# Autonomous Navigation using Gravity Gradient Measurements

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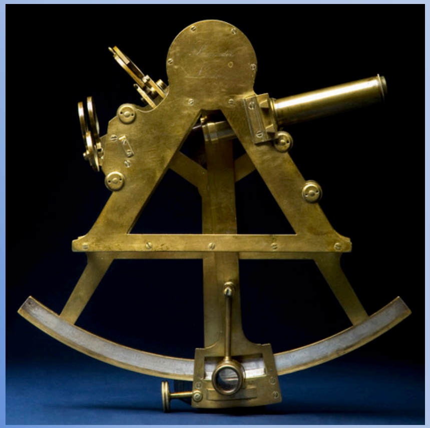
13th April 2017
Student Research Symposium

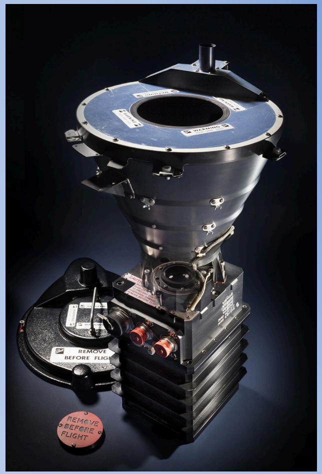

Where?

The art of navigation was first developed with the need to explore sea. Earliest records show that navigation is older than 1000 B.C.

On 21st Dec 1968, with the launch of Apollo 8 mission, we pioneered Space Navigation.



When?



# Satellite Navigation Systems

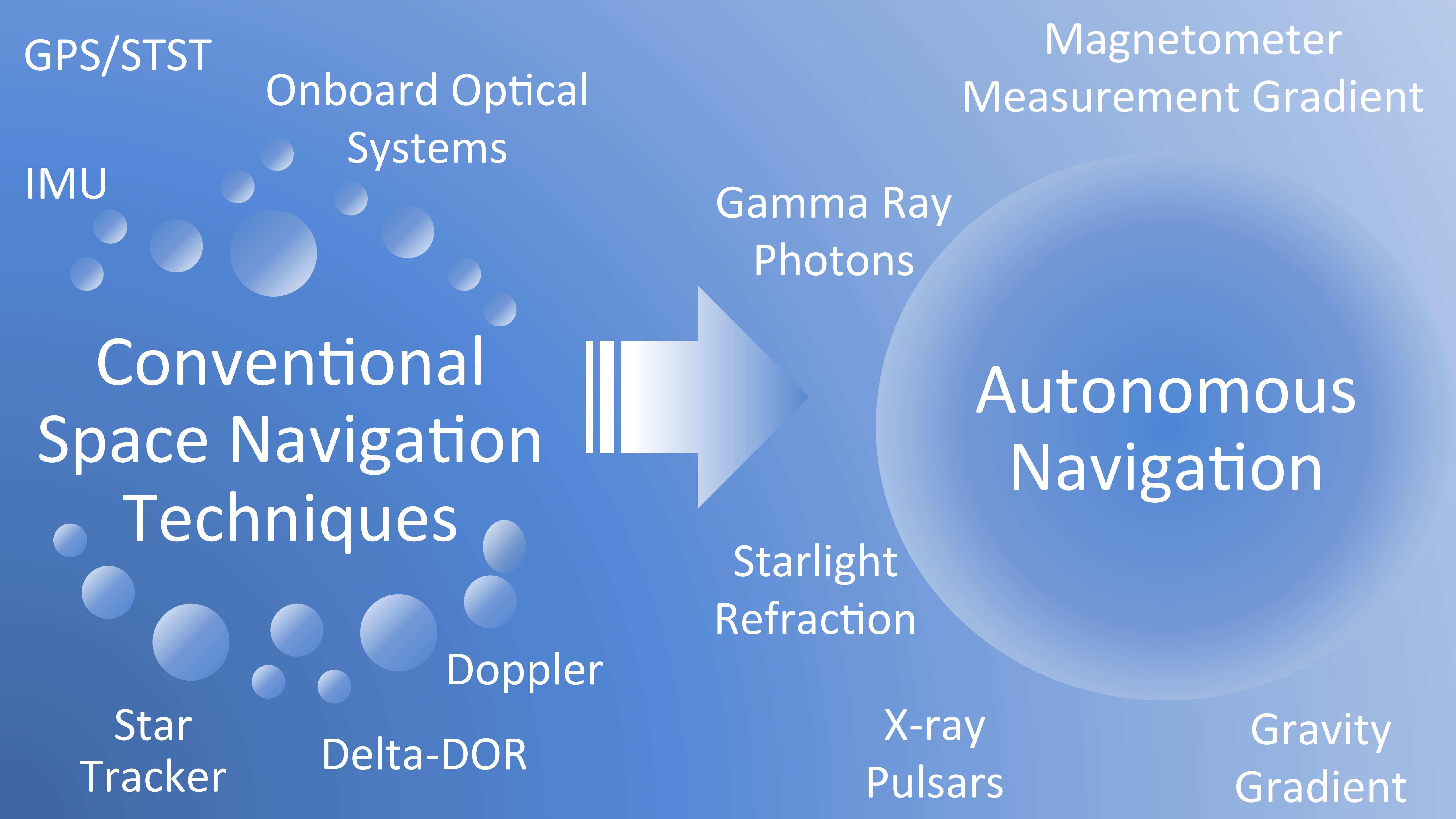
| Global Navigation Satellite System (GNSS) | | Regional Navigation Satellite System (RNSS) | |
|---|---|---|---|
| Operational | In Development | Operational | In Development |
| 1. GPS (USA)<br>2. GLONASS (Russia) | 1. Galileo (EU)<br>2. Compass (China) | 1. NAVIC (India)<br>2. BeiDou (China) | 1. QZSS (Japan) |

