of this variability. Figure 13 shows the simplified procedure has little or no discernable dependence on the reinforcing ratio, contrary to Hansen (1961).

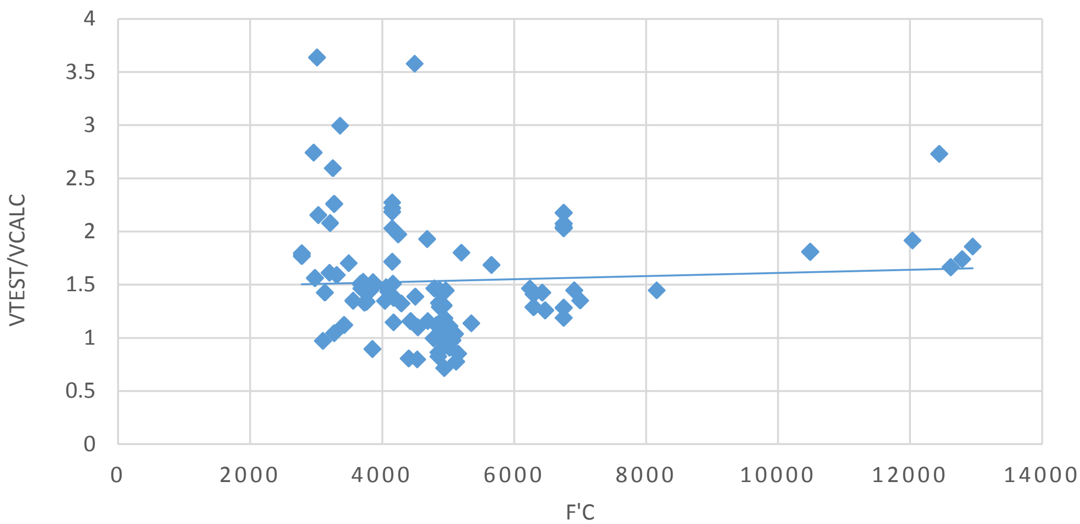

Figure 11 Experimental to Calculated ratio vs Strength of Concrete

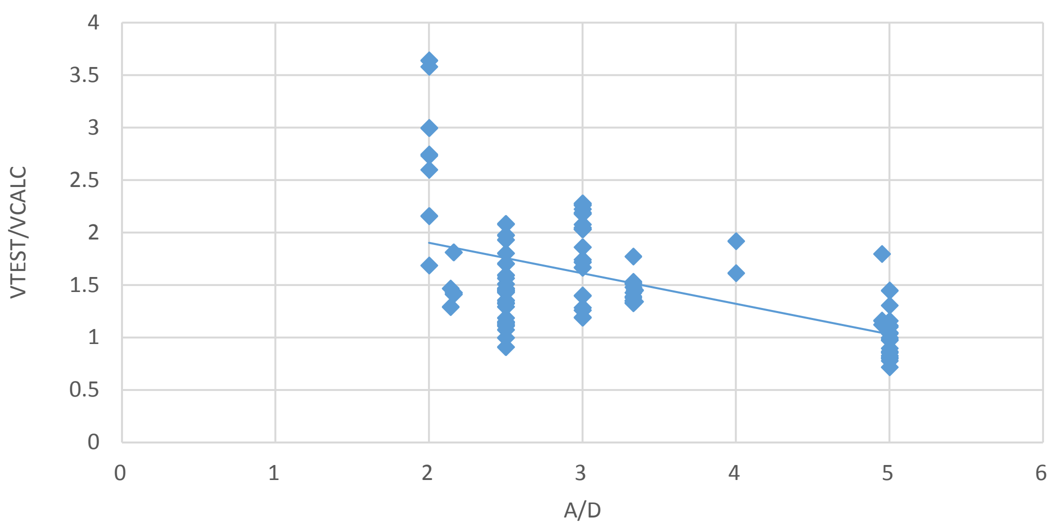

Figure 12 Experimental to Calculated vs Shear span

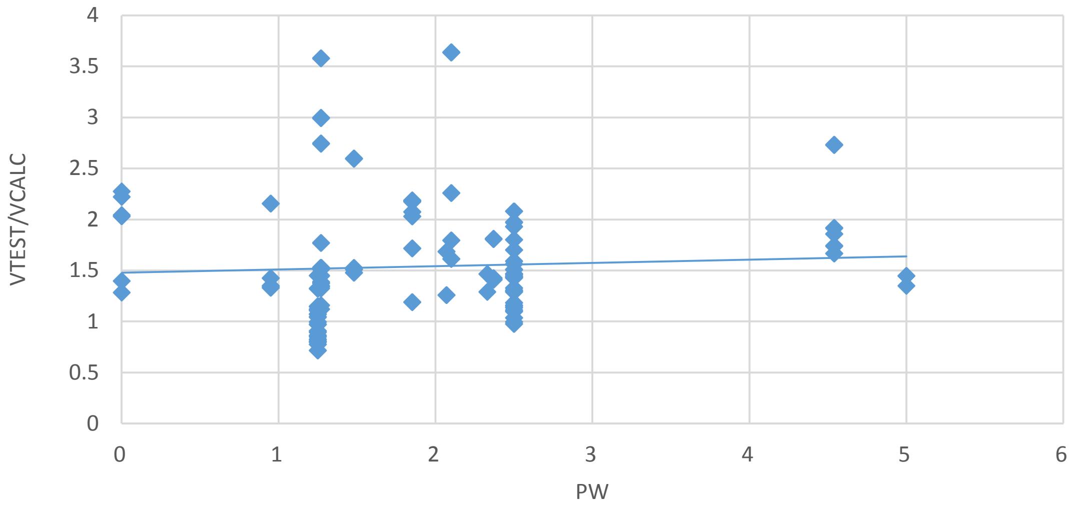

Figure 13 Experimental to Calculated vs Reinforcement Ratio

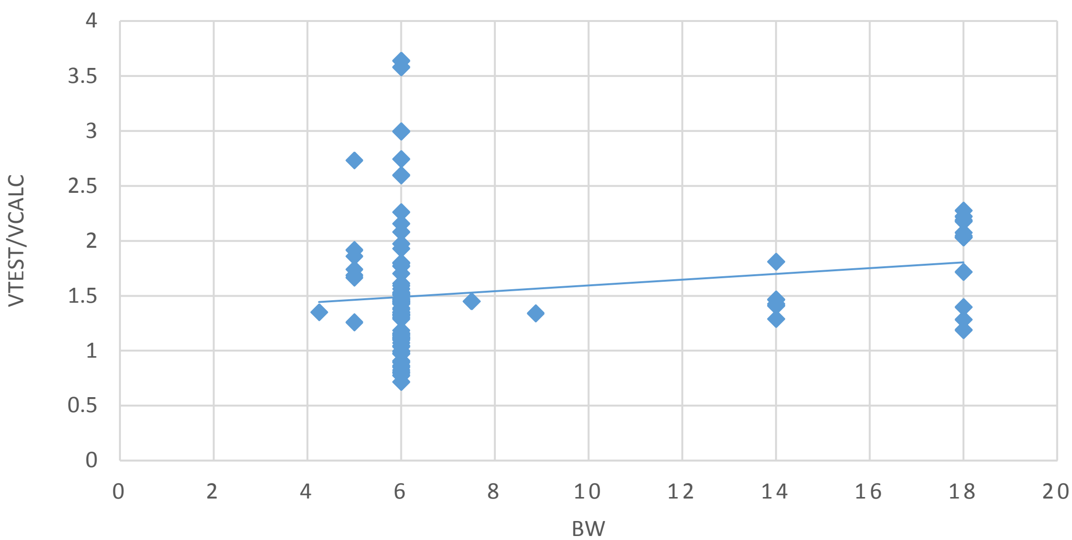

Figure 14 Experimental to calculated vs Beam Width

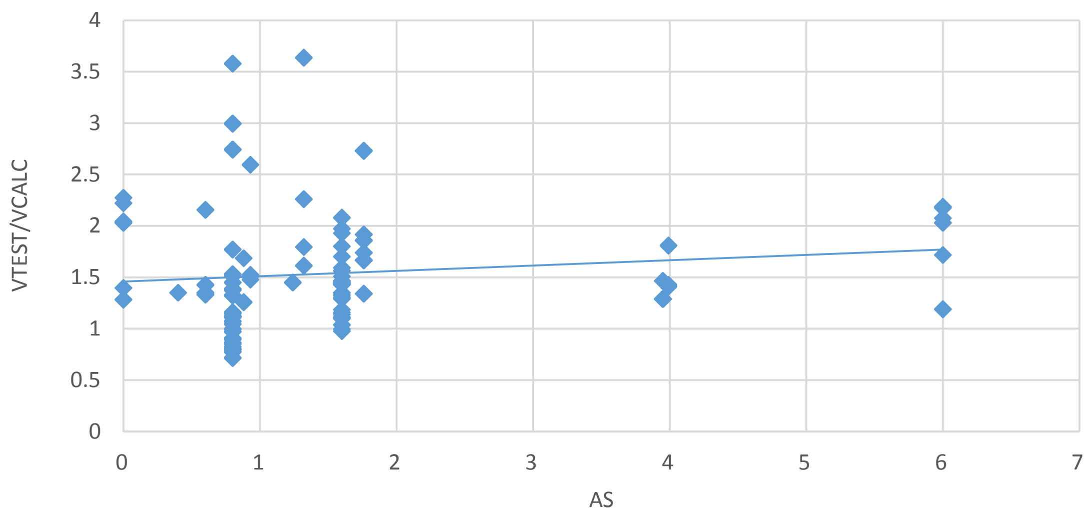

Figure 15 Experimental to Calculated vs Area of Steel

In general there does not seem to be a particular parameter that affects the prediction of the simplified method. While all these plots show values that have greater variability, majority of the values are higher than 1.0. This means that even though the simplified method is an approximation, it is a reliable approach to take if a budget allows for a more conservative concrete beam.

4.3 Shear Database Analysis Summary

From investigating the shear database we have seen that a large number of samples concludes to better understanding of how lightweight beams act. Either by using the general procedure or the simplified procedure the ratio of experimental to calculated value, which will be defined later as the professional factor, is consistent with all parameters. Two differences can be seen between the two. One is that using the simplified method causes higher ratios or professional factors. This is understandable because the simplified method is more of an approximation than using the general method, Therefore more conservatism is built into the procedure. The second difference that can be seen is there is more variability between data points using the simplified method. Even though ratios are higher the difference between those ratios are greater. This is also due to the approximation that the simplified method uses. The Mean and Standard Deviation from the shear database can be seen in Table 3 for each design method.

The professional factor is very important in doing a reliability analysis. It shows how reliable the design procedure is. If we cannot get a good approximation of how different beams will perform based on calculated code values than we don't understand shear enough to even perform a reliability analysis. The database shown earlier from Paczkowski only has minimal data looking at only a few different variables. With this larger database we can see how the professional factors vary between all the different variables.

Table 3 Bias and Standard Deviation for Shear Database

	Mean	Standard deviation
General Procedure	1.14	0.37
Simplified Procedure	1.43	0.55

5. STRUCTURAL RELIABILITY METHODS

