Diffraction analysis for blackbody calibrations

The NIST LBIR Facility offers blackbody calibration as a calibration service. Correcting measurements for diffraction effects is an integral part of data analysis. In this talk, we point to how the effects are assessed.

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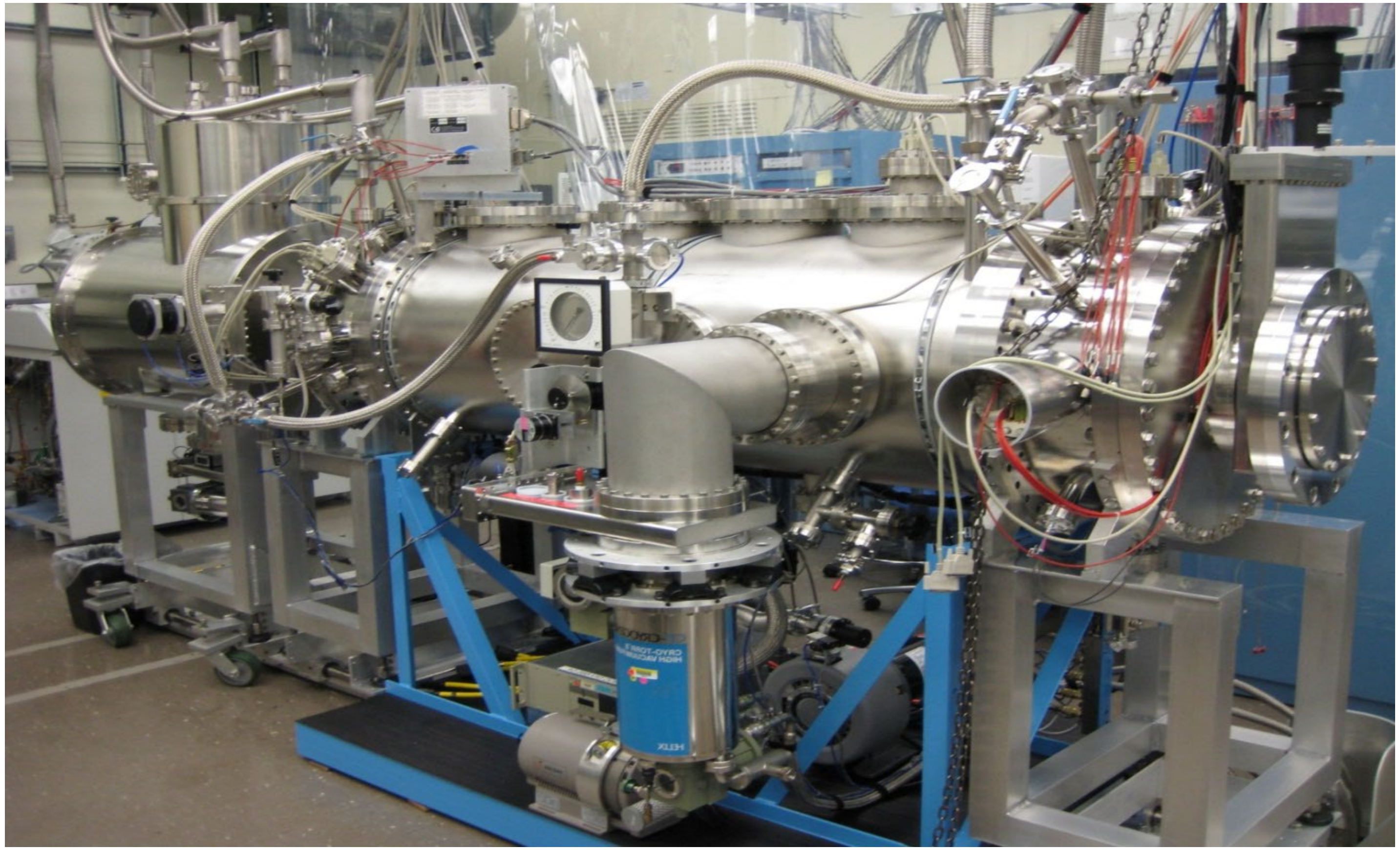
Sensor Science Division, NIST

Broadband Calibration Chamber (BCC)

NIST Low-Background Infrared (LBIR) Facility



"Blackbody calibration" measures *radiance temperature*

It involves the entire Planckian spectrum

$$L(\lambda,T)=\frac{c_{1L}}{n^2\lambda^5\left(\exp\left(\frac{h}{n\lambda\, c\, kT}\right)-1\right)}$$

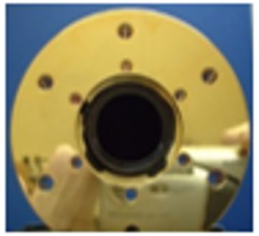
$$L(T) = \int_0^{\infty} d\lambda\, L(\lambda,T)$$

$$L(T) = \frac{2\pi^5}{15}\frac{k^4T^4}{c^2h^3}\frac{1}{\pi} = \sigma T^4\frac{\cos(\theta)}{\pi}$$

The Broadband Calibration Chamber calibrates customer blackbodies using an *Absolute Cryogenic Radiometer (ACR, a.k.a. Active-Cavity Radiometer)*

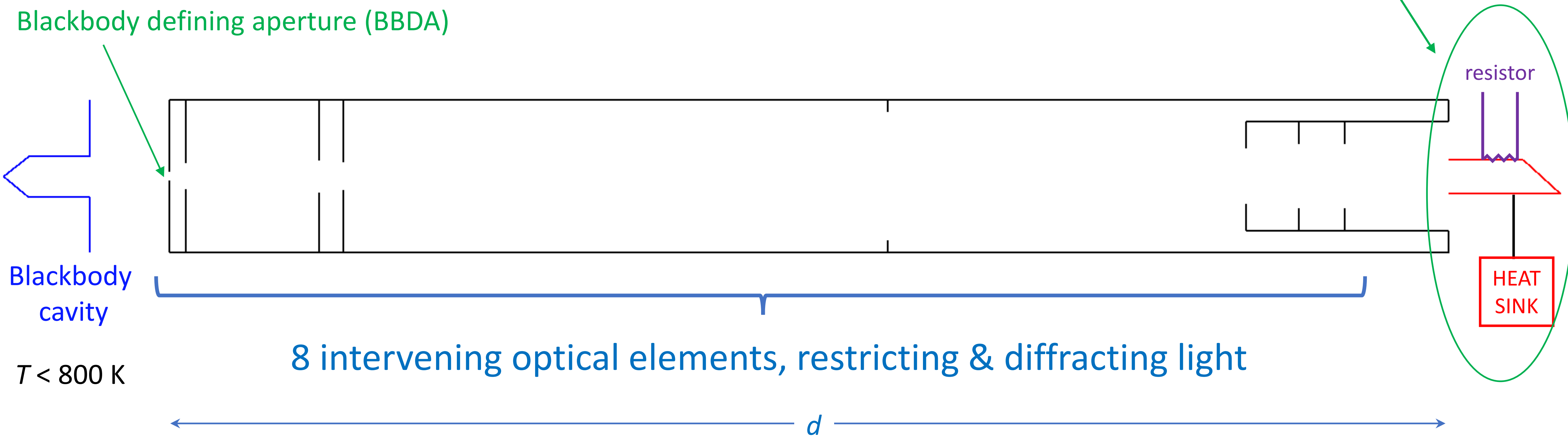
Blackbody calibration, chamber model:

NIST Primary Optical Watt Radiometer (POWR)

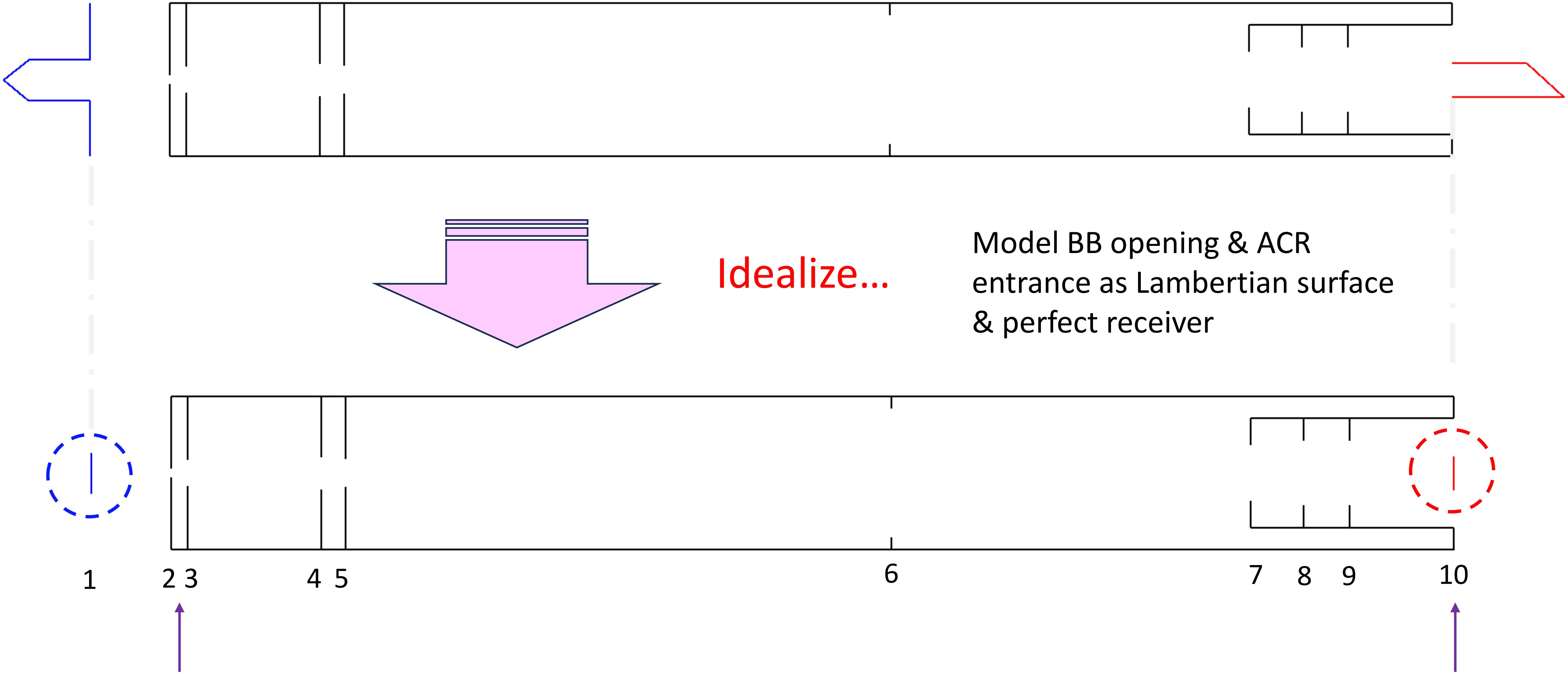


Active-Cavity Radiometer (ACR)
(held at constant, liquid-He temperature, shuttered, compares Resistive (ohmic) heating power to optical power received)

