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# STRING INDUCED SPACE COMPACTIFICATION*

Peter G. O. Freund, Phillial Oh and James T. Wheeler
The Enrico Fermi Institute
and the Department of Physics
University of Chicago, Chicago, IL 60637

## ABSTRACT

Motivated by the possibility of a finite theory of gravity provided by superstrings in ten space-time dimensions, we analyze the problem of space compactification in the context of string dynamics. Such analysis is hampered by conceptual and technical problems, stemming from the existence of the quantum string's own graviton mode on the one hand, and from Witten's observation of anomalies in a not specially chosen curved space-time on the other hand. Still, in the context of a classical local field presentation of string theory à la Nambu and Hosotani, supplemented by gravitational and Kalb-Ramond interactions, we are able to find solutions with space compactification. It is the antisymmetric tensor zero modes that dictate this compactification towards three space-time dimensions for ordinary strings or towards four or five space-time dimensions for superstrings.

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## 1. INTRODUCTION

Physics is replete with theories constructed to embody certain aesthetically highly appealing general principles (often of a geometric nature), which then get used in contexts quite different from those contemplated by their proponents. In the new contexts they solve a surprisingly new range of problems. A familiar case in point is that of gauge theories and of spontaneous symmetry breaking, which when used for weak interactions, lead to their unification with electromagnetism and ultimately to the respectable renormalizability of the electroweak quantum theory.

Higher dimensional unified theories[1] have also taken on new problems, particularly that of a consistent quantum theory of gravity. Already non-renormalizable in four space-time dimensions, a quantum theory of gravity (whether in four or more dimensions) stands a chance to be successful if it could only be made finite, i.e., devoid of divergences. This does not happen in any of the gravity or supergravity theories in 4-dimensions, and there are reasons to suspect (though not yet rigorous arguments to prove) that it won't happen in higher dimensional supergravities either. It has been suggested that a finite quantum theory of gravity may energe from superstring theory.[2] So far as is known, superstrings themselves have a consistent, Lorentz invariant, quantum theory only in a critical number of space-time dimensions $d_c=10$.[3] They, therefore raise anew the need of preferential spontaneous space-compactification towards our 4-dimensional world.[4] It is this problem that we shall consider in the present paper.

Traditionally, string theories are formulated in flad $d_c$-dimensional Minkowski space-time. This complicates the issue, since the gravitational metric-field does not even appear at the classical level. Of course the quantum theory develops its graviton, but compactification is then hardly a simple phenomenon. One could formulate string theory in curved space-time[5] where compactifying tendencies could then be readily analyzed. Such formulations raise, as we shall see, problems both of a technical and conceptual nature. After all, strings were considered precisely to avoid introducing an ordinary gravitational field with its supposedly incurable divergences at the quantum level. To boot, strings anyway grow their own graviton beyond the graviton introduced "by hand", and one could readily end up with two candidates for the function of "true" graviton! Obviously a self-consistency criterion of some sort is required. It has further been argued[7] that only curved space-times of a very special kind and dimensionality can accommodate string theories free of ghosts, e.g., space-time manifolds like $R^4 \times G$ where G is a Lie group of rank 22 in the nonsuperysmmetric case.

Assume that somehow or other one could tackle all these problems (more on this in section 2). One would then still be far from having a method for finding any compactifying tendencies in the theory. In their classical form string theories are hardly suited for such a task. After all, assume that instead of strings we had point particles with, say, a $\delta$-like interaction with each other. When coupled to gravity no compactification will ensue. Only upon going over to a field theoretic description in terms of one or more scalar fields with quartic self-interaction could one meaningfully explore the conditions for compactification. In the same way for strings, a field theoretic description is needed. Rather than invoke the nonwieldy functional formulations[8] we shall extract a field theory for strings from the Hamilton-Jacobi-like approach of Nambu[9] and Hosotani[10] suitably generalised to d>4 dimensions. Even this description, complex as it



may be, only deals with free strings. To include the necessary interaction will will couple the generalization of Hosotani's version of the theory to the natural mediator for string interactions, namely a Kalb-Ramond field.[1] As an afterthought we then throw in a gravitational and a scalar field, thus completing the zero mode sector of closed string excitations. Fortunately the seeming arbitrariness of this procedure is alleviated at the supersymmetric level, where the Kalb-Ramond field brings the graviton and some further Bose and Fermi fields automatically along, as members of its supermultiplet. Once in possession of a field formulation of string theory we can analyze the compactification. Not surprisingly the antisymmetric tensor fields play a crucial role and we find, in the non-supersymmetric case with only a rank two Kalb-Ramond field, compactification towards 3 space-time dimensions. In the non-chiral supersymmetric version this is corrected by the appearance of a rank three field and compactification towards 4 space-time dimensions. In the chiral supersymmetric version things may be more complex. Anyway, finding these solutions will be the main body of our paper. To the extent that the compactification radius is large compared to the "typical" string size, such a local field theoretic description may not lack realism.

