Random Motion and the Diffusion of Brine Shrimp: A Project Based Learning Unit

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RANDOM MOTION AND THE DIFFUSION OF BRINE SHRIMP: A PROJECT BASED LEARNING UNIT

by

Rebecca Atkins

A report submitted in partial fulfillment of the requirements for the degree of

MASTER OF MATHEMATICS

UTAH STATE UNIVERSITY
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I. Introduction

Over the years, there have been several trends, philosophies, and ideas about how mathematics should be taught. These have included progressive movements, essentialist movements, drill and practice, teacher-centered methodologies, student-centered methodologies, and others. However, despite these various instructional trends, research has found that mathematics teachers have basically taught the same way for years, with little or no variation with respect to the changing trends (Ward, 2001). For many students, the traditional teaching method employed in their mathematics classes is typically centered on the teacher and includes lecture in which the teacher provides the necessary definitions and notations and does several sample problems. The teacher then assigns the students similar problems from the textbook for in-class practice and homework that is graded and collected the following day. This teaching method tends to increase students' computation and algorithmic ability, but does little to promote deep, conceptual understanding.

Research has found that teachers “tend to teach in the manner that they were taught during their own educational experiences” (White-Clark, DiCarlo, & Gilchriest, 2008). Because I was taught using the traditional methods described above, I felt the need to explore what other methods for teaching mathematics are currently being researched and recommended. This paper will discuss project-based instruction, a teaching method based upon constructivist learning theory. An overview of the defining characteristics of project-based instruction will be given and results from research on the effectiveness of project-based learning will be provided. I will then describe a project-based unit that I have designed for a middle-school mathematics class, the design of which has been influenced by recommendations from and results of previous research.
II. Constructivist Learning Theory

The learning theory of constructivism can be defined as "the philosophy or belief that learners create their own knowledge based on interaction with their environment, including their interactions with other people" (White-Clark et al., 2008). By this belief, individuals are thought to construct their own knowledge through an interplay between existing knowledge and beliefs and new knowledge and experiences (Yilmaz, 2008). The views of constructivism, therefore, contrast sharply with the idea that learners are "empty vessels" waiting to be filled, a learning philosophy that encourages the actions found in traditional lecture and textbook-based mathematics classrooms. Instead, constructivists view learners as intellectually active individuals who can pose questions, solve problems, and construct theories and knowledge (Yilmaz, 2008).

If learners are truly active individuals that can learn through experiences and construct their own knowledge, then perhaps traditional teaching methods are not the best ways to promote mathematical learning and understanding. Thus, a pedagogy has emerged that is based upon the ideas of constructivism. Constructivist pedagogy is especially informed by the ideas of John Dewey, William James, Jean Piaget, Lev Vygotsky, and Jerome Bruner, as well as other cognitive psychologists and scientists (Yilmaz, 2008). Teachers using constructivist pedagogy attempt to create classroom environments where the thought process is valued more than the answer, and overall goals focus on individual students developing deep understanding of the subject matter (Ward, 2001). Several teaching methods can help to achieve the goals of constructivist pedagogy and are based upon the ideas of constructivism; specifically, that learners can construct their own knowledge through interactions and experiences with their environment and other people. These methods include, but are not limited to, project-
based learning, inquiry and discovery learning, cooperative learning, and teaching with technology.

Studies show that an increasing number of high school students do not see the utility of mathematics. In a survey conducted in 2000, only 61 percent of American high school seniors agreed to the statement that “math is useful” (Gainsburg, 2008). This number was down 12 percent from a similar survey conducted 10 years earlier. Other surveys indicate that many students in elementary, middle, and high schools today “have not mastered basic concepts and principles, cannot apply what they learn to everyday life, and frequently see school as irrelevant” (Krajcik et al, 1994). One possible way to help students see the usefulness of the math that they are learning in school is to present students with problems in a real-life context, as is done in the constructivist-based teaching method of project-based instruction.
III. Project-Based Learning

Project-based learning (PBL) is a model that focuses student learning around a driving question or problem about which core concepts within the curriculum can be integrated (Krajcik et al, 1994). While addressing the central driving question, students engaged in PBL are involved in authentic experiences such as experimental design, data collection and analysis, problem-solving, inquiry, and decision making that culminates in realistic "artifacts" or products such as presentations, written reports, or physical models (Thomas, 2000; Solomon 2003). Throughout the learning process, the role of the teacher is to introduce relevant content before and during the lesson as well as to guide and advise students rather than to directly manage student work (Solomon, 2003; Hmelo-Silver, 2004). There is not one universally accepted model of PBL, but the following are defining characteristics typically found in learning activities described as project-based learning.

First, in project-based learning, students are confronted with authentic problems that are both central to the curriculum and do not have a predetermined solution (Thomas, 2000; Solomon, 2003). This means that in PBL, projects do not serve to illustrate examples of concepts previously taught or to provide extra practice. Rather, students learn the central concepts of the curriculum through the project. Also, the driving questions in a PBL learning activity cannot be so constrained as to restrict students from developing their own solution methods. Students must be granted a certain degree of autonomy, as well as more choice and responsibility than is typically seen in traditional teaching methods (Thomas, 2000).

Second, PBL activities involve students in investigations. These investigations could take the form of experimental design, problem-solving, decision-making, or model
building, with the requirement that existing knowledge is connected to newly constructed knowledge throughout the inquiry processes of the investigations (Thomas, 2000). Throughout the PBL experience, students ask and refine questions, explore ideas, make predictions, conduct experiments, analyze data, and draw conclusions while gaining new knowledge (Krajcik et. al, 1994). By the end of the PBL experience, students will have generated artifacts that represent the solution process and solution based upon these investigations.

Lastly, the driving question around which a PBL experience is formed must be an authentic, real-world problem (Solomon, 2003). The problem must be worthwhile, in that it contains important content and reflects what actual practitioners of the given discipline might encounter, while also being feasible as a source for student learning (Krajcik et al, 1994). The success of PBL can depend largely on the chosen question, as well as to the tasks and roles given to students to carry out during the learning process. According to Erickson, when choosing problems, teachers should make sure that they are genuine problems that reflect the goals of school mathematics, situations that consider students’ interests and experiences, appropriate content considering students’ prior knowledge, and of a difficulty level that will challenge the students without discouraging them (1999).

This last factor also leads to a second challenge for the teacher implementing project-based instruction. When presenting students with problems that are challenging and do not immediately lend themselves to a certain solution strategy, teachers need to be willing to let students struggle without offering suggested methods. However, teachers also need to provide students with sufficient guidance to keep them interested in the problem and on-task (Erickson, 1999).
One specific type of project-based learning that often appears in research and literature is problem-based learning. Problem-based learning contains all of the previously described aspects of project-based learning, and also has some of its own defining characteristics. While project-based learning can rely on the teacher to provide some necessary information through direct instruction both before and during the investigation process, problem-based learning is specifically characterized by a large amount of self-directed learning on the part of the student (Hmelo-Silver, 2004). Problem-based learning was originally developed by instructors at a medical school in Canada in the 1970s and has since been implemented in other higher education settings, as well as in elementary, middle, and high school programs (Savery, 2006). Project and problem-based learning are so similar that some researchers do not make a distinction between the two. Accordingly, in reviewing the research involving project-based learning, studies based on problem-based learning methods will also be discussed.

Another program with a PBL design is Expeditionary Learning. Expeditionary Learning has its roots in the Outward Bound program, which is a service-based education program that is often characterized by wilderness adventures (Thomas, 2000). The learning “expeditions” in Expeditionary Learning involve inquiry, community building, and character development (Udall & Mednick, 1996). Expeditionary learning emphasizes all of the above described elements of project-based learning, and it differs from other PBL based experiences in that it tends to involve more fieldwork, teambuilding exercises, and community service than is seen in the typical PBL classroom.
IV. Literature Review

Much of the research on PBL has focused on specific types of instructional strategies included under this heading, such as problem-based learning. While problem-based learning at the middle and secondary levels has mainly been applied in mathematics and science classrooms, it has been demonstrated to bring about positive results in other content areas as well. For example, in a study conducted by Stepien et al. (1993) a course in American Studies for sophomores employed a problem-based instructional approach. The problem students faced involved both historical and ethical issues regarding post-surrender relations with Japan in the Second World War. When compared to a control class of sophomores who studied the same material in a non-problem-based atmosphere, students in the problem-based class showed equal or greater understanding of the factual content of the unit.

Research in the area of problem-based instruction in mathematics classrooms indicates that this approach can lead to several positive results. According to the National Mathematics Advisory Panel [NMAP], presenting students with real-world problems improves their performance on assessments that include other real-world, open-ended problems requiring modeling and reasoning skills (2008). However, when using a primarily problem-based instructional approach, other mathematical skills such as “computation, simple word problems, and equation solving are not improved” (NMAP, 2008).

Problem-based instruction may also play a role in affecting students’ future course selections. Research indicates that students who have enrolled in classes where the teacher regularly used problem-based learning activities tend to enroll in more math
and science classes later on, when compared to students in classes taught through traditional, teacher-centered methods (Erickson, 1999).

Another program that has participated in a large amount of research evaluating the effectiveness of project-based learning is Expeditionary Learning Outward Bound (ELOB) schools. These schools are ones in which Expeditionary Learning programs have been adopted in all classes for all grade-levels and content areas. The following studies are all summarized in an ELOB publication (2001).

In 1999, the American Institutes for Research evaluated and compared 24 models of school reform and rated them on positive effects on student achievement. Of the 24 reform models, researchers found that only eight models, including ELOB schools, were shown to have positive effects on student academic achievement. Only three of the models were given a higher ranking than ELOB schools. These three models, which focused on more direct instructional methods than are seen in ELOB, had all been implemented in schools for an average of 15 years prior to when the studies took place. At the time of the studies, the ELOB model had only been in use for five years. According to the criteria for the reviews, a model had to be in use for at least 10 years to receive a higher rating than what the ELOB model received. Despite the fact that ELOB was a fairly new approach, in reviewing this model, researchers found that positive results as far as improvement on statewide standardized tests were seen across all subject areas in schools where Expeditionary Learning was employed.

Also in 1999, in *What Works in the Middle: Results Based Staff Development*, a National Staff Development Council Report, a two-year study of programs that demonstrate an impact on student achievement in middle school is described. This study found that in middle schools that were implementing Expeditionary Learning, student
achievement on standardized tests in math and reading improved significantly when
compared to non-ELOB schools in the district and state. This research also found that in
ELOB schools, student attendance, engagement, and attitude about school improved, and
parental involvement increased.

In 2000, a researcher from Brown University conducted an evaluation of the
academic achievement of students at two specific schools that began using Expeditionary
Learning in 1993. The researcher, Polly Ulichny compared test scores of these schools
with other schools in their districts as well as to particular schools with similar student-
body populations. The first school she studied was King Middle School in Portland,
Maine. When compared to the other middle schools in its district, higher percentages of
the students at King came from low socio-economic backgrounds or had limited English
proficiency. In 1990, before the implementation of Expeditionary Learning, King Middle
School’s scores on the Maine Educational Assessment (MEA) were lower than the
average state and district scores. However, in the 1998-1999 school year, MAE scores
from King Middle School were above the state average in six content areas and were at
the average level for the state in the seventh content area. King Middle School has seen a
steady rise in scores on the MAE since the implementation of Expeditionary Learning in
1993 in every content area except Social Studies. At the second school that Ulichny
studied, Rafael Hernandez School, a K-8 Spanish bilingual school in Boston,
Massachusetts, similar results were observed. Results similar to Ulichny’s research are
reported for numerous schools in a variety of states.

It is important to note that all of the above studies were reported in ELOB
publications. The studies were conducted by third-party researchers not connected with
ELOB, so the results should not necessarily be suspected of bias towards the
Expeditionary Learning program. However, it is possible that these studies were chosen for publication because of their positive results, and studies with neutral or negative outcomes were not included in the publication. Regardless, the studies do show the possibility of academic achievement gain, as measured by standardized tests, in schools where a project-based curriculum has been adopted.

Perhaps one of the “best” studies of project-based learning in terms of experimental design was conducted by Jo Boaler of Stanford University. This study’s experimental design is unique in that it was a multi-year longitudinal study comparing two schools that were similar in almost all aspects except for method of instruction. The details of the study are given in Boaler’s book *Experiencing School Mathematics* (2002) as well as in an article from Education Week (1999). Boaler conducted a three-year longitudinal study of 300 students at two secondary schools in England. The two schools had similar student populations but differed in their approaches to teaching mathematics. The “traditional” school used a whole class, teacher-centered approach to mathematics instruction that was characterized by the use of textbooks and frequent tests. The second school was project-based and mathematics instruction consisted mainly of students working on open-ended projects in groups. Textbooks were rarely used, and students worked on their own and were given a lot of choice in their mathematics learning. The project-based school used these methods until January of the third year of the study, when the school switched to more traditional methods in order prepare students for a national exam that was taken a few weeks later.

Throughout the study, Boaler watched approximately 100 one-hour lessons at each school, interviewed students and teachers, gave questionnaires to students, and analyzed student performance on the national exam taken in the third year, as well as
responses on exams that she created herself. Prior to the study, students at each school had received similar mathematics instruction. Project-based instruction was not employed at the second school before the first year of the study. On a range of mathematics tests at the beginning of the study, students at the two schools showed similar mathematical understanding and ability. On a national examination in mathematical proficiency that was given at the beginning of the study period, the majority of the students at both schools scored below the national average.

In assessments given each of the three years of the study, students at the project-based school scored as well or better than students at the traditional school on items that required rote knowledge of mathematical concepts. On the national examination given in the third year of the study, more students at the project-based school received a passing score, and three times as many students at this school than at the traditional school received the highest possible score on the examination. Students at the PBL school outperformed students at the traditional school on conceptual questions requiring more thought and creative application of several mathematical concepts at one time.

Along with differences in achievement, the students at the two schools showed differences in their views of mathematics when interviewed by Boaler. For example, students at the traditional school felt that math was a very rule-bound subject that required lots of memorization. Students said, “In math[s] you have to remember, in other subjects you can think about it,” and “In math[s], there’s a certain formula to get to, say from a to b, and there’s no other way to get to it, or maybe there is, but you’ve got to remember the formula, you’ve got to remember it,” (Boaler, 2000, p. 43).

Students at the project-based school seemed to see math as something they could think about and reason through. When asked to compare their project-based learning to
traditional instruction they had received before the study began, one student said “It’s an easier way to learn because you’re actually finding things out for yourself, not looking for things in the textbook...like, if we got an answer they would say, ‘You got it right.’ Here you have to explain how you got it...I think it helps you,” (Boaler, 2000, p. 62). Another student said, “You’re able to explore more, there’s not many limits, and that’s more interesting” (Boaler, 2000, p. 68).

Overall, research on project-based learning seems to be positive. Students in classrooms where PBL instructional strategies are often used score better on exams requiring conceptual understanding or application, and as well or better on exams requiring simple knowledge and skills. Project-based learning also seems to have a positive impact on student attitudes toward school.
V. The Brine Shrimp Movement Lab

The project-based learning unit I have created for a middle school mathematics classroom is based upon the “Population Dispersal Effect of Random Movement” lab developed by Dr. Jim Powell and Dr. Jim Haefner for their Applied Mathematics in Biology class at Utah State University. In this lab, students are introduced to the diffusion equation, and asked to use the solution to this partial differential equation to determine whether or not the diffusion equation and random motion is an appropriate model for the dispersal of brine shrimp. For more information on the mathematical background and motivation for this lab, see “Leading Students to Investigate Diffusion as a Model of Brine Shrimp Movement” (Kohler, Atkins, Haefner & Powell, 2008) in Appendix A.

