Hydraulic Performance of Piano Key Weir with Linear and Curvilinear Profiles of Outlet Keys

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Hydraulic Performance of Piano Key Weir with Linear and Curvilinear Profiles of Outlet Keys

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Abstract With rising demand for storing water in reservoirs, many spillways require modification by optimizing their shape for enhancing their discharge capacity. A Piano Key Weir (PK weir) is a modified weir and it is installed at the dams to augment the discharge capacity of the weir. Due to different geometrical configurations of PK weirs, the hydraulic performance of the weir is still in evolving research. In this study, hydraulic performance of two different models of PK weir is studied experimentally. All the configurations of both the PK weir are same except the shape of the outlet keys. The outlet key of the first model is linear whereas for the second model a curvilinear ogee shape is adopted. Ten discharges between 0.0075-0.03 m³/s and ratio of head over crest to weir height between 0.17-0.58 m were considered for each model. Water surface profiles were plotted along longitudinal and transverse directions at various sections of the PK weir. The effect of the weir geometry on discharging capacity was studied and it is found that the PK weir with curvilinear shaped outlet key is hydraulically more efficient than the linear shaped key. The coefficient of discharge of PK weir with curvilinear shaped outlet key is 12.41-18.15% more than the linear shaped key.

Keywords: Curvilinear, Hydraulic-performance, Piano Key Weir, Water flow profile

1. Introduction

A Piano Key Weir (PK weir) is currently designed to increase the overflow discharge capacity of dams. It is one of the types of a modified version of the labyrinth weir structures. It is preferred over labyrinth weirs in rehabilitation purpose for existing dams. It increases the discharge capacity of the dams. An universal design technique of PK weirs is not available now due to the complexity of its shape (Ribeiro et al. 2012). It is essential to estimate the coefficient of discharge of the PK weir with high clarity in order to ensure a safe and cost-effective design of the PK weir. The precision of the coefficient of discharge estimation is very important since it is required to deal with the dams' overflow capacity. A PK weir is considered to be a novel and realistic alternative for dam rehabilitation and construction of new projects with limited space.

Hydrocoop (France), in partnership with three institutions, Biskra University (Algeria), the Hydraulic Laboratory in Electricite de France (France) and IIT Roorkee (India) proposed the PK weir at the turn of the 21st century to replace the long-established weir so that it can augment the discharging capacity of the weir (Erpicum et al. 2017; Lemperiere and Ouamane 2003). The PK weir was originally developed by Blanc and Lemperiere (2001) to improve the performance of labyrinth-type weirs. At equal head, initial model tests showed that a PK Weir enhances discharge capacity by four times that of traditional frontal weirs (Lemperiere and Ouamane 2003). The 1st prototype PK weir was constructed in France at the Goulours Dam (Laugier 2007). The 2nd PK weir was built on the St. Marc Dam, whereas the 3rd was installed on Gloriettes Dam (Bieri et al.2009). The design procedure was first reported by Machiels et al. (2011b). Historical reviews on the evolution from the labyrinth to the PK weir are summarized by Schleiss (2011) and Lemperiere et al. (2011).

PK Weir's geometry is complicated since it is regulated by a vast number of parameters. In certain laboratories, a naming standard was created particularly to combine the notations (Pralong et al. 2011). The major geometric parameters are \( W_i \) and \( W_o \), the width of the inlet and outlet keys, respectively, \( P \) is the height of weir, \( P' \) is the depth below inlet key, \( W_u \) is the width of one cycle or PKW unit, \( N_u \) is the total number of PK weir units, \( B \) is the total stream wise crest length, the lengths \( B_p \) and \( B_l \) of the upstream and downstream overhangs, \( B_k \) is the base length and \( T \) is the thickness of wall. The width of a PK weir unit \( W_u \) is equal to \( W_i + W_o + 2T \) and the total width \( W \) of the weir is equal to \( N_u \times W_u \). The developed crest length \( L_u \) of a PKW unit is equal to \( W_u + 2B \) and the total developed crest length \( L \) of the weir is calculated as \( N_u \times L_u \) (Erpicum et al. 2017). All the geometrical parameters are shown in the Figure 1.
To determine the discharge capacity of the PK weir, many researchers have performed a series of both experimental and numerical research (Erpicum et al. 2017). Ribeiro et al. (2012) carried out a series of experiments on numerous PKWs with upstream and downstream overhangs (Type-A) and identified the characteristics that had the greatest impact on the flow. They came up with a head–discharge relationship for PK weir by expressing the enhanced discharge ratio of PKW in relation to a rectangular sharp-crested weir. Nowadays extensive and systematic model studies have yielded more elaborated design equations with reasonable accuracy within the parameters given by the researchers (Cicero & Delisle 2013; Crookston et al. 2018 and Kumar et al. 2019). PK weir are classified according to the presence or absence of overhangs in the upstream and downstream of the weir (Lemperiere et al. 2011), if upstream and downstream, both part of weir contains overhang, and the overhangs are identical then that is called type A PK weir. When only upstream of the PK weir Includes overhang then that is called the type B PK weir. Whereas PK weir only downstream overhang is called as Type C PK weir. When piano key weir does not contain any overhangs, it has only an inclined floor then that is called type D PK weir. Kabiri-Samani and Javaheri (2012) conducted a series of experiments and derived equation for coefficient of discharge for all types of PK weir (Type A, B, C & D) for both free and submerged condition.