## 2. GRAVITATING STRINGS

As was already mentioned in the introduction, string theories are usually formulated over flat Minkowski space in the critical number $d_c$ of dimensions.[6] For simplicity, here we will concentrate on the nonsupersymmetric case for which $d_c=26$.[12] So in 26-dimensional Minkowski space we are considering a Lorentz invariant theory of linearly extended objects: strings. The surprise is that in the closed string sector the spectrum of states contains a graviton. Such a state is not optional, but rather its masslessness is intimately related to the gauge invariance that rids the theory of ghosts. But once a graviton appears, its couplings are bound to follow a general covariant pattern. In the Scherk limit[13] of small strings (Regge slope $\alpha' \to 0$, string tension $T \to \infty$) this graviton survives and interacts as in Einstein's theory. For $\alpha' \neq 0$ there are[13] higher order terms ($R^2$, $R_{\mu\nu}R^{\mu\nu}$, $R^3$,...) suppressed by corresponding powers of $\alpha'$. Now in Einstein's theory the graviton's field is related to the space-time metric $g_{\mu\nu}$, but here all is started in flat space-time. So space-time geometry must somehow get "renormalized". How this can happen has been beautifully described by Thirring[14] in a somewhat different situation. Not only is space-time affected but so are measuring devices, and all told, observers using these devices will never get to the underlying flat metric, but will only measure the curved one.

The trouble with all this is that at the classical level we have no handle on space-time geometry, the graviton first appeared as a state in the quantum theory and then as a quantum field. Were we to consider the string gravitating from the start, we would introduce a graviton field by hand (in the supersymmetric context this would defeat the purpose of the string theory, namely to make quantum gravity finite). Even in this case the string would still develop its own graviton and one would have to ensure somehow that the input graviton and the string-dynamical one do not clash (this has been noted independently by J. Schwarz). It appears that a self-consistency requirement ought to be enforced: that the two gravitons be identical, or, that they mix and produce one effective massless graviton,.... We have glossed over an important problem here. Were we to start from a gravitating string, would the no-ghost theorem still hold? Witten[9] has recently addressed this question with some startling results. Specifically he finds that in a curved space-time the ghost-eliminating Virasoro algebra[6] is lost in general due to the conformal anomaly.



Viewing the string theory as a 2-dimensional $\sigma$-model, one can reinstate the Virasoro algebra by first adding a Wess-Zumino term and then going to a non-trivial fixed point of the $\beta$-function.[7] When this can happen, depends on the value of the integer N that multiplies the Wess-Zumino term and in the simplest case of N=1 one finds that the Virasoro algebra is restored when space-time is a manifold of the type $R^4 \times G$ where $R^4$ is 4-dimensional Minkowski space-time and G a Lie group of rank 22. The ordinary 26-dimensional flat theory is recovered for $G=R^{22}$, the product of 22 abelian groups each isomorphic to R. But new possibilities open. A simple example is G = O(44) which has rank 22 in which case $R^4 \times O(44)$ has dimension 950. The spectrum of this theory is readily obtained by Witten with the result that it is identical to the spectrum of the ordinary string theory in 26-dimensions, 22 of which have been compactified into a "most symmetric" 22-torus with all 22 radii the same. This degenerate compactification does indeed involve a gauged O(44) symmetry[15] and its spectrum after an adjustment of scales is identical (mass-formulas, O(44) multiplicities,...) to that of the 950-dimensional string theory just mentioned. So we have a 950-dimensional string theory on an $R^4 \times O(44)$ space-time and a 26-dimensional one on $R^4 \times T^{22}$ most symmetric and they have identical Hilbert spaces. Moreover there is circumstantial evidence[7] that these 950- and 26-dimensional theories have identical S-matrices. The supersymmetric case is expected to be quite different since the additional symmetry obtained for most symmetric toroidal compactification is found to be global rather than gauged.[7,15]

For us the moral of all this is that one should not be too dogmatic about the critical dimension of space-time, as it may change when the world-manifold ceases to be flat.