## Current GNSS

1. Strong Dependence on Ground-Based Infrastructure $\Rightarrow$ Low Accuracy
2. Range Limitation, Constant Maintenance Requirement, & Continuous Tracking $\Rightarrow$ Unsuitable for Beyond Earth Exploration Missions

## Autonomous Navigation

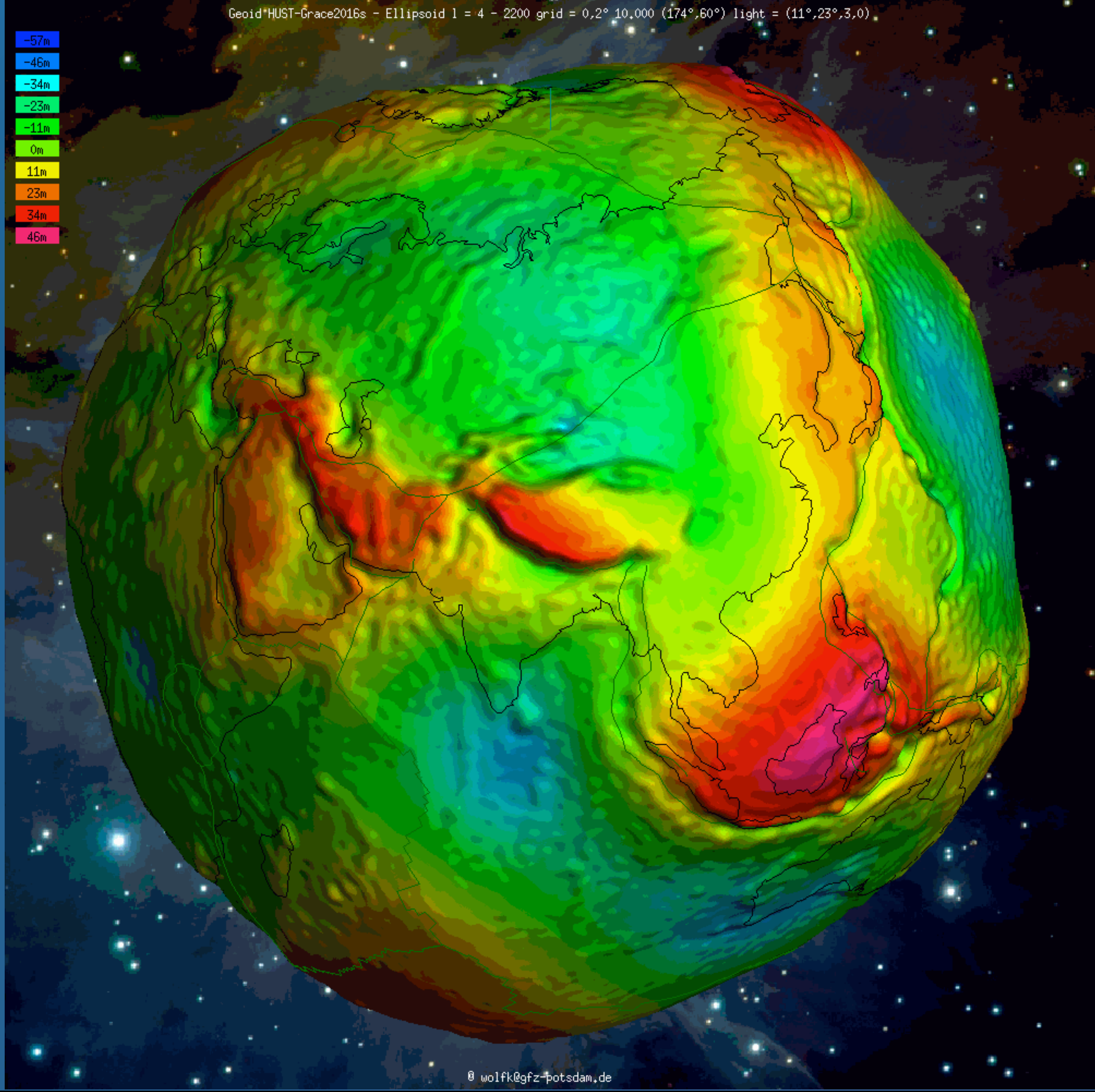
1. Self-Contained & Passive System $\Rightarrow$ Enhanced Accuracy
2. Autonomous, Resistant to Signal Blockage & Spoofing $\Rightarrow$ Suitable for Beyond Earth Exploration Missions