While experiencing this lab, students are first asked to collect data on the displacement over time for individual shrimp in order to compute mean squared displacements over time. This enables students to calculate an estimate for the diffusion coefficient, D, for the brine shrimp. Students then collect data on shrimp arrivals in certain areas after being released in the center of a Petri dish. These observed arrival numbers are compared to the predicted arrival values given by the solution to the diffusion equation when calculated using the students’ estimate for the diffusion coefficient, as well as other necessary parameters such as the number of shrimp in the dish, size of the area in which arrivals are being counted, and the distance from the center of the Petri dish to the specific area. Based upon their results, students can decide if the diffusion model used is appropriate for the situation. Again, refer to Appendix A for further details on the lab procedures.
During the summer of 2008, I worked extensively with data collected in this brine shrimp lab in order to investigate questions on how variations in the methods of data collection would affect the results of the lab. These questions included how large of a time step should be used when collecting data on the displacement over time for individual shrimp, what length of time should be used to record the displacement of each shrimp, and how many individual shrimps' displacements should be recorded in order to get an accurate estimate of the diffusion coefficient. I also performed a sensitivity analysis on the parameters used to calculate predicted arrival rates based on the solution to the diffusion equation.

The results of all of these investigations, including related figures, can be found in the results section of the paper in Appendix A. Actually experiencing the data collection processes required for this lab, becoming familiar with the procedures, and exploring the above questions helped me to work out the details for my adaptation of this lab to a project for a middle school mathematics classroom.
VI. Brine Shrimp Random Motion Unit

Based upon the lab developed by Dr. Powell and Dr. Haefner, I have created a project-based learning unit designed for use in an Algebra I class. Following a general description of the unit, including rationales and connections to the defining characteristics of PBL, all of the necessary materials for the unit are included in this paper.

In the Brine Shrimp Random Motion unit I have created, students first learn basic information about brine shrimp and explore the concept of randomness. On Day 1, students are introduced to the idea of predicting the location of an object that moves according to a random process through a coin-tossing activity. Without actually having to use the solution to the diffusion equation, students are told that their mission is to determine whether or not baby brine shrimp move randomly. Students are told that mathematicians have developed an equation that will predict how many brine shrimp will be in a given area at a certain time if they really are moving randomly, and taking the role of mathematical biologists, students must decide if this equation accurately predicts how the shrimp move. The unit culminates with students giving oral and written presentations of their results.

I chose to design this Brine Shrimp Random Motion unit for an Algebra I class for several reasons. Part of the curriculum generally included in Algebra I involves collecting, recording, and analyzing bivariate data with the use of scatter plots and lines of fit (see Utah Core Curriculum at http://www.schools.utah.gov/curr/core/corepdf/Mth7-12.pdf). All of these concepts are taught to students experiencing the brine shrimp lab. While these concepts could be taught in several ways, many of which are probably simpler and less time-consuming for the instructor, I believe the opportunity to collect data from live, moving organisms will not only teach the above concepts but also
introduce students to the excitement and challenges often associated with the data
collection performed by mathematical biologists.

Algebra I students are often only faced with variables which have linear
relationships. Both the actual values that students observe for the shrimp arrival numbers
and the predicted values from the solution to the diffusion equation for the number of
shrimp in a given area over time do not produce a linear relationship. The opportunity to
see a non-linear relationship arising from data that they actually collected could serve as a
helpful transition for students from linear relationships in Algebra I to dealing with more
non-linear data in Algebra II.

The unit that I have created fits many of the defining characteristics of project-based
learning. The problem that the students face, determining whether or not brine
shrimp actually move randomly by validating a mathematical model, is central to the
curriculum. As already stated, the unit teaches the necessary curriculum requirements
about bivariate data collection, analysis, and interpretation. The unit also reviews several
skills, such as plotting and naming points, finding distances, averages, areas, and ratios.
Generally, PBL projects are not used to serve primarily as examples of previously taught
material or extra practice of skills already taught, but review and practice in conjunction
with teaching new material may be beneficial. Students are given the opportunity to
decide how to apply their previously acquired knowledge, in connection with new
knowledge, in order to solve problems.

The problem students are faced with in this unit does not have a pre-determined
solution, which is another defining characteristic of project-based learning. Different
groups of students will undoubtedly obtain different results, as they will use different
shrimp to calculate their mean squared displacements, use different lines of fit in
determining the value of the diffusion coefficient, have different numbers of shrimp in
the dish when counting arrivals, and will count arrivals in different sections of the Petri
dish.

Students will be allowed to interpret their results for themselves. Some will
conclude that even though the predicted and observed arrivals are not exactly the same,
the brine shrimp are moving randomly. They could decide this based on knowledge that
predicted results are theoretical and will not line up with actual results perfectly (as they
see in the coin-tossing activity). They will also know that their data collection was not,
and could not be, perfectly accurate as they were dealing with moving creatures that are
often hard to count and follow. Other students may conclude the shrimp do not move
randomly because the actual and predicted results are so different that the equation could
not be a valid model of the shrimp movement. Students will have the freedom to
interpret their results however they wish, as long as they have data and logical arguments
to back-up the assertions they make.

The figures on the next page show the qualitative data results obtained from two
different trials of this experiment that could lead to varying conclusions on whether or not
the equation, and random motion in general, is an appropriate model of the shrimp
movement. Each figure shows the predicted values for the number of shrimp in the given
area based on the solution to the diffusion equation, as well as the numbers that were
actually observed. The error bars in the figures denote one standard deviation above and
below average for four sections, all equidistant from the center of the dish, in which
shrimp arrivals were recorded.
Qualitative results for observed and predicted shrimp arrivals from data collected by research group in the summer of 2008.

Qualitative results for observed and predicted shrimp arrivals from data collected at a workshop for mathematics and science teachers, January 2009.
The Brine Shrimp Random Motion unit is perhaps not as open-ended as most project-based learning projects because there is certain information that the groups must collect during the process of determining whether or not the brine shrimp move randomly. For example, in order to correctly use the solution to the diffusion equation to predict shrimp arrivals, students must have calculated the diffusion coefficient and obtained values for the number of shrimp in the dish, the size of the area that they are observing, and the distance from the center. Students cannot decide whether or not to collect this information, it is required. However, I do not plan to simply tell students that they need these quantities. Students are asked to think of the factors that would affect the number of shrimp in a given area at a certain time after they had left the center of the dish. Thinking of these quantities for themselves provides motivation for the students to collect these values.

Throughout the unit, students have varying degrees of choice and freedom to decide how to get the information that they need. After discussing what information is needed in order to estimate the diffusion coefficient (displacements of many individual shrimp over time), students are allowed to decide how to collect this data. During the arrival data collection procedure, students are told how to collect the data but are allowed to choose what area in which to count the shrimp arrivals and how many areas to count in. And, after being told what information they will have to use to estimate the total number of shrimp that are in the dish when counting the shrimp arrivals, the groups have to decide what mathematical knowledge and skills they will use to determine this value. I have tried to balance teacher-directed instruction with student-directed choice throughout the unit.
Recall that another defining characteristic of project-based learning is that it involves students in investigations. These investigations can involve experimental design, decision making, and problem-solving. Students experience all of these in the Brine Shrimp Random Motion unit when they decide how to collect the individual shrimp displacement data, what line of fit to use to calculate the diffusion coefficient, what area to count shrimp arrivals in, and how to estimate the total number of shrimp in the Petri dish.

The last defining characteristic of project-based learning is that it incorporates an authentic problem that reflects what actual practitioners in the discipline might encounter. Even though whether or not brine shrimp move randomly is not a problem of importance to the mathematical or biological community, the problem that students are faced with in this unit is similar to what mathematical biologists may be faced with. They have to collect data in order to test it against a proposed model and then either accept or reject the model. As Erickson (1999) recommends, the problem that students are confronted with in this unit reflects the content goals of school mathematics while challenging students to learn and incorporate new information.

While my Brine Shrimp Random Motion unit may not perfectly employ all of the characteristics of project-based learning, I believe that it gives students a good opportunity to investigate an authentic, open-ended problem while learning new mathematical concepts and reviewing and applying previously learned skills.

This unit has not been tested with middle school students. However, the lab portions of it were used in a teaching workshop for a group of middle school and high school math and science teachers. The teachers had varying levels of mathematical understanding, and none were familiar with the brine shrimp random motion lab prior to
the workshop. By leading teachers through the lab portions of this unit, I was able to
gain valuable insights into how to best conduct, organize, and explain parts of the lab.
Feedback and comments from the teachers, as well as observations that I made
throughout the experience, all contributed to the formation of the unit that I present here.
VII. Unit Materials

The following are all of the materials that I have created for use with the Brine Shrimp Random Motion unit. The unit covers 6 class days. Each day includes a lesson plan, a homework assignment that either reviews prior knowledge in preparation for the next day’s activities, introduces concepts necessary for the next class day, or reinforces learned material from previous class periods. The materials also include any task sheets and example sheets necessary for the teaching of the unit.
Homework # 1: Introducing Brine Shrimp

Directions: After reading the articles on brine shrimp, answer the following questions using complete sentences.

1. What is the scientific name for brine shrimp?

2. What are brine shrimp more commonly known as?

3. Brine shrimp are a major food source for what animals?

4. What is the average size of adult brine shrimp?

5. If conditions in the environment are not good, such as when the water is too cold or the oxygen level is too low, are baby brine shrimp still born?

6. After a baby brine shrimp is born, what is it called when it is in its first stage of life?

7. What do brine shrimp eat?

8. How long does it take for a baby brine shrimp to become an adult?
Brine shrimp

Commonly known as the "Sea Monkey"...

Many people who visit the Great Salt Lake don't even notice the most numerous inhabitant of this ecosystem. The brine shrimp, *Artemia franciscana*, thrives here regardless of the salty water and fluctuating water temperatures. Although small, they serve as an essential food source for millions of birds that breed or stopover at the Great Salt Lake during migration, and, in recent years, these shrimp support a multi-million dollar commercial harvest. Who would have thought that a shrimp industry could develop in the Great Basin desert?! The Utah Division of Wildlife Resources has undertaken the duty of monitoring, researching, and protecting *Artemia* in the Great Salt Lake in order to preserve this vital resource.

What is a brine shrimp?

Brine shrimp are crustaceans that inhabit salty waters around the world, both inland and on the coast. It is currently accepted that one species, *Artemia franciscana*, is the only brine shrimp species that inhabits the Great Salt Lake, but there is discussion of genetic and life-history variability that could result in more species being discovered (see Parthenogenesis and shrimp genetics). The average adult male brine shrimp is 0.3-0.4 inches long, and the average female is 0.4-0.5 inches long. They feed by directing food towards their mouth via a series of undulating appendages and digest food through a simple digestive tract. In the process, they ingest a lot of salt water, which must be excreted through gills called "branchia". They can survive in water with salinities ranging from 30–330 g/l (3% to 33% salinity).

Around 5 million birds, representing around 250 species, use the Great Salt Lake annually. A variety of these birds feed on brine shrimp, either exclusively or opportunistically, to fuel long migrations.
Based on physiological traits, scientists believe that brine shrimp were originally a freshwater species that adapted to saline water. Predation by fish restricted the periods of time when brine shrimp were present in Lake Bonneville. The present day Great Salt Lake is too salty for fish and provides an optimal habitat for brine shrimp.

**Where do baby brine shrimp come from?...**

A method commonly utilized by wildlife for withstanding poor environmental conditions is timing reproduction to match food availability. Most animals attempt to time the birth or hatching of their young when food is prevalent and temperatures are moderate. Brine shrimp take this to the extreme, by either giving live birth if conditions are suitable or creating eggs that can remain viable (un-hatched but alive) for hundreds, or thousands, of years. These tough, coated eggs are called cysts.

**The life cycle**

(Courtesy of the U.S. Geological Survey).

Whether baby brine shrimp hatch from a cyst or are born live, in its first free-swimming period it is called a nauplius (plural: nauplii). The rate at which it develops through the rest of the stages in its life cycle is affected by salinity, water temperature, and food availability. The algae on which brine shrimp feed is most abundant at the end of winter, and Artemia attempt to time cyst hatching with the highest food availability. This occurs when water temperatures reach 4 °C (48 °F), typically by February or March. The emerging nauplii feed on the abundant algae, providing energy for the 12-24 molting stages a brine shrimp goes through to reach maturity, a process that takes 2-3 weeks depending on food availability and temperature.
Lesson Plan – Brine Shrimp Random Motion Unit – Day 1

* Note – Prior to beginning the unit, assign Homework #1: “Introducing Brine Shrimp”

Objectives for Homework #1 and Day 1:
A. The student learns basic facts about brine shrimp. (Simple Knowledge)
B. The student distinguishes between examples and non-examples of random processes. (Construct a Concept)
C. The student develops a working definition for random processes. (Comprehension and Communication).
D. The student can explain, in his or her own words, the relationship between theoretical probability and actual results as it applies to random processes. (Comprehension and Communication)

Materials Needed:
1. Cut out scenarios from Example Sheet #1 for each group
2. Coin and “marker” (any small item) for each student
3. “Heads or Tails” task sheet for each student
4. Video segment of brine shrimp moving in a Petri dish.
5. Homework #2 sheet for each student.
6. (Optional) Live brine shrimp in different life stages, including adults, for student observation and motivation.

Classroom arrangement: Groups of 4 or 5

-Beginning of Class-

1) Have student desks arranged in groups of 4 or 5, with scenarios from Example Sheet #1 cut out in an envelope in the center of each group. Have students sort scenarios into the examples and non-examples. Have students make conjectures as to how the examples are similar to each other and different from the non-examples. Instruct the group scribe to write down the decision rule. Scribes then share rules with the class.

2) Ask students the question, “If I tell you that something is random, what does that mean?” Have each student write down his or her ideas individually for one minute. Then have the groups discuss their ideas to form a list of ideas of what it means for something to be random. (If discussion is lagging, add the sub-question: If something happens randomly, can we predict what is going to happen?)

3) Have one student come to the board to act as a scribe. Choose one student from each group to share the group’s ideas while the scribe makes a class list of ideas on the board. Lead a short whole-class discussion on the similarities and differences between the ideas shared. Connect these ideas back to the examples and non-examples. Lead students to see that in the examples, outcomes were not affected by factors other than chance. If ideas such as “if something is random, you can’t predict it” come up, save discussion until after the next activity.
**-Heads or Tails Activity-**

4) Give each student a “Heads or Tails” task sheet, one coin, and one marker.

5) Ask students the questions “Is flipping a coin a random process?” In other words, what determines whether the coin lands “heads” or “tails”? Keep the discussion short and lead students to decide that coin-tossing is a random process.

6) Instruct each student to place his or her marker on the 0 position of the number line at the top of the page. Instruct students to flip their coin once, and move their marker one unit to the right if it landed “heads” and one unit to the left if it landed “tails.” Have students mark the original placement of their coin and the placement after Toss 1 on the number lines in the “My Marker” column on the task sheet. (Note - Model each step on a document-cam or overhead projector. )

7) Give students the total number of students in the class and ask them to predict how many of them got “heads” on the first toss and how many got “tails.” Draw the original placement and predicted placement after Toss 1 in the “Expected for Class” column of the task sheet. (Note – If there is an odd number of students, the teacher’s marker should be included. A total even number will help students more easily make a half-and-half prediction for placement after Toss 1). Now record the original placement and actual placement after Toss 1 in the “Actual for Class” column.