Many different aspects of structures are usually considered static or deterministic values. Meaning if a designer were to order 4000 psi concrete from the plant, Designers assume they would receive concrete exactly the strength they ordered. That concrete would then be assumed to act exactly the same as the same strength of concrete you ordered a month ago. This false idealization is completely short sided. Almost all factors of structural design are random variables. For example most concrete ordered today has a produced strength well above that which is specified. The reliability of any structure is its ability to fulfill the design purpose for some specified lifetime by incorporating variability of engineering parameters (Nowak 2013). Using this technique, engineers can assess the probability that structures will fail their design purpose.

In this study, the resistance of a component is based on several different random variables. The variables that are used to find resistance are categorized by three different factors

- Material Factor M: Strength of materials, Modulus of elasticity.
- Fabrication Factor F: Variability in geometry due to casting in place or manufacturing differences
- Professional Factor P: This reflects the accuracy of the design model for each system

A Mathematical model can be built which represents the resistance of a system.

$$R = R_n MFP \tag{13}$$

Where $R_n$is the nominal shear resistance calculated by AASHTO 2012, which can also be expressed as V_n as stated earlier.

In this analysis, a bias factor for the fabrication and materials can be found based on results of the study. Once these values are calculated they can be combined with the professional factor which will be found from database results of multiple tests. The equations used to combine these factors is:(Scott 2010)

$$\lambda_R = \lambda_P \lambda_{FM} \tag{14}$$

$$COV_R = \sqrt{COV_P^2 + COV_{FM}^2} \tag{15}$$

Using these values an actual bias and coefficient of variation of the design procedure can be found.

The nominal value of a random variable, which is any variable that is used in design equations is considered the value that is used to design. Therefore, the nominal value for a 16 in. width beam would be 16 in.. A bias and coefficient of variation can then be applied to that nominal value. The bias is then a factor increasing or decreasing the nominal value based on fabrication and material factors as shown in equation 11. After applying a bias factor to a nominal value a true average or mean value of the variable is found. Taking a mean value and multiplying that by a coefficient of variation will obtain a standard deviation of the random variable. This has to be done for each random variable. For deterministic variables, the nominal value is treated as the mean value with a standard deviation of zero.

Each variable will also have its own unique distribution. An extremely common distribution that can describe many phenomenon even outside of engineering is the normal distribution. An example of a normal distribution PDF is shown in Figure 16.

