Autonomous Distributed Control Algorithm for Multiple Small Spacecraft during Simultaneous Close Proximity Operations

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Overview

- Introduction
- Overview of Spacecraft Model
- Novel LQR/APF Control Algorithm
  - Artificial Potential Function (APF)
  - Spacecraft Convergence using LQR instead of APF
  - Adaptation of APF-based Collision Avoidance
- Docking Performance Evaluation and Sample Simulation
- Conclusion (Questions and Discussion)
Development of an autonomous distributed control algorithm for multiple spacecraft in close proximity operations is examined.

This research aims to give a contribution to the control of multiple spacecraft for emerging missions, which may require gathering, rendezvous, and docking.

A control algorithm is proposed which combines the efficiency of the Linear Quadratic Regulator (LQR) and the robust collision avoidance capability of the Artificial Potential Function method (APF). The LQR control effort serves as the attractive force toward goal positions, while the APF-type repulsive functions provide collision avoidance for both fixed and moving obstacles.

The multiple spacecraft close proximity control algorithm gave promising results in simulations involving multiple spacecraft maneuvers.
Spacecraft Models

- 6 DOF Dynamic Spacecraft Model
- Perturbations
  - Gravity Gradient depends on spacecraft mass distribution
  - Non-Symmetric Earth (J2-J4 coefficients)
  - Atmospheric Drag
  - Third Body effects due to Sun and Moon
  - Solar Radiation Pressure
  - Thrusters for translational control – Mass variation
  - Momentum Exchange Device (MED) for attitude control
- Model Validation and Visualization via STK
  - Refer to AIAA Modeling and Simulation Technology (MST) Conference (Hilton Head, SC on 22 Aug 2007)
Control Algorithms

- Artificial Potential Functions
  - Translational Geometry
  - Obstacle Avoidance

- Linear Quadratic Regulator (LQR)
  - Control efficiency versus time of maneuver
  - Incorporates linearized relative dynamics

- Model/Sensor Uncertainty
- Actuator Limitation
Control Algorithms: Development Assumptions

- Control algorithms were evaluated as developed
  - Efficiency of control effort
    - $\Delta v$ based on changes in velocity commanded
  - Duration of maneuver
    - Limited due to operational need for rendezvous/docking maneuver
  - Precision upon approach of goal
    - Maneuver termination as a relative range is achieved
    - Within 2 mm is good enough for docking…limited by sensors

- Assumptions:
  - Initial spacecraft relative position within 1 km
  - Initial relative velocity is neutral (0 m/s)
  - Actuation limited due to thruster response
  - 1 sec simulation rate (allowing future hardware implementation)
Control Algorithms: Artificial Potential Function (APF)

- Potential function can be Lyapunov based function
- Convergence follows the negative gradient (negative rate of change)

Potential function: \( V = V_{\text{goal}} + V_{\text{obs}} \)

\[
V_{\text{goal}} = \frac{\lambda}{2} \cdot (\vec{r}_c - \vec{r}_\text{goal})^T \cdot Q \cdot (\vec{r}_c - \vec{r}_\text{goal})
\]

\[
V_{\text{obs}} = A \cdot e^{-(\vec{r}_c - \vec{r}_{\text{obs}})^T \cdot (\vec{r}_c - \vec{r}_{\text{obs}}) / \sigma}
\]
Control Algorithms: Spacecraft APF

APF Control Block Diagram

Desired (r, v)

Spacecraft States (r, v)

Sensors

Relative Frame

Σ

Residuals

Attractive APF

1/Δt

Thrusters

Actuation Command (a)

Obstacle (r, v)

Sensors

Relative Frame

Repulsive APF

vc

vco
Control Algorithms: Refined APF with collision avoidance

- **Goal potential**
  - Velocity damping relationship required for position
    - Goal position approach precision
    - Actuator limitations (thruster firings)

\[
V_{\text{goal}} = \frac{\lambda}{2} (r)^2 \quad \text{with} \quad r = \left| \left( \vec{r}_c - \vec{r}_{\text{goal}} \right) \right| \quad \& \quad \lambda = \frac{1}{r}
\]

\[
\vec{V}_c = -k \frac{\nabla V_{\text{goal}}}{\left| \nabla V_{\text{goal}} \right|} \quad \text{with} \quad k = \left( \frac{r_{\text{init}}}{r_{\max}} \right) \cdot v_{\max} \cdot \left( 1 - e^{-b_g V_{\text{goal}}} \right)
\]

\[
\vec{a}_c = \frac{\vec{v}_c - \vec{v}_{\text{act}}}{\Delta t}
\]
Control Algorithms: Refined APF with collision avoidance

- **Obstacle potential**
  - Gaussian based to allow for direct incorporation of uncertainties
  
  \[ V_o = \lambda_o \left( e^{-\frac{r_{co}^2}{2\sigma^2}} - e^{-\frac{D_o^2}{2\sigma^2}} \right) \]
  
  with \( \lambda_o = \left( \frac{r_{init}}{2} \right) \left( e^{-\frac{L_o^2}{2\sigma^2}} - e^{-\frac{D_o^2}{2\sigma^2}} \right)^{-1} \)