8) Lead students through predicting the placements of the markers after Toss 2 (based on the predicted placement after Toss 1, not actual results). When there is an odd number of markers at a single spot, do not divide the number exactly in half but continue to work with whole numbers, choosing to send one extra marker either to the right or left. Have students work in their groups to finish the “Expected for Class” column. (For a sample of possible student work, see page 31).

9) Have students toss their coin 4 more times moving their marker in the correct direction and marking it’s position in the “My Marker” column.

10) Have a student act as a scribe at the document cam or overhead to record the actual results for the class as a whole. (Perhaps the most efficient way to do this would be to ask “After toss 2, how raise your hand if your maker was at -5...-4...-3...and so on.”)

11) At the conclusion of the activity, lead a short discussion including questions such as:
   - Were our actual results exactly what we predicted?”
   - Do the differences (any time it was not 50% heads 50% tails) mean that coin-tossing is not random? (Theoretical predictions are different than actual results)
   - In view of this activity, do you want to change or add anything to your earlier ideas of what it means for something to be random?
-Introduce Brine Shrimp Unit-

12) Remind students that in Homework #1, they read about brine shrimp. Tell students that for the next several days, they are going to be mathematical biologists, whose mission is to determine whether or not baby brine shrimp move randomly. Mathematicians have used ideas similar to what we used during the Heads or Tails activity to develop an equation that will predict where members of a group of brine shrimp will be if they do move randomly starting at a particular place.

13) Show a short video clip of brine shrimp moving in a Petri dish to introduce the students to the size and movement of the shrimp. After the video, ask students: “What would it mean for the brine shrimp to be moving randomly?” and “Do you think it looks as if the shrimp are moving randomly?” and “If they do not move randomly, what might be guiding their movement? (Why might they choose to go in one direction rather than another?)” Have them write responses individually on a piece of paper.

14) Tell students that their job is to continue the work that mathematicians have done (the equation they have developed) to determine whether the brine shrimp are moving randomly. We will need to see if the equation accurately predicts how the brine shrimp move. If time remains, have students discuss ideas from the questions above in groups or as a whole class. Collect responses so that you can determine if students understand basic concepts of random movement.

Assign Homework #2: “Plotting Points and Finding Distance” to be due the next class day.
Example Sheet #1: Random Processes

**Directions:** Cut out each of the following scenarios. On the back of the examples write “example” and on the back of the non-examples write “non-example.” Cut out the scenarios and place them in an envelope for each group.

**Non-Examples: Not random**

When entering the house after being outside all afternoon, my pet cat Joycie goes straight to the kitchen where her food bowl is.

Matthew decided to ride his bike to school instead of walking Tuesday morning because he was late.

When dividing the class into two teams to play soccer, Coach Jones let Mike and Carly be team captains. They got to take turns choosing people to be on their team. Carly chose Shayla first, because she is the fastest runner in the class. Mike chose Derrek first, because they are best friends.

**Examples: Random**

Cindy’s family could not decide where to go for dinner, so they each wrote their favorite restaurant on a piece of paper. They folded the papers and put them in a large bowl. Cindy’s father mixed the papers and then chose one to decide where they would go to eat.

Brooke was playing The Farming Game with her friends. On her turn, she rolled the die and it landed on “5”. Accordingly, Brooke moved her piece ahead 5 spaces.

When dividing the class into two teams to play dodge ball, Coach Carter flipped a coin for each student. Tails meant the student was on the red team, and heads meant that the student was on the blue team.
Heads or Tails?

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<th>Actual For Class</th>
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</thead>
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<td><img src="image" alt="Actual For Class" /></td>
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</tbody>
</table>

Name: ____________________________

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30
Heads or Tails?

Name: Student

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<th>Toss</th>
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<th>Expected For Class</th>
<th>Actual For Class</th>
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</thead>
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<tr>
<td>Toss 5</td>
<td></td>
<td>1 4 9 9 4 1</td>
<td>2 3 9 10 3 1</td>
</tr>
</tbody>
</table>
Homework # 2: Plotting Points and Finding Distance

Name: _______________________

- Remember all points are named as \((x, y)\). The x-axis is the horizontal axis and the y-axis is the vertical axis.

1. Plot and label the points \((-4, 2)\), \((0, 5)\), \((-1, -9.5)\), and \((-6, 0)\) on the graph below.

2. What is the formula used to find the distance between the points \((x_1, y_1)\) and \((x_2, y_2)\)?

3. Find the distance between the following pairs of points:

   A. \((1, 1)\) and \((4, 5)\)

   B. \((-2, 0)\) and \((6, -4)\)

   C. \((3, -1)\) and \((-5, 2)\)
4. Find the distance between points A and B.

5. If you live at 100 North 300 East, and your friend lives at 600 North 1500 East, what is the straight line distance (in blocks) between your homes? Assume all blocks are the same size. Drawing a picture may help.
Lesson Plan – Brine Shrimp Random Motion Unit – Day 2

Objectives for Homework #2 and Day 2:
A. The student reviews previously learned skills of plotting points and naming points. (Algorithmic Skill)
B. The student reviews how to find the distance between two points. (Algorithmic Skill)
C. The student can recall the definition of diffusion. (Simple Knowledge)
D. The student decides what information is necessary and what method to use to collect data to determine the displacement of individual shrimp over time. (Application)

Materials Needed:
7. Video segment of brine shrimp moving in a Petri dish.
8. Brine shrimp two days old.
10. One overhead projector.
11. Pipettes
12. Great Salt Lake water (or equivalent) in proper concentration.
13. One polar graph paper used for arrival rates data collection.
14. Homework #3: Slope Review for each student

Classroom arrangement: Groups of 4 or 5

-Beginning of Class-

1) Instruct students to go over Homework #2 in their groups and attempt to resolve any differences in answers. After a few minutes, when most groups seem finished, share answers with students so that they can check their work.

-Review Goal of Brine Shrimp Unit and Introduce Equation details-

2) Review the goal: Students need to use mathematicians’ equation to determine whether or not brine shrimp move randomly. If they do, the equation will predict where the brine shrimp are, just like they could predict where coins would be during the activity the day before.

3) Explain that specifically, the equation will predict how many shrimp to expect in some area of the Petri dish at a given time if all the shrimp start in the same place and spread out. Demonstrate this process. Drop shrimp in center of Petri dish and let students watch them spread out, with polar graph paper underneath Petri dish.

4) Ask students what things would effect how many shrimp would be in a given area of the Petri dish at a certain time. (Possible answers: time since the shrimp entered the dish, size of area, distance from center, total number of shrimp, how fast they spread out/move) If necessary, refer back to Heads or Tails activity to help
students think of these parameters. Tell students that in order for the equation to predict the number of shrimp in an area, we have to find each of those values to put into the equation. Today we will focus on how quickly they spread out.

-Individual Movement Procedure-

5) Ask students to again look at the shrimp in the dish. Are they all going the same speed, and is the speed constant over time for an individual shrimp? (Students should notice that they are not.) Since they all are going at different speeds, ask students if they want a good way of summarizing the speed of all of the shrimp together, what could they use? (an average).

6) Introduce the concept of diffusion (objects spreading out) and tell students that for the equation we need to know how quickly the shrimp spread out from where they started. Ask students to discuss in groups: If we want to know how quickly the shrimp are spreading out, what do we need to know? (Where they start, where the end, how long it took them to get there, etc.)

7) To summarize, to figure out how quickly the shrimp are spreading out, we need to find how far the shrimp go (straight line distance from where they started) in short intervals of time (since it is not constant for each shrimp). Give groups 10 minutes to brainstorm how they could get this information. Tell them they will have access to shrimp in a Petri dish, overhead projectors, and can ask for other supplies that they may need. While groups discuss, move often from group to group to give help and ask guiding questions if needed.

8) Let groups share method with class and then give groups 5 more minutes to revise plans if desired. Then have groups create a plan for how they will record their data (use “Data Collection Sheet for Individual Shrimp Movement” as an example if necessary).

*Note – As groups share plans, make note of any materials you need to collect in order for them to carry out their proposed data collection procedure the next day. Give them the responsibility of getting these materials when possible.

Assign Homework #3: “Slope Review” to be due the next class day, along with any necessary preparations to collect their individual movement data.
Polar Graph Paper – For Arrival Rates Data Collection
Homework # 3: Slope Review

Name: __________________________

5. The line with the equation \( y = mx + b \) is a straight line with slope _____ and y-intercept ____.

6. Recall that the slope of a line is the rise over the run. Write the formula for finding the slope of the line that contains the points \((x_1, y_1)\) and \((x_2, y_2)\).

7. Find the slope of the line containing the points (-7, -3) and (-4, -5).

8. Find the slope of the line containing the points (-4, 4) and (3, 4).

9. Classify the following lines as having positive, negative, zero, or undefined slope. Write your answer on the line below the graph.

![Graphs of lines with different slopes]
10. On the following graph, draw a line that has positive slope.

11. Find the slope of the following lines. Write your answer on the line below the graph.
Lesson Plan – Brine Shrimp Random Motion Unit – Day 3

Objectives for Homework #3 and Day 3:
A. The student will review how to find the slope of a line given two points on the line. (Algorithmic Skill)
B. The student will review how to find the slope of a line given a graph of that line. (Algorithmic Skill)
C. The student will collect and record bivariate data. (Algorithmic skill)
D. The student graphs bivariate data on a scatter plot and interprets the results. (Algorithmic Skill and Comprehension and Communication)

Materials Needed:
1. Brine shrimp approximately three days old.
2. Great Salt Lake water (or equivalent) at the proper concentration.
3. Petri dish for each group
4. Rectangular coordinate graph transparency for each group.
5. Pipettes.
6. One overhead projector for each group and overhead markers for each group.
7. Stop-watch or timer for each group.
8. Rulers.
9. (Optional) Data Collection Sheet for Individual Shrimp Movement, if students did not create their own data collection methods.
10. (Optional) Mean Squared Displacement versus Time data sheet for each student to record the group’s data.
11. (Optional) Parent volunteers to assist and supervise.
12. (Optional) Any other materials necessary for students to collect data using the methods they chose the previous day.
13. Homework #4 sheet for each student.

Classroom arrangement: Groups of 4 or 5

Before class, set up the necessary number of overhead projectors around the classroom, so that projectors can project on whiteboards, projector screens, blank walls, or large white pieces of paper. Set up equipment area containing items listed above.

-Beginning of Class-

1) Have students discuss/review in their groups for 3 minutes what data they need to collect from the shrimp and why. Have one group share to make sure all students remember what was discussed the previous day and what the overall goal is.

-Data Collection (Actual procedure may vary based upon student ideas and plans)-

2) Within student groups, assign a group leader and 2 materials collectors. The materials collectors are in charge of getting the necessary materials for the lab, and the group leader is responsible for making sure that all group members are on-task. Have materials collectors gather materials so that groups can start data collection.
3) During data collection, each group needs:
- One Timer to call out every 5 or 10 seconds (time length dependant upon ability of students) for 80 seconds.
- One or two Shrimp Watchers to call out coordinate locations of shrimp at appropriate time intervals.
- One Recorder to write down the coordinate locations of the shrimp at the appropriate times.

Each group should record the path of approximately 10 shrimp. Have students rotate responsibilities so that each gets the opportunity to be the Timer, Shrimp Watcher, and Recorder.

Note: Some tips to make the data collection process go smoothly:
- For instructions on hatching brine shrimp and necessary salt water concentration, see Appendix A.
- Before placing shrimp in the Petri dish, there needs to be a small amount of Great Salt Lake water in the dish (barely covering the bottom of the dish).
- Rather than putting the shrimp in the Petri dish and then carrying them to the overhead projector, bring the shrimp in a pipette to the overhead projector and then put them in the dish.
- Have students get shrimp from an area of the dish that has a low concentration of shrimp. Smaller numbers of shrimp make this exercise easier.
- Students do not have to follow shrimp that start near the origin. As long as the starting position is recorded, the correct displacements can be calculated.
- Some shrimp will hit the edge of the Petri dish before the 80 seconds have elapsed. Once a shrimp hits the edge, its path should no longer be recorded as its motion can no longer be considered random. Having some data tracks shorter than others does not affect the results.
- If the student is unable to name the coordinate location of a shrimp at one time interval, have them leave that time blank and continue on at the next time interval.
- If the Shrimp Watcher loses track of which shrimp he or she is following because it entered a large group of shrimp, they should choose a shrimp to follow that is in the same place as the shrimp that they were previously following.
- Students will come up with their own ideas to make this process work for them. Some groups like to have one person follow the shrimp at a time. This person calls out the coordinate locations at the given times. Others like to have two Shrimp Watchers, who alternate calling out the coordinate locations. Or, two Shrimp Watchers can also divide the work by having one call out the x-coordinate and one call out the y-coordinate. Students may also like to use a pencil to mark the location of the shrimp (using numbers to keep the locations in order) throughout the 80 seconds and then name to coordinate locations after the 80 seconds have elapsed. This can be difficult if the shrimp is often in the same place, as numbers tend to overlap.
- Let students be creative and experiment with their own methods.
**Data Analysis**

4) Each student in the group should take two tracks of data and calculate the displacements and squared displacements in each direction. When these calculations are complete, the group can find the mean squared displacements in each direction at each time and then find the total mean squared displacement at each time ($< r^2 > = < x^2 > + < y^2 >$). In order to complete the homework assignment, each student needs to have a copy of the mean squared displacement data.

**Scatter Plots**

5) Once students have their group’s data, ask questions about the data such as “What is happening to the mean squared displacement over time?”, “Is the change constant?, etc. Lead students to see that having the data in the table is not the easiest way to see how the two variables relate to each other.

6) Introduce scatter plots as a way to visually observe the relationship between two variables. Use the following data set as an example. Plot the first few points and have students plot the rest on their own paper. Check work together as a whole class, having students come to the board to plot remaining points. Label the axes. Discuss the relationship between the variables as evident by the scatter plot.

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<th>4</th>
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Assign Homework #4 – Each group member should make a scatter plot of the mean squared displacement data versus time. For ease of comparison, have all students have time on the x axis and means squared displacement on the y axis.
Data Collection Sheet for Individual Shrimp Movement

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Mean Squared Displacement versus Time

Group Members:

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<th>Time (sec)</th>
<th>Mean Squared Displacement $&lt;r^2&gt;$ (cm²)</th>
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</table>
Homework # 4: Scatter Plot – Mean Squared Displacement versus Time

Name: ________________________
Group Members: ________________________

Directions: Make a scatter plot of the data that your group collected. Be sure to label the axes.
Lesson Plan – Brine Shrimp Random Motion Unit – Day 4

Objectives for Homework #4 and Day 4:
A. The student creates a scatter plot for real data that he or she collected. (Algorithmic Skill)
B. The student distinguishes between examples and non-examples of variables with an approximately linear relationship. (Construct a Concept)
C. From scatter plots, the student classifies the relationship between two linearly related variables as having positive, negative, or zero correlation. (Comprehension and Communication)
D. The student draws and estimates the equation of a line of fit for the scatter plot that he or she created from the collected data. (Algorithmic Skill)
E. The student uses the slope of the line of fit for the collected data to estimate the diffusion coefficient for the brine shrimp. (Algorithmic Skill)

Materials Needed:
1. Scatter plots from Example Sheet #2 cut out for each group.
2. A ruler for each student.
3. Finding the Diffusion Coefficient task sheet for each student.
4. Homework #5 sheet for each student.

Classroom arrangement: Groups of 4 or 5, with a ruler at each desk.

-Beginning of Class-
1) Have group members compare scatter plots that they created as homework. Because they came from the same data, the plots should be the same. Have students work together to correct any mistakes.