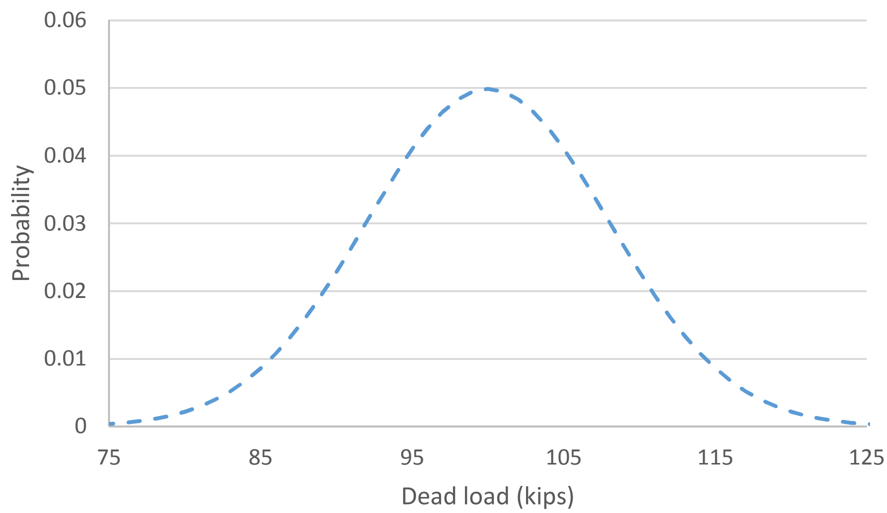

Figure 16-Normal distribution PDF curve produced from excel

Other types of distributions that will be used for this paper are lognormal distributions and Extreme Type I distributions. Each different type of distribution requires a transformation from normal random variables to lognormal or Type I variables. This will have to be done each time for each separate variable in our design equation. Examples of Lognormal and Extreme Type I distributions are shown in Figures 17 and 18.

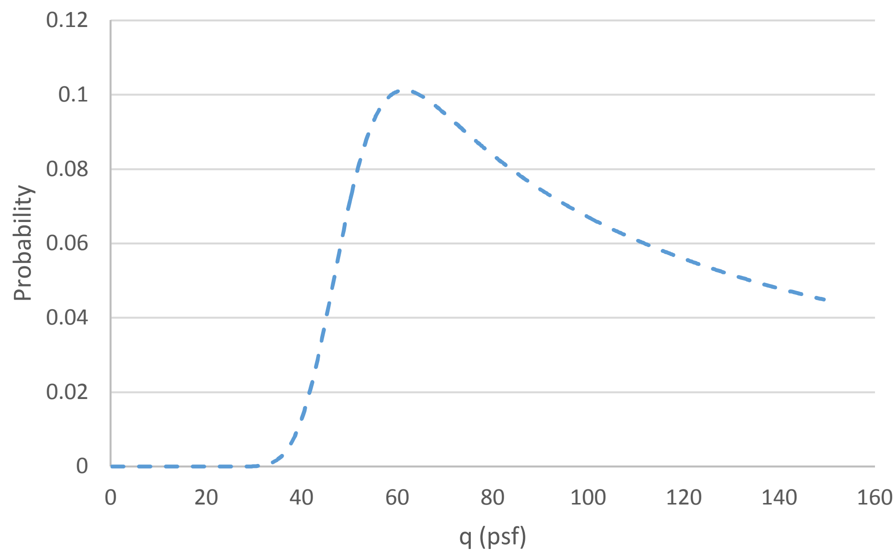

Figure 17 - Lognormal PDF Distribution curve produced form excel

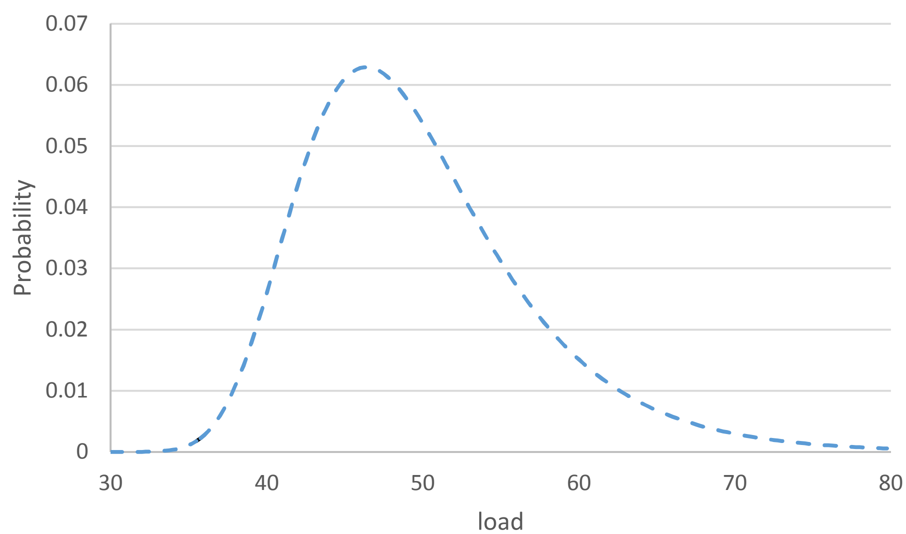

Figure 18 - Extreme Type I PDF distribution curve produced form excel

To run a simulation or reliability analysis a method of analysis has to be chosen. Introduced by Hasofer and Lind in 1974, the simplest way to find reliability can be found by:

(16)

This equation referred to as the general definition of reliability index, is limited to a normal distribution for both resistance and load. The equation is derived from Figure 19 adapted from Reliability of Structures second edition by Nowak and Collins. The shortest distance from the origin of reduced variables to the line $g(Z_R,Z_Q) = 0$ is considered the reliability index (β). Where the function $g(Z_R,Z_Q) = 0$ is the limit state function (Nowak and Collins 2013). Anything plotted below the limit state function is considered a failing conditions. Whereas plotted points above are safe values.