Blackbody defining aperture (BBDA)



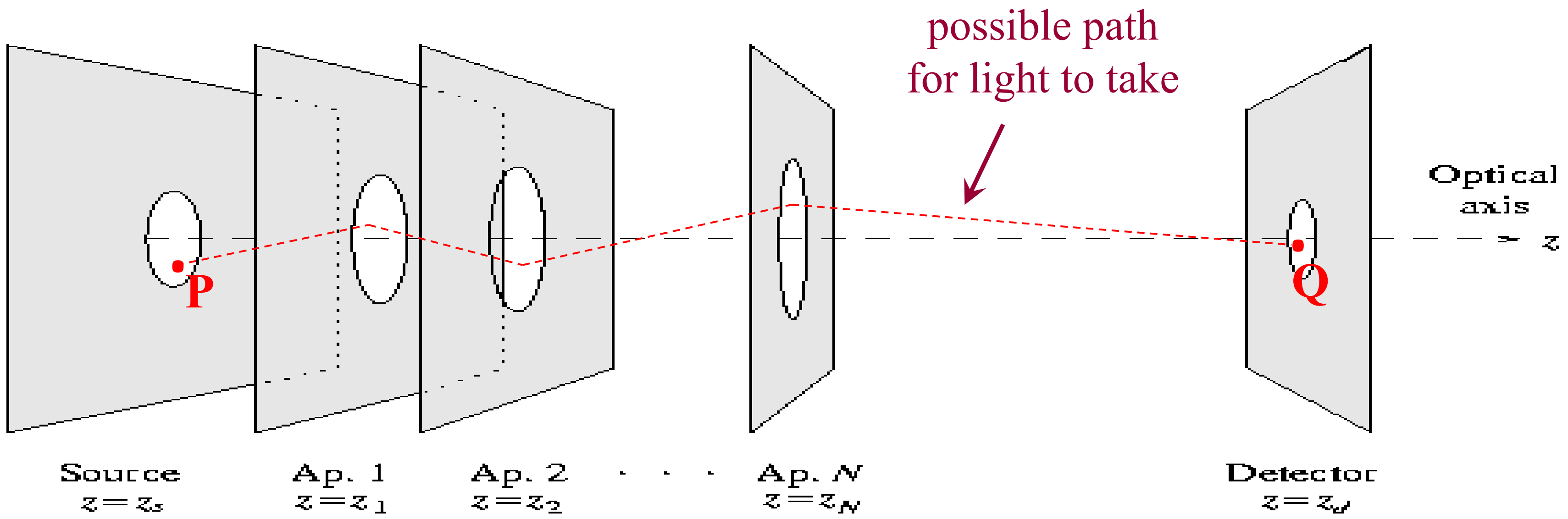
Blackbody calibration, chamber model:



Model BB opening & ACR entrance as Lambertian surface & perfect receiver

Note: Apertures 2 & 10 are precision apertures.

Consider an optical system, and the *wave propagation* of a light field *u* from P to Q:



Repeated application of Kirchhoff's diffraction formula gives

$$u(\mathbf{Q}) = \frac{1}{(i\lambda)^N} \int_{A_1} dx_1\, dy_1 \ldots \int_{A_N} dx_N\, dy_N\; G(\mathbf{P}, \mathbf{x}_1) \cdots G(\mathbf{x}_N, \mathbf{Q}) \cdot \underline{\exp[i\textstyle\sum_k \delta l_k(\mathbf{x}_k)]}$$

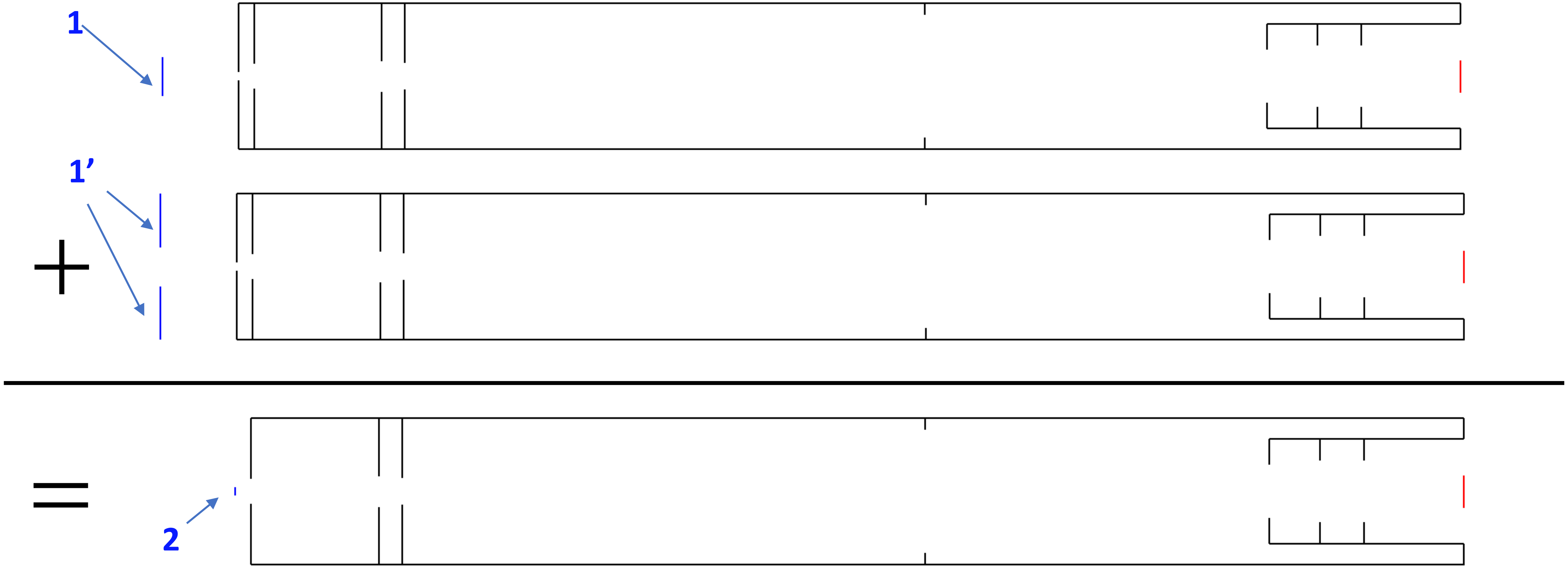
For powered optics
Not used here

Numerical computational cost $\propto 1/\lambda^3$ *
Dense sampling required at small λ, so… → Approximate models at small λ
Numerical methods used at large λ

* See Rubin et al., Appl. Opt. **57**, 788 (2018)

Flux conservation:* having Aperture 2 be an ideal Lambertian emitter equivalent to having *entire plane* of BB opening with equivalent radiance.

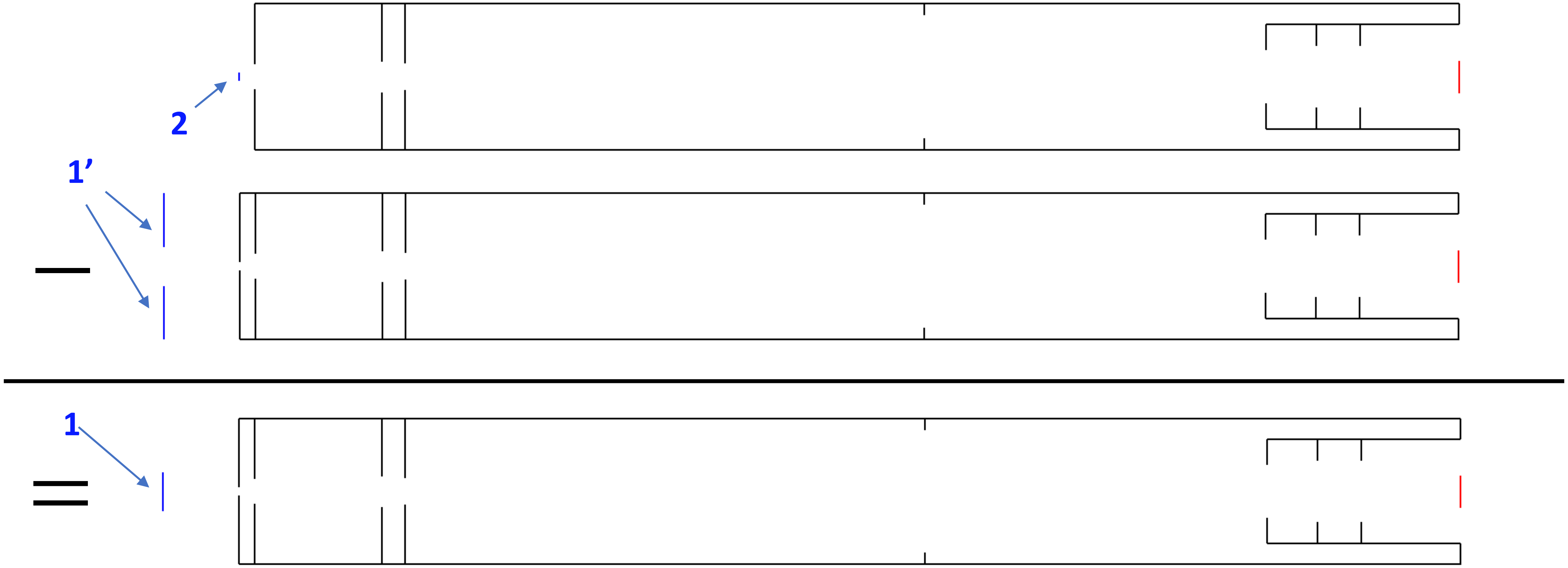
Call the BB opening, **1**, and its complement, **1’**



Functionally, one has **1 + 1’ = 2**

* J. W. Goodman, *Introduction to Fourier Optics*

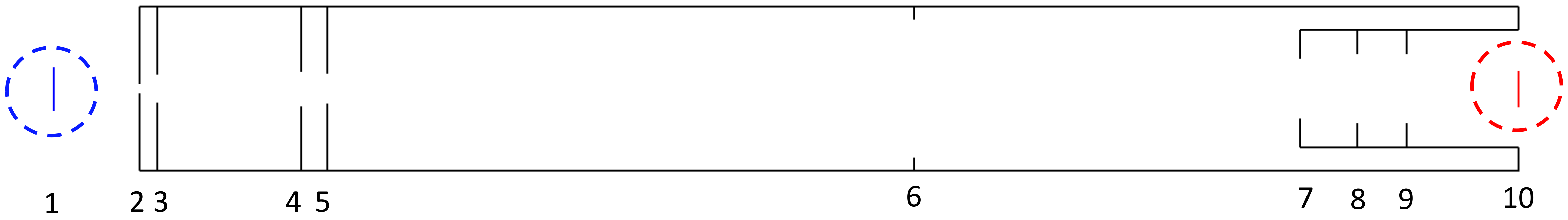
Flux conservation:* having Aperture 2 be an ideal Lambertian emitter equivalent to having *entire plane* of BB opening with equivalent radiance.