-Scatter Plots and Lines of Fit-
2) Give each group all of the scatter plots from Task Sheet #2 (except for the last three). Have them sort the scatter plots into the examples and non-examples. Have students individually develop a conjecture about how the examples are alike, and how they differ from the non-examples. After everyone has a conjecture, have students share their conjectures in their groups to form one group conjecture.

3) Conduct a large-group question and discussion session in which each group shares their conjecture. Ask questions to lead students to explain their rationales and compare their conjectures in meaning and wording. Work towards developing a whole-class generalization. When a consensus is reached, introduce the terms linear and non-linear association between two variables. Connect these terms to wording students used.

4) After students understand the concept of a linear association between two variables, give them the last three scatter plots on Example Sheet #2 and have them classify...
these scatter plots as either having a linear or non-linear relationship between the variables.

5) Lead discussion on positive, negative, and zero correlation asking students to give examples of variables that would have each type of relationship.
   o Possible answers include childrens' ages and heights (positive correlation), car weight and fuel efficiency (negative correlation), age and number of letters in last name (zero correlation).

6) Demonstrate that when the relationship between two variables is approximately linear, we can draw a line through the points on the scatter plot as a way to summarize the data and the relationship between the variables. This line also gives us the ability to predict unknown values in our data set. Have students each practice drawing a line of fit on one of the scatter plots with approximately linear association from Example Sheet #2. Point out that there are many different fit lines that can be drawn on a scatter plot.

**-Estimating the Diffusion Coefficient-**

7) Have students draw a line of fit on the scatter plots that they created for their displacement vs. time data. As a group, the students need to then choose one plot with line of fit that they think best summarizes the data. This is the plot that they will use to estimate the diffusion coefficient, D.

8) Have a student remind the class why they collected data on the displacement of the shrimp over time. If necessary, review the word diffusion. Then explain that when mathematicians want to know how quickly something spreads out, or diffuses, they use a diffusion coefficient. Explain that the diffusion coefficient is one number that summarizes the diffusion of a group of organisms. One of the variables in the equation that the mathematicians have developed that they need to validate is this diffusion coefficient, D, so they need to find D for their shrimp. Tell them that D is equal to one fourth of the slope of their fit line.

9) Have the students work through the Finding the Diffusion Coefficient task sheet numbers 1 through 4.

**-Predicting Shrimp Arrivals-**

10) Remind students that the equation that the mathematicians have developed predicts how many shrimp will be in a given area at a certain time after they start moving from the center of the Petri dish, provided that they are moving randomly. Connect this to what we did on Day 1 with the coin tossing activity.
11) Tell students that on the next class day, they will place shrimp in the dish and count the number of shrimp in certain areas at given times and compare these numbers to the predicted numbers given by the equation. To compare, they will make graphs of the number of shrimp in a given area vs. time. Have students predict the shape of this graph, drawing their prediction along with an explanation in number 5 on the task sheet.

12) While students are working, move about the room and find 3 or 4 students with different ideas. Ask if they would be willing to share their predictions and explanations with the class. Lead a short discussion of the predictions and rationales and attempt to come to a class consensus as to the shape of the graph of the number of shrimp in a given area versus time.

Possible student predictions and explanations:

A) ~  
B) ~  
C) ~

Explanations:

A) At first there will be no shrimp in the area, because they will all be in the center. Then, as the shrimp spread out, there will be more because more shrimp are getting further from the center.

B) At first there will be no shrimp in the area. Then, as they spread out, more and more will be in the area. As the shrimp keep spreading out, shrimp will leave the area until there are none left.

C) At first there will be no shrimp in the area. As they spread out the number of shrimp in the area will increase. As they keep spreading out past the area, the number will decrease but there will always be some there if they are moving randomly. Shrimp will be moving back and forth, not always away from the center.

13) If time remains, explain the procedure for counting the shrimp arrivals on the following class day. Collect students' task sheets, recording the value of D for each group before returning the task sheets the following day.

Assign Homework #5: Areas to be due the following class day.
Example Sheet #2: Linear Relationships

Directions: Cut out each of the following scatter plots. On the back of the scatter plots with a positive linear relationship between the variables, write “Example.” On the back of the scatter plots with a non-linear relationship between the variables write “Non-example.” Leave the back of the scatter plots with a negative linear relationship or no linear relationship blank.

Examples: Positive Linear Relationship
Non-examples: Non-linear relationship

Leave blank: Negative Linear Relationship
Leave blank: No linear relationship
Finding the Diffusion Coefficient

Name: ____________________  Group Members: ____________________

1. Find the equation of your group’s line of fit.

2. What does the slope of the line of fit tell you?

3. Using your line of fit, predict the mean squared displacement of the shrimp at 95 seconds. Show your work.

4. Estimate the diffusion coefficient, D, for the brine shrimp. What are the units of D?

-Shrimp Arrivals in an Area versus Time-

5. Draw your prediction here:  

   Explanation:

   New Prediction:  

   Explanation:
Homework #5: Area

Name: ________________________________

Directions: Answer the following questions. Show all of your work and clearly mark your answers.

1. What is the ratio of the area of the small rectangle to the area of the large rectangle?

2. If the large rectangle has an area of 48 units\(^2\), what is the area of the small rectangle?

3. Suppose your little brother takes your homework sheet and starts making dots all over the above rectangles in no particular pattern. If he makes 12 dots in the small rectangle before you stop him, about how many dots do you think he made in the large rectangle? Explain.

4. Find the area of the circle.
Lesson Plan – Brine Shrimp Random Motion Unit – Day 5

Objectives for Homework #5 and Day 5:
A. The student reviews how to find the area of a circle. (Algorithmic Skill)
B. The student will review ratios related to areas. (Algorithmic Skill)
C. The student collects and records data with two variables, time and number of shrimp in area. (Algorithmic Skill)
D. Given the number of shrimp in a section of the Petri dish, the student will decide how to use areas, ratios, or any other mathematical knowledge to estimate the number of shrimp in the whole Petri dish. (Application)

Materials Needed:
1. How Many Shrimp? task sheet for each student.
2. Rulers
3. Calculators
4. Brine Shrimp approximately three days old.
5. Great Salt Lake Water (or equivalent) at the proper concentration.
7. Petri dish and pipettes for each group.
8. Polar graph paper transparency for each group.
9. Three overhead projectors
10. Transparency markers for each group.
11. Stop-watch or timer for each group.
12. Data Collection Sheet for Shrimp Arrivals for each group.
13. Homework #6 sheet for each student.
14. (Optional) Parent volunteers to assist and supervise.

Classroom Arrangement: Groups of 4 or 5

Before class, set up the overhead projectors so that each can project onto a whiteboard, projector screen, blank wall, or large piece of white paper. Set up equipment area containing items listed above. Have one overhead projector with graph paper transparency and Petri dish ready to demonstrate procedure.

-Beginning of Class-

1) In groups, give students 4 minutes to discuss their thought process on #3 on Homework #5. Collect Homework #5.

-Shrimp Arrivals Data Collection-

2) Explain procedure for counting shrimp arrivals. With one region on the transparency outlined, place shrimp in dish and have students count the number of shrimp in the area every 10 seconds. Specifically describe how to place shrimp in Petri dish (explained below).
3) Lead short discussion with students based upon their observation of the demonstration. Is counting shrimp in one region accurate enough? What are other possibilities?

- Possible answers: Count shrimp in multiple sections, all the same distance from the center, and average the number of shrimp in the regions at each time.

Groups can decide how many regions to count (also dependant upon group size).

4) Have students recall from Day 2 the factors that they said would affect the number of shrimp in a given area (time since the shrimp entered the dish, size of the area, distance from the center to the area, total number of shrimp in the dish, how quickly they spread out). Make a list on the board, reminding students that they have already calculated D, but will need all of these other values to put into the equation to see how well it predicts how many shrimp are in the section.

5) Refer back to the shrimp in the demonstration dish. They should be evenly distributed in the whole dish now. Ask students if it is possible to count all of the shrimp at one time? Outline a sector of the polar graph and ask if we could count the shrimp just in that sector. Tell students that they will have to use that information to estimate how many shrimp are in the entire dish. Divide the sector into three or four sections and explain the procedure for counting the number of shrimp in the sector.

6) Give each group the Data Collection Sheet for Shrimp Arrivals and give each student the How Many Shrimp? task sheet. Depending on the number of groups, have two or three groups collect data at one time, while the other groups are filling in information such as the area of the section and the distance from center on the Data Collection sheet, as well as start the How Many Shrimp? task sheet. Groups that have not yet collected data can use a variable to represent the number of shrimp in the sector when determining how to estimate the number of shrimp in the entire dish. After they have the data, the correct value can be used. When the first groups are done collecting data, they will work on the task sheets, and the remaining groups will collect their data to finish their task sheets.

7) During the Shrimp Arrivals data collection process, each group needs:
   - One Timer/Recorder to call out “time” every 10 seconds and record the number of shrimp in each section.
   - Three or four (depending on group size) Counters to count the number of shrimp in their section whenever time is called.
During the Estimating the Total Number of Shrimp data collection process, each group needs:

- One Recorder to call out "now" and record the number of shrimp in each region. As this is not dependant on time, it does not need to be done in certain time intervals.
- Three or four Counters to count the number of shrimp in their section of the sector whenever "now" is called.

Note: Some tips to make the data collection process go smoothly:

- Shine a lamp on one corner of the dish containing the brine shrimp. The shrimp are attracted to the light so this will cause an area of high concentration to form. For this exercise, high numbers of shrimp are best, so shrimp should be taken from this area.
- Before placing the shrimp in the Petri dish, there needs to be a small amount of Great Salt Lake water in the dish (barely covering the bottom of the dish).
- Have students mark their polar graph transparencies so that the areas in which they are counting the shrimp are outlined. Numbering the areas also helps, as students can call off the number of shrimp in their areas in sequential order each time to aide the Recorder.
- For the Shrimp Arrivals procedure, the shrimp need to be released all at once, as close to the center of the dish as possible. Place the transparency on the projector with the Petri dish on top. Everyone should be in place and ready before the shrimp enter the dish. With the Pipette held vertically above the center of the dish, and the tip barely touching the water, one drop of shrimp should be released, and the Timer should start timing as soon as the shrimp enter the dish. This may take practice!
- If the drop of shrimp is far away from the center, very spread out, or does not contain many shrimp, have the students empty their Petri dish and try again.
- Let students choose which section to count the shrimp arrivals in. A variety of sections (with varying distances from the center) will give differing results to compare. Sections very close to the center should not be used, because the shrimp may start in those sections. Sections very close to the edge of the Petri dish should not be used because when shrimp hit the edge, they are more likely to turn around and return to those sections than they otherwise would be.
- When counting, students should count only how many shrimp are in their section right when the recorder says "time." They do not need to keep track of how many shrimp have entered and left the area during the 10 second interval.
- To ensure that the shrimp are evenly distributed in the dish before counting them to determine the total number of shrimp, gently stir the shrimp with a pipette. Make sure the water has stopped moving before counting the shrimp.
8) After all data has been collected, have students concentrate on finding the area of the section, distance from the center, and the number of shrimp in the dish. Before they leave, have each group write down their values for $D$, the number of shrimp, the area of the section in which they counted shrimp arrivals, and the distance from the center of the dish to that section. Students must hand in this work. Tell them that on the next class day, using their calculated information for $D$, number of shrimp, area, and distance from center, you will give them the predicted shrimp arrivals based upon the equation. They will then compare this to their actual observed numbers.

Assign Homework #6: Scatter Plots and Lines of Fit

Before the next class day, use Excel to calculate the predicted number of shrimp in the section every 5 seconds given by the solution to the diffusion equation and each group’s values for the diffusion coefficient, area, length, and number of shrimp.

$$N_{\text{section}} = \frac{NA}{4\pi D t} e^{-\frac{L^2}{4Dt}}$$

In the above equation, $N$ is the total number of shrimp in the dish, $A$ is the area of the section, $D$ is the diffusion coefficient, and $L$ is the length from the center of the circle to the section.
How many shrimp?

Name: ____________________________
Group Members: ______________________

As a group, you will find, on average, how many shrimp there are in one sector of the Petri dish (excluding the outermost ring). On the graph below, shade the sector in which your group will count the shrimp.

Use the number of shrimp in that region to estimate the number of shrimp in the entire dish. Show all of your work!
Data Collection Sheet for Shrimp Arrivals

Group Members:

- Shrimp Arrivals -

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<thead>
<tr>
<th>t</th>
<th>Section 1</th>
<th>Section 2</th>
<th>Section 3</th>
<th>Section 4</th>
<th>Average</th>
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<td>180</td>
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</tbody>
</table>

Length from the center of circle to the middle of the section = __________ cm

Area of section = __________ cm²
-Estimating the Total Number of Shrimp-

Number of shrimp in 1/8 of the dish (excluding outer ring):

<table>
<thead>
<tr>
<th></th>
<th>Section 1</th>
<th>Section 2</th>
<th>Section 3</th>
<th>Section 4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
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<td>Trial 2</td>
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</table>

Average =
1. The following table shows the largest vertical drops on nine roller coasters in the United States and the number of years after 1988 that the roller coasters were opened.

<table>
<thead>
<tr>
<th>Years since 1988</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>11</th>
<th>12</th>
<th>12</th>
<th>13</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Drop (ft)</td>
<td>151</td>
<td>155</td>
<td>225</td>
<td>230</td>
<td>306</td>
<td>300</td>
<td>255</td>
<td>255</td>
<td>400</td>
</tr>
</tbody>
</table>

Data from: Glencoe McGraw-Hill Algebra 1

a) Draw a scatter plot for the data. Label the axes.

b) Is there positive, negative, or zero correlation between the variables? What does this correlation mean?
c) Draw a line of fit on the scatter plot from part a). Find the equation of the line of fit in slope-intercept form.

d) What does the slope of this line tell you about the roller coasters? What does the y-intercept tell you?

e) If the trend continued in a similar way, what would you expect would be the size of the largest vertical drop of a roller coaster that opened in 2008?
Objectives for Homework #6 and Day 6:
A. The student interprets the slope and y-intercept of a line of fit through data as they connect to real-world situations. (Comprehension and Communication)
B. The student predicts y-values given x-values using an appropriate line of fit through bivariate data. (Algorithmic Skill)
C. The student plots two data sets, including one that they collected, on one graph and interprets the results. (Algorithmic Skill and Comprehension and Communication)
D. The student creates a verbal and written presentation summarizing work done throughout the unit and interpreting overall results. (Comprehension and Communication)

Materials Needed:
1. Arrival rates predicted by the solution to the diffusion equation calculated using each group’s parameters (one copy for each student).
2. Graphing Shrimp Arrivals task sheet for each student.
3. One Graphing Shrimp Arrivals transparency for each group.
4. One Scatter Plot – Mean Squared Displacement vs. Time transparency per group.
5. Two transparency markers of different colors for each group.
6. Presentation and Lab Report Instructions for each student.
7. Colored pencils and rulers.
8. Calculators

Classroom arrangement: Groups of 4 or 5

-Beginning of Class-
1) Instruct groups to get their Data Collection Sheet for Shrimp Arrivals and calculate the average number of shrimp in the sections at each time.

-Plotting Observed and Predicted Arrivals-
2) While groups are calculating averages, give each student a sheet of the predicted values for his or her group’s parameters, as well as a Graphing Shrimp Arrivals task sheet. Have students graph both the predicted arrivals and the observed values on the task sheet and answer the questions.