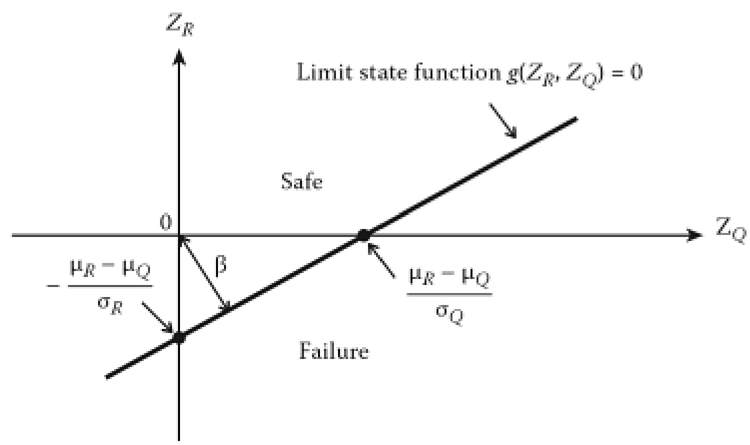

Figure 19-Reliability index, shortest distance to origin. Adapted from Nowak & Collins 2013

For normal distributions can be directly related to probability of failure by:

(17)

Table 4 gives examples for probability of failures compared to its corresponding reliability index

Table 4-Reliability index and probability of failure

Reliability index β and probablility of failure P_f	
P_f	β
10^{-1}	1.28
10^{-2}	2.33
10^{-3}	3.09
10^{-4}	3.71
10^{-5}	4.26
10^{-6}	4.75
10^{-7}	5.19

The probability of failure for the common target reliability of 3.5 is $2.33*10^{4}$

Other simulations that can be used are: First-order, second-moment reliability, The Hasofer-Lind reliability index, Rackwitz-Fiessler procedure, as well as the Monte Carlo Method. The latter method is the most common because it is a relatively easy analysis that can be programmed easily in most programming software as well as excel. The Monte Carlo method will be used for this analysis. Excel will also be used to run the simulations. Two different ways to obtain the reliability can be done the first is to do millions of simulations and compare the number of failures to the number of simulations. This method takes large amounts of computing power for the large limit state of shear in reinforced concrete. The other method which will be used for this analysis is to plot all limit state values produced on a normal probability plot and then find the y intercept of the curve fit. This can be seen in Figure 23.

6. STRUCTURAL LOAD MODELS

Structural loads can be defined as two different load types: Transient and Permanent. Permanent loads are loads that remain on the structure for extended periods of time. These consist of structural weight as well as other permanent objects added to the structure. Transient loads consist of movable loads on the structure. These could include vehicular traffic, pedestrians, as well as Wind, water, and earthquake loads. Each load type has specific factors that are applied to them depending on duration, type, severity of damage to structure, or large variations in the load. Live loads and Dead loads control for short to medium spans whereas environmental loads come into effect with larger spans (Paczkowski 2010). This study will be focusing on spans from 20 ft. to 100 ft. because R/C beams are only economical in shorter spans (Barker and Puckett 2007). Therefore a load combination of dead and live load will represent well.

Dead load in this case will be comprised of two different components. The structural material (DL_2) being the weight of cast in place concrete being used to calculate resistance for the section. This study will be using the nominal value of 120 pcf as stated before with bias and coefficient of variation displayed in Table 5. The other aspect of deal load is the nonstructural wearing surface (DL_3) applied to the top of the bridge deck. The nominal value of the wearing surface is being estimated as 140 pcf with statistical parameters displayed in Table 5 (NCHRP 655). Dead load will be individually calculated based the section size for each span length. The distribution type will be assumed as a normal distribution for all types of dead load. This assumption is common among many research programs, Nowak & Collins, Paczkowski, Galambos, Rakoczy.

Table 5-Statisitical Parameters for Dead load Adapted from NCHRP-655

Dead Load Component	Bias Factor λ	Coefficient of Variation, V
DL_1	1.03	0.08
DL_2	1.05	0.1
DL_3	1.00*	0.25
DL_4	1.05	0.10

Live load will be produced by vehicles moving along to bridge. Two different cases of vehicles will be used for the study. The first being lane load of basic traffic, which is modeled by a 640 psf distributed load along the girder. Moment and shear diagrams from the distributed load were produced at every 10 foot increment to be used in analysis. The second type is the AASHTO LRFD HS20 truck load. Load distribution of the HS20 truck load is shown in Figure 20 (NCHRP 368). In this case, load envelopes were modeled in SAP 2000 finite element program to find moment and shear envelopes. These models were computed for the case of 1, 2 and 3 span bridges with lengths varying from 20" to 100" in increments of ten feet. A representative shear and moment envelope are presented in Figure 21. For truck loads, an impact factor must be applied to the moment and shear calibrated by Missouri Transportation In statute (2010) factor that will be used to find nominal loads is 1.33 described by AASHTO LRFD. The impact factor will be described as a random variable with a mean value of 1.1 and a standard deviation of 0.08 (Paczkowski 2010).

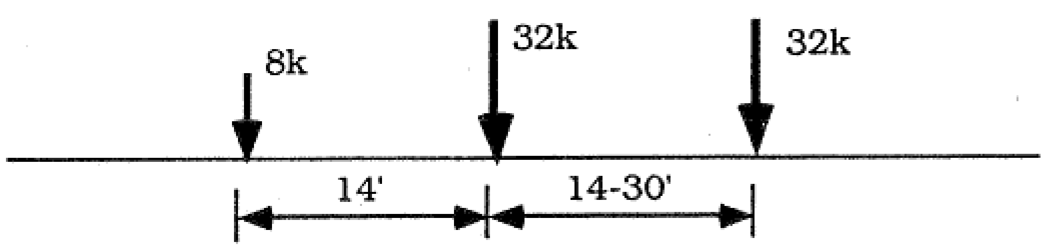

Figure 20-HS20 truck model from NCHRP 368 Nowak 1999

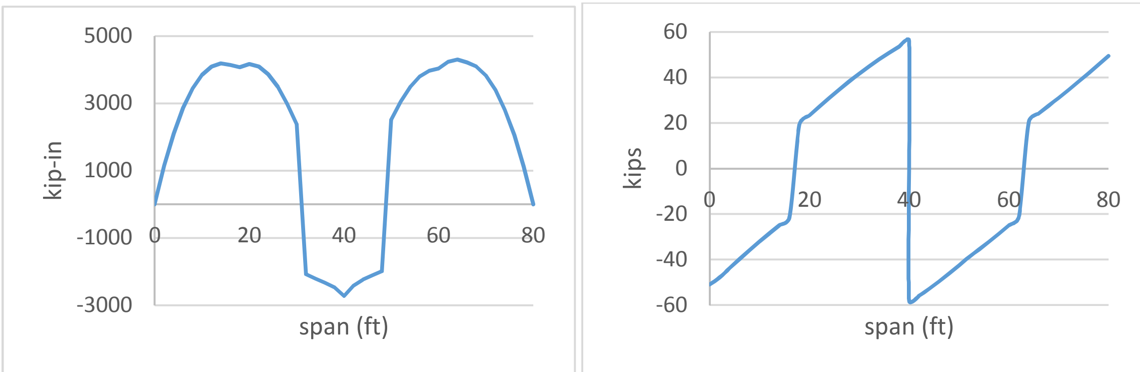

Figure 21-Shear (right) and Moment (left) envelopes for a 40 ft 2 span bridge due to an HS20 Truck

In this study we are only looking at a single interior girder in a bridge system for this case we need to distribute the loads form live load to all the girders. This will be done with distribution factors from AASHTO LRFD. Table 4.6.2.2.2b-1 for moment and Table 4.6.2.2.3a-1 for shear are used to calculate the distribution factor. For this a few assumptions had to be made: One design lane is loaded, only interior girders are being analyzed, girder spacing is between 3.5 and 16 feet, the thickness of the slab is between 4.5 and 12 inches, the span length is between 20 ft and 240 ft, and the number of beams in the bridge has to be greater than 4. Live load bias and coefficient of variation of truck load and lane load are assumed to have the statistical parameters. A bias of 1.25 will be used for both shear and moment however a coefficient of variation of 0.14 will be used for shear and 0.16 for moment. The distribution type for all live loads will be modeled as an Extreme Type I (Nowak 1993).

