THEMIS Spacecraft Multi-Body Stability Analysis

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THEMIS

- **Time History of Events and Macroscale Interactions during Substorms (THEMIS)**
  - Two-year mission with 5 identical probes to study violent colorful eruptions of Auroras (substorms)
  - First five-satellite NASA constellation
  - Fifth NASA's Explorer Program medium-class (MIDEX) mission
  - Managed by the Explorers Program Office at Goddard Space Flight Center
  - The University of California, Berkeley's Space Sciences Laboratory and Swales Aerospace (ATK) built the THEMIS probes
  - Swales Aerospace (ATK) built the probe carrier and separation systems
  - Launched February 17, 2007
• Three inner probes ~10 Earth radii (RE) from Earth monitor current disruption onset

• Two outer probes (20 and 30RE) monitor plasma acceleration due to lobe flux dissipation

• THEMIS probes line up over North America once every four days

• Magnetic field lines map phenomena occurring at the inner spacecraft to the ground arrays, where they can be observed as nightside auroral displays and geomagnetic perturbations
• The array of spacecraft and ground observations will enable researchers to pinpoint when and where substorms begin, thereby distinguishing between models that begin with current disruption in the near-Earth magnetotail and those that begin with magnetic reconnection in the distant magnetotail.
THEMIS Mission

An advanced weapon and space systems company
THEMIS Deployed Configuration

- 2X ~5 m long axial EFI booms
- 4X~20 m long Radial EFI booms
- 2 m long FGM boom
- 1 m long SCM boom
THEMIS Mass Properties

• Arbitrary non-zero central body products of inertia relative to radial boom locations
• Radial wire booms offset from the central body center of mass axial location
• Asymmetry of axial booms from central body center of mass axial location
• Non-orthogonal radial wire booms
Stability Requirements and Validation

• Requirements
  – Passive spin stabilized
    - Nominal and off-nominal configurations
    - 2 to 26 RPM
  – Probe Z-axis to maximum principal axis (PA offset) < 1 degree
  – Effective spin axis to transverse inertia ratio > 1.04

• Validation
  – Analytical analysis
    - Expanded Dr. Pankow’s/UCB spin-axis inertia based derivation
  – Simulation
    - Coupled multi-rigid body Simulink simulation
\[ I_{ZZ0} = I_{ZZ0,cb} + \sum_{i=1}^{n_r} I_{ZZ0,r_i} + \sum_{i=1}^{n_a} I_{ZZ0,a_i} \]

\[ = I_{ZZ} \sin^2 \beta \cos^2 \alpha + I_{YY} \sin^2 \alpha + I_{ZZ} \cos^2 \beta \cos^2 \alpha \]

\[ - I_{XY} \sin \beta \sin 2\alpha - I_{XZ} \sin 2\beta \cos^2 \alpha + I_{YZ} \cos \beta \sin 2\alpha \]

\[ \sum_{i=1}^{n_r} m_{r_i} \left[ \left( \overrightarrow{r}_{cm \rightarrow hp_{r_i},X} \right)^2 + \left( \overrightarrow{r}_{cm \rightarrow hp_{r_i},Y} \right)^2 + L_{r_i}^2 + 2L_{r_i}r_{cm \rightarrow hp_{r_i},XY} \right] + \]

\[ \sum_{i=1}^{n_a} m_{a_i} \frac{1}{\left( 1 - \frac{\omega_{spin}^2}{\Omega_{n_i}^2} \right)} \left( \overrightarrow{r}_{cm \rightarrow m_{a_i},XY} \right)^2 \]
Spin-Axis Inertia Contributions Examples

Central Body Spin Axis Inertia

Wire Boom Spin Axis Inertia

Axial Boom Spin Axis Inertia

Total Spin Axis Inertia
General Stability Criteria

First Derivatives = 0

\[ I_{YZ}^P - \sum_{i=1}^{n_S} F_{r_i} m_{i_j} y_{cm \rightarrow hp_i}^P z_{cm \rightarrow hp_i}^P - \sum_{i=1}^{n_S} F_{a_i}^2 m_{a_i} y_{cm \rightarrow m_{a_i}}^P z_{cm \rightarrow m_{a_i}}^P = 0 \]

\[ - I_{XZ}^P + \sum_{i=1}^{n_S} F_{r_i} m_{i_j} x_{cm \rightarrow hp_i}^P z_{cm \rightarrow hp_i}^P + \sum_{i=1}^{n_S} F_{a_i}^2 m_{a_i} x_{cm \rightarrow m_{a_i}}^P z_{cm \rightarrow m_{a_i}}^P = 0 \]