-Final Presentation and Lab Report-
3) Give each student the instruction sheet for the Presentation and Lab Report, as well as the necessary transparencies and markers. Have students work on presentations and reports for the rest of the class. Both will be due the next class day, so any unfinished work will be assigned as homework. Move about the room as students are working, leading groups to discuss their interpretations of the observed and predicted results as necessary. Students may need to be reminded of the Heads or Tails activity: What they predicted was not exactly what happened, but coin tossing is a random process.
Graphing Shrimp Arrivals

Name: ______________________
Group Members: ______________________

Directions: Plot the shrimp arrival numbers that the equation predicted using your group’s parameters values. Connect these data points with a smooth curve. In a different color, plot the actual arrival numbers that your group observed. Be sure to label both axes and include a legend. Then answer the questions.

1. From the numbers of shrimp that you actually observed in the section, how many of the data points were above the value that the equation predicted for that time?

2. How many of the observed data points were below the value that the equation predicted for that time?
3. Were there any times that what you observed was exactly what the equation predicted?

4. From your graph, do you think that the equation did a good job predicting how many shrimp would be in the section at each time?

5. Does this evidence make you think that the shrimp are actually moving randomly? Why or why not?
Group Presentation and Individual Report

Name: ______________________

Group Members: ______________________________________

Directions: Using the work that you have done, complete the following tasks.

-Group Presentation-

1. On the transparencies provided, make a copy of the Mean Squared Displacement versus Time scatter plot, including the line of fit, and the Shrimp Arrivals graph for the data that your group collected.

Divide the work among all members of your group.

**Mean Squared Displacement versus Time Scatter Plot**

**Task:**
- Plot mean squared displacement data points __________________________
- Draw line of fit (same line used to calculate D) __________________________
- Label axes and identify slope and D value __________________________

**Shrimp Arrivals Graph**

**Task:**
- Plot predicted values; connect with smooth line __________________________
- Plot actual observed values __________________________
- Label axes and make legend __________________________

2. Choose two members of your group to present your graphs to the class on our next class day.

Mean Squared Displacement vs. Time Scatter Plot presenter: __________________________

Shrimp Arrivals Graph presenter: __________________________

Page 1 of 3
Individual Report

Directions: Answer the following questions. Use complete sentences with correct spelling, grammar, and punctuation.

3. What does it mean for a process to be random?

4. What does the term “diffusion” mean? Why did you need to find the diffusion coefficient for the brine shrimp?

5. Did creating a scatter plot help you to learn anything about the data that you collected? Why or why not?

6. During the second day of data collection, how did your group estimate the total number of shrimp in the Petri dish? (Do not just tell me the calculations that you did, but explain why you did the calculations.)
7. Do you think that your data collection was perfect and flawless? If not, what parts of the data collection might have contained errors? Be specific.

8. If you were to do the data collection processes again, how would you change your methods to make them easier or to give more reliable results?

9. Based on your data collection, calculations, and results, do you think that the equation that the mathematicians developed is an appropriate way to model the movement of the brine shrimp? Do you think that the baby brine shrimp move randomly? Explain clearly.

10. Your scatter plot of Mean Squared Displacement versus Time and your Graphing Shrimp Arrivals task sheet need to be turned in with this report.
VIII. Conclusion

While the unit materials for my Brine Shrimp Random Motion unit are complete, there are still improvements I could make and items that I could add to make the materials better for my own use, as well as to benefit other teachers who may use the unit. In the future, I can create sample solution pages and scoring rubrics for all of the homework assignments, task sheets, and the final report. I can also test this unit with middle school students and make necessary revisions based on their experiences and feedback. Finally, I can share the unit and materials through publication, possibly in a teacher trade journal such as *Mathematics Teaching in the Middle School*.

In the teaching of mathematics, I believe that there is no single method that will most effectively teach all students all of the time. Direct instructional strategies are best for teaching certain objectives and skills. Other objectives can most effectively be taught through more student-centered methods based upon constructivist learning theory, such as project-based learning. My experience researching PBL and designing this unit have helped me to increase my knowledge of this research-based teaching practice. It is my hope that through using this unit, as well as other PBL lessons and units that I will design in the future, I will improve my mathematics teaching.

Throughout my teaching career I want to continue to research teaching methods and read about learning activities that other teachers have applied successfully, to continue to build upon the base of tools and strategies that I can use to help my students improve their mathematical abilities and understanding.
References


Leading Students to Investigate Diffusion as a Model of Brine Shrimp Movement

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December 17, 2008

Abstract

Integrating experimental biology laboratory exercises with mathematical modeling can be an effective tool to enhance mathematical relevance for biologists and to emphasize biological realism for mathematicians. This paper describes a lab project designed for and tested in an undergraduate biomathematics course. In the lab, students follow and track the paths of individual brine shrimp confined in shallow salt water in a Petri dish. Students investigate the question, “Is the movement well-characterized as a 2-dimensional random walk?” Through open, but directed discussions, students derive the corresponding partial differential equation, gain an understanding of the solution behavior, and model brine shrimp dispersal under the experimental conditions developed in class. Students use data they collect to estimate a diffusion coefficient, and perform additional experiments of their own design tracking shrimp migration for model validation. We present our teaching philosophy, lecture notes, instructional and lab procedures, and the results of our class-tested experiments so that others can implement this exercise in their classes. Our own experience has led us to appreciate the pedagogical value of allowing students and faculty to grapple with open-ended questions, imperfect data, and the issues of modeling biological phenomena.

Keywords: Project-based learning, undergraduate education, brine shrimp, Artemia franciscana, diffusion, random walks

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†Department of Mathematics and Statistics
‡Department of Biology
1 Introduction

It is becoming increasingly clear that mathematics teaching must evolve. Mathematical pedagogy should train future scientists to reason inductively, discern patterns in complex data, apply mathematics in novel empirical contexts, and communicate results clearly to non-mathematicians. Teaching must therefore address multiple aspects of cognition, and students should prepare written and verbal presentations consistent with the standards and format of general scientific communication. In this paper, we describe a class project designed for a biomathematics course in which students integrate experiments and models to investigate the applicability of diffusion for understanding the dispersal of brine shrimp.

The brine shrimp diffusion project provides students with the experience of dealing with the imperfections and realities of data collection, parameter estimation and model validation, thus narrowing the gap between instruction and research practices. Our goal for this paper is to provide enough mathematical, biological and pedagogical content so that a mathematics professor teaching applied mathematics, differential equations, or statistics (or a biology professor teaching a quantitative, or modeling-oriented course) has sufficient information to comfortably undertake this investigation in his or her class.

The paper proceeds with a general introduction to our pedagogical framework and educational approaches, the course in which we have implemented the project, and our motivation for addressing diffusion and movement. A methods section describes both the mathematical background and biological basics for conducting the discussions and experiments required for the exercise. Teaching notes are interspersed throughout. In the results section, model parameterization and validation results are analyzed and presented from our own experiments to help the implementation of the project. A summary of student results is also included to illustrate the level of reasonable variability which may be expected from student-collected data. A concluding discussion section summarizes our experiences and findings regarding best classroom practices and possible adaptations and extensions of this movement activity.
1.1 Pedagogical Framework

One important component of any instructional design is the consideration of how students will interact with the mathematical content. Some educators believe that the central focus of teaching should be on the transfer of knowledge, while others emphasize that students need to be actively engaged in the construction of knowledge for themselves. The scheme displayed in Table 1 allows us to take both perspectives into account with definitions that describe the various cognitive activities that pertain to learning mathematics (Cangelosi, 2003). We use this scheme for organizing lesson sequences, articulating specific learning objectives, selecting appropriate teaching strategies, and evaluating the achievement of our learning goals. For example, while direct instructional strategies (such as lecture) are appropriate for accomplishing simple knowledge or algorithmic skill objectives, inquiry instructional strategies are most effective for attaining conceptual, discovery, application and creative thinking objectives (Cangelosi, 2003).

To achieve this wide array of cognitive objectives we engage students in project-based learning. The project-based learning (PBL) model focuses on driving questions or problems requiring core concepts of the curriculum (Krajcik et al., 1994). While addressing the central driving question, students engaged in PBL are involved in experimental design, data collection and analysis, problem-solving, inquiry, and decision-making that culminates in realistic products such as presentations, written reports, or physical models (Thomas, 2000; Solomon, 2003).

Throughout the learning process, the role of the teacher is to introduce relevant content before and during the lesson as well as guide and advise students rather than directly manage student work (Solomon, 2003; Hmelo-Silver, 2004). There is not one universally accepted model of PBL, but the following are defining characteristics typically found in learning activities described as project-based learning.

First, students are confronted with authentic problems that are both central to the curriculum and do not have a predetermined solution (Thomas, 2000; Solomon, 2003). This means that in PBL, projects do not serve to illustrate examples of concepts previously taught or to provide extra practice. Rather, students learn the central concepts of the curriculum through the
project. Also, the driving questions in a PBL learning activity cannot be so constrained as to restrict students from developing their own solution methods. Students must be granted a certain degree of autonomy, as well as more choice and responsibility than is typically seen in traditional teaching methods (Thomas, 2000).

Second, PBL activities involve students in investigations. These investigations could take the form of experimental design, problem-solving, decision-making, or model building, with the requirement that existing knowledge is connected to newly constructed knowledge throughout the inquiry processes of the investigation (Thomas, 2000). Throughout the PBL experience, students ask and refine questions, explore ideas, make predictions, conduct experiments, analyze data, and draw conclusions while gaining new knowledge (Krajcik et al., 1994). By the end of the PBL experience, students will have generated artifacts that represent the solution process and solution based upon these investigations.

Lastly, the driving question around which a PBL experience is formed must be an authentic, real-world problem (Solomon, 2003). The problem must be worthwhile, by containing important content and reflecting what actual practitioners of the given discipline might encounter, while also being feasible as a source for student learning (Krajcik et al., 1994). The success of PBL can depend largely on the chosen question, as well as on the tasks and roles given to students to carry out during the learning process. According to Erickson, when choosing problems, teachers should make sure that they are genuine problems that reflect the goals of instruction, situations that consider students’ interests and experiences, appropriate content considering students’ prior knowledge, and of a difficulty level that will challenge the students without discouraging them (Erickson, 1999). This last factor also leads to an additional challenge for the teacher implementing problem-based instruction. When presenting students with problems that are challenging and do not immediately lend themselves to a certain solution strategy, teachers need to be willing to let students struggle without offering suggested methods. However, teachers also need to provide students with sufficient guidance to keep them interested in the problem and focussed (Erickson, 1999).
Many resources on undergraduate mathematics teaching have recently been published that support the general integration of education research findings and trends into undergraduate mathematics instruction. The reader may find some of the following resources useful in the endeavor to improve teaching. The CUPM Curriculum Guide makes recommendations for undergraduate programs and courses in mathematics and outlines the need for action due to the increasing scientific and technological demands of our society (Pollatsek et al., 2004). For ideas to get students working together see Cooperative Learning (Rogers et al., 2001). Krantz’s book How to Teach Mathematics, Second Edition, emphasizes the traditional lecture technique, but also contains informative appendices by various mathematics education researchers that address reform-oriented teaching strategies (Krantz, 1999). Some researchers have specifically studied the implementation of inquiry strategies for differential equations concepts, and one example of the results of such research is the formulation of specific types of pedagogical tools (Rassmussen and Marrongelle, 2006). The book Math&Bio 2010 offers a variety of articles on the issues specific to successfully integrating biology and mathematics (Steen, 2005). Following up on that report, a recent special issue from the journal PRIMUS Problems Issues and Resources in Mathematics Undergraduate Studies on “Integrating Mathematics and Biology” contains several articles with specific examples of curricular links and teaching approaches from other institutions (Ledder, 2008).

1.2 Introduction to the Class

The class project presented here is part of a Utah State University class for advanced undergraduate or beginning graduates entitled “Applied Mathematics in Biology” (AMB). It was originally developed as part of a program to introduce mathematical modeling elements into existing biology laboratory classes. The program aimed to counteract the negative reactions of biology students to required mathematics and statistics courses by introducing mathematical components throughout the biology curriculum. Thus, we developed modeling modules for courses from introductory freshman biology through advanced physiology and genetics courses.
In addition we developed a capstone course, AMB, for students in USU’s BioMath Minor as well as those wanting more exposure to the interface of mathematics and laboratory experimentation. The motivation was to confront students with the experience of doing mathematical biology in the real-world context.

AMB is open to both biologists with minimal mathematics (often only a year of calculus) and mathematicians with little or no biology experience. This openness has disadvantages and advantages, as it implies wide differences in expertise. Lecture-based instruction is particularly difficult as content background challenges only part of the class. However, it provides a natural test situation for project-based learning, and permits a rare opportunity for both sets of students to engage in a real-world situation: collaborating with scientists from disparate disciplines.

The course content consists of a series of projects that integrate laboratory exercises and model development and analysis, team-taught by a biologist (Haefner) and a mathematician (Powell). We have taught it to as few as 5 students and as many as 12. The overall learning goals are: (1) developing multiple mathematical models from general biological questions (e.g., What size of prey should a fish eat?), (2) developing an experimental design appropriate to selecting among the models, (3) estimating parameters and statistically testing model predictions with the data, (4) selecting the best of several models, (5) experience in handling biological material and dealing with messy, noisy data, (6) presenting basic tools to analyze the model behaviors (e.g., nullclines, stability analysis), and (7) creating a learning environment that fosters social skills needed for integrative research. The specific topics covered were chosen to address these goals, not to cover any particular canon of knowledge. Some of the topics covered are: epidemiology games (Powell et al., 1998), fluids flowing from containers, yeast population growth, optimal foraging in fish, and spatial movement of brine shrimp.

### 1.3 Why Diffusion Models?

Almost all biology occurs in a spatial context; almost every process, from propagation of nerve impulses and oxygenation of tissue to the spread of forests after the last ice age, involves
movement in space. At times and small enough scales this movement may seem very directed, as
when ants follow pheromone trails or squirrels move seeds; at others it is clearly random, as in the
sprint and tumble of *E. coli*, the foraging of ladybird larvae, or the dispersal of pollen in the wind.
Even directed movement can be apparently random if the cues which give it direction are
stochastic from an observer's standpoint. Depending on the interaction between movement
mechanisms, spatial heterogeneity and biological process, models must often account for the
effects of movement to faithfully describe the natural world.

Among the most celebrated spatial models are those with a diffusive term modeling random
spatial movement, including Fisher's 1937 discussion of the spread of advantageous mutations
into a population, Skellam's 1951 description and analysis of invasion following a population
introduction, and Turing's 1952 use of diffusion models and their instabilities to provide a theory
for pattern formation (as with stripe and spot patterns in the hides of animals) (Fisher, 1937;
Skellam, 1951; Turing, 1952). Since these classics there have been thousands of papers and many
books written about the effect of random movement on biological dynamics. Berg's 1993 book is
an excellent starting reference on the diffusion equation, which is the canonical PDE description
of the random transport of populations of entities (either chemical or organismal) (Berg, 1993). A
special issue of Ecology in 1994 (Miller, 1994) devoted to spatial modeling is a useful survey
from an ecological perspective, and Turchin's 1998 book gives practical advice on connecting
diffusion models to animal movement (Turchin, 1998).

While hands-on labs are fun for students, their real value comes through an inherent link to
important mathematics and the degree to which they facilitate learning. Hence, choosing
mathematical and biological content that is relevant in current research is essential. The project
described here concerns testing diffusion theory and models of spatial movement in small aquatic
organisms. In a project-based, discovery framework we want students to see how random motion
in space leads to the heat equation, be able to connect movement observations with parameters,
understand the qualitative behavior of solutions and their relationship to observations, and test
model predictions against observational reality.
2 Methods

Our goal with this project is not to push students through a series of programmed steps, but very few students have been introduced to partial differential equation (PDE) movement models. We begin with the general observation that animals and plants move in space and provide some historical background, similar to that outlined above. We initiate a discussion of patterns of animal movement. This can be sophisticated if certain types of biologists are in the class, but in any case, everyone has watched ants or ladybird beetles or seen fish swim, so there is always a common background. Through directed discussion we lead the students to the role of randomness in searching decisions. From this, a probabilistic derivation follows naturally, and then to the continuous time system and PDEs. Details of this derivation are presented below.