Previous investigations indicate that almost all the studies till date considered the linear inlet outlet keys of PK weir. Limited literatures are available on the discharge coefficient for the linear and curvilinear outlet keys of PK weir. The present study deals with the experimental result of discharge coefficient for the linear and curvilinear outlet keys of PK weir.

2. Methodology

2.1 Geometry of models

In this paper, two models of Type-A PK weir with different shape of outlet keys have been studied. The first model of piano key weir with three cycle consists of linear shape of both inlet and outlet keys (indicated as L-PKW). All the parameters of the second model are similar as L-PKW except the shape of the outlet key which is curvilinear and indicated as CL-PKW. The curvilinear shape of the outlet key was selected based on waterway
experimental station (WES) standard spillway shapes given by U.S. Army corps of engineers USBR (1987). The curvilinear profile of the outlet key slope of CL-PKW was selected as ogee downstream profile based on WES standard spillway shapes. The equation given by the WES for the ogee shape is as follows.

\[ X^n = k H_d^{n-1} Y \]  

(1)

Where (X, Y) are the coordinates of points on the outlet key with the origin at highest point on key, \( H_d \) is the design head including the velocity head, \( k \) and \( n \) are the constants depending upon the slope of the upstream face. The values of \( k \) and \( n \) given by WES for vertical upstream face is 2 and 1.85 respectively. The reverse curve radius \( r \) has been taken as half of the design head and the internal angle made is 60\(^\circ\). Figure 2 shows a schematic diagram of the curvilinear profile. This form is developed from the lower surface of a free-falling nappe and is recognized for its efficiency and safety in passing flows (Savage & Johnson 2001). At the bottom of the main slope, a reverse curve was added to smoothly direct the falling jet downstream. To obtain the slope profile for the model with curvilinear outlet keys, the design head required to be fixed above the crest. Most of these weirs constructed or built till date have a total design head to weir height ratio of less than one, because PKWs are very efficient at lower heads (Crookston et al. 2019). Therefore, for the present study, the profile of the outlet slope of the curvilinear model were designed with \( \frac{H_d}{p} \) equal to 0.25.

Figure 2. Profile of outlet key for (a) CL-PKW (b) L-PKW

For the calculation of discharge coefficient, the general formula for a rectangular sharp crested weir were used (Kabiri-Samani and Javaheri 2012, Kumar et al. 2019) i.e.,

\[ Q_{PKW} = \frac{2}{3} C_{PKW} W \sqrt{2 g H^3} \]  

(2)

Where \( Q_{PKW} \) is discharge, \( H \) is upstream head above the crest of the weir, \( W \) is width of weir (in transverse direction) and \( C_{PKW} \) as discharge coefficient of PK weir. Ribeiro et al., (2012) calculated the discharge enhancement ratio \( r \) between PK weir discharge \( Q_{PKW} \) and a sharp crested weir discharge \( Q_s \) and has been defined as:

\[ r = \frac{Q_{PKW}}{Q_s} = \frac{C_{PKW} L \sqrt{2 g H^3}}{C_d W \sqrt{2 g H^3}} = f \left( \frac{L}{W} \right) \]  

(3)

Where \( W \) is total width of the PK weir. The discharge coefficient \( C_d \) of the sharp crested standard weir can be assumed as almost constant with \( C_d = 0.42 \) (Hager and Schleiss 2009). Anderson (2011), Anderson and Tullis (2011, 2012, 2013) conducted experimental studies on various PK weir shapes, and Crookston et al., (2018) later developed an empirical formula for the coefficient of discharge. While comparing their study with Eq. (2), \( C_{PKW} \) comes out to be

\[ C_{PKW} = \left[ 1 \left( a_1 + b_1 \frac{H}{P} + c_1 \frac{H}{P} \right) + d_1 \right] \left( \frac{L}{W} \right) \]  