With these remarks we can then go on to a specific, if complex, local field theory to explore how compactification can result in string theory.

## 3. HOSOTANI'S FIELD THEORY IN d-DIMENSIONS

The world-line of a point particle in a curved space-time is a geodesic, the action just the length of the world line. As such the action has support on a curve, and, even with some interaction included, could hardly be expected to drive space-compactification. Switching to a field theoretic description in terms of one or more scalar fields (depending on the number of particle species) compactification can be obtained. A part of this field description, essentially its phase, also a scalar field, obeys the Hamilton-Jacobi equations in the classical limit. So, already at the classical level a field description with support over all of space-time is available in the Hamilton-Jacobi formalism.

The world-sheet of a string is a minimal surface. This time the action has support on a two-dimensional manifold. The formalism[10] provides a description in terms of scalar fields, somewhat analogous to the Hamilton-Jacobi formalism for point particles, and again usable, after some interaction has been included, to study space-compactification. We describe here this formalism directly in d-dimensions following step-by-step Hosotani's 4-dimensional arguments. For simplicity we consider the case of a string moving in d-dimensional Minkowski space (general relativity will be included in the next section). We start from the Nambu-Goto action[6]

$$I_X = -\frac{1}{2\pi\alpha'}\int d^2\tau\,\sqrt{-v_{\mu\nu}^{\,2}} \tag{3.1a}$$

with

$$v_{\mu\nu} = \frac{\partial(x_\mu,x_\nu)}{\partial(\tau_1,\tau_2)} \qquad \mu,\nu=1,\dots,d \tag{3.1b}$$

The equations of motion are

$$\frac{\partial(x^\mu,\hat{v}_{\mu\nu})}{\partial(\tau_1,\tau_2)} = 0 \tag{3.2a}$$



where

$$\hat{v}_{\mu\nu} = v_{\mu\nu}/\sqrt{-v_{\alpha\rho}^2} \tag{3.2b}$$

Now consider a set of solutions $x^\mu(\tau^1,\tau^2,S^3,\dots,S^d)$ of eqs. (3.2) that depend on d-2 parameters $S^3,S^4,\dots S^d$. As the $\tau$'s and the parameters range over a domain D of $R^d$, the d coordinates $x^\mu$ range over a domain $\bar{D}$, the solution providing a map

$\lambda:\ D \to \bar{D}$

Assuming $\lambda$ to be invertible we can regard the $\tau^1,\tau^2,\ S^3,\dots,S^d$ as functions of the $x^\mu$

$\lambda^{-1}:\ \bar{D} \to D$

With this observation we can write the $v_{\mu\nu}$ of eq. (3.1b) in the form

$$v_{\mu\nu} = \frac{1}{\chi}\,{}^*W_{\mu\nu} \tag{3.3a}$$

where

$$W^{\mu\nu} = \frac{1}{(d-2)!}\,\varepsilon^{\mu\nu\mu_3\dots\mu_d}\,W_{\mu_3\dots\mu_d}$$

$$W_{\mu_3\dots\mu_d} = \varepsilon_{\bar{\mu}_3\dots\bar{\mu}_d}\,\frac{\partial S^{\bar{\mu}_3}}{\partial x^{\mu_3}}\cdots\frac{\partial S^{\bar{\mu}_d}}{\partial x^{\mu_d}} \tag{3.3b}$$

$$\chi = \varepsilon^{\mu_1\mu_2\dots\mu_d}\,\frac{\partial\tau^1}{\partial x^{\mu_1}}\,\frac{\partial\tau^2}{\partial x^{\mu_2}}\,\frac{\partial S^3}{\partial x^{\mu_3}}\cdots\frac{\partial S^d}{\partial x^{\mu_d}}$$

and barred greek indices run from 3 to d.

The action

$$I_H = -\int d^dx\,\sqrt{W_{\mu_3\dots\mu_d}\,W^{\mu_3\dots\mu_d}} \tag{3.4}$$

is then readily shown to lead to field equations, which upon use of the relation (3.3) turn out to be precisely the string equations (3.2). $I_H$ is the d-dimensional Hosotani action. It presents a complex but local classical d-dimensional field theory, equivalent as just explained to the free (2-dimensional) string theory governed by $I_{NG}$. $I_H$ is to be viewed as a purely classical stand-in for $I_{NG}$. The corresponding quantum theory is certainly quite ill-defined as of now. This will not deter us from further embellishing this theory to get a semblance of string interactions.