Bridges are designed according to the Strength I load combination:

$$1.25DL_2 + 1.5DL_3 + 1.75(LL + IM) \leq \emptyset_v(V_c + V_s) \quad (18)$$

Where $\emptyset_v$ is the resistance factor for shear in RC beams. From this we can form an equation for the required nominal resistance. (Paczkowski 2010).

$$R_n = \frac{1.25DL_2 + 1.5DL_3 + 1.75LL}{\emptyset_v} \quad (19)$$

7. RESISTANCE MODELS

The resistance will be computed as our as described earlier from AASHTO 2012. Values for load have been previously modeled and selected based on section, span length and number of spans. The next step is to select section values for the following: girder spacing (8 ft), flange height (7.5 in), beam width (14 in), and number of beams (6). The assumptions for these values are based on recommendations from Design of Highway Bridges by Barker and Puckett (2007). Average flange heights used in design are from 7 inches to 7.5 inches, an example depth of 7.5 will be used for all sections analyzed. The girder spacing recommendations are at an 8 ft. girder spacing. This will be assumed as an appropriated girder spacing for our analysis. The number of beams will be set to six and assume that no load of the barriers will be transferred to the inside girders. The last assumed value will be the beam width which will be analyzed at 14 inches and 20 in. these widths will allow plenty of room for longitudinal reinforcement to be placed in the bottom of the girder. Figure 22 will show sections of design girder and design bridge.

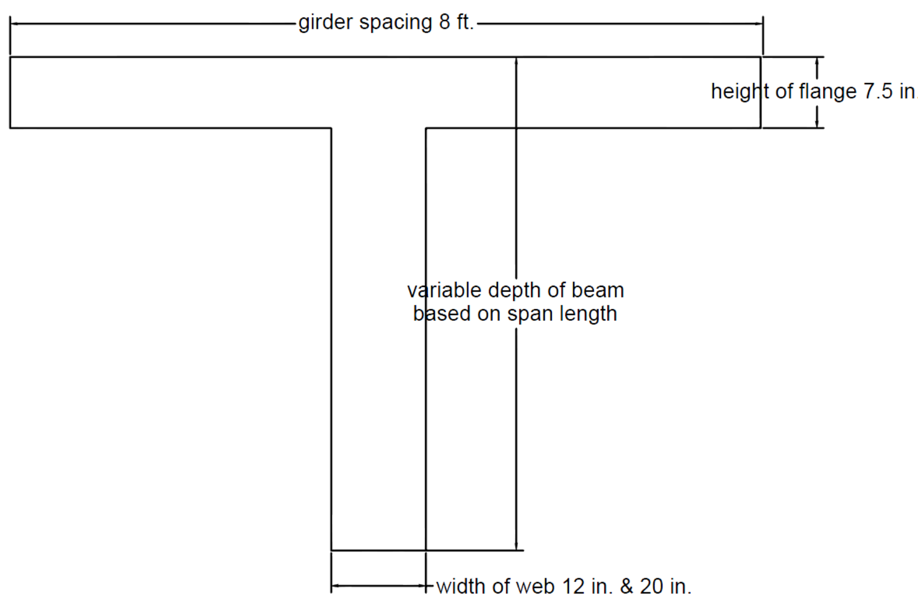

Figure 22 - T-beam section Investigated

After section values have been set to nominal values. Values for steel properties are set at 29,000 ksi for modulus of elasticity and 60 ksi for yield strength of steel. A span length and number of spans is now chosen. For this study analysis of only 2 and 3 span bridges are being analyzed, for spans from 20 ft. to 100 ft. Most T-beam bridges being built today are short span bridges so reliability analysis over 120 ft. are not applicable to today's design standards.

Once preliminary values are set for the analysis, the depth of the T-beam is decided based on Table 2.5.2.6.3-1 from AASHTO LRFD shown by Table 6. The equation used to calculate the depth of the T-beam section is based on span length is:

$$D_{min} = 0.065L \tag{20}$$

Values calculated for D_{min} will be rounded up to the nearest inch because of constructability. The next step is to find the critical section for shear and choose an area of reinforcing steel. This will be done by using the Service I condition area approximation:

$$A_s \approx \frac{M}{f_s jd} \tag{21}$$

Where M is the factored moment M_u at the critical section due to all loads, $f_s = .6f_y$, $j = .875$, and $d = dv$

Because A_s and d_v are both unknown values this process is an iterative approach.

The moment associated with the critical section is also based on dv so iteration is required between $d_v, A_s, M_u, a,$ and d_e. This can get extremely tedious by hand so the equations are programmed into an excel sheet and iterated multiple times.

Table 6-Traditional Miminum Depth for Constant Depth Superstructures. AASHTO 2012

Superstructure		Minimum Depth (Including Deck) When variable depth members are used, values may be adjusted to account for changes in relative stiffness of positive and negative moment sections	
Material	Type	Simple Spans	Continuous Spans
Reinforced Concrete	Slabs with main reinforcement parallel to traffic	$\frac{1.2(S+10)}{30}$	$\frac{S+10}{30} \geq 0.54$ ft.
	T-Beams	$0.070L$	$0.065L$
	Box Beams	$0.060L$	$0.055L$
	Pedestrian Structure Beams	$0.035L$	$0.033L$
Prestressed Concrete	Slabs	$0.030L \geq 6.5$ in.	$0.027L \geq 6.5$ in.
	CIP Box Beams	$0.045L$	$0.040L$
	Precast I-Beams	$0.045L$	$0.040L$
	Pedestrian Structure Beams	$0.033L$	$0.030L$
	Adjacent Box Beams	$0.030L$	$0.025L$
Steel	Overall Depth of Composite I-Beam	$0.040L$	$0.032L$
	Depth of I-Beam Portion of Composite I-Beam	$0.033L$	$0.027L$
	Trusses	$0.100L$	$0.100L$

Once these nominal values are all calculated, the critical section is applied to each load type and the corresponding shear and moment is calculated for the critical sections of each support. It has been found that for multi span bridges, the maximum moments and shears at all critical section occur in negative moment by the inner supports. It has also been seen observed that truck loads govern all load combinations in shorter spans. Therefore, a critical section is selected from truck loads based on the maximum strain (. Corresponding sections from other loads are selected for nominal values. Area of shear reinforcement (must be calculated using factored loads divided by the resistance factor (. The equation used to calculate has been derived from AASHTO LRFD combining equations 3,4,5,7,8,9,10, and 11 from above.

$$A_v = \frac{\frac{V_u}{\emptyset_v} - .0316\left(\frac{4.8}{1 + 750\left(\frac{\frac{|M_u|}{d_v} + \frac{|V_u|}{\emptyset_v}}{E_s A_s}\right)}\right)\sqrt{f_c'}\,b_v d_v}{f_y d_v cot\left(29 + 3500\left(\frac{\frac{|M_u|}{d_v} + \frac{|V_u|}{\emptyset_v}}{E_s A_s}\right)\right)}$$

This gives a complex but closed form equation for A_v.