Functionally, we have **1 = 2 – 1'**

* J. W. Goodman, *Introduction to Fourier Optics*

Computer-assisted identification of major diffraction effects



INPUT GEOMETRY (radii, intervening distances in mm)

```
nap
10
12.000          0.0 1 0
   64.000
 0.040          0.0 1 0
   13.586
 7.515          0.0 1 0
  108.217
 9.487          0.0 1 0
   19.050
 8.222          0.0 1 0
  440.817
37.949          0.0 1 0
  290.513
16.465          0.0 1 0
   42.412
18.975          0.0 1 0
   37.401
18.975          0.0 1 0
   83.725
 9.996          0.0 1 0
```

Varies from 0.04 mm to 2.56 mm

1035.721 mm

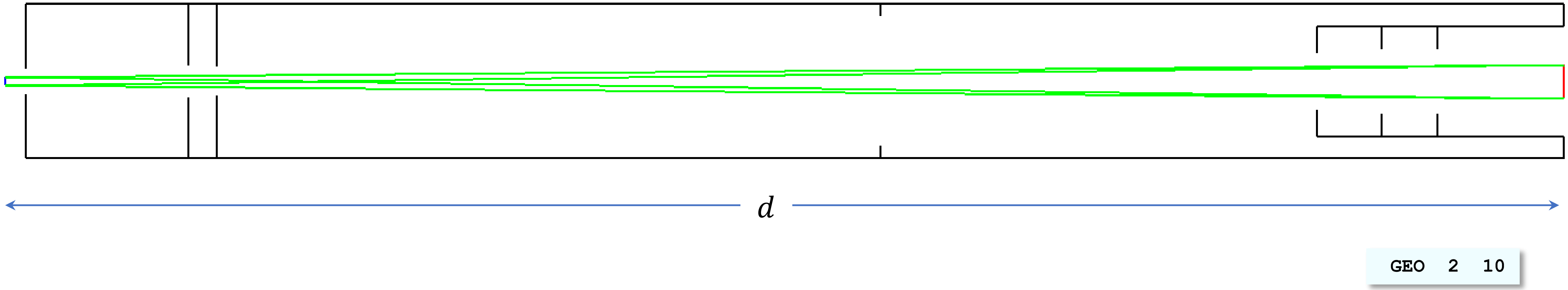
PRESCRIBED CALCULATIONS

```
GEO  2  10
SAD  1  2  10
SAD  2  5  10
OFB  1  2  5  10
SAD  2  7  10
OFB  1  2  7  10
XBP  2  4  5  10
XBP  2  5  7  10
XBP  2  7  8  10
XBP  2  7  9  10
UFB  1  2  3  10
SPC2  4  6  F     4.923819   5.045854   -4.371048   9.996000
SPC2  5  6  T     3.677691   3.794041   -4.371048   9.996000
SPC2  5  9  F     3.677691   3.794041   -9.996000   9.996000
```

very small, neglected* (SPC2 lines)

* Size of effect smaller than shortcomings of models for other effects

Combination 2-10 defines geometrical throughput and expected power.



$$\Phi = L\cdot\frac{\pi^2}{2}\left\{d^2+R_{\mathrm{BBDA}}^2+R_{\mathrm{ACR}}^2-\left[\left(d^2+R_{\mathrm{BBDA}}^2+R_{\mathrm{ACR}}^2\right)^2-4R_{\mathrm{BBDA}}^2R_{\mathrm{ACR}}^2\right]^{1/2}\right\}$$

$$= L\cdot\frac{\pi^2}{2}\left(\frac{4R_{\mathrm{BBDA}}^2R_{\mathrm{ACR}}^2}{d^2+R_{\mathrm{BBDA}}^2+R_{\mathrm{ACR}}^2+\left[\left(d^2+R_{\mathrm{BBDA}}^2+R_{\mathrm{ACR}}^2\right)^2-4R_{\mathrm{BBDA}}^2R_{\mathrm{ACR}}^2\right]^{1/2}}\right)$$

$$\cong L\cdot\left(\frac{\pi R_{\mathrm{BBDA}}^2\cdot\pi R_{\mathrm{ACR}}^2}{d^2}\right)$$

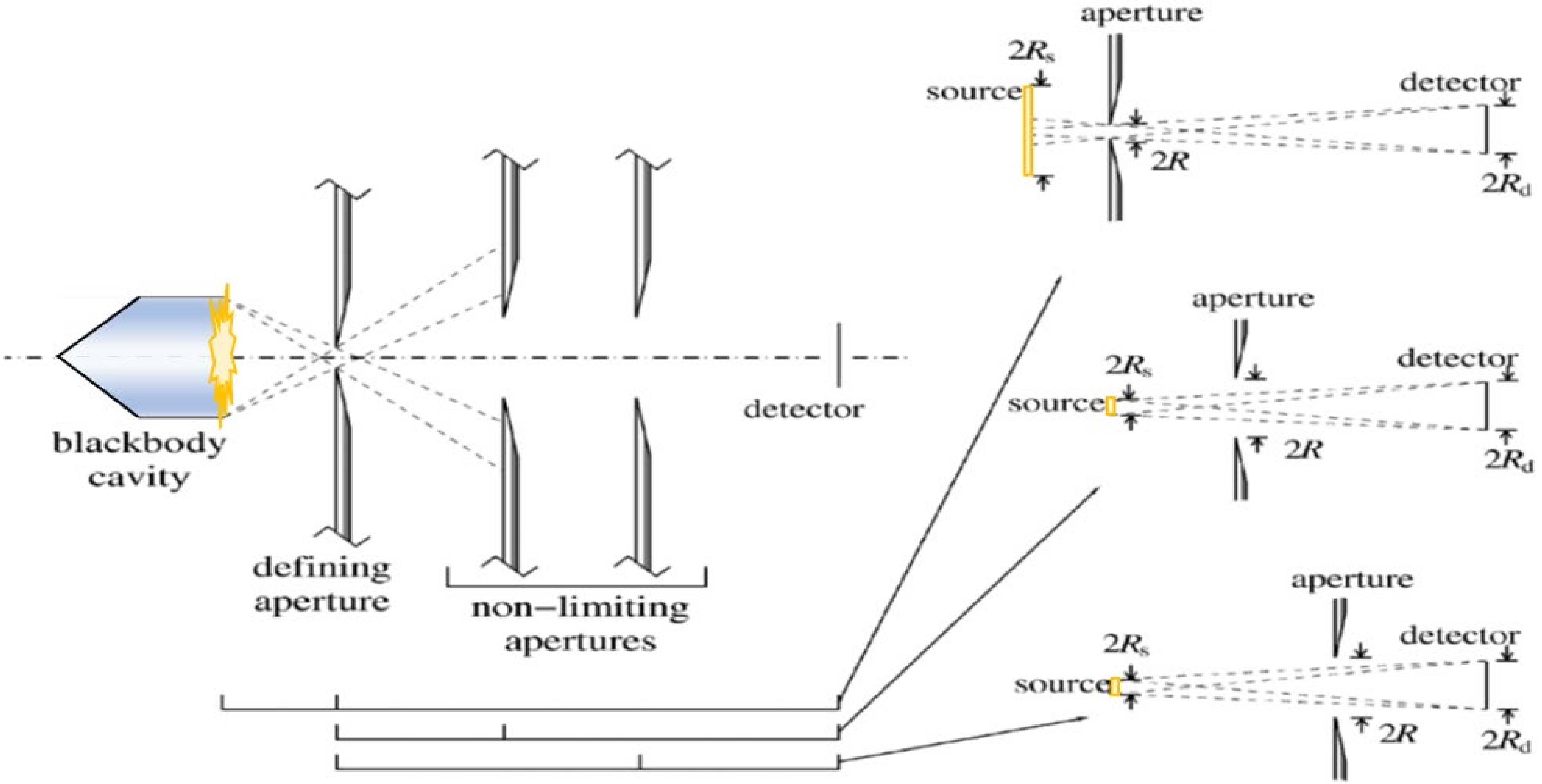
Modeling SAD Subsystems

Largest effects can be described by identify three-element source-aperture-detector (SAD) subsystems *

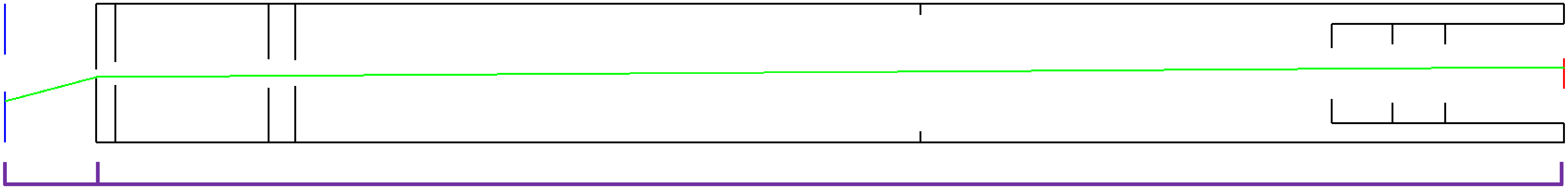
* See JOSA A **33**, 1509 (2016), and references therein

Example:

Five element systems, and three SAD subsystems exhibiting main diffraction effects.