Once the basic mathematical framework is available, more directed discussion gets the class to study small animals either swimming or crawling. The fundamental biological assumptions in a diffusion model (e.g., no individual interactions, no effect of external environment) are elicited from the students and considered in the context of choosing an experimental organism and protocols. We introduce the brine shrimp and their biology, and discuss experimental objectives and constraints imposed by available materials and space. Biological and experimental details are presented below. Depending on the timing of class periods, it can be profitable to have these discussions in a class period before the experiments are performed, allowing students more input into variations of the basic experimental design.

Students must revisit the mathematical model and discuss how essential parameters will be estimated given available protocols. Generally speaking, the easiest way to do this is by observations of individual movement trajectories and analysis of mean-squared displacements; details on why this works and how students can be mathematically responsible for regression are presented below. However, this is not nearly the only way to arrive at diffusion parameters. Presenting parameterization as a problem to be solved by the group is essential to student involvement, and in our experience also essential to student internalization of mathematical concepts. The issue of numbers of replicates and the role of sensitivity analysis should be
discussed at this point also; a formal sensitivity analysis is presented below to provide a context that will allow instructors to usefully guide this discussion.

The project follows the two basic steps of all mathematical biosciences: parametrization and validation/falsification. Here we present the most common form that the brine shrimp diffusion project takes, (1) students follow individual shrimp and record positions and times to estimate a diffusion coefficient $D$, and (2) students count numbers of shrimp in regions over time for comparison with analytic predictions. All students do both activities in small groups; sizes of groups are discussed below. If preparations are limited, it is possible for some groups to do (2) before (1), but all students should do both. We generally frame the entire project as a question, “Do brine shrimp move randomly?” and present the mathematical framework for population predictions as a consequence of random motion as a ‘null’ model which students may or may not choose to reject.

2.1 Mathematical Methods

At a population level, what are the consequences of assuming that shrimp move completely at random? What follows is a summary of the mathematical background needed for investigating diffusive aspects of brine shrimp movement. A lattice version of random movement, leading to the diffusion model, is presented and discussed pedagogically. Lecture notes are provided for solution behavior and linear regression for finding diffusion constants (as well as many other parameters in the rest of AMB). For model validation, we develop diffusion predictions for an initial point release of shrimp, and summarize the appropriate sensitivity coefficients for parameters in the point release model to give instructors a notion of where student measurement efforts may be most fruitfully focussed.

2.1.1 Derivation of Diffusion Equation

There are many ways to derive the diffusion equation. In advanced engineering and mathematics classes a flux-based derivation is often used. A difficulty with the flux-based derivation is that it depends on a number of relatively advanced mathematical concepts for both particular steps and
general understanding. For example, the notion that gradients point directly up hill, and therefore random motion will, on average, create a flux of individuals down hill (parallel to the negative gradient) often loses students. Applying the Divergence Theorem, to pass from fluxes through boundaries to divergence of flux inside the area, may not be in the front of most students’ minds; only rarely do biology students take multi-variable calculus. Consequently the flux-based derivation becomes one of many slick, elegant presentations that students will accept, but may not understand. We generally opt for a more constructive approach, based on random walks.

Imagine space partitioned into a grid of (small) rectangles of width $\Delta x$ and breadth $\Delta y$. Let $N(x, y, t)$ denote the number of individuals in the rectangle centered at the point $(x, y)$ at time $t$. Suppose that during each step of time, $\Delta t$, individuals in each box choose to move either left or right, either forward or backward, with no bias. We ask students then to predict the number of individuals at $(x, y)$ one step in the future. After some discussion they realize that individuals will all have come from directly neighboring cells in a ‘+’ pattern, each of those neighbors contributing on average one quarter of the individuals originally resident. Thus,

$$N(x, y, t + \Delta t) = \frac{1}{4} [N(x - \Delta x, y, t) + N(x + \Delta x, y, t) + N(x, y - \Delta y, t) + N(x, y + \Delta y, t)].$$

(1)

It is important to connect with densities ($P$ measured in organisms/area), so that numbers of organisms in arbitrary areas can be analyzed. For suitably small cells, $N \approx P \Delta x \Delta y$, with the degree of the approximation improving as the rectangles become smaller. Then (1) holds for the density function, substituting $N = P \Delta x \Delta y$ and cancelling terms.

We seek an equation which holds at a single $(x, y, t)$ location. Accordingly we expand $P$ (legitimately a smooth function, unlike $N$) in a Taylor series,

$$P(x \pm \Delta x, y, t) = P(x, y, t) \pm \Delta x P_x + \frac{1}{2} \Delta x^2 P_{xx} + \cdots,$$

$$P(x, y \pm \Delta y, t) = P(x, y, t) \pm \Delta y P_y + \frac{1}{2} \Delta y^2 P_{yy} + \cdots,$$

$$P(x, y, t + \Delta t) = P(x, y, t) \pm \Delta t P_t + \cdots,$$

where we have adopted subscript notation for partial derivatives and all functions are evaluated at
Substituting into (1) gives

\[
P(x, y, t) + \Delta t P_t + \cdots = \frac{1}{4} \left[ P(x, y, t) + \Delta x P_x + \frac{1}{2} \Delta x^2 P_{xx} + P(x, y, t) - \Delta x P_x + \frac{1}{2} \Delta x^2 P_{xx} \\
+ P(x, y, t) + \Delta y P_y + \frac{1}{2} \Delta y^2 P_{yy} + P(x, y, t) - \Delta y P_y + \frac{1}{2} \Delta y^2 P_{yy} + \cdots \right].
\]

Students get excited canceling the obvious terms, leading to

\[
\Delta t P_t = \frac{1}{4} \left[ \Delta x^2 P_{xx} + \Delta y^2 P_{yy} \right] + \cdots
\]

If \( \Delta x = \Delta y \) (that is, movement is isotropic, or step size is equal in both directions) and we take

the distinguished limit \( \Delta t, \Delta x, \Delta y \to 0 \) while \( D = \frac{\Delta x^2 + \Delta y^2}{4 \Delta t} \) is held constant, we arrive at the

diffusion equation

\[
P_t = D \left[ P_{xx} + P_{yy} \right].
\]

For future reference, note that the diffusion constant can be interpreted

\[
D = \frac{\Delta x^2 + \Delta y^2}{4 \Delta t} = \frac{\text{Mean Squared Distance Moved}}{4 \cdot \text{Time to Move That Far}}.
\]

This derivation, in our experience, invites student discussion and participation and is also

amenable to experimentation – students can put a pile of coins in the center of a square grid and

start flipping each coin (twice) and moving it left, right, up or down as indicated. A few iterations

convey the behavior of the diffusion equation.

\subsection*{2.1.2 Solutions and Behavior}

At this point we generally exhibit a solution for discussion,

\[
P(x, y, t) = \frac{1}{4\pi D t} e^{-\frac{x^2+y^2}{4Dt}}.
\]

This is the fundamental solution, which is found using similarity and transform methods in PDE

and applied math classes. In a multivariate calculus setting this solution is often given as an

applied example in partial differentiation and the utility of higher partial derivatives (e.g. testing

to see that (3) is actually a solution to the diffusion equation). In all classes we discuss behavior
of the solution, including convergence to the delta function as time tends to zero and what this
means from an observational and biological perspective. We also make some connection with the
statistical background. Since (3) can be viewed as the product of two normal distributions for
independent $x$, $y$ variables,

$$\frac{1}{4\pi Dt} e^{-\frac{x^2+y^2}{4Dt}} = \frac{1}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}} \frac{1}{\sqrt{4\pi Dt}} e^{-\frac{y^2}{4Dt}},$$

each with variance $\sigma^2 = 2Dt$, the variance of sums of squares of these variables is additive,

$$E(x^2 + y^2) = \sigma^2 + \sigma^2 = 4Dt. \tag{4}$$

This means that the characteristic area covered by the population profile grows linearly with time,
while the peak density decreases inversely with time.

### 2.1.3 Regression to Find Diffusion Constants

Discussion of population variance following point release is important not only because it
increases understanding of solution behavior, but also because this provides an avenue by which
researchers estimate $D$ for real populations (Turchin, 1998). Since the expectation of squared
distances increases linearly (4), it follows that if squared displacements of several individuals are
averaged and observations of mean squared displacement are plotted in time they should produce
a line with slope $4D$ passing through the origin. Students must have a method to determine the
most likely slope from imperfect data – a special case of linear regression. Students at many
levels can benefit from a discussion of regression with general linear models, and its basis in
linear algebra, in order to figure out the slope of a regression line passing through the origin. Most
linear algebra books, and most linear algebra and basic statistics classes, seem to skip over this
crucial piece of mathematical technology, so we present the approach in detail, following the
discussion given in Lay (2003).

Generally, if a model

$$x = a_1 f_1(t) + a_2 f_2(t) + \cdots + a_m f_m(t)$$
is to be fit to data \( \{(t_j, x_j)\}_{j=1}^{n} \), then the system of equations

\[
x_j = a_1 f_1(t_j) + a_2 f_2(t_j) + \cdots + a_m f_m(t_j)
\]

must be solved for the coefficients \( a_i \). The system can be written in matrix form

\[
\bar{x} = \bar{F} \bar{a},
\]

where \( \bar{x} \) is the vector of observations, \( \bar{a} \) is the vector of coefficients, and \( \bar{F} \) is called the design matrix, with rows \( (i) \) given as the individual model functions \( f_j(t_i) \) evaluated at \( t_i \). The system is overdetermined when \( n > m \), and so solutions normally only exist in the least squares sense.

Following Lay (2003), the system

\[
\bar{F}^T \bar{x} = (\bar{F}^T \bar{F}) \bar{a}
\]

is the orthogonal projection of these equations onto the space spanned by the model functions; therefore

\[
\bar{a} = (\bar{F}^T \bar{F})^{-1} \bar{F}^T \bar{x}
\]

is the 'best' solution in the least-squares sense. The matrix \( (\bar{F}^T \bar{F})^{-1} \bar{F}^T \) is called the pseudo-inverse. (Just a note for mathematicians – correlations among the model functions across \( t_i \) appear as near-collinearity of columns in the design matrix, which leads to poor conditioning in the inverse.) We generally spend a lecture and a computer lab on this subject in the AMB class since determining parameters from experimental data is probably the single most important technical proficiency required for biomathematics students. Most math students, even those with advanced linear algebra experience, have not seen the connection between projections, least squares solutions, pseudo-inverses, regression and parameter estimation, so this content is invariably an eye-opener.

In the case of fitting a line with intercept at the origin, the design matrix is a vector,

\[
\bar{F} = (t_1, t_2, \ldots, t_n)^T,
\]

so the slope is just

\[
\hat{a}_1 = \frac{(t_1, t_2, \ldots, t_n) \cdot (r_1, r_2, \ldots, r_n)^T}{(t_1, t_2, \ldots, t_n) \cdot (t_1, t_2, \ldots, t_n)^T},
\]

13
where the $r_j$ are mean squared displacements at time $t_j$.

### 2.1.4 Arrival Predictions for Validation

An overall theme in AMB is validation/falsification of parametrized models in the arena of independent experiments. Ideally students construct the validation experiments themselves, often on the basis of situations we have discussed in class. One example involves release of a population and then subsequent observation to determine whether dispersal assumes a Gaussian profile, but typically this involves a lot of counting and it is difficult to count rapidly enough to capture an accurate snapshot of the spatio-temporal process. Another example is observations of arrival density in a small target area following a 'point' introduction, simulating arrival of exotic or diseased populations in a conservation area at some distance from an initial introduction. This is a pedagogically interesting, since it can be attached to a variety of stories (e.g. at what rate should national park managers expect arrival of deer with chronic wasting disease if the park is some distance from a point source of infection) and leads to a good discussion of pulse behavior in the diffusion equation. Will there (can there?) be a maximum in the number arriving? If so, when should it be expected and how long until it fades away? These questions can be illuminated by considering the asymptotic cases: at time zero there should be no arrivals, and as time goes on to eternity the population should become evenly dispersed at low density (or zero density in an infinite space). Somewhere in between, then, must be a maximum rate of arrival (which can lead to more discussion in a PDE context, since diffusion solutions can have no internal extrema).

A little preliminary experimentation reveals that 'point' release of a population is impossible. Via either discussion (beginning with the statistical nature of the diffusion solution and estimating the variance of shrimp following point release) or instructor revelation we introduce an extension of (3),

$$P(r, t) = \frac{N_0}{2\pi(\sigma_0^2 + 2Dt)} e^{-\frac{r^2}{2(\sigma_0^2 + 2Dt)}},$$

(5)

where $\sigma_0$ characterizes the spatial extent of the initial release and $N_0$ is the number of individuals
released. The predicted number of individuals in a counting area, $A$, would be

$$N_A(t) = \int_{A} \int P(r, t) \, dA = \frac{N_0}{2\pi} \int_{L_1}^{L_2} \int_{\alpha}^{\beta} e^{-\frac{r^2}{2(\sigma_0^2+2Dt)}} \frac{r \, dr \, d\theta}{\sigma_0^2 + 2Dt},$$

where $A$ is an annular region contained between the angles $\alpha$ and $\beta$ with $L_1 \leq r \leq L_2$. The substitution

$$u = \frac{r^2}{2(\sigma_0^2 + 2Dt)}$$

gives

$$N_A(t) = \frac{N_0}{2\pi} \left[ e^{-\frac{L_1^2}{2(\sigma_0^2+2Dt)}} - e^{-\frac{L_2^2}{2(\sigma_0^2+2Dt)}} \right].$$

(6)

Alternatively, the prediction

$$N_A(t) \approx N_0P(L, t) = \frac{N_0A}{2\pi(\sigma_0^2 + 2Dt)} e^{-\frac{L^2}{2(\sigma_0^2+2Dt)}}$$

(7)

is reasonable for a class that is not up to the double integration, provided the radial extent of the area $A$ is small. An interesting problem for more advanced students is to determine the order of accuracy of the approximation. Letting $L_1 = L - \Delta$ and $L_2 = L + \Delta$ in (6),

$$N_A = N_0 \left[ e^{-\frac{(L-\Delta)^2}{2(\sigma_0^2+2Dt)}} - e^{-\frac{(L+\Delta)^2}{2(\sigma_0^2+2Dt)}} \right]$$

$$= N_0 \left( \frac{\beta - \alpha}{2\pi} \right) \frac{d}{dL} \left[ e^{-\frac{L^2}{2(\sigma_0^2+2Dt)}} \right] + O(\Delta^2)$$

$$= N_0 \left( \frac{\beta - \alpha}{2\pi} \right) 2L\Delta e^{-\frac{L^2}{2(\sigma_0^2+2Dt)}} + O(\Delta^2).$$

Since the annular region has area

$$A = \frac{\beta - \alpha}{2\pi} \pi [(L + \Delta)^2 - (L - \Delta)^2] = 2(\beta - \alpha) L\Delta + O(\Delta^2)$$

we have

$$N_A(t) = \frac{N_0A}{2\pi(\sigma_0^2 + 2Dt)} e^{-\frac{L^2}{2(\sigma_0^2+2Dt)}} + O(\Delta^2),$$

giving the accuracy of (7).
2.1.5 Sensitivity Analysis

Students soon realize that, in addition to their estimate of \( D \), model validation requires measurements of \( N_0, L \) and \( \sigma_0 \), each of which is likely to have some error attached. These errors are important to understand for both the students and for the instructors, since they give some notion of where to place experimental emphasis and what may cause divergence of results. A good exercise in multivariate calculus is to determine the relative sensitivity of (7) using the linear approximation

\[
\frac{\Delta N_A}{N_A} \approx \frac{D}{N_A} \frac{\partial N_A}{\partial D} \frac{\Delta D}{D} + \frac{\sigma_0}{N_A} \frac{\partial N_A}{\partial \sigma_0} \frac{\Delta \sigma_0}{\sigma_0} + \frac{L}{N_A} \frac{\partial N_A}{\partial L} \frac{\Delta L}{L} + \frac{N_0}{N_A} \frac{\partial N_A}{\partial N_0} \frac{\Delta N_0}{N_0},
\]

where the sensitivity coefficients are

\[
\epsilon_D = -\frac{D t (-L^2 + 2\sigma_0^2 + 4Dt)}{(\sigma_0^2 + 2Dt)^2},
\]

\[
\epsilon_\sigma = \frac{\sigma_0^2 (L^2 - 2(\sigma_0^2 + 2Dt))}{(\sigma_0^2 + 2Dt)^2},
\]

\[
\epsilon_L = -\frac{L^2}{\sigma_0^2 + 2Dt}, \quad \text{and} \quad \epsilon_N = 1.
\]

We suggest that students include such a sensitivity analysis in their lab reports along with an interpretation of the numerical values of these coefficients based on their experimentally determined parameters, as these results have implications for the design of the validation procedure. (Our results are reported in Section 3 and Figures 9 and 10.) While many other mathematical extensions of this lab project are possible, we proceed now with the necessary biological background.