Statistical parameters on all resistance values are presented in Tables 7-9. The distribution of these values are all normal with the exception of the yield stress of steel which is lognormal. Modulus of elasticity can be modeled as either normal or lognormal but for this study is being modeled as a normal distribution with a mean value of 30,000 and coefficient of variation of 0.0327 (Mansour et al. 1984).

This study will only look at lightweight concrete 4000 and 5000 psi because these are common for cast in place concrete strengths. Comparison to other strengths and ordinary concrete can be found in Table 7.

Table 7-Parameters for Concrete Strength from Nowak And Collins 2013

Statistical parameters for strength of concrete				
	Compressive Strength		Shear Strength	
Property f'c	Bias Factor, λ	Coefficient of variation, V	Bias Factor, λ	Coefficient of variation, V
Lightweight concrete				
3000	1.38	0.155	1.38	0.185
3500	1.33	0.145	1.33	0.175
4000	1.29	0.14	1.29	0.17
4500	1.25	0.135	1.25	0.16
5000	1.22	0.13	1.22	0.155
5500	1.2	0.125	1.2	0.15
6000	1.18	0.12	1.18	0.145
6500	1.16	0.12	1.16	0.145
Ordinary concrete				
3000	1.31	0.17	1.31	0.205
3500	1.27	0.16	1.27	0.19
4000	1.24	0.15	1.24	0.18
4500	1.21	0.14	1.21	0.17
5000	1.19	0.135	1.19	0.16
5500	1.17	0.13	1.17	0.155
6000	1.15	0.125	1.15	0.15
6500	1.14	0.12	1.14	0.145

Table 8-Parameters for yield strength of Reinforcing Steel from Nowak and Collins 2013

Statistical parameters for reinforcing steel bars		
Bar size	Bias Factor, λ	Coefficient of variation, V
#3	1.18	0.04
#4	1.13	0.03
#5	1.12	0.02
#6	1.12	0.02
#7	1.14	0.02
#8	1.13	0.025
#9	1.14	0.02
#10	1.13	0.02
#11	1.13	0.02
#14	1.14	0.02
reccommend	1.13	0.03

Two different reinforcing steels are used in AASHTO LRFD shear design, bias factors and coefficient of variations for the yield strength of the steel vary with bar size. Most longitudinal reinforcing is on the range of a number 8 to 14 bar and shear reinforcement using an average bar size of number 4. This analysis will be using the recommended values of bias of 1.13 and a coefficient of variation of 0.03 (Rakoczy and Nowak 2013).

Table 9-Parameters for Fabrication Factors from Rakokzy 2013

Statistical parameters for fabrication factor		
Item	Bias Factor, λ	Coefficient of variation, V
Width of beam, cast in place	1.01	0.04
Effective depth of a R/C beam	0.99	0.04
Effective depth of prestressed concrete beam	1	0.025
Effective depth of slab, cast in place	0.92	0.12
Effective Depth of slab, plant cast	1	0.06
Column width and breadth	1.005	0.04
Area of reinforcement, As, Av	1	0.015
Spacing of shear reinfrcement	1	0.04

Factors for fabrication with a bias of 0.99 and coefficient of variation of 0.04 are used to modify . Assuming these values are directly related to the effective depth of concrete section.

8. MONTE CARLO ANALYSIS

Once nominal values for all variables are established, a statistical based analysis can be performed. A Monte Carlo analysis can be performed on the nominal values with 2 x 10^4 simulations. Once all variables are simulated, β, θ, and ε_s are calculated by applying the Service I load factors to M_u and V_u. V_n can now be expressed as a load resistance. Using the limit state function $g(V_n, V_u) = V_n - V_u$ we compare V_n to an unfactored shear force. If the limit state function is below zero then a failure has occurred. Once all values are recorded for every simulation a reliability index can be found. A curve fit is attached to the data and the y intercept of the curve fit is recorded as the reliability index for that simulation. An example of one simulation is shown in Figure 23.

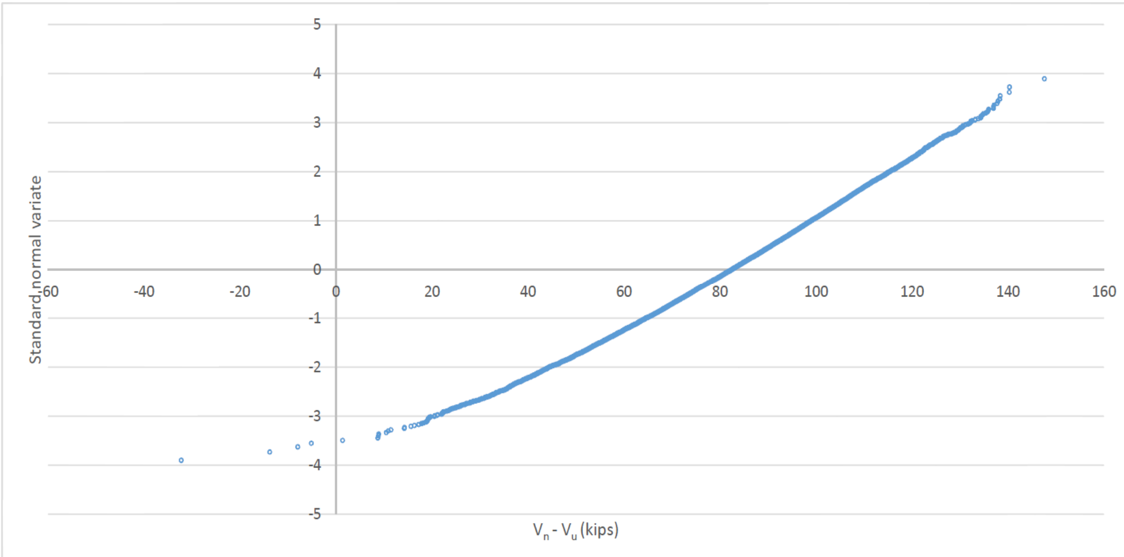

Figure 23-Reliability simulation example

Simulations for every span length and resistance factor are executed to produce an envelope for the resistance of a 2 and 3 span girder based on resistance factors 0.75 to 0.9 in 0.05 increments.