- Obstacle region if influence is position and velocity dependent

\[ D_o = d_o \left( L_o + D_{stop} \right) \quad \text{with} \quad D_{stop} = \frac{\bar{v}^2}{4a_{max}} \]

- standard deviation: \( \sigma = \frac{\left( L_o + D_o \right)}{3} \)
LQR/APF Control Block Diagram

- Desired (r, v)
- Spacecraft States (r, v)
- Sensors
- Obstacle (r, v)
- Sensors
- Relative Frame
- Collision Avoidance APF
- Residuals
- LQR
- 1/Δt
- Actuation Command (a)
- Thrusters
- a_{LQR}
- v_{co}
- a_{co}
- Relative Frame
Control Algorithms: LQR/APF Controller

- LQR iterative optimal feedback yields $\vec{a}_{LQR} = - K_{LQR} \vec{x}$

- Collision avoidance is based on damping components of velocity and acceleration

$$\vec{v}_{co} = \frac{\vec{r}_{co} \bullet (\vec{v} + \vec{v}_{offset})}{r_{co}} \left( \frac{\vec{r}_{co}}{r_{co}} \right)$$

and

$$\vec{a}_{co} = \frac{\vec{r}_{co} \bullet \vec{a}_{LQR}}{r_{co}} \left( \frac{\vec{r}_{co}}{r_{co}} \right)$$

- LQR/APF:

$$\vec{a} = \vec{a}_{LQR} - \sum_{co=0}^{n} \left( \left( k_v \vec{v}_{co} / \Delta t \right) + k_s k_a \vec{a}_{co} \right)$$

where

$$k_v = \begin{pmatrix} e^{-\left(r_{co}^2/(2\sigma^2)\right)} & e^{-\left(D_o^2/(2\sigma^2)\right)} & e^{-\left(L_o^2/(2\sigma^2)\right)} & e^{-\left(D_o^2/(2\sigma^2)\right)} \end{pmatrix}^{-1}$$

$$k_a = \exp\left(-d_a \left(r_{co} - L_o\right)\right) \quad \text{and} \quad k_s = 1 - \exp\left(-d_a r_{cg}\right)$$
Performance Evaluation: Close Proximity Operations

- Multiple spacecraft close proximity ops
- Convergence, Rally, and Rendezvous
- Docking
Performance Evaluation: Docking

- Docking maneuvers require precise convergence to the outer boundary of a Target spacecraft while avoiding collision

- Collision avoidance of stationary obstacles and moving spacecraft

- Asterisk (*) indicates thrust actuator saturation

<table>
<thead>
<tr>
<th>Far Docking</th>
<th>LQR/APF</th>
<th>APF</th>
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</thead>
<tbody>
<tr>
<td>Far with Obstacle</td>
<td>$\Delta v = 4.1792$</td>
<td>$\Delta v = 3.7513 \ast$</td>
</tr>
<tr>
<td>RSW [0, 1000, 0]</td>
<td>$t_d = 1530$</td>
<td>$t_d = 1478$</td>
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<tr>
<td>Far with Obstacle</td>
<td>$\Delta v = 4.5243$</td>
<td>$\Delta v = 4.1084 \ast$</td>
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<tr>
<td>RSW [412,-812,-412]</td>
<td>$t_d = 1870$</td>
<td>$t_d = 1488$</td>
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<tr>
<td>Far with Obstacle</td>
<td>$\Delta v = 4.7083$</td>
<td>$\Delta v = 5.2832 \ast$</td>
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<td>$t_d = 1719$</td>
<td>$t_d = 1549$</td>
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<tr>
<td>Far with Obstacle</td>
<td>$\Delta v = 4.9268 \ast$</td>
<td>$\Delta v = 5.2024 \ast$</td>
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<tr>
<td>RSW [1000, 0, 0]</td>
<td>$t_d = 1520$</td>
<td>$t_d = 1602$</td>
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<tr>
<td>Far with Obstacle</td>
<td>$\Delta v = 3.6151$</td>
<td>$\Delta v = 3.8136 \ast$</td>
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<td>RSW [0, 0, 1000]</td>
<td>$t_d = 1678$</td>
<td>$t_d = 1496$</td>
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<tr>
<td>Far with Obstacle</td>
<td>$\Delta v = 3.0789$</td>
<td>$\Delta v = 5.1804 \ast$</td>
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<tr>
<td>RSW [707, 707, 0]</td>
<td>$t_d = 1463$</td>
<td>$t_d = 1509$</td>
</tr>
</tbody>
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Performance Evaluation: Far Docking (2nd Chase Spacecraft)

- LQR/APF:

![Graphs showing velocity and control signal](image1)

- APF:

![Graphs showing velocity and control signal](image2)
Performance Evaluation: STK Visualization
Conclusions

- Develop a novel, robust, and effective LQR/APF control algorithm which can be applied to a wide variety of emerging spacecraft servicing missions.

- Build confidence and provide path for spacecraft close proximity operation, on-orbit assembly and reconfiguration missions.

- Preparation for ground testing on the NPS AMPHIS
  - 2D dynamics of orbital flight (3 DOF)
  - Communication and synchronization refinement
Questions?

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