GPS/STST
Onboard Optical Systems
IMU
Conventional Space Navigation Techniques
Star Tracker
Delta-DOR
Doppler
Magnetometer Measurement Gradient
Gamma Ray Photons
Autonomous Navigation
Starlight Refraction
X-ray Pulsars
Gravity Gradient

Conventional Space Navigation + Gravity Gradiometry = Autonomous Space Navigation

The Gravity Gradiometry has been in use since mid 20th century, mostly for Marine Navigation & the survey of Mineral/Oil Fields.

However, the space application of the Gravity Gradiometer has been very limited.




Source- International Center for Global Earth Models (ICGEM), the model used is HUST-Grace2016s, with Orion Nebula in the background.

The Gravity Gradient Tensor ($\nabla g$) is defined as the second order derivative of the gravitational potential $U$-:

$$\nabla g_{ij} = \frac{\partial^2 U}{\partial r_i \partial r_j} \quad , \quad i,j = X,Y,Z$$

$r$ is Position vector

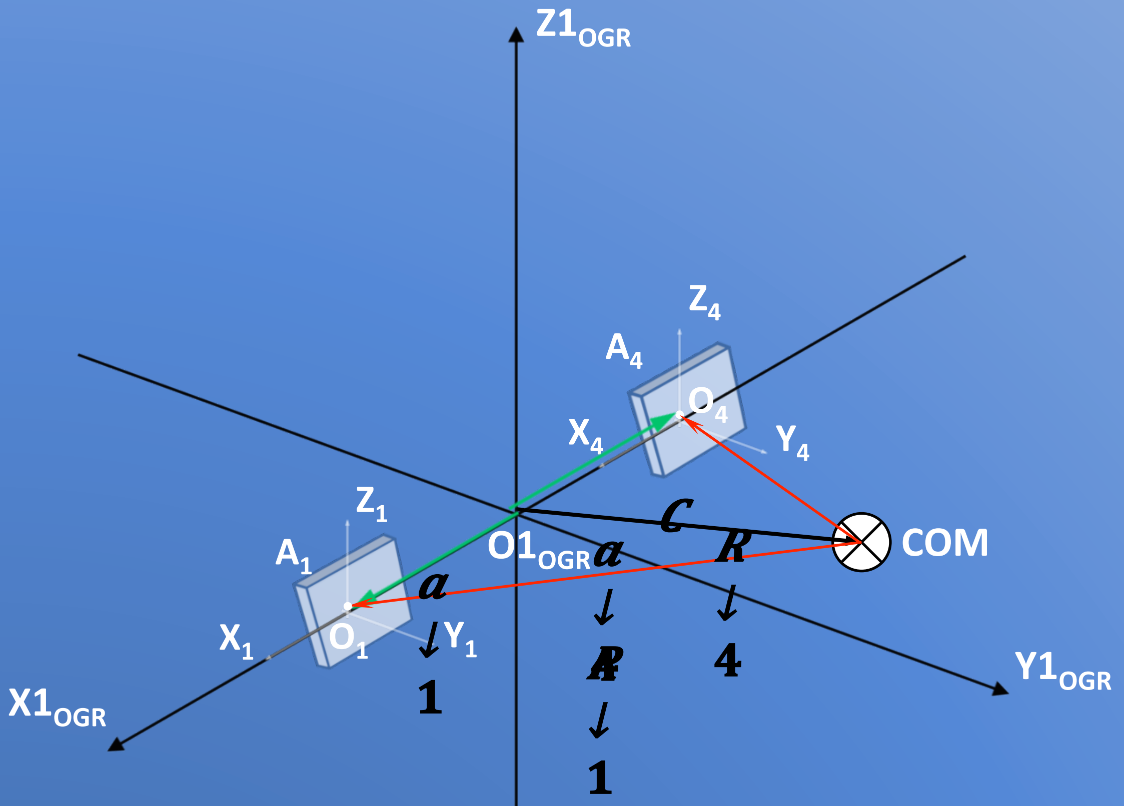
$$\nabla g = \begin{bmatrix} \nabla g_{XX} & \nabla g_{XY} & \nabla g_{XZ} \\ \nabla g_{XY} & \nabla g_{YY} & \nabla g_{YZ} \\ \nabla g_{XZ} & \nabla g_{ZY} & \nabla g_{ZZ} \end{bmatrix}$$ (Cesare S., 2008)

**Artist's view of the GOCE satellite (image credit: ESA-AOES MediaLab)**

# History of Gravity Gradiometer Instruments(Richeson J.A., 2008)

| Gradiometer | Developer | Noise, 1-$\sigma$ Eö | Data Rate,sec |