2.2 Biological Methods

Although any small crustacean would serve for this exercise, those from the group of brine or fairy shrimp (family Anostraca) are readily available. We have successfully used the Great Salt Lake (GSL) brine shrimp (Artemia franciscana, Crustacea, Branchiopoda), also known as “Sea Monkeys” shown in Figure 1. Below, we briefly describe their natural history and procedures for
hatching and raising them to a size appropriate for this exercise. Additional information is available online as described in Appendix A.

### 2.2.1 Natural History

As their common name suggests, brine shrimp are adapted to high saline conditions and *A. franciscana* survives well in the GSL whose salinity ranges from 12–27% (seawater is 3.5%). Adults are approximately 10 mm long and feed primarily on GSL algae such as the flagellate *Dunaliella viridis*, but the species varies with season. Reproduction in *A. franciscana* is sexual with either live birth of first instar nauplii (i.e., first larval stage) or the production of cysts that are capable of prolonged diapause. The cysts are the “eggs” one buys in pet stores. Optimal reproduction occurs at 14–17% salinity, which in the GSL occurs in the south arm of the lake where persistent winds drive adults and floating cysts towards freshwater inflows and reduced salinity. The life span of a typical female is 6–9 months and she can produce about 8 broods per year. Once born or hatched, 15–16 molts by nauplii are required for maturity. Additional life history information is available from resources listed in Appendix A.

### 2.2.2 Hatching the Shrimp from Cysts

The easiest method of obtaining the appropriate size class of brine shrimp nauplii for this laboratory exercise is to hatch the shrimp from commercially available cysts. Optimal hatching success occurs in actual GSL water diluted to about 1/3 of original salinity (c. 6–10%). In the absence of the real thing, InstantOcean®, at appropriate concentration, has a proper balance of ions. If neither is available, hatching will occur in a solution of 60 g/liter of table salt and tap water that has been exposed to the air for 3 days (to ‘de-gas’ and remove fluoride and chlorine).

In a common household 9×13 inch (23×33 cm) baking dish add about 1 inch (2.5 cm) of salt water and pour the cysts onto the surface so that about 1/2 to 1/3 of the surface is covered with the brown cysts. Cover the dish with plastic kitchen wrap to prevent evaporation, using a small beaker or juice glass for support (Figure 2). Early instar nauplii are delicate and undue turbulence can increase mortality so that aeration by atmospheric diffusion alone is best. Very
gentle aeration with an aquarium bubbler pump and stone is possible, but avoid too much
turbulence. Expect about 50% hatching success. The dish can be left at room temperature in
typical office ambient light intensities and cycles. No other incubation equipment is needed.
Allow three days to obtain sufficient numbers for this exercise.

The experiment uses second nauplii shrimp which is the first feeding stage, so a food
source is not needed if the animals are used soon after hatching. They are able to feed on bread
yeast cells if you wish to keep them alive longer than a few days or attempt to grow them through
additional nauplii stages. Because brine shrimp adults are easily viewed with the unaided eye,
have elaborate, beautiful anatomy, and interact with each other in small containers (especially
males and females), the affective domain of learning (Table 1) can be addressed by providing
students the opportunity to simply observe adults and nauplii in small containers. If dissecting
microscopes or an overhead projector are available this experience is greatly enhanced.

2.2.3 Items Needed for the Labs

The required materials for both lab procedures are:

1. A supply of second nauplius brine shrimp larvae (other members of the family Anostraca
can be used);
2. Blunt, wide-aperture pipettes (plastic pipettes cut to an aperture of about 3–5 mm are best),
   3 per group;
3. 150 mm by 15–20 mm deep Petri dishes, 2 per group (glass preferred, or free of scratches if
   plastic);
4. Overhead transparency gridded with x-y coordinates (see Figure 3a), 1 per group;
5. Overhead transparency marked in concentric circles (see Figure 3b), 1 per group;
6. Small custard dishes or Petri dishes for transferring, 2 per group;
7. Overhead projector (ideally, 1 per group) or a light box or a multimedia projector that
displays flat surfaces;
8. Stop watch or timer accurate to seconds, 1 per group;
9. (Optional) Video camera on a tripod to record movements;
10. (Optional) Adults for behavioral observation and student motivation.

Appendix A lists further references and websites with resources pertaining to this exercise.
2.2.4 Individual movement procedure

Before the exercise begins, the dish of second instar nauplii should be brought into the classroom and covered with foil, leaving one corner exposed. A bright light shined on the exposed corner will cause the shrimp to congregate so that high concentrations can be extracted for the arrival densities experiment. A dense cloud of organisms will form in 30–60 minutes.

To observe and record individual shrimp movement, the bottom of a petri dish should be barely covered with salt water of the same salinity as that used to incubate the cysts. Place the gridded transparency (Figure 3a) on the overhead projector and the dish on top of that centered on the origin of the grid. Use pipettes to deposit a small drop containing approximately with only a few shrimp in the dish. If neither a projector nor light box is available shine a small desk lamp obliquely on the Petri dish. Be aware that the shrimp are positively phototactic and will be attracted to the light.

After the shrimp are added to the Petri dish, one student times and records positions while the other calls out the $x$ and $y$ coordinates for an individual shrimp for as long as possible, at five or ten second intervals (quicker is better but also more difficult, see Results below). When the shrimp nears the edge of the dish, observation should cease. This is repeated for 6–10 shrimp. The more shrimp and the longer the time traces, the better the estimates of $D$ (see Section 3.1 below) – best results seem to be obtained by following a shrimp starting at the center of the Petri dish for 60–90 seconds. Old style overhead projectors produce heat which will affect movement rates, so restarting the experiment with new water and shrimp every 5–8 minutes is advisable. This is not a problem if you are using a video camera connected to a classroom digital projector.

2.2.5 Arrival densities procedure

As discussed above, this validation experiment simulates arrival in a spatial area after point release of invaders. Again, add salt water to barely cover a petri dish; center the dish on top of the transparency with concentric rings (Figure 3b) on the overhead projector. Use the pipette to extract a dense sample from the culture dish. Being careful to acquire as little water as possible,
use the pipette to withdraw about 2–3 ml of shrimp culture.

Count accuracy is improved if an overhead projector is used to project the image on a wall.

If a video camera is being used, start the camera. Carefully deposit the contents of the pipette in the center of the arena (Figure 4). When releasing the shrimp, the pipette should be held vertically so as to minimize advective bias in the initial conditions. The student timer should note and record the area that the initial placement of shrimp occupies, by estimating the approximate radius of the population after release. This is an important parameter in the model ($\sigma_0$ above).

We have found that a group of 5 students is optimal. One student times the intervals between counts (10 seconds) and records the values reported verbally by 4 student observers. Each student is assigned one region of the grid and reports the number of shrimp in the region when commanded by the timer. We repeat the observations at least twice using 4 regions equally near to the center of the grid and, in a separate experiment, 4 regions farther from the center. The near region should be outside the zone of the initial conditions, and the far region should not be too near the wall of the Petri dish. Students will want to design the experiment to last long enough to distinguish the projected pulse; in our experiments we continued for two to three minutes.

### 2.3 Assessment Strategies

Due to the interactive nature of project-based learning, continual assessment of student learning is required to guide instructional strategies. For formative assessment, we listen carefully to student contributions during class discussions. Ideas offered by students are frequently incorporated into our presentation of new information; we ask many leading, open-ended questions to ensure that central ideas are eventually articulated by students and based on their individual experiences.

During the class period, it is important for the instructors to circulate to the various groups, listen to questions raised and problem-solving strategies. Generally this is not passive; carefully framed questions and observations keep students engaged and facilitate their nascent ideas. We require a project from small groups containing a mix of biologists and mathematicians. We observe both intra-group learning (mainly math students teaching biologists) and inter-group learning, since we
require on-going progress reports for the entire class. This presents students with alternative
approaches to the same problem. When time permits, the whole class shares and discusses
approaches to problems arising in data collection or model evaluation.

The target question, “How much like a random walk is the brine shrimp movement?” forces
students to synthesize their understanding of the diffusion model in a written report. Individual or
group reports are checked for mathematical accuracy to see that students have made true
statements, correct calculations, and performed appropriate algorithms. Work is also evaluated for
appropriate incorporation of mathematical equations and expressions as well as data and plots
into a narrative that relays an acceptable understanding of the model and its applicability in the
particular case of our experimental set-up. These latter aspects of mathematical writing are
introduced not only in the diffusion lab; throughout the course students prepare reports to hone
their written communication talents. Two main deficiencies that continually arise are: (1)
intelligent and well-formed writing that surrounds the equations, and (2) figure and table captions.
A surprising number of students either do not read or do not conform to writing standards in
journal articles. Early in the term we collect preliminary versions of reports which we correct and
return; hence we expect the final product to be high quality.

3 Results

There are three flavors of results achieved using the brine shrimp diffusion experiment. In a direct
sense there are estimates of diffusion constants and comparisons of arrival observations with
predictions from both our students and ourselves. We present results both from our students’ labs
in Spring, 2008 as well as results obtained by the authors in Summer, 2008 to give potential
instructors some feeling for what to expect from student results. Our analysis of the experiment is
presented to help instructors evaluate, interpret, and anticipate student results, but not necessarily
as a model of student results. There are procedural results to present as well; during our Summer
experiments we performed extensive videotaping of experiments so that we could ascertain the
degree of error produced by human observation and also to provide a sufficiently extensive data
set that bootstrapping would reveal the distribution of parameter results. The purpose of this is not
to suggest that videotaping is a more effective way to collect data (although it is), but rather to
reassure potential instructors that direct observation (which thoroughly engages students in a way
that videotaped data does not) is 'good enough.' Finally there are the qualitative student responses
to the lab, based on observations and reflections on what students have gained from this activity.

3.1 Analysis of the Experiment

3.1.1 Estimation of diffusion constant

Figure 5 depicts observed mean squared displacements of 12 shrimp sampled at both five and ten
second intervals. Several different slopes can be determined depending on the length of the data
set considered. This is a good example of the real world of messy data that students must
confront. To determine how often the position of each shrimp should be recorded during the
individual movement procedure, Figure 5 shows the effects of collecting data every 5 seconds and
every 10 seconds on the estimate of the diffusion constant, $D$. We conclude that the time interval
used does not have a large effect on the estimate of $D$, and the appropriate time interval should be
determined based upon the ability of the students conducting the data collection. In both our and
our students’ experience, five second intervals was about the limit sustainable for calling out and
recording positions, but excitement is maintained; ten second intervals is very easy to keep up
with but about the time spacing that begins to feel tedious because not enough is happening.

A second experimental design constraint is the length of time that students should follow an
individual shrimp. As mentioned in Section 2.2.4, observation of an individual shrimp should
cease when it comes in contact with the edge, as its movement can no longer be considered
random. However, each shrimp will hit the edge after a different length of time, and accordingly
longer data tracks will begin sampling only slower shrimp. Figure 6 shows the effect of the total
length of data collection on the estimate of $D$. We conclude that 80 to 90 seconds is an
appropriate length of time to record the positions of shrimp that do not contact the edge before
that point. In taking a large number of data samples, we have found that approximately 75% of
the data tracks for individual shrimp displacement will contain information for up to 90 seconds
and results seem fairly stable up to that time.

The last experimental design consideration is the number of data tracks, \( N \), required to
produce reasonable results. Through bootstrapping both the real-time data and the video data we
obtained the results shown in Table 3. Results vary between class and author data due to variance
in experimental conditions; experiments were performed in different rooms, at differing
temperatures and light conditions, with shrimp of different broods, leading to diffusion constants
almost twice as large. With only ten observations the standard deviation of estimates hovers
around 28% and decreases inversely with \( \sqrt{N} \), so that at 40 observations the standard deviation is
approximately 14% of the estimate of \( D \). Statisticians would no doubt tell us that \( N \) should be
much larger; on the other hand, to get the standard deviation down to 10% would require on the
order of 80 observations \((28^2/10^2 \approx 7.8)\). As discussed below, sensitivity of the validation
experiment to \( D \) is relatively low. Since ten observations require 15-20 minutes, a reasonable goal
given sufficient class time, would be 20 observations with an anticipated error of 20% in estimates
of \( D \).

3.1.2 Arrival densities procedure

Both students and the authors observed arrival numbers in two counting areas (far and near,
\( L = 1.83 \text{ cm and 3.65 cm, respectively} \)). The goal of the experiment is to determine whether or
not brine shrimp are following a direct prediction of the diffusion model. Students immediately
realize that \( D \) is not the only parameter required; while other parameters come from direct
measurement, the total number of shrimp released, \( N_0 \), must be determined directly by counting.
We compared direct counts with stop-action counts of video data to ascertain the error in human
counting. Using our real-time data, \( N_0 \) was calculated with a 3% error and 15% error in the two
separate trials when compared to exact counts from the video recordings. Typically, real-time
numbers were lower than numbers found when using the videotaped data, but since the sensitivity
of the predictions to error in \( N_0 \) is direct, these errors are relatively trivial from the standpoint of
validation/falsification.

Figure 8 shows observed shrimp arrivals with the predicted arrivals based on the diffusion model. The figure shows that variability in counts at the arrival area is obviously high, of the same order as the expected number. Figure 7 shows the arrival of shrimp at two different regions for both student real-time data and numbers gathered from a video recording of the same trials. This figure indicates that counting at arrival areas is unlikely to be the source of variability; the arrival process itself is inherently stochastic and counting error seems to contribute little to overall variability. Most students, in fact, looking at the raw data, are convinced that the diffusion model is a failure and that there is no clear pulse of arrival. However, when they plot model predictions their perceptions reorganize. As often happens, inflicting a predicted curve on observational data brings out elements of the global behavior which might otherwise be missed. The variability makes it difficult to definitively validate the diffusion model (particularly given its sensitivity to the measurements, see below), but the model’s ability to organize the overall behavior of the observations makes it difficult to falsify as well.