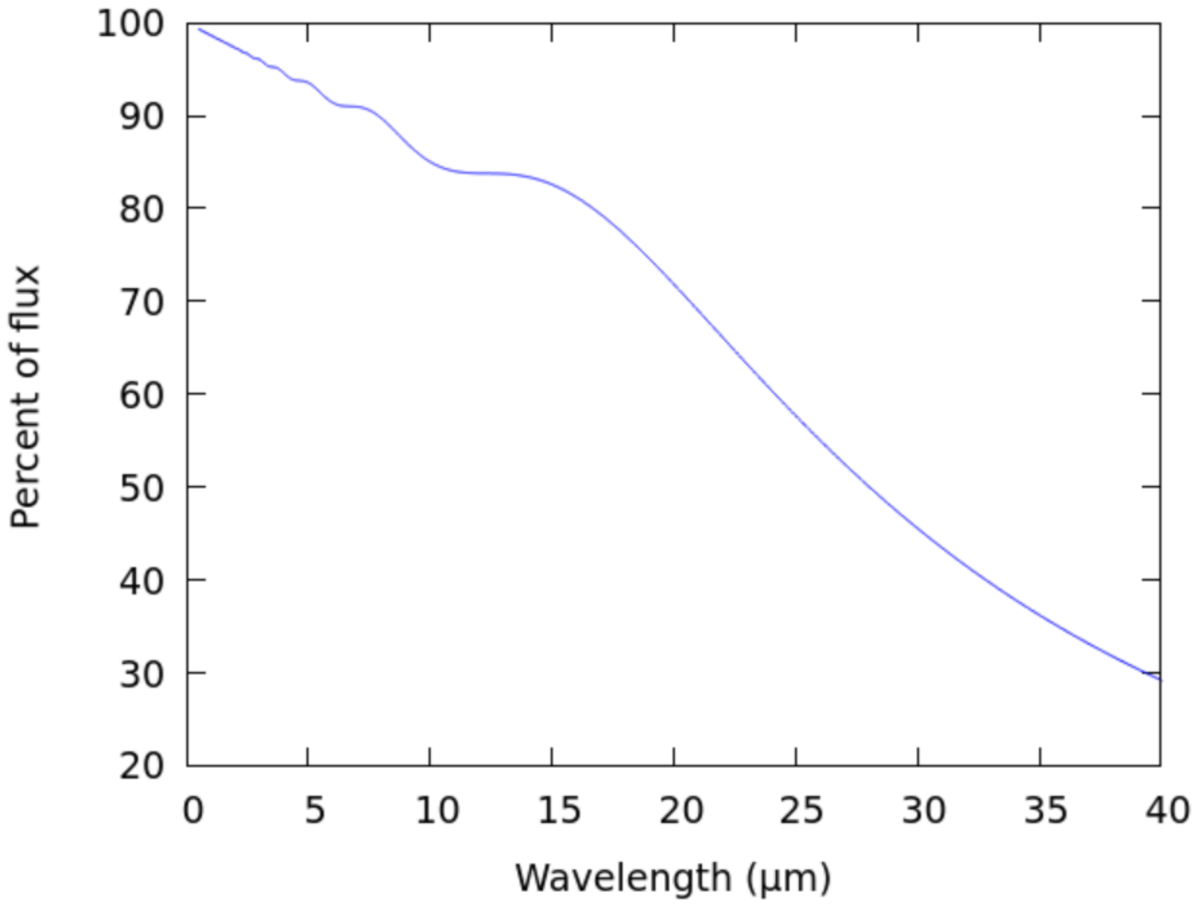
SAD Combination 1-2-10 leads to a diffraction loss (estimated by 1'-2-10 "false" light).



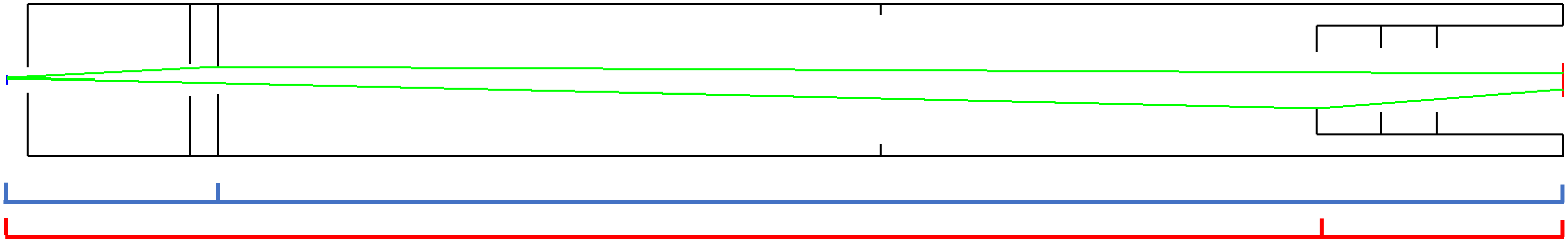
SAD 1 2 10

SAD subsystem scales spectral power at ACR by a factor shown:

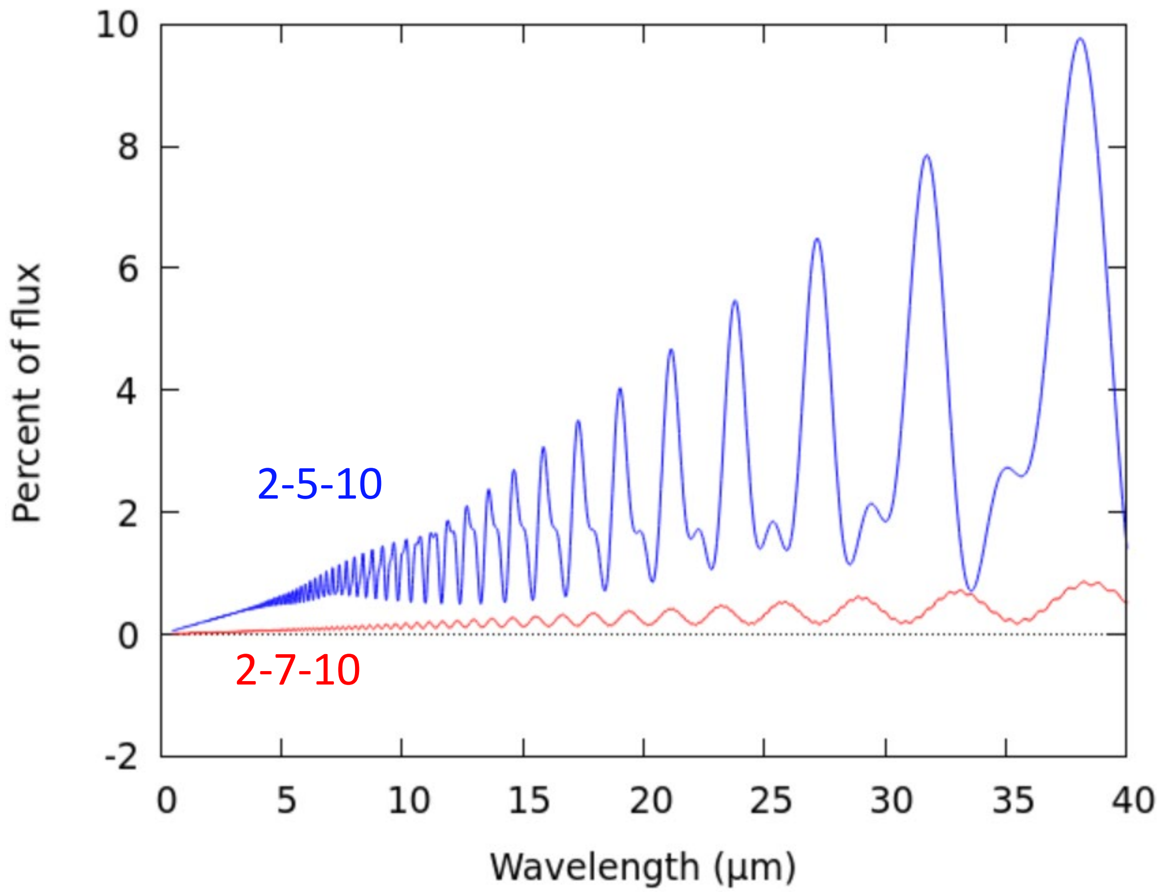
SAD effects given by Wolf's formula for integrated flux + accounting for "extended source."



SAD Combinations 2-5-10 and 2-7-10 leads to first-order diffraction gains.