|---|---|---|---|
| Rotating Accel. GGI | Bell Aerospace/Textron | 2(Lab.),10 (Air) | 10 |
| Rotating Torque GGI | Hughes Research Lab | 0.5(Goal) | 10 |
| Floated GGI | Draper Lab | 1(Lab.) | 10 |
| Falcon AGG | LM/BHP Billiton | 3 | Post Survey |
| ACVGG | Lockheed Martin(LM) | 1 | 1 |
| 3D FTG | LM/Bell Geospace | 5 | Post Survey |
| FTGeX | LM/ARKeX | 10(Goal) | 1 |
| UMD SGG (Space) | Univ. of Maryland | 0.02(Lab.) | 1 |
| UMD SAA (Air) | Univ. of Maryland | 0.3(Lab.) | 1 |
| UWA OQR | Univ. of Western Australia | 1(Lab.) | 1 |
| Exploration GGI | ARKeX | 1(Goal) | 1 |
| HD-AGG | Gedex/UMD/UWA | 1(Goal) | 1 |
| Electrostatic GGI | European Space Agency | 0.001(Goal) | 10 |
| Cold Atom Interfer. | Stanford Univ./JPL | 30(Lab.) | 1 |


**Illustration of EGG system onboard GOCE (image credit:ESA,ONERA)**




The objective is to use 6 accelerometers arranged on a distance of 1 meters, on three mutually perpendicular baselines, as shown in the figure. (Cesare S., 2008)

Assuming that all perturbations, except drag are negligible. A simple acceleration Measurement Model (ECI frame) can be defined as-:

$$a_i = a_{grav}(r_{Sc}) - a_{grav}(r_{Sc} + R_i) + a_{drag}(r_{Sc}, V_{Sc}) + \omega \times (\omega \times R_i) + (\omega \times R_i) + 2\omega \times R_i + R_i$$

