A Simple Time Synchronization Scheme for Satellite Clusters in Formation Flying

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Objective

• To formulate a time synchronization strategy for satellites in formation.

• Analyze and establish the feasibility by simulation.

Need

• For navigation among satellites in formation.

• For precise electronic steering of the beams of the satellites to synthesize very large antenna structures.
Contents

• Synchronization in Fireflies
• Proposed ‘Transmit & Listen’ method
• Calculation and correction of the clock offsets
• Simulation results by Matlab
• Conclusion
Synchronization in Fireflies

• *Synchronization in Fireflies is a self-organized process.*

• *Fireflies influence each other.*

• *Emit flashes periodically and receptive to flashes from others*
Synchronization in Fireflies...

- **Pulse Coupled Oscillator Model.**

- Synchronization accuracy is governed by the propagation delay in the line-of-sight.
‘Transmit & Listen’ method

- Whenever A wants to synchronize with respect to B, it transmits a pulse which is reflected back by B.
- B also periodically transmits pulses.
‘Transmit & Listen’ method

Counter starts at $t_A$

\[ t_{AA} = n_{AA} T = \frac{2d}{c} \]
\[ t_{BA} = n_{BA} T = \tau + \frac{d}{c} \]
\[ t_c = t_{AA} / 2 = \frac{d}{c} \]
Calculation and correction of the offsets

The Clock Offset ($\tau$) equals,

$$\tau = \left( n_{BA} - \frac{n_{AA}}{2} \right) T$$

Next transmission of A is delayed (or advanced) by $\tau$ or a fraction of it ($k\tau$)
### Single step and progressive correction offsets

- **2nd cycle offset:**
  \[
  t_{A,2} = t_A + NT + \tau \\
  = t_A + NT + \left( n_{BA} - \frac{n_{AA}}{2} \right) T
  \]

- **2nd cycle offset:**
  \[
  t_{B,2} = t_B + NT \\
  = t_A + NT + \tau \\
  = t_A + NT + \left( n_{BA} - \frac{n_{AA}}{2} \right) T
  \]

\[\text{Assumption: Clock Frequencies of A and B are same}\]

- **pth cycle offset:**
  \[
  t_{A,p} = t_A + NT + k\tau \\
  t_{B,p} = t_B + NT \\
  = t_A + NT + \tau
  \]

- **2nd cycle offset:**
  \[
  = t_A + NT + \left( n_{BA} - \frac{n_{AA}}{2} \right) T
  \]

- **pth cycle offset:**
  \[
  = t_A + NT + \left( n_{BA} - \frac{n_{AA}}{2} \right) T
  \]

**Offset in 2nd cycle is,**
\[
(1-k)\tau
\]

**Offset in pth cycle is,**
\[
(1-k)^{p-1}\tau
\]
Synchronization with Unequal Clock Frequencies

**Clock Periods**: $T$ for B and $T + \delta$ for A

$$
t_{A,2} = t_A + N(T + \delta) + k\tau \\
t_{B,2} = t_B + NT \\
= t_A + NT + k\tau + N\delta \\
= t_A + NT + \tau
$$

**Offset in the $n^{th}$ cycle,**

$$
\tau_n = (1 - k)^{n-1} \tau_1 - \left\{ (1 - k)^{n-2} + (1 - k)^{n-3} + \cdots + (1 - k) + 1 \right\} N\delta
$$

This leads to,

$$
\tau_n = (1 - k)^{n-1} \tau_1 - \frac{1}{k} N\delta
$$

**Result**: Unequal frequencies lead to a fixed offset at the start of each cycle
Synchronization with Unequal Clock Frequencies

1. Initiate synchronization.
2. Calculate the offset ($\tau$) and apply progressive correction by factor $k$.
3. Repeat step 2 in each cycle and monitor $\tau$.
4. If $\tau$ is constant after say 50 cycles, add an **additional correction (i.e. Look-ahead correction)** of $k\tau_n$ (which is equal to $N\delta$) in timing the subsequent pulses of A.
5. This brings the offset to $\tau_n = (1 - k)^{n-1} \tau$, for $n > 50$. 

$$\tau_n = (1 - k)^{n-1} \tau$$
Simulation results by Matlab

Typical Formation of 4 nano-satellites around a mother satellite (B)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>Clock Period (nSec)</strong></td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Clock freq (GHz)</strong></td>
<td>10</td>
</tr>
<tr>
<td><strong>Initial Offset (Counts)</strong></td>
<td>0</td>
</tr>
<tr>
<td><strong>Node Status</strong></td>
<td>M</td>
</tr>
</tbody>
</table>

*Legend: M – Mother, C - Child*
Simulation results

Identical Clocks, with only an initial offset

Freq=10GHz, K=0.1

Progressive Correction

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.6</th>
<th>0.5</th>
<th>0.4</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offset</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>
Simulation results

Unequal Clock Frequencies, AND with an initial offset

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Legend: M – Mother, C - Child
Conclusion and future work

• ‘transmit and listen’ method formulation is effective to time-synchronize in a formation.

• Feasible to remove the offsets due to initial mismatch and also the clock frequency differences.

• To set up an indoor UWB network with a distance of 10-20 m and conduct the experiments with 10 GHz clock.
Thank You