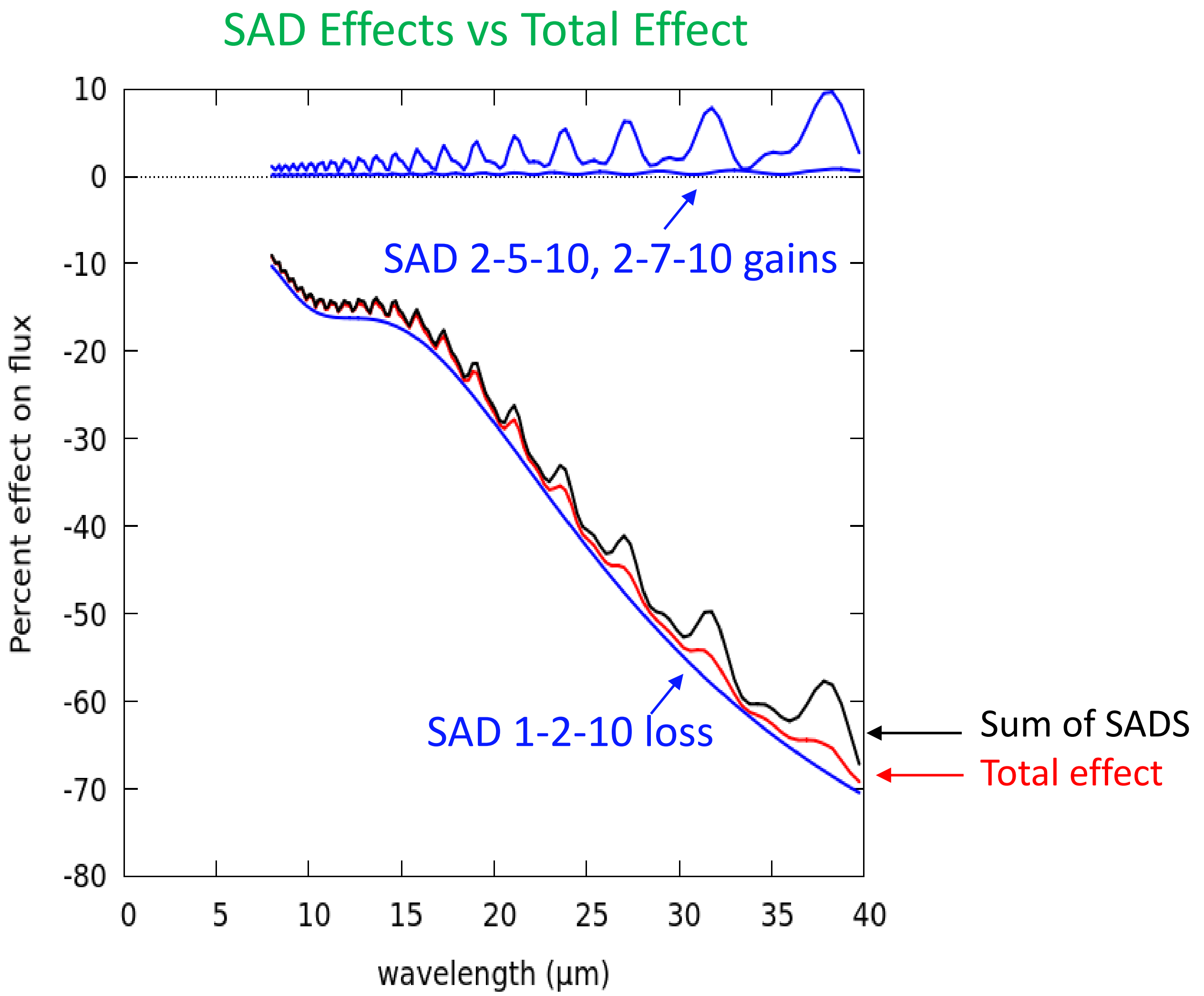
SAD subsystems increase spectral power at ACR by amounts shown:

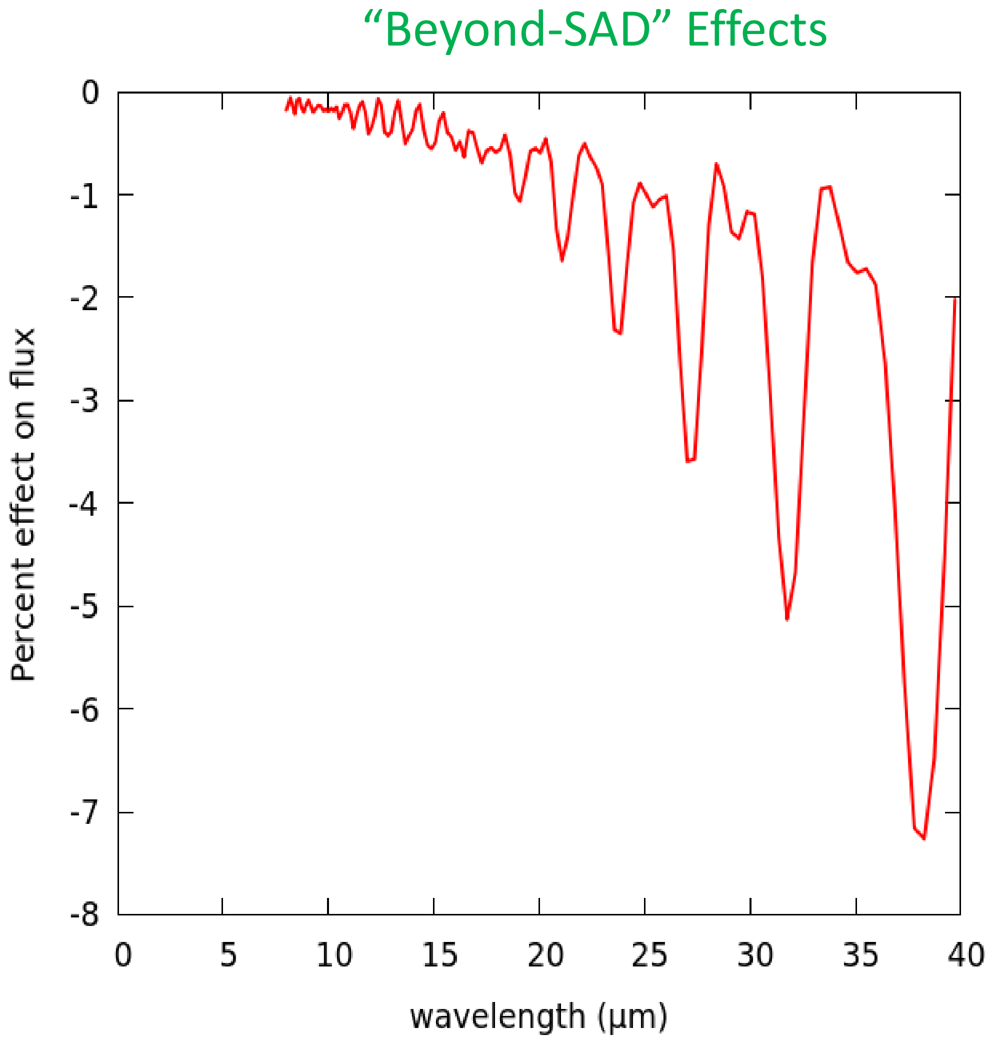


SAD	2	5	10
SAD	2	7	10

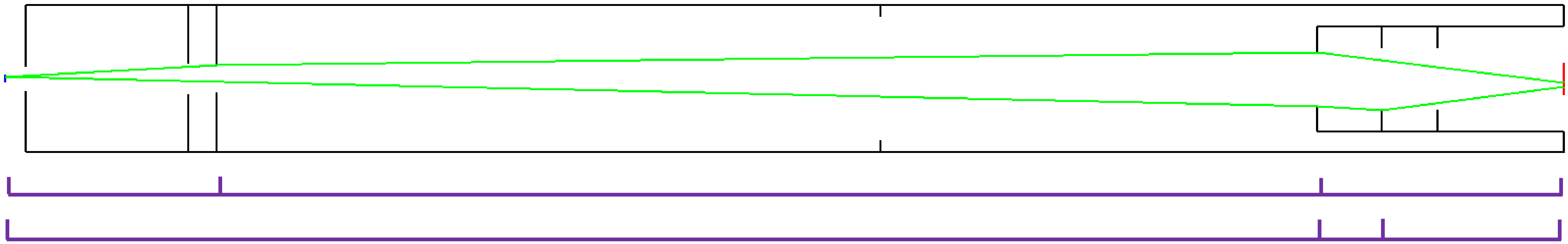
Combination of SAD results versus total effect

SAD	1	2	10
SAD	2	5	10
SAD	2	7	10





Combinations 2-5-7-10, 2-7-8-10, … leads to 2nd-order diffraction gains.

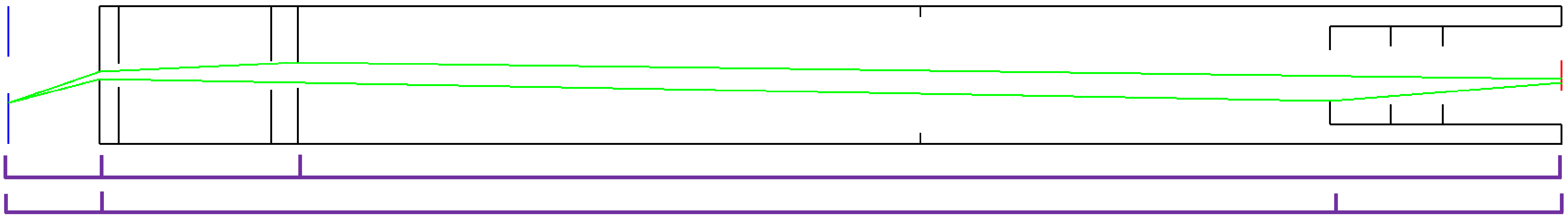


XBP	2	5	7	10
XBP	2	7	8	10
XBP	2	7	9	10

These gains vary roughly as λ^2.* Also, vignetting can also affect such gains.

* Coefficient of λ^2 found for several 2nd-order effects in a manuscript (to be submitted)

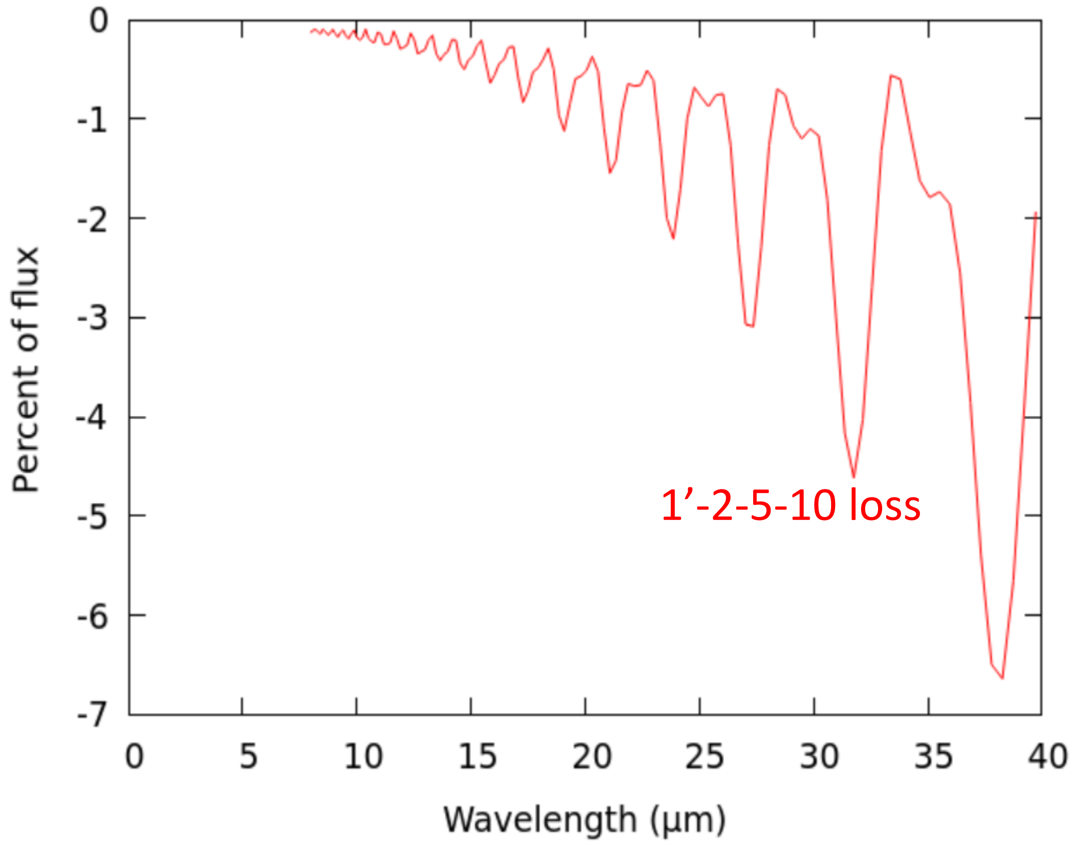
Combinations 1’-2-5-10 and 1’-2-7-10 account for effective decreases of 2-5-10 and 2-7-10 gains.