The accelerometer model can now be written as-:

$$a_i = -(\nabla g - [\Omega^2] - [\Omega])R_i + 2[\Omega]R_i + R_i + D$$

where term $(-\nabla g)\ R_i = a_{grav}(r_{Sc}) - a_{grav}(r_{Sc} + R_i)$,

$[\Omega] = \begin{bmatrix} 0 & -\omega_Z & \omega_Y \\ \omega_Z & 0 & -\omega_X \\ -\omega_Y & \omega_X & 0 \end{bmatrix}$ is the cross-product matrix, and

$D$ is the acceleration due to non-gravitational forces, like Atmospheric Drag.

Assuming ideal case the 3 OAGRFs are coincident, we get: $C_1 = C_2 = C_3 = C$

The vectors $R_i$ and its derivatives can thus be expressed as-:

$$R_i = A_i - C, \quad R_i = -C, \quad R_i = -C$$

Rewriting the equation for $a_i$, we get-:

$$a_i = -(\nabla g - [\Omega^2] - [\Omega])(A_i - C) + 2[\Omega](-C) - C + D$$

$$\Rightarrow a_i = -(\nabla g - [\Omega^2] - [\Omega])A_i + (\nabla g - [\Omega^2] - [\Omega])C - 2[\Omega]C - C + D$$

To isolate the Perturbation (Drag) and Gravity Gradient Tensor, we define following two modes-: (Cesare S., 2008)

1. Common-Mode Acceleration measured by the accelerometers $A_i$ , $A_j$ -:

$$a_{c,ij} = 1/2\,(a_i + a_j)$$

$$\Rightarrow a_{c,ij} = -(\nabla g - [\Omega^2] - [\Omega])A_{c,ij} + (\nabla g - [\Omega^2] - [\Omega])C - 2[\Omega]C - C + D$$

where $A_{c,ij} = 1/2\,(A_i + A_j)$

To isolate the Perturbation (Drag) and Gravity Gradient Tensor, we define following two modes-: (Cesare S., 2008)

2. Differential-Mode Acceleration measured by the accelerometers $A_i$, $A_j$ -:

$$a_{d,ij} = 1/2\,(a_i - a_j)$$

$$\Rightarrow a_{d,ij} = -(\nabla g - [\Omega^2] - [\dot{\Omega}])A_{d,ij}$$

where $A_{d,ij} = 1/2\,(A_i - A_j)$

Now, if the accelerometer $A_i$, $A_j$ belong to the same OAG (ij = 14, 25, 36), then $A_{c,ij}=0$, and $A_{d,ij}=A_i$

1. Common-Mode Accel. $\Rightarrow a_{c,ij} = (\nabla g-[\Omega^2]-[\Omega])C-2[\Omega]C-C+D$
2. Differential-Mode Accel. $\Rightarrow a_{d,ij} = -(\nabla g-[\Omega^2]-[\Omega])A_i$

Assuming $C=0$, i.e. COM of the Spacecraft is coincident with the center of all 3 OAGs.

Thus, ignoring terms $(\nabla g-[\Omega^2]-[\Omega])C$, $2[\Omega]C$, $C$, we get-:

$$a_{c,ij}=D$$

$$a_{d,ij}=-(\nabla g-[\Omega^2]-[\Omega])A_i$$

Hence, using the common-mode, the non-gravitational force like drag, can be measured, while using the differential-mode, the GGT can be measured.

Results have been obtained for an Orbit defined as-:

Altitude = 400 km.