Second Derivatives < 0

\[
A_{eff, \alpha} = I_{YY}^P + \sum_{i=1}^{n_S} F_{r_i} m_{i_j} (z_{cm \rightarrow hp_i}^P)^2 + \sum_{i=1}^{n_S} F_{a_i}^2 m_{a_i} (z_{cm \rightarrow m_{a_i}}^P)^2
\]

\[
C_{eff, \alpha} = I_{ZZ}^P + \sum_{i=1}^{n_S} m_{i_j} \left( F_{r_i} (y_{cm \rightarrow hp_i}^P)^2 + L_{r_i} \frac{(y_{cm \rightarrow hp_i}^P z_{cm \rightarrow hp_i}^P)^2}{\sqrt{(x_{cm \rightarrow hp_i}^P)^2 + (y_{cm \rightarrow hp_i}^P)^2}} \right) + \sum_{i=1}^{n_S} F_{a_i}^2 m_{a_i} (y_{cm \rightarrow m_{a_i}}^P)^2
\]

\[
A_{eff, \beta} = I_{XX}^P + \sum_{i=1}^{n_S} F_{r_i} m_{i_j} (z_{cm \rightarrow hp_i}^P)^2 + \sum_{i=1}^{n_S} F_{a_i}^2 m_{a_i} (z_{cm \rightarrow m_{a_i}}^P)^2
\]

\[
C_{eff, \beta} = I_{ZZ}^P + \sum_{i=1}^{n_S} m_{i_j} \left( F_{r_i} (x_{cm \rightarrow hp_i}^P)^2 + L_{r_i} \frac{(x_{cm \rightarrow hp_i}^P z_{cm \rightarrow hp_i}^P)^2}{\sqrt{(x_{cm \rightarrow hp_i}^P)^2 + (z_{cm \rightarrow hp_i}^P)^2}} \right) + \sum_{i=1}^{n_S} F_{a_i}^2 m_{a_i} [(x_{cm \rightarrow m_{a_i}}^P)^2]
\]

\[
\frac{C_{eff, \alpha}}{A_{eff, \alpha}} > 1
\]

\[
\frac{C_{eff, \beta}}{A_{eff, \beta}} > 1
\]
Dynamic Simulation

- **Simulink models**
- **Up to 9 bodies**
  - One 6-DOF rigid bus
  - Four wire boom pendulums with 2-DOF joints at bus interface
  - Two axial boom pendulums with 2-DOF joints at bus interface
  - Two slosh pendulums with 2-DOF joints at pivot
- **Two rigid body magnetometer booms**
  - Treated as rigidly attached to the central body
- **Equations of motion verification**
  - ADAMS
  - SimMechanics
Stability Validation

• Analyses
  – Analytical and simulation-based spin axis to transverse axis inertia ratio

\[
\frac{C_{eff}}{A_{eff}} = \frac{\omega_{nut}}{\omega_{spin}} + 1
\]

  – Simulation-based principal axis and center of mass offsets for multi-body configurations

• Configurations
  – Stowed, partially deployed, and deployed configurations
  – Off-nominal configurations (e.g., one boom fails to deploy)
  – Variations of radial boom length, axial boom length, and central body inertias
Sample Results

- Magnetometer booms deployed, all other booms stowed (single rigid body) (SBN)
- Fully deployed (central body + 4 radial booms + 2 axial booms configuration) (MBN)
- Magnetometer booms deployed, three radial booms fully deployed, one radial boom stowed, and both axial booms deployed (central body + 3 radial booms + 2 axial booms configuration) (MBON)

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Effective Inertia Ratio</th>
<th>Ref</th>
<th>Sim</th>
<th>PA (deg)</th>
<th>CM (in)</th>
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<tr>
<td></td>
<td>α</td>
<td>β</td>
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<tr>
<td>SBN</td>
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<td>1.6</td>
<td>1.3</td>
<td>1.6</td>
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<tr>
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<td>1.6</td>
<td>1.9</td>
<td>1.9</td>
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<tr>
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Summary

- Analytical effective inertia ratio stability criteria were derived for application to the THEMIS spacecraft.
- Results apply to a spinning central body with flexible radial and axial booms.
- The central body can have unequal transverse inertias, non-zero products of inertia, and offset of the radial booms from the center of mass Z location.
- Analytical and simulation-based effective inertia ratios were computed for limiting configurations for design validation.
- The THEMIS probes were shown to be stable for sufficiently large uncertainty in mass properties to finalize the radial and axial boom designs.
Acknowledgements

• Dr. David Pankow, University of California at Berkeley, provided the starting point for this work through his unpublished manuscript entitled “SMEX/FAST Spin Axis Stability Discussion and Review,” and helpful feedback during THEMIS reviews

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• Dr. Darrell Zimbelman, NASA Goddard Space Flight Center, provided FAST spacecraft hi-fidelity simulation results for Simulink model verification

• Dr. Kenneth London, Swales Aerospace (ATK), assisted with the MSC.ADAMS validation simulations

• Dr. Carl Hubert, Hubert Aeronautics, recommended computing the effective inertia ratio from simulation results
• All instruments are powered on and continue to function well and are collecting science

• The EFI spin-plane and axial booms are completely deployed on THEMIS C (P2), D (P3), and E (P4), while those on THEMIS B (P1) and A (P5) will remain stowed until their mission orbit placement is completed.