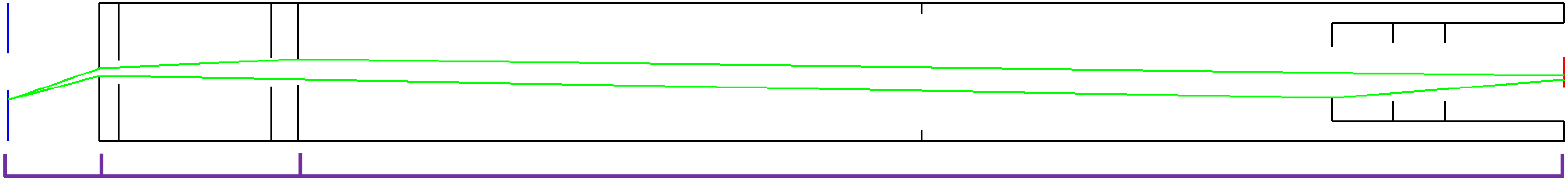
OFB	1	2	5	10
OFB	1	2	7	10

Example:

1’-2-5-10 loss is shown



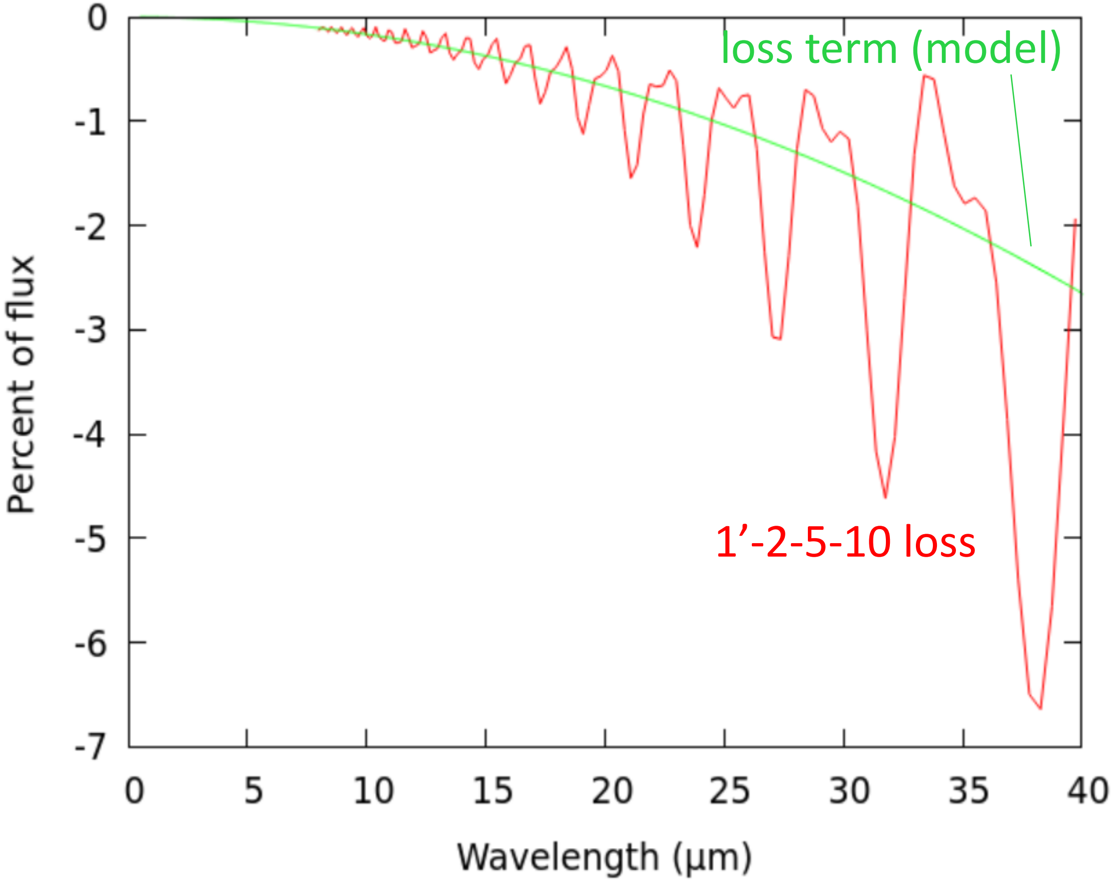
Combinations 1’-2-5-10 and 1’-2-7-10 account for effective decreases of 2-5-10 and 2-7-10 gains.



OFB	1	2	5	10
OFB	1	2	7	10

One part of the decrease is given by estimating 1’-2-5-10 false light alone. One can model this with a formula, avoiding lengthy numerical calculations.*

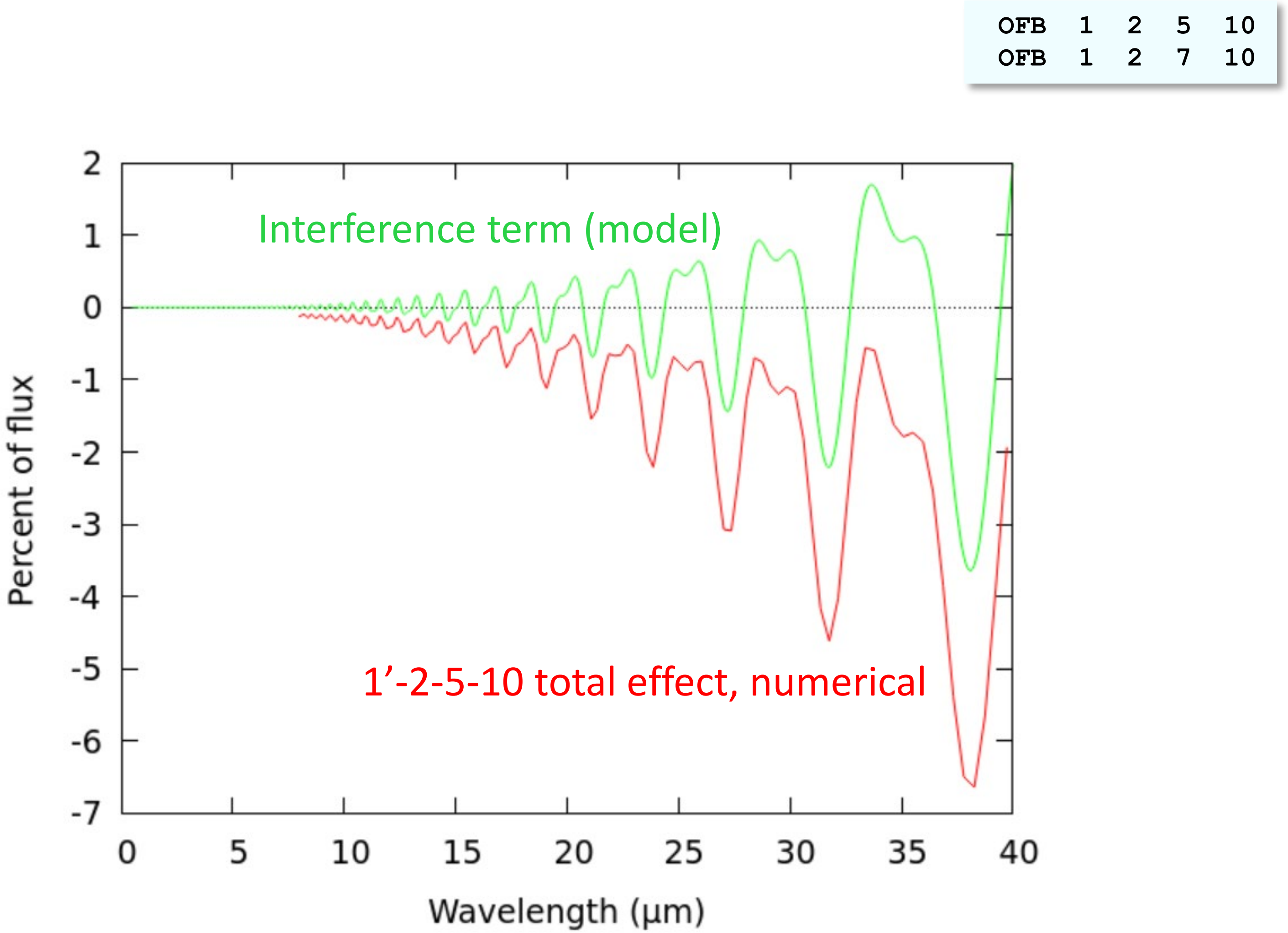
* Coefficient of λ^2 found for several 2^{nd}-order effects in a manuscript (to be submitted)



Combinations 1'-2-5-10 and 1'-2-7-10 account for effective decreases of 2-5-10 and 2-7-10 gains.

1'-2-5-10 and 1'-2-10 false lights interfere.
One can find a formula to model this,*
avoiding lengthy numerical calculations.

* Derived in partially completed manuscript.