Eccentricity = 0.01

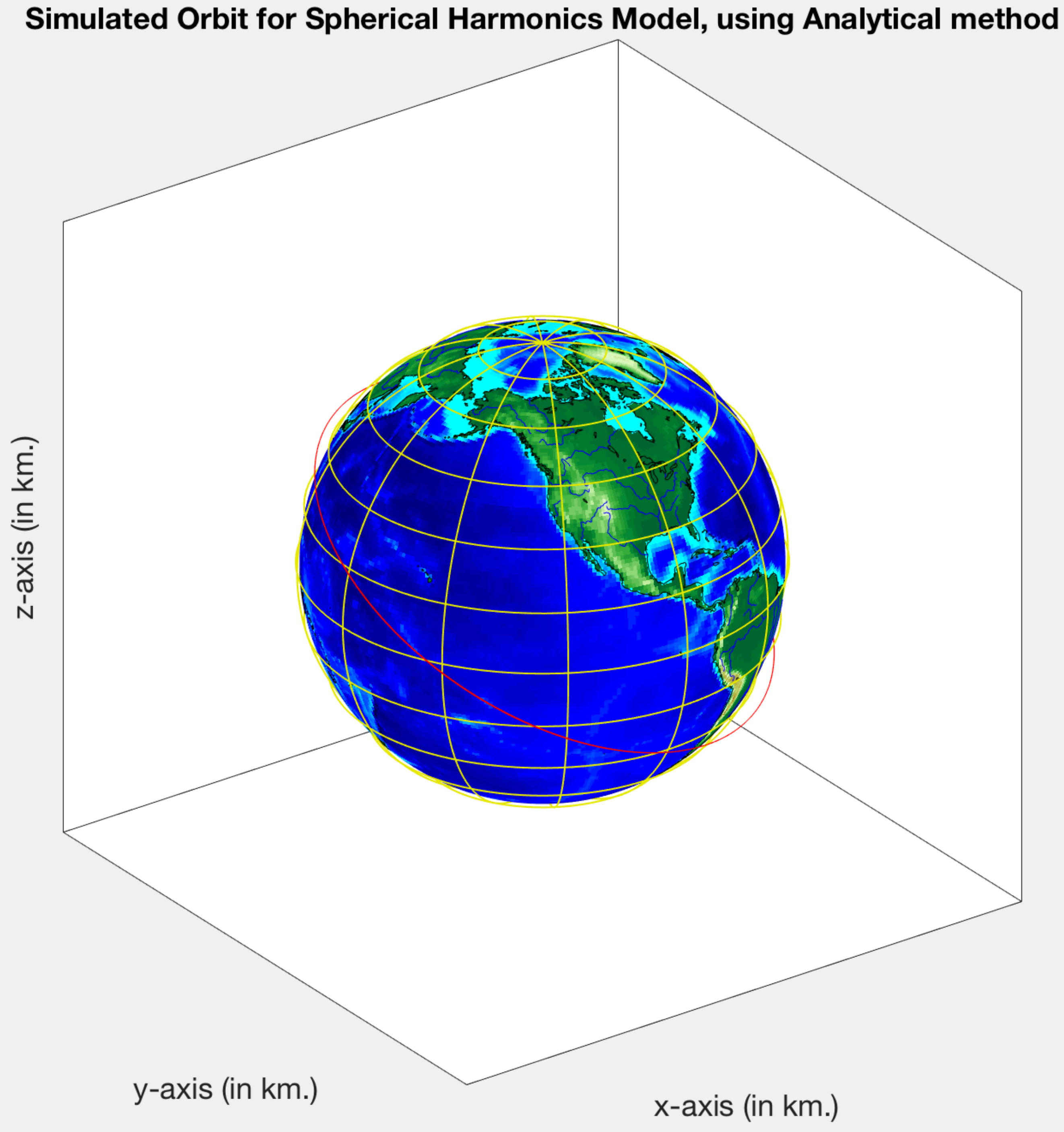
Inclination = pi/6 rad.