As shown by Kalb and Ramond[11], this interaction between strings is to be mediated by an antisymmetric rank-two tensor field $A_{\mu\nu}$. At the same time we also want to couple the system to gravity $g_{\mu\nu}$ and to complete the set of zero modes of the string[13] to a massless scalar field $T(x)$. General covariance and gauge invariance (for the Kalb-Ramond field) dictate a lagrangian (after some convenient field redefinitions and Weyl transformations) of the form:

$$I = -\frac{1}{16\pi G}\int dx^d \sqrt{-g}\,\Big[R + \Lambda + \alpha e^{AT} F_{\mu\nu\rho}F^{\mu\nu\rho} + \beta\,\frac{\epsilon^{\mu_1\mu_2\cdots\mu_d}}{\sqrt{-g}}\,A_{\mu_1\mu_2}\,W_{\mu_3\cdots\mu_d} + \gamma e^{CT}\sqrt{W_{\mu_3\cdots\mu_d}\,W^{\mu_3\cdots\mu_d}} - \frac{\delta}{2}\,g^{\mu\nu}\partial_\mu T\partial_\nu T\Big] \qquad (3.5)$$

Here $\alpha$, $\beta$, $\gamma$, $\delta$, $\Lambda$, A, C are constants. The exponential appearance of the scalar field is caused by field redefinitions in the usual way[16]. The cosmological term has been introduced for generality. The metric conventions we use are those of ref.17. Finally, $F_{\mu\nu\rho} = \partial_\mu A_{\nu\rho} + \partial_\nu A_{\rho\mu} + \partial_\rho A_{\mu\nu}$.

In the next two sections we shall deal with space compactifying solutions of the field equations derived from this lagrangian. A few remarks are in order at this point. First it seems as though we had quite a choice in writing down down our lagrangian. Of course we "wanted" gravity, and standard arguments[11] dictate the Kalb-Ramond field . In the supersymmetric case things turn out neater. The Kalb-Ramond field there brings along gravity and further Bose



and Fermi fields simply on account of supersymmetry. So, once we settle on the Kalb-Ramond field, the rest follows directly.

At a deeper level we must inquire what the precise relationship of $I_H$ is to $I_{NG}$, or of I to interacting strings as studied in dual models. It is more usual to deal with an action in terms of some "field functional" $\psi(x^\mu(\tau^1, \tau^2))$ as proposed by Kaku and Kikkawa[8], or its supersymmetric generalization by Green and Schwarz[8]. The trouble is, such functional approaches are quite unwieldy and it is far from clear how they are to lead to space-compactification. A more geometrical approach has been proposed by Freund and Nepomechie[16] as a Kaluza-type lagrangian for a U(1) bundle over loop space. This lagrangian has the correct Sherk $\alpha' \to 0$ limit including gravity, a Kalb-Ramond and scalar fields, and could conceivable be used also for $\alpha' \neq 0$ (this was noted independently also by R. Nepomechie, private communication). Whereas the geometry is clearer and one has a simpler overall picture, it is not at all obvious what the quantum theory of this lagrangian would replicate that of an interacting string. We shall, therefore, go along and explore space compactification on the basis of action I of eq. (3.5).

## 4. SPACE COMPACTIFICATION WITH PASSIVE STRINGS

Before we go into more detail let us remark that the action I eq. (3.5) contains a gravitating antisymmetric tensor field of rank two. It is well known that this entails an automatic compactification mechanism[4] towards three space-time dimensions. The world manifold is then of the form $M_3 \times M_{d-3}$ with $M_3$ 3-dimensional anti-de Sitter space and $M_{d-3}$ a compact (d-3)-dimensional manifold (actually we can even flatten $M_3$ using the cosmological term in $L$, admittedly in an "unnatural" way). We thus see that with the action I compactification is preferential towards three space-time dimensions. This is "phenomenologically" obviously unacceptable, but then so is the theory of ordinary nonsupersymmetric strings. Were we to deal with type IIa superstrings[2] instead, then there would be a rank three antisymmetric tensor and compactification towards four space-time dimensions would again become feasible as in 11-d supergravity.