(4)

Cicero and Delisle (2013) conducted experimental studies on the various piano key weir and proposed the equation for the coefficient of discharge.

\[ C_{PKW} = \frac{3}{2} \left[ a_2 + a_3 \left( \frac{H}{P} \right) + a_4 \left( \frac{H}{P} \right)^2 + a_5 \left( \frac{H}{P} \right)^3 + a_6 \left( \frac{H}{P} \right)^4 \right] \]  

(5)
Where, $a_1, b_1, c_1, d_1, a_2, a_3, a_4, a_5, a_6$ are the empirical coefficients and can be found in the literature. Eq. (5) and (6) has been used to plot the graph in Figure 6.

2.2 Experimental Setup

All the experiments were conducted in the Hydraulics laboratory of Civil Engineering Department at Indian Institute of Technology, Roorkee. Two PK weir models were installed in a flume of length of 15 m, width of 0.39 m and height of 0.5 m, respectively. The flume has a 13 m long glass side wall for observing the flow over the weir and visual observation over the weir. A rectangular tank of length 2.5 m, width 1.15 m, depth 1.0 m was provided at the end of the flume for the storage of water and recirculation of flow. A pump was installed from the tank to the flume inlet for supplying the water. A control valve and an electro-magnetic flow meter were installed on the pump to control the flow and measure the discharge, respectively. At the entrance of flume honeycomb grid walls were installed to minimize the disturbances in the flow. A flow straightener and wave suppressor were also provided at the entrance of the flume to suppress cross current and surface disturbances respectively. A point gauge of ±0.0001 m least count was installed to measure the water level in the flume. The experimental set up is shown in the Figure 3.

The PK weir model made of 0.006 m thick acrylic sheet and installed at the midway of the flume. To measure the water level, the point gauge system was placed at 2$P$, i.e., 0.210 m, upstream from the outflow crest. Ten discharges between 0.0075 to 0.03 m$^3$/s were used at each model of PK weir. The range of $H/P$ ratio for linear and curvilinear models were 0.19-0.63 and 0.17-0.58 respectively. After the flow is uniform and constant for at least 20 minutes, the point gauge was utilized to measure the water level for each discharge. At least three water level readings were measured across a section to ensure that a steady state flow had been reached. The minimum value of $H$ is kept higher than 0.03 m to eliminate the scale effects as mentioned by Erpicum et al., (2016).

### Table 1. Geometrical parameter of PK weir models

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Parameters</th>
<th>Linear model (m)</th>
<th>Curvilinear model (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Channel width ($W$)</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>2</td>
<td>Side wall overflowing length ($B$)</td>
<td>0.254</td>
<td>0.254</td>
</tr>
<tr>
<td>3</td>
<td>Inlet key width ($W_i$)</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>Outlet key width ($W_o$)</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>5</td>
<td>Inlet key overhang length ($B_i$)</td>
<td>0.064</td>
<td>0.064</td>
</tr>
<tr>
<td>6</td>
<td>Outlet key overhang length ($B_o$)</td>
<td>0.064</td>
<td>0.064</td>
</tr>
<tr>
<td>7</td>
<td>Crest thickness ($T$)</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>8</td>
<td>Weir cycle number ($N$)</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
3. Results and Discussion

3.1 Water surface profile over PK weir

For both the models i.e., L-PKW and CL-PKW water surface profiles were analyzed. In the PK weir, there are two types of keys that is outlet key and inlet key, the water surface profile was analyzed on both the keys separately. At every section of PK weir, the water surface profile is not same. In this paper, the water depth at Centre of each outlet and inlet keys were taken and analyzed. Measurement points for water flow profile plot were taken from 0.75 m upstream to 1.05 m downstream from the upstream crest of the piano key weir at 0.05 m of interval and at each section, a minimum of three readings were noted and average value were plotted for the greater accuracy.

3.1.1 Water surface profile over outlet key

The Water flow profile at the center of the outlet key for the different discharges are shown in the figure 4. It shows constant head in the upstream to the start point of the piano key weir and then small drop of head at the start of the weir crest. Figure 4 shows that the head for the L-PKW is more as compared to the CL-PKW for the same discharge. In the far downstream, the flow is more gradual and steadier in the CL-PKW as compared to the other.