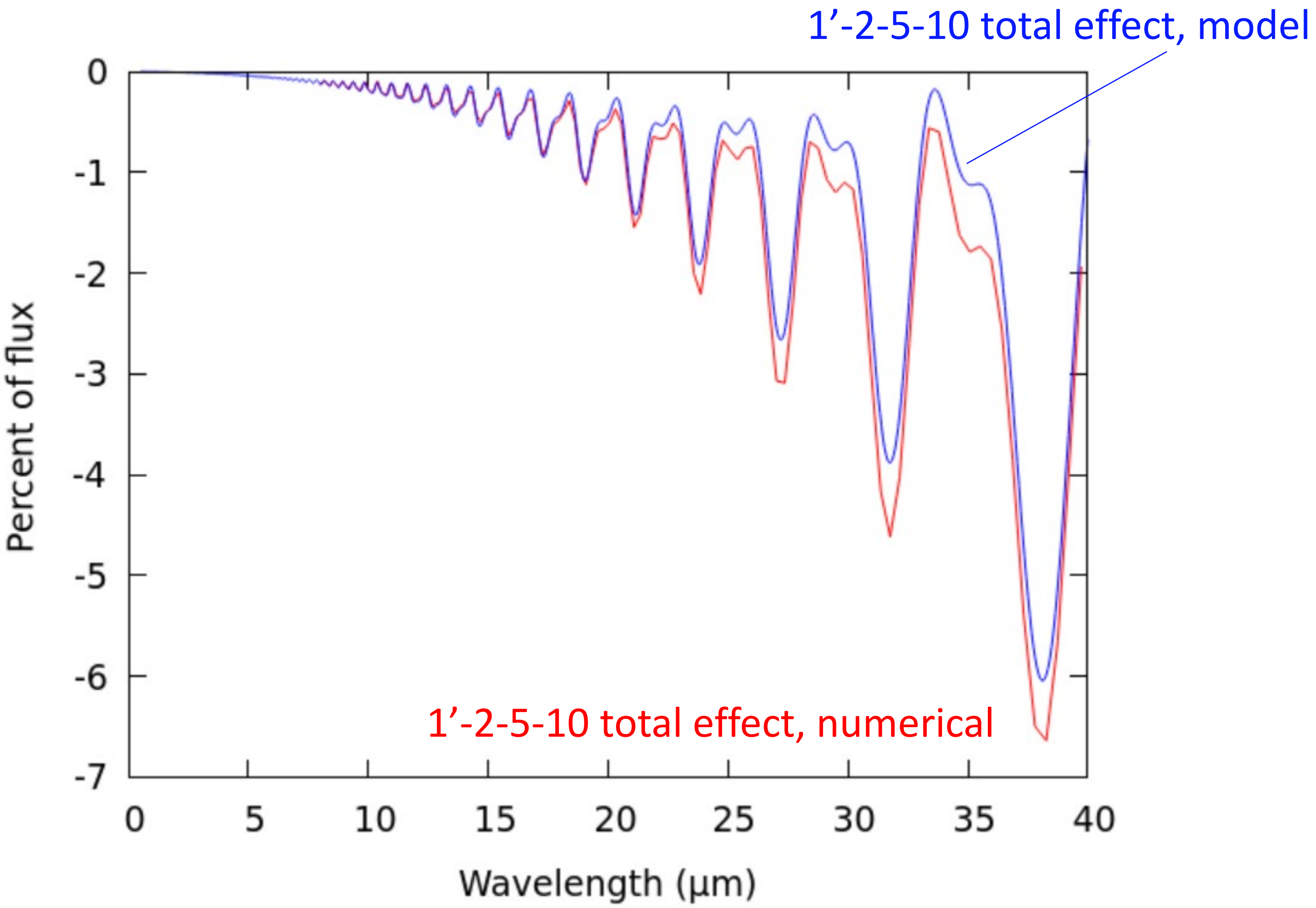
Combinations 1’-2-5-10 and 1’-2-7-10 account for effective decreases of 2-5-10 and 2-7-10 gains.

These effects are the largest “beyond-SAD” corrections for end-to-end propagation of light.

As an example, the 1’-2-5-10 effects are shown as found numerically and by model formulas.

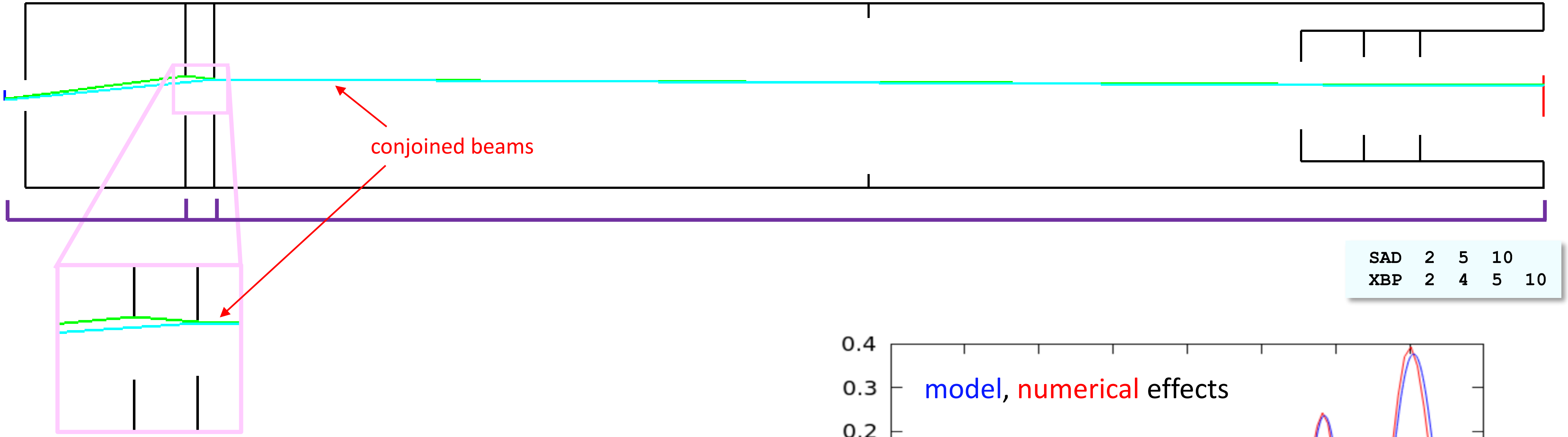
long calculations

instantaneous



OFB	1	2	5	10
OFB	1	2	7	10

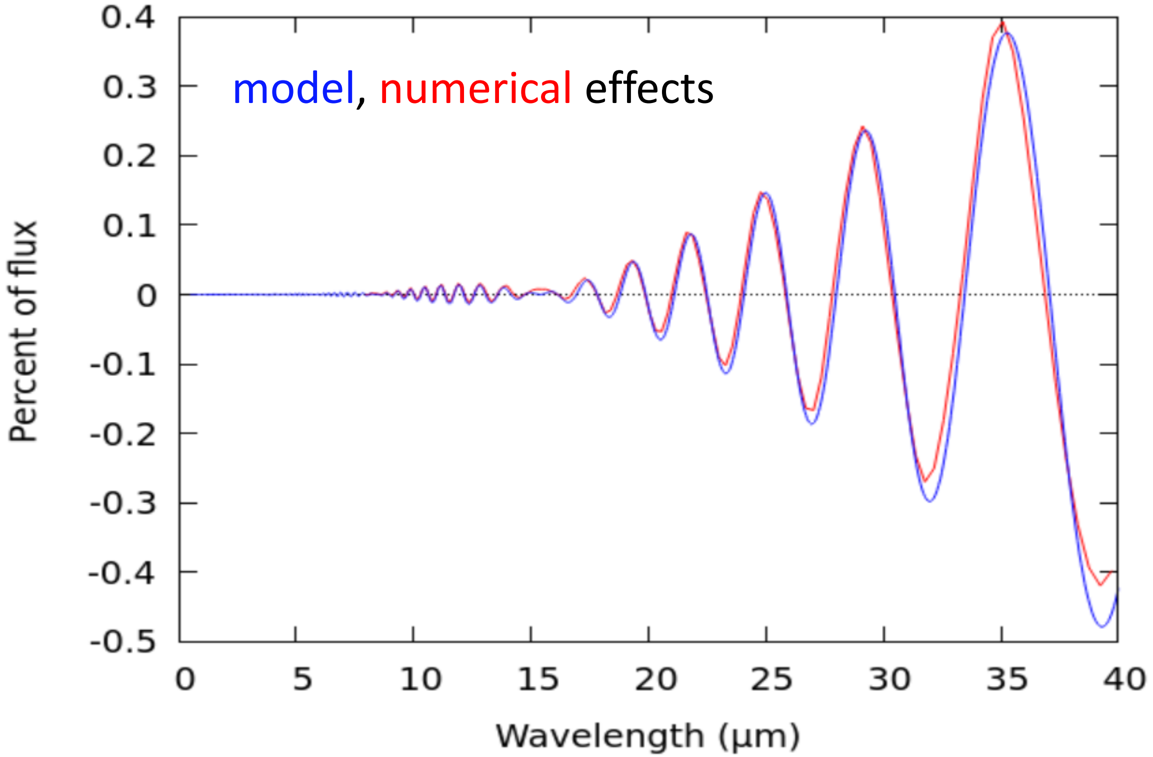
Proximity of Apertures 4 & 5 causes 2-5-10 and 2-4-5-10 interference to be considerable.



SAD	2	5	10	
XBP	2	4	5	10

The path-length difference for 2-5-10 and 2-4-5-10 light causes oscillatory behavior in the interference, as found numerically and by a model formula.

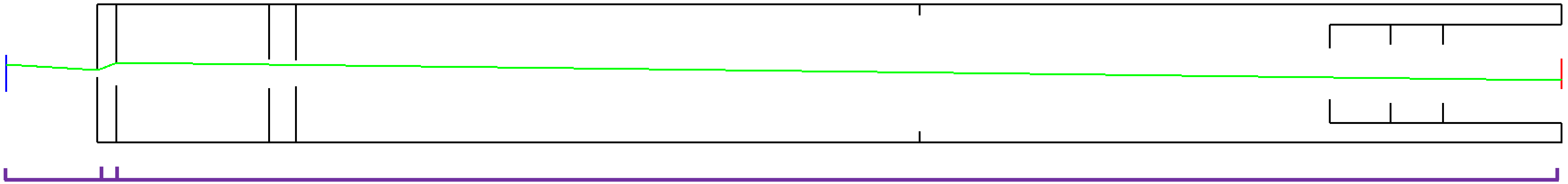
Finite extent of Aperture 2 leads to an envelope function modeled using theory of Bessel functions. *



* Formula derived in a partially completed manuscript.