Returning to the action I, let us emphasize that the preferential compactification towards three space-time dimensions which we just noted corresponds to genuine solutions in which however the string fields $S^3, \dots S^d$ are all constant and the scalar field T is set to zero. So in some sense the string is passive and plays no role in the solution. Essentially what happens is that the now supersymmetric string, by its geometric nature selects a Kalb-Ramond field to mediate its interactions. This Kalb-Ramond field without any further reference to its string-source then develops the vacuum expectation value that drives preferential compactification to three dimensions. To better appreciate this argument let us notice that it amounts to dealing with strings as point particles. This is allowed if the "typical" string size is smaller than the compactification radius. But for a ground state, small strings, indeed strings shrunk to a point, are expected to dominate. In this point-like Scherk limit[13] all that remains of string theory is a local field theory of the fields $g_{\mu\nu}$, $A_{\mu\nu}$, T, and compactification proceeds as just described. We note the generality of this argument. Thus for superstring theory we expect the corresponding supergravity theory (its Scherk limit) to determine compactification. For membranes the interaction agency should be a rank three antisymmetric tensor field and preferential compactification towards 4-d space-time would be expected.

In the next section we explore solutions in which the string fields $S^{\tilde{\mu}}$ participate. Still directed by the Kalb-Ramond field, such solutions again compactify to 3-d, yet they "actively" involve the strings.



## 5. SPACE COMPACTIFICATION WITH ACTIVE STRINGS

While accepting the leading role played by the Kalb-Ramond field in dialing the reduced space-time dimensionality, we now search for solutions of the field equation for the action I which actively involve the strings. We start by writing these equations down:

$$6\alpha\partial_\rho(\sqrt{-g}\, e^{AT}F^{\mu\nu\rho}) - \beta\epsilon^{\mu\nu\rho_3\cdots\rho_d} W_{\rho_3\cdots\rho_d} = 0$$

$$[\gamma\partial_\mu(\sqrt{-g/W^2}\, e^{CT} W^{\mu\nu_4\cdots\nu_d}) + \frac{\beta}{3}\epsilon^{\mu\rho\sigma\nu_4\cdots\nu_d}F_{\mu\rho\sigma}].$$

$$\cdot\, \epsilon_{\bar{\bar{\sigma}}\bar{\sigma}_4\cdots\bar{\sigma}_d}\, \partial_{\nu_4} S^{\bar{\sigma}_4}\dots\partial_{\nu_d} S^{\bar{\sigma}_d} = 0 \qquad (5.1)$$

$$\frac{\delta}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\; g^{\mu\nu}\partial_\nu T) + A\alpha e^{AT}F^2 + \gamma C e^{CT}\sqrt{W^2} = 0$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R - \frac{\Lambda}{2} g_{\mu\nu} = -3\alpha e^{AT}(F_{\mu\alpha\beta}F_\nu{}^{\alpha\beta} - \frac{1}{6} g_{\mu\nu}\, F^2)$$

$$-\frac{\gamma(d-2)}{2} e^{CT}\,(\frac{1}{\sqrt{W^2}} W_{\mu\nu_4\cdots\nu_d} W_\nu{}^{\nu_4\cdots\nu_d} - \frac{1}{d-2}\; g_{\mu\nu}\sqrt{W^2})$$

$$+\frac{\delta}{2}\,[\partial_\mu T\partial_\nu T - \frac{1}{2} g_{\mu\nu}(g^{\rho\sigma}\partial_\rho T\partial_\sigma T)].$$

Here $W^2 \equiv W^{\mu_3\cdots\mu_d}W_{\mu_3\cdots\mu_d}$, $F^2 \equiv F^{\alpha\beta\gamma}F_{\alpha\beta\gamma}$ .

We now make an Ansatz in which the Kalb-Ramond field strengths are still proportional to the volume form on an $M_3$ space-time, with the d-dimensional manifold $M_d = M_3 \times M_{d-3}$ and $M_{d-3}$ a compact Einstein manifold. For simplicity we assume $M_3$ conformally flat, and everything manifestly $M_3$-Lorentz invariant. But we excite the $S^{\tilde{\mu}}$ and T. fields. In detail our Ansatz reads

$$\dot{g}_{\mu\nu} = \left(\begin{array}{c|c} e^{2\phi(r)}\eta_{mn} & 0 \\ \hline 0 & g_{\bar{m}\bar{n}}(y) \end{array}\right)$$

$$F^{\mu\nu\rho}(x) = \begin{cases} \dfrac{\epsilon^{mnp}}{\sqrt{-g_3}} F(r) & \text{for } \mu,\nu,\rho = m,n,p \\ 0 & \text{otherwise} \end{cases} \qquad (5.2)$$

$$T(x) = T(r)$$

$$W_{\mu_3\mu_4\cdots\mu_d} = \sqrt{g_{d-3}}\, \partial_{[\mu_3} S(r) \epsilon_{123\,\mu_4\cdots\mu_d]}$$