A bias factor and a coefficient of variation is also found from the Monte Carlo analysis by comparing the nominal resistance to the calculated resistance. The bias changes based on the resistance factor used in the analysis The bias is shown in Figures 24 and 25 with a range from 1.25 – 1.5 and coefficient of variation shown in Figures 26 and 27 shows a steady decrease in variation with span length.

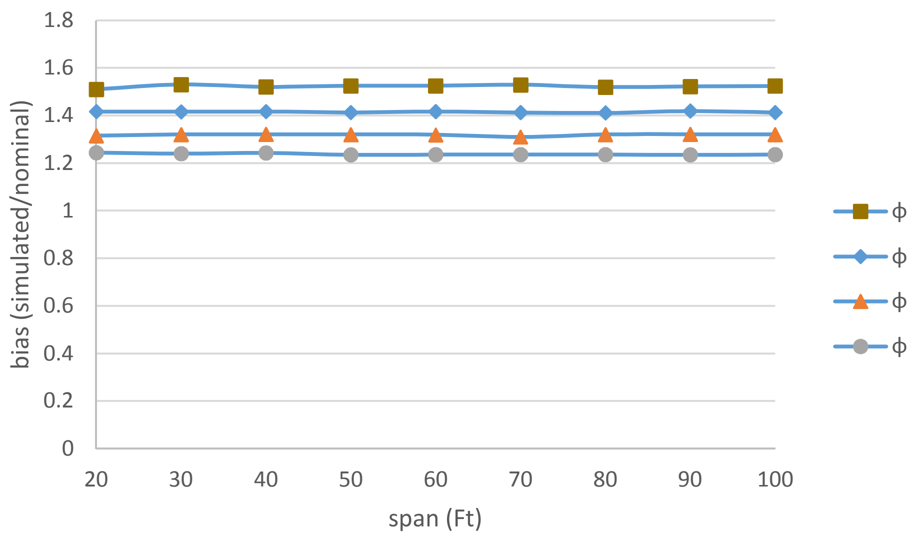

Figure 24-Bias for 2 span girder & 4000 psi concrete

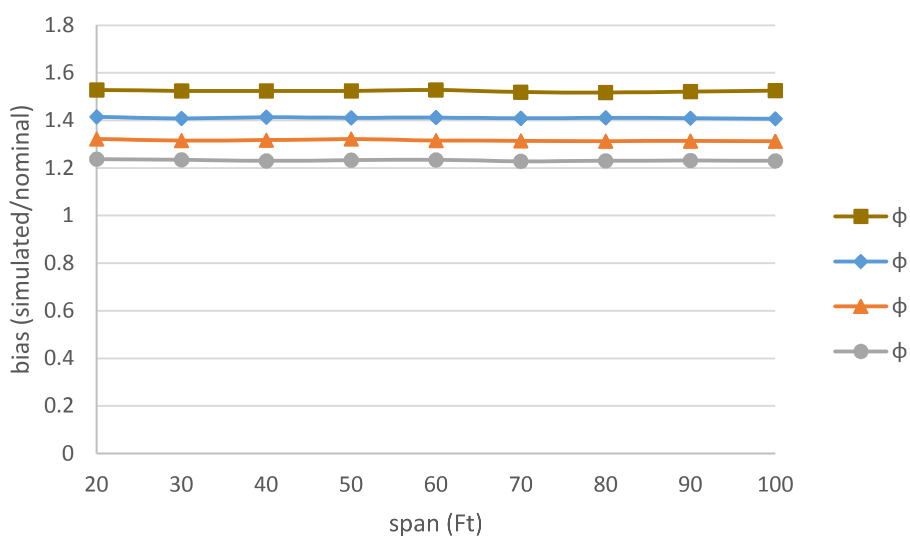

Figure 25-Bias for 2 span girder & 5000 psi concrete

The differences in the bias between 4000 psi concrete and 5000 psi concrete are very minimal. The plots show that with change in concrete parameters the bias remains relatively constant and doesn't increase or decrease with span length.

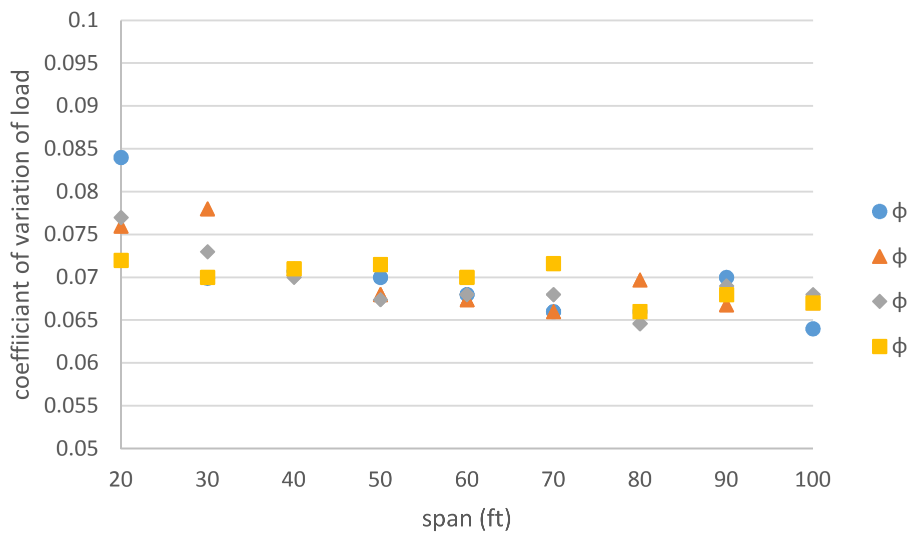

Figure 26-COV of 2 span 4000 psi concrete

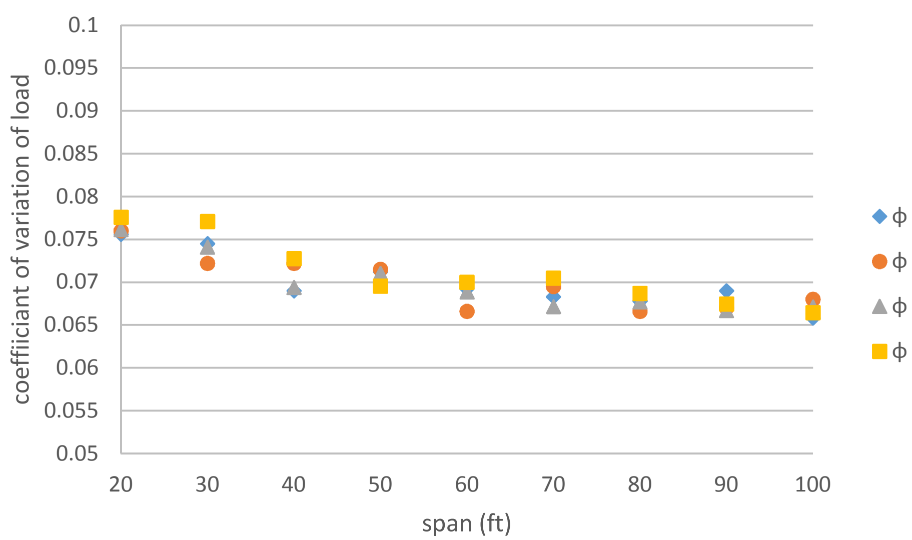

Figure 27-COV of 2 span 5000 psi concrete

The coefficients of variation found in this analysis are comparable to each other as the span length increases. Large variations in coefficients are found in lower spans and as span increases coefficients get smaller and more consistent with each other.