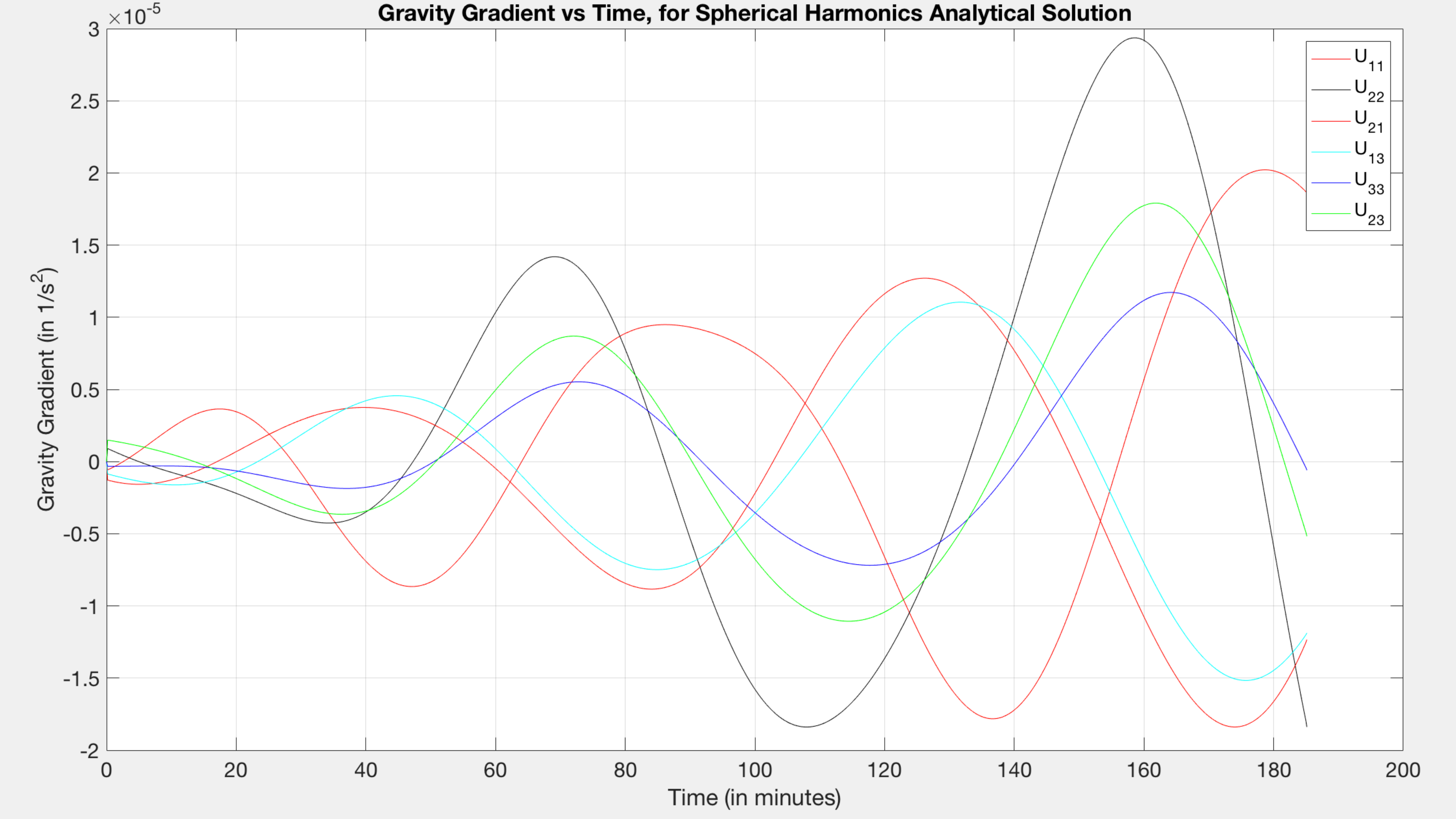
Right Ascension of the Ascending Node = pi/6 rad.

Argument of Periapsis = pi/2 rad.

True Anomally = 0 rad.

Results have been shown for an ideal Gravity Gradiometer Measurement Model, using 3x3 Spherical Harmonics Gravity Model





Gravity Gradient vs Time, for Spherical Harmonics Analytical Solution
×10^-5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Gravity Gradient (in 1/s^2)
0
20
40
60
80
100
120
140
160
180
200
Time (in minutes)
U11
U22
U21
U13
U33
U23

However, we can never have perfect measurements.

Hence, there is always a need for-:

Future work includes-:

i. Formulate the Measurement Model with appropriate error model,
ii. Implement Kalman Filter for Orbit Determination, and
iii. Complete Covariance Analysis using techniques like Monte Carlo or Linear Covariance analysis
iv. Identify various Error Sources, and determine the contribution of each.

# Backup Slide

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