3.1.3 Parameter sensitivities

To assess elements of the experimental protocol, we performed a sensitivity analysis on selected parameters. For nominal measurements of \( L = 1.4 \text{ cm}, \sigma_0 = .5 \text{ cm}, D = 2.6 \times 10^{-2} \text{ cm}^2/\text{sec} \) the peak of the arrival pulse occurs at \( t \approx 134.3 \text{ sec} \), at which time, from Equations 9–11, the elasticities are

\[
\epsilon_D = .11, \quad \epsilon_\sigma = .01, \quad \epsilon_L = -2.23.
\]

Recalling that \( \epsilon_N = 1 \) (Equation 11), we see that accuracy at the pulse is more than twice as sensitive to measurement of \( L \) as measurements of \( N_0 \). A 5% error in estimating the population size will create a five percent error in measurements of population at the peak of the pulse; a 5% error in estimating \( L \) produces an 11% error. Measurements at the pulse will be relatively insensitive to errors in either \( D \) or \( \sigma \). Earlier (e.g. \( t = 15 \))

\[
\epsilon_D = 4.17, \quad \epsilon_\sigma = 3.18, \quad \epsilon_L = -13.5.
\]
While only a small fraction of the population will have arrived at this time, this indicates that measurements of the initial ramp of the pulse may be very sensitive to parametric error, particularly in measurements of $L$. Figure 9 shows the sensitivity of the model to changes in the parameters over the entire length of data collection. We discuss these kinds of results in class in the context of experimental design. For example, “If we were to repeat the experiment, how would you change the observational techniques?” Or, these discussions can profitably occur before the students collect data. For the instructor’s benefit, designing an arrival area with $L = 2$ cm is probably optimal for balancing the desire for a clear peak relatively, but not too, early (as opposed to the broad, flat peak illustrated on the right hand side of Figure 8) and low sensitivity to measurement errors.

### 3.2 Evaluation of Student Learning

Our evaluation of what students have learned from this activity comes from our assessment of students during the course, student course evaluations and an emailed questionnaire to the course instructors and recent students. Results from these samples are summarized in Table 1, Column 3. A summary of the main teaching components that lead to these outcomes is provided in Table 2. Students reported they enjoyed learning through the hands-on labs and collaboration with peers and instructors. Course instructors were impressed with the creativity in student explanations and the ease with which the project raised mathematically and scientifically important issues for students. For example, students discussed the effect of a no-flux boundary (the edge of the Petri dish) and raised concern about trying to balance where ‘removing outliers’ becomes ‘misrepresenting data.’ The challenges and rewards of using personally collected data from living creatures helped students gain a deeper appreciation for the process of fitting and evaluating a mathematical model. As one student said, “It was the first time I had used collected data to parameterize a PDE model. I learned the difficulty of adding a spatial element to a mathematical model. I learned that it is hard to keep data collection consistent and accurate, especially when observing something that moves quickly and unpredictably like brine shrimp.”
4 Discussion

The brine shrimp lab is an example of a project-based learning experience for students that kindles interest in advanced mathematics and provides a vehicle for friendly collaboration among math, biology and math education faculty. Our experience confirms that in addition to being a tractable lab (without requiring sophisticated equipment), the project is also mathematically reasonable as a vehicle for studying the classical diffusion model, and consistent with reform efforts in improving mathematics instruction. The biological preparation is relatively simple, making it feasible to incorporate into classes in PDEs, vector calculus, mathematical modeling, advanced statistics classes, and stochastic processes. Each of these specific classes would emphasize different aspects of the exercise as appropriate to the subject matter. For example, a statistics class might emphasize goodness-of-fit tests to a Gaussian distribution, a PDE class might develop error terms to approximations to flux rates, or a class on stochastic processes might devote more attention to the interface of individual probabilistic events and average stochastics of ensemble behavior.

A key component to the instructional effectiveness of this project is the classroom environment that fosters learning because of the communication and interaction style among the professors and students. The instructors of the course employ a variety of strategies for creating a community of inquiry in the discussions (compare with (Goos, 2004)), and predominantly these strategies are realized through the communication style in the class. By avoiding the typical Initiate-Response-Evaluate cycles that dominate traditional classroom communication, the instructors create an environment in which students no longer fear but embrace failure and are invested in gathering and interpreting experimental results (Cangelosi, 2003). The inherent uncertainty of a biological experiment puts students and faculty on equal footing for discussion as lectures can not. Hence, classroom discourse is enhanced since conversations arise naturally addressing the variability in the methodology and findings. Students engage in reasoning-level (rather than memory-level) question-and-answer sessions throughout the project. Ideas offered by students are valued, frequently followed, and not judged by the instructor.

Another essential component to the success of this project is the direct experience of
gathering data from living organisms. Presenting students with prepared, published, or
hypothetical data fails to engage them in scientific inquiry. Exploring computer simulations of
random walks can help involve students in conjecturing and inductive reasoning, yet the
simulation approach still greatly diminishes the investment and connection students feel when
working with real brine shrimp. Possibly, this is because we are deeply wired to try to understand
the motion of other agents in the real world, which ‘hooks’ students. The time and energy
necessary to gather data gives students ownership and motivation. The novelty of handling living
things in a mathematical context inspires curiosity leading to the creative examination of the
relationship between diffusion solutions and observational reality. Mathematics changes its
meaning from an abstract logical game to a sense-making language adding predictability to
erratic situations.

There is a trade-off between the didactic and discovery nature of this project. The discovery
process takes time, is messy, inexact, and likely to be uncomfortable territory for a didactic
researcher/professor used to making polished presentations of carefully edited essential
information and choice examples. Especially at the start of a new unit of instruction, the initial
biological question is vague, “Do guppies forage optimally? Do shrimp move randomly?” These
types of questions usually elicit long periods of silence. One has to be careful that expert input is
held until the class has had a chance to struggle. Less information and mathematical knowledge
will be delivered, but discovery-based teaching involves students more directly in the activities of
our profession. Our hope is that readers will try this or similar exercises in their classes, enjoy the
incorporation of project-based learning in their own teaching, and that our efforts will save time in
class preparation.
References


Educational Psychology Review 16(3), 235–266.


A Resources

An excellent source for many of the items needed is either a colleague from the local biology department or:

Carlolina Biological Supply Company
2700 York Road
Burlington, NC 27215
www.carolina.com
voice: 800-334-5551

The item references below are from CBSC Catalog 77, 2008.

1. 150 mm Petri dishes: local chemistry stores or CBSC (glass: item FA-74-1164, plastic: FA-74-1254)

2. brine shrimp: local pet store or CBSC: hatching kit (everything you need in one package: eggs, pump, salt: FA-14-2214); eggs only: FA-14-2240; adults for optional behavioral observation: FA-14-2230)

3. Large plastic pipettes: CBSC: FA-73-6988

4. Information on the GSL and brine shrimps:
   
   
   

5. PDF files for the templates can be downloaded from:
   
   http://www.math.usu.edu/~kohler

6. A Java simulation of Artemia movement, growth, and reproduction:
   
   http://www.captain.at/artemia-simulation.php
7. Webpage for “Applied Mathematics in Biology” USU Biol/Math 4230:

http://cc.usu.edu/~jhaefner/AMBcourse.html

8. Additional laboratory exercises for biomathematics:

http://cc.usu.edu/~jhaefner/BioMathLab.html
<table>
<thead>
<tr>
<th>Domain</th>
<th>Learning Level</th>
<th>Examples of learning objectives students attained through this project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognitive</td>
<td><strong>Construct a concept</strong></td>
<td>Provided examples of diffusion or random motion in nature such as movement of ions, dispersal of particles, loss of heat.</td>
</tr>
<tr>
<td></td>
<td>Use inductive reasoning, distinguish between examples and non-examples.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Discover a relationship</strong></td>
<td>Observed, explained and quantified patterns in the shrimp movement. Related the mathematical model to these observations and discovered the complexity involved in relating experiment and theory. Invented and refined experimental protocols for data collection such as how to estimate $N_0$.</td>
</tr>
<tr>
<td></td>
<td>Use inductive reasoning, discover relationships among concepts.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Simple knowledge</strong></td>
<td>Recalled the PDE for diffusion and the correct units for the diffusion coefficient.</td>
</tr>
<tr>
<td></td>
<td>Recall a specified response (not multistep) to a specified stimulus.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Comprehension and Communication</strong></td>
<td>Integrated quantitative findings and mathematical formulae in a written report of experimental results. Used the appropriate commands and made meaningful plots using MATLAB.</td>
</tr>
<tr>
<td></td>
<td>Extract and interpret meaning, use the language of mathematics.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Algorithmic skill</strong></td>
<td>Systematically gathered data for each experimental procedure. Estimated parameters for the arrival densities validation using linear regression.</td>
</tr>
<tr>
<td></td>
<td>Recall and execute a multistep procedure.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Application</strong></td>
<td>Developed a prediction for the existence of a peak in the arrival rate results based on PDE solution. Invented plausible explanations for the effects of Petri dish edges on the randomness of observed shrimp movement.</td>
</tr>
<tr>
<td></td>
<td>Use deductive reasoning, decide if at all mathematical content is relevant.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Creative thinking</strong></td>
<td>Provided novel explanations for discrepancies between data and model predictions.</td>
</tr>
<tr>
<td></td>
<td>Use divergent reasoning to view mathematical content in unusual, novel ways.</td>
<td></td>
</tr>
<tr>
<td>Affective</td>
<td><strong>Appreciation</strong></td>
<td>Expressed an interest in quantifying observations made about the cute and likable shrimp.</td>
</tr>
<tr>
<td></td>
<td>Believe mathematical content has value.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Willingness to try</strong></td>
<td>Included an analysis of the sensitivity of parameters in the lab report.</td>
</tr>
<tr>
<td></td>
<td>Choose to attempt a mathematical task.</td>
<td></td>
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</table>
Table 2: Summary of the instructional procedures and our recommendations for implementation.

<table>
<thead>
<tr>
<th>Instructional Procedures</th>
<th>Recommendations</th>
</tr>
</thead>
</table>
| **1. Derivation of the model**
Introduce the derivation and solution of the diffusion equation as a 2-dimensional random walk through lecture and discussion. Lead students to the question of the project, “Is diffusion a good model for brine shrimp movement?” and begin a discussion of appropriate experiments. | Emphasize the construct a concept and comprehension & communication learning levels. |
| **2. Individual Movement Lab**
Allow students to get comfortable with the equipment and handling of the shrimp, also allow some flexibility in the way they chose to collect data keeping the goals of the procedure in mind. | Emphasize discover-a-relationship level learning. Encourage students to make connections and raise issues regarding the experimental procedure and mathematics. |
| **3. Regression**
Lecture on the mathematical framework for fitting a line to the data gathered during the individual movement lab. Introduce and review required MATLAB commands as students find their diffusion coefficients. Discuss advantages of pooling the data and provide a way for groups to share results. | Students practice applying the algorithms on their own and check results with one another. |
| **4. Model Validation**
Through discussion lead students through the qualitative behavior expected in the model for the arrival density experiment. Also, present the mathematical formulation. Conduct the arrival density lab to collect data and estimate parameters. | Allow students to deductively reason through the qualitative behavior and mathematics. Emphasize the application learning level. |
| **5. Follow up discussion and assessment**
Follow up with a discussion of findings and clarify the assigned report. Discuss the optional parameter sensitivity analysis and other issues the experiments raised to be addressed in the written reports. | Promote and reward divergent thinking. Follow through on suggestions and ideas students raise through their experience. |
Table 3: Results of bootstrapping the data collected from the students in the class, and from our own experiment in real time and the video-recording. Samples of size $N$ were randomly selected from the total tracks collected. The mean diffusion coefficient $D$ and standard deviation (in cm$^2$/sec) were computed from repeated randomly selected samples.

For video data:

<table>
<thead>
<tr>
<th>$N$</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>all 43</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean $D$</td>
<td>0.0250</td>
<td>0.0250</td>
<td>0.0250</td>
<td>0.0250</td>
<td>0.0250</td>
</tr>
<tr>
<td>SD $D$</td>
<td>0.00721</td>
<td>0.00499</td>
<td>0.00410</td>
<td>0.00348</td>
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For class data:

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<th>30</th>
<th>40</th>
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<tr>
<td>mean $D$</td>
<td>0.0413</td>
<td>0.0414</td>
<td>0.0412</td>
<td>0.0412</td>
<td>0.0414</td>
<td>0.0413</td>
</tr>
<tr>
<td>SD $D$</td>
<td>0.0113</td>
<td>0.00995</td>
<td>0.00794</td>
<td>0.00693</td>
<td>0.00623</td>
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</table>

For the original real time data we collected:

<table>
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<th>all 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean $D$</td>
<td>0.0298</td>
<td>0.0299</td>
<td>0.0299</td>
</tr>
<tr>
<td>SD $D$</td>
<td>0.0101</td>
<td>0.00830</td>
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</tr>
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</table>

For the class data and the data we collected:

<table>
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<th>30</th>
<th>40</th>
<th>50</th>
<th>all 71</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean $D$</td>
<td>0.0378</td>
<td>0.0379</td>
<td>0.0378</td>
<td>0.0377</td>
<td>0.0378</td>
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</tr>
<tr>
<td>SD $D$</td>
<td>0.0113</td>
<td>0.00934</td>
<td>0.00767</td>
<td>0.00655</td>
<td>0.00592</td>
<td></td>
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</tbody>
</table>
Figure 1: Second instar nauplius of *Artemia franciscana*. The animal is about 1.5mm in length. Photograph by Kevin Johnson was downloaded from http://ut.water.usgs.gov/shrimp/ accessed October 22, 2007, Used with permission.
Figure 2: Diagram of the baking dish set up used to hatch brine shrimp. The support beaker prevents the plastic wrap from collapsing on the saline water and shrimp.
Figure 3: Transparencies needed to measure shrimp movement (not actual size). (a) a grid for recording individual shrimp movement, (b) circles for measuring arrival densities. The URL for actual size templates is listed in Appendix A.
Figure 4: Digital still images showing release and dispersal of shrimp within 10 seconds of the arrival densities procedure. The Petri dish image projected via overhead projector is approximately 5 feet in diameter. Individual shrimp, appearing as specks in these images, are easy to detect in part due to their movement.
Figure 5: The mean squared displacements for shrimp movement are shown above when data were collected at 5 and 10 second intervals. Each point represents the average of roughly 12 shrimp.
Figure 6: The estimated diffusion coefficients are pictured as a function of the duration of data collection. The points are computed from approximately 12 shrimp in the first 100 seconds, but data became unavailable as shrimp hit the edges of the dish so points after 100 seconds arise from averages of 11 down to 3 shrimp.
Figure 7: Pictured above is the number of shrimp counted in each region from our visual inspection in the arrival densities procedure for near (top) and far (bottom) areas. We compare this to the more careful method of counting shrimp in still images of the video recorded data.
Figure 8: Predicted and observed arrival densities at near (top panels) and far (bottom panels) regions for real time (left panels) and video taped (right panels) data. Symbols are data, solid line are prediction from Equation 7. Error bars are ±1 standard deviation.
Figure 9: The parameter sensitivity coefficients from Equations 9–11 are shown. The parameter values for $L = 1.4$ cm, $\sigma_0 = .5$ cm, $D = 2.6 \times 10^{-2}$ cm$^2$/sec are used as a base measures in the expressions for each coefficient.
Figure 10: The predicted arrivals from Equation 7 are plotted here for 10% variations in each parameter to show sensitivity to parameters.