Analysis of 4000 and 5000 psi concrete with a professional factor of 1 applied to the analysis is shown in Figures 28 and 29. The reliability indices are plotted on graphs to compare simulated reliability to the target reliability of 3.5. It can be seen that with the way variables are calculated in this analysis and using a varying depth of T-beam the reliability increases with the length of span.

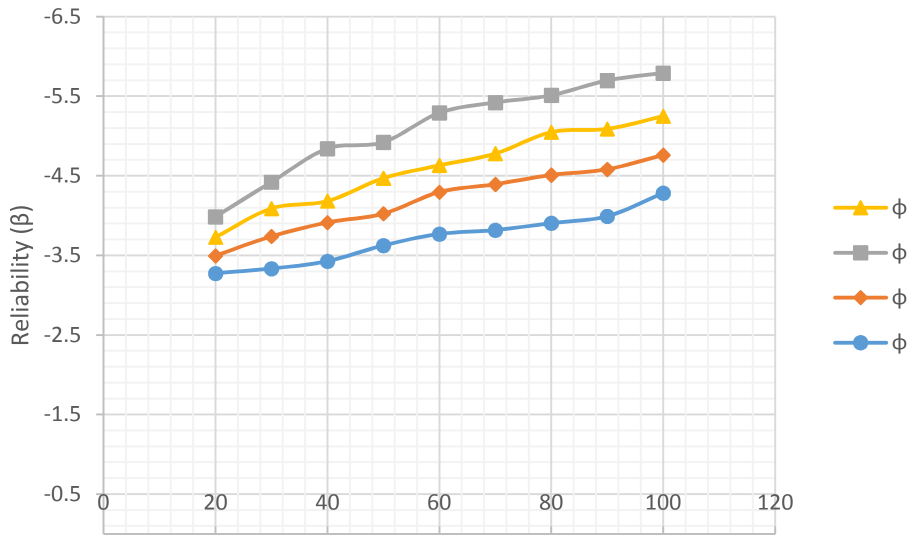

Figure 28-Reliability indices for 2 span 4000 psi concrete

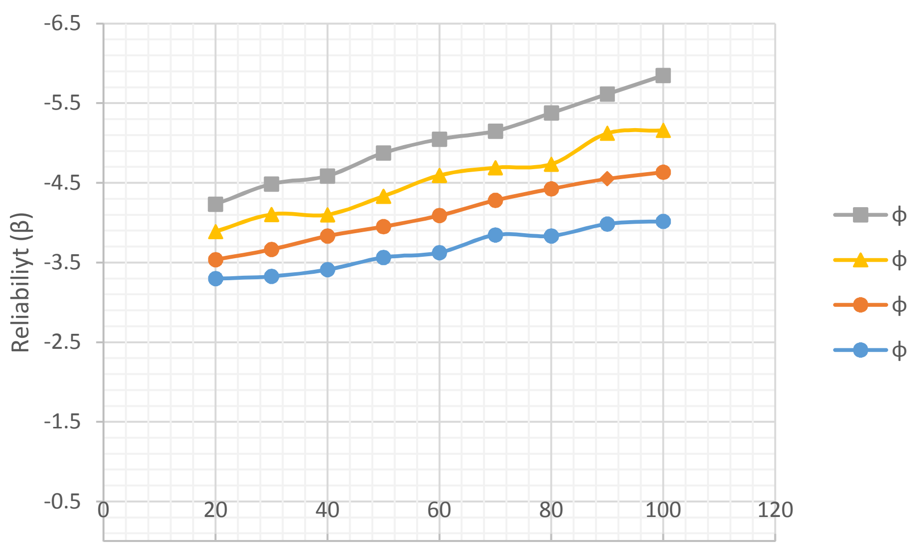

Figure 29-Reliabiliyt indices for 2 span 5000 psi concrete

It can also be seen that to meet the target reliability of 3.5 for all span lengths a resistance factor of 0.8 should be used. Previously calibrated in NCHRP 368 in 1999 by Nowak the resistance factor was set at 0.7. After extensive analysis for multispan bridges the resistance factor can be raised for lightweight concrete in shear using the general procedure AASHTO 2012.

9. CONCLUSION

A calibration of the AASHTO LRFD code has been established with reductions applied for lightweight concrete. By looking at almost a hundred lightweight concrete beam tests we have seen how the ratio of experimental to calculated shear values change with the change of different variables. Calculating the shear in two different ways using the general method and the simplified method has shown that using the approximate simplified method is a reliable approach that can be used in the case of an easy calculation.

Loads were produced using a finite element software to create a truck load envelope on multiple span bridges. Maximum shear values were recorded and then corresponding moments were recorded as well. Based on loads applied design nominal values were set to appropriate values based on AASHTO LRFD. These values are designed and chosen with minimal unintentional safety factors. A reliability analysis using the Monte Carlo method was performed with the most up to date statistical parameters of load and resistance used in the resistance equation as well as for loads applied to the bridge. Bias values were calculated and shown to correspond to the resistance factor, however COV of the analysis was shown to correlate to the span length used. Once reliability indices were found it, a resistance factor of 0.8 was show adequate for the design value.

Many things could be changed in this analysis. A larger database of lightweight beams could be looked at to see if the outcomes of this analysis are still accurate with even more tested beams. Another change that should be made is changing different span widths in the reliability analysis. Lowering the span width will cause less concrete to resist shear and should be controlled more by reinforcement steel. Different analysis procedures to find the area of longitudinal reinforcement should be used to see if the approximation used is an appropriate approximate. This analysis only looks at interior beams but exterior beams can control a bridge. Alternate analysis could look at the difference between exterior and interior girders and which controls based on span length. Alternative studies to come can use the updated information provided in this report as base points for many studies to come. A study on prestressed could be easily adapted from the procedures used in this analysis. Most variable for R/C girders and Prestessed girders are the same. Variables that pertain to prestressed girders would just be included in the current equations used for reinforced concrete.

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