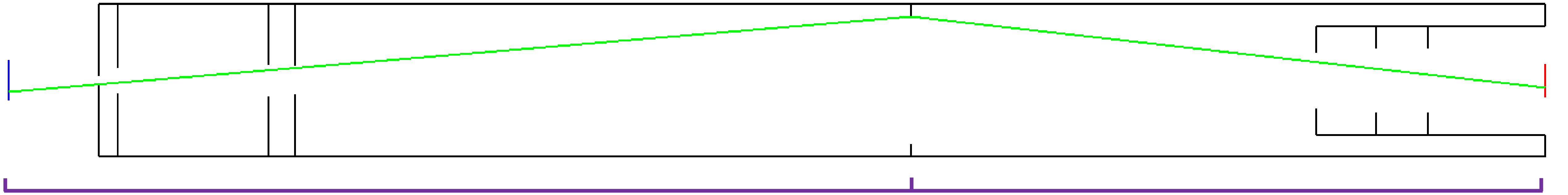
Sundry additional "beyond-SAD" corrections…

1-2-3-10 2^{nd}-order diffraction leads to a small accidental gain.



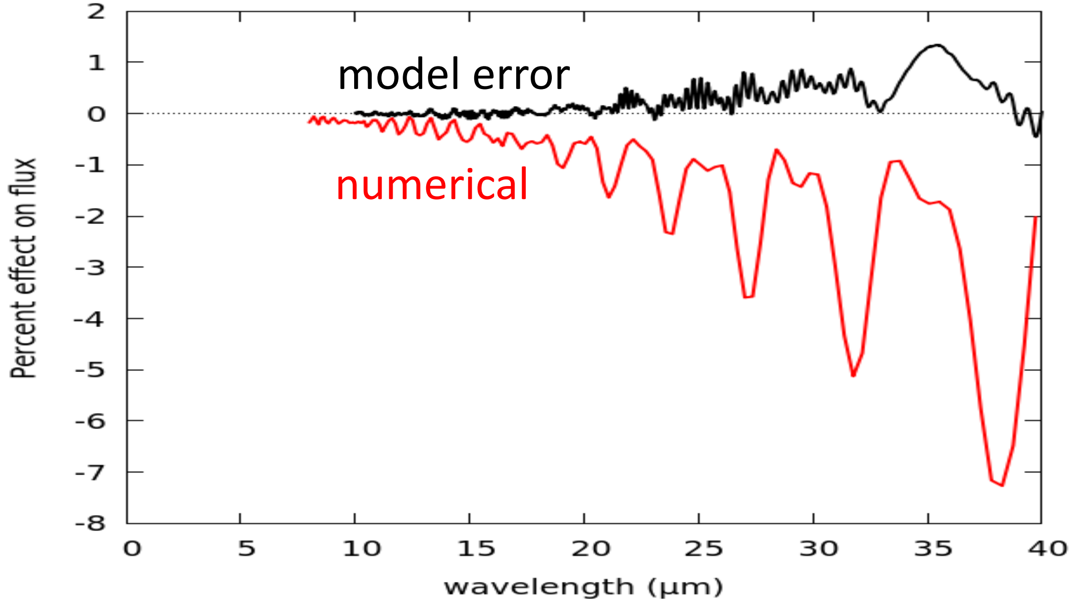
UFB 1 2 3 10

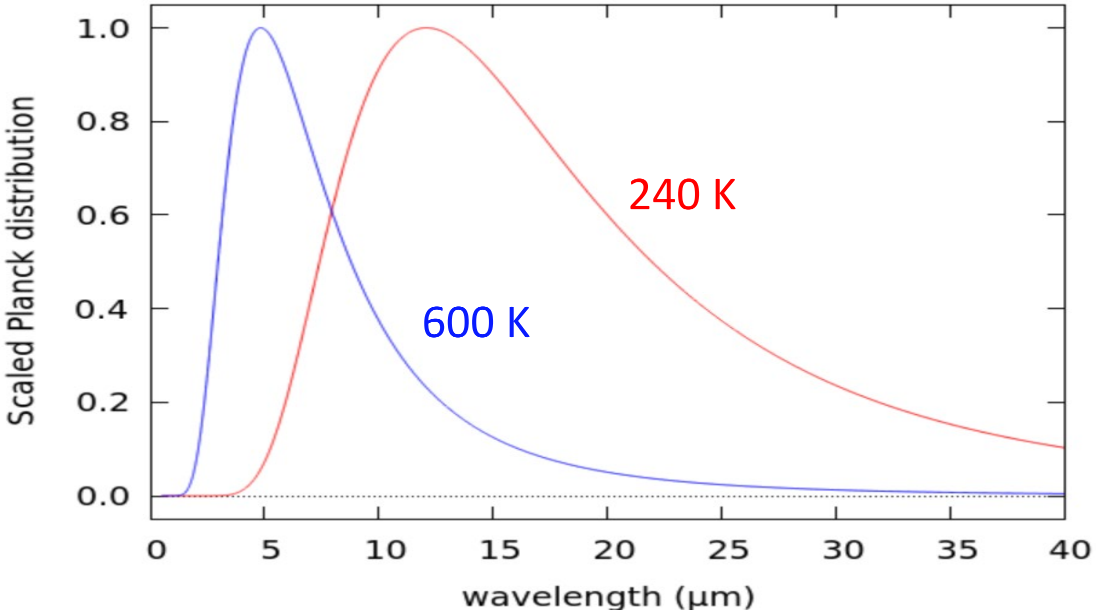
For larger Aperture 2, even the very large Aperture 6 can lead to a small 1^{st}-order gain.



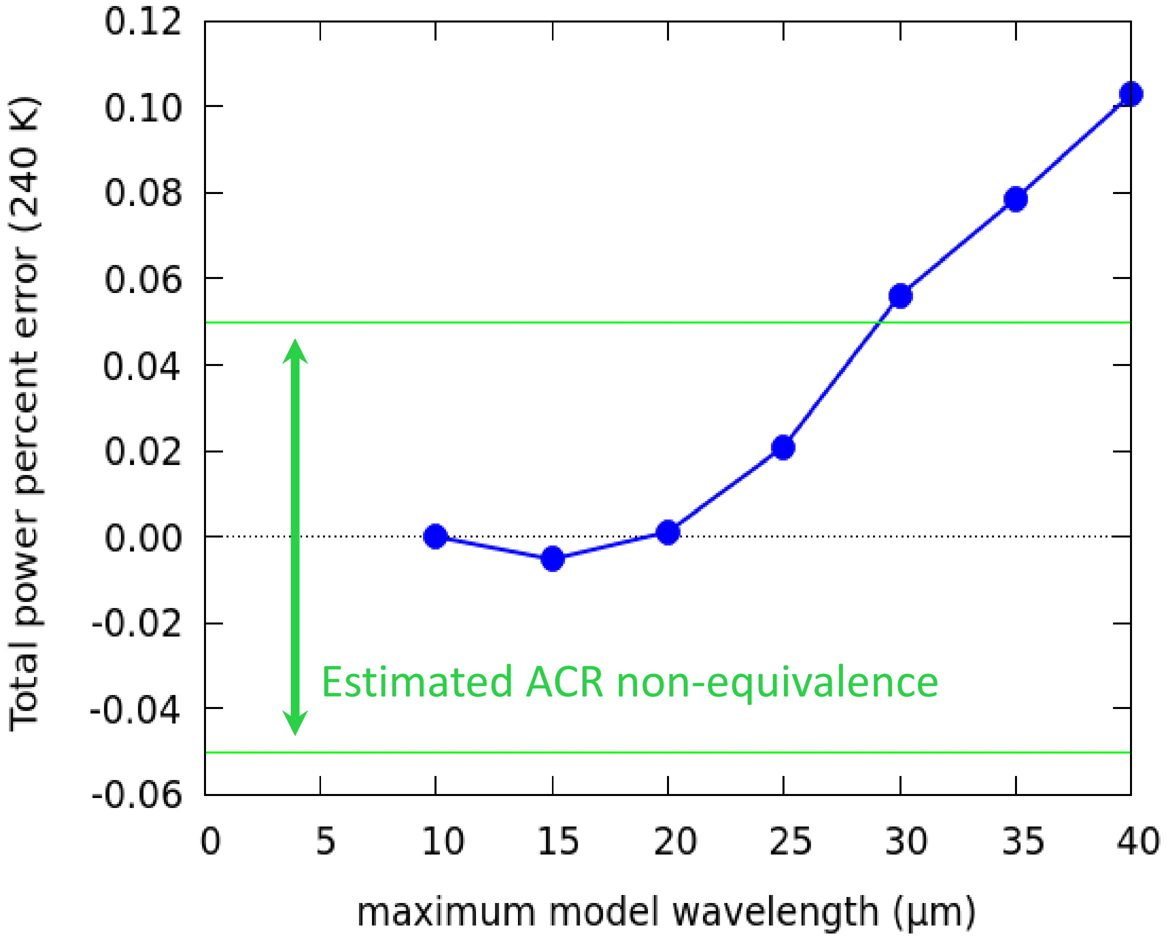
SPC1 1 6 10 5.592497 7.017125 -4.371048 9.996000

Beyond-SAD effects





Error in total power estimate vs model-numerical crossover λ



Model errors worst for *low temperatures* and *small apertures*.
Worst-case scenario shown ($T = 240$ K, $R_{\mathrm{BBDA}} = 0.04$ mm).

Path forward

This improves times for diffraction calculations for BB calibrations.

Estimate: < 60 s (20 threads) wall-clock time for each defining aperture.

0.04 mm $< R_{\mathrm{BBDA}} < 2.56$ mm, $R_{\mathrm{ACR}} \cong 10$ mm, $d \cong 1.04$ m

Prescription:

(1.) Identify diffraction effects
(2.) Model calculations
(3.) Numerical calculations: large λ, sparse sampling
(4.) Identify numerical-model crossover
(5.) Improve numerical sampling

Diffraction effects on spectral power or total power (vs Blevin's effective wavelength)