Here $r^2 = (x^2)^2+(x^3)^2-(x^1)^2$, $x^1$ is the time-like coordinate, latin indices run from 1 to 3, barred latin indices from 4 to d, $g_3 = \det g_{mn}$, $g_{d-3} = \det g_{\bar{m}\bar{n}}$, and $(y^1,\ldots y^{d-3}) \equiv (x^4,\ldots x^d)$. Finally $g_{\bar{m}\bar{n}}(y)$ is the metric of a compact (d-3)-dimensional Einstein manifold and $\eta_{mn}$ is the 3-dimensional Minkowski metric. The Ansatz for $W_{\mu_3\cdots\mu_d}$ is integrable. In terms of the scalar fields $S^3, S^4,\ldots S^d$ it amounts to $S^3 = S(r)$, $S^4 = \int \sqrt{g_{d-3}}\, dy^1$, $S^5 = y^2$, $S^6 = y^3,\ldots S^d = d^{d-3}$. The special form of $S^4$ can at will be switched to $S^5, S^6,\ldots,$ or $S^d$. Inserting this Ansatz into the field equations we find the solution

$$e^{2\phi(r)} = \frac{\ell^2}{r^2}, \quad F(r) = \tilde{F} r^{CE}, \quad T(r) = E \ln r + K,$$

$$S(r) = \tilde{S} r^{-CE}, \; E = \sqrt{2}\left(\delta + 3C^2 \frac{\alpha\gamma^2}{(d-2)\beta^2}\right)^{-1/2},$$

$$\Lambda = (d-2)/\ell^2, \quad A = -2C, \quad R_{d-3} = -(d-3)/\ell^2, \quad \tilde{F} = \frac{\gamma C E e^{CK}}{2\ell\beta((d-2)!)^{1/2}}, \quad \tilde{S} = \frac{6\alpha e^{-2CK}}{\beta(d-2)!}\tilde{F}$$

where $R_{d-3}$ is the scalar curvature of the small (d-3)-manifold (correct sign for compactification, i.e. negative in our convention), $\ell$ and K are integration constants, and for simplicity units have been chosen such that $16\pi G=1$.

$M_3$ thus has the metric $g_{mn} = \frac{\ell^2}{r^2}\eta_{mn}$ $r^2 > 0$. A more natural presentation of this metric is obtained following a coordinate transformation ($16\pi G=1$).



$$x^1 = e^\rho \sinh\theta$$
$$x^2 = e^\rho \cosh\theta \cos\psi$$
$$x^3 = e^\rho \cosh\theta \sin\psi$$

so that the line element on $M_3$ becomes

$$ds_3^2 = -\ell^2(d\theta)^2 + \ell^2(d\rho)^2 + \ell^2(\cosh\theta)^2(d\psi)^2$$

This solution as compared to the one in the previous section involves the strings directly, but compactification is still, on the main, governed by the anti-symmetric tensor field.

## 6. CONCLUSIONS

If, for reasons of quantum finiteness in the presence of gravity, the fundamental theory should present itself in string form and be supersymmetric, it would then involve ten space-time dimensions and require the compactification of six space-like dimensions. Given the Minkowskian formulation of string theory we found this to involve certain technical and conceptual problems. Going beyond these problems, we considered a "stand-in" local field theory, found compactification towards the number of space-time dimensions (3 in the nonsupersymmetric, 4 or 5 in the supersymmetric), dictated by the rank of the antisymmetric tensor fields mediating string interactions and/or their supersymmetric partners. We found explicit classical compactifying solutions in such a theory.

One more question is raised by all this. Once we contemplate a theory of extended objects, why stop at one-dimensional strings and not go on to two-dimensional membranes, or even higher dimensional objects? In the limit in which those objects are much more extended in one direction than in all others, they will appear as strings, their higher dimensionality reflected, in some inner attributes, leading to internal symmetry.[18] Just like compactified extra dimensions of space can be the source of internal symmetries, so can extra dimensions of objects that move in this space. While this is expected to lower the critical dimension[18] for the theories of extended higher dimensional objects, and thus reduce the size of the Kaluza-like symmetries, we get these extra symmetries from the higher dimensionality of the objects. As was pointed out above, for membranes (bags) a natural compactification towards four space-time dimensions is expected (on account of the corresponding Kalb-Ramond-like field having rank three). Though a first step towards a theory of membranes, with a local interaction had already been taken,[19] it remains to be seen whether, presumably supersymmetric, theories of objects extending in two or more dimensions are consistent and/or needed.

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