<table>
<thead>
<tr>
<th></th>
<th>Total crest length (L)</th>
<th>1.884</th>
<th>1.884</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Height of weir (P)</td>
<td>0.105</td>
<td>0.105</td>
</tr>
<tr>
<td>11</td>
<td>Approach flow depth (P')</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td>12</td>
<td>Inlet key slope (S_i)</td>
<td>29.06°</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>Outlet key slope (S_o)</td>
<td>29.06°</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>H/P ratio</td>
<td>0.19-0.63</td>
<td>0.17-0.58</td>
</tr>
</tbody>
</table>

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3.1.2 Water surface profile over Inlet key

The water surface profile at the center of inlet key for different discharges are shown in Figure 5. It shows constant head in the upstream to the start point of the piano key weir and then small drop of head at start of the weir crest. Figure 5 shows that the head for the L-PKW is more as compared to the CL-PKW at the same discharge. In the far downstream, the flow is more gradual and steadier in the CL-PKW as compared to the other. In the downstream when water is falling from the inlet key the air entrainment takes place between piano key weir and flow nappe, which is more in L-PKW compared to CL-PKW.
3.2 Discharge Characteristics

The discharge capacity of piano key weir can be described using the conventional formula for the weir which is mentioned in Eq. (2). The coefficient of discharge for the different discharges were calculated using equation (2) and plotted with head to weir height ratio, as shown in the Figure 6. From the figure it is clear that the discharge coefficient for the L-PKW is less as compared to the CL-PKW. For low head $C_{PKW}$ is very high for CL-PKW model as compared to the L-PKW but as head increases the value of $C_{PKW}$ for both the models nearing the same value for the given range of the data in the Table 1. As $H/P$ tends to 0.6, the $C_{PKW}$ value for both the models are almost equal.

For the given range of data, the relative increase in the coefficient of discharge for CL-PKW is 12.41-18.15% which is shown in the Table 2. Present study shows the similar trend as in literature by Cicero and Delise (2013) and Crookston et al. (2018) [Eq. (4) and (5)] that is plotted in Figure 6.
Table 2. Relative increase in $C_{PKW}$ of CL-PKW

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Discharge (m$^3$/s)</th>
<th>$C_{PKW}$ of L-PKW [1]</th>
<th>$C_{PKW}$ of CL-PKW [2]</th>
<th>Relative increase in Percentage ((\frac{[2]-[1]}{[2]}\times100))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0075</td>
<td>2.30</td>
<td>2.81</td>
<td>18.15</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
<td>2.10</td>
<td>2.54</td>
<td>17.33</td>
</tr>
<tr>
<td>3</td>
<td>0.0125</td>
<td>2.00</td>
<td>2.36</td>
<td>15.41</td>
</tr>
<tr>
<td>4</td>
<td>0.015</td>
<td>1.90</td>
<td>2.23</td>
<td>14.97</td>
</tr>
<tr>
<td>5</td>
<td>0.0175</td>
<td>1.80</td>
<td>2.11</td>
<td>14.69</td>
</tr>
<tr>
<td>6</td>
<td>0.02</td>
<td>1.71</td>
<td>2.00</td>
<td>14.25</td>
</tr>
<tr>
<td>7</td>
<td>0.0225</td>
<td>1.65</td>
<td>1.95</td>
<td>15.54</td>
</tr>
<tr>
<td>8</td>
<td>0.025</td>
<td>1.62</td>
<td>1.86</td>
<td>13.00</td>
</tr>
<tr>
<td>9</td>
<td>0.0275</td>
<td>1.56</td>
<td>1.79</td>
<td>12.70</td>
</tr>
<tr>
<td>10</td>
<td>0.03</td>
<td>1.53</td>
<td>1.74</td>
<td>12.41</td>
</tr>
</tbody>
</table>

From the Figure 6, it is also clear that both the models show the similar trend for the coefficient of discharge. From Table 2, it is apparent that as head increases the CL-PKW for both the models decreases that’s why the PK weir is most suitable for the low head flow. Also, from Table 2, CL-PKW is more efficient than the L-PKW model.

The flow passing through a PK weir is three-dimensional, with a longitudinal jet flow over the upstream and downstream crests and a lateral jet flow over the sidewalls. The longitudinal and lateral jets pouring into the outlet key are near to the walls at low upstream heads. These jets spring merge with each other in the downstream area of the outlet key as the head increases.

This interaction of jets downstream of the outlet crest decreases the discharging capacity of outlet key. As a result, the nappe interference may be blamed for the considerable decrease in efficiency of both PKW models as head increases. It has been also shown in the literature that lateral crests submergence because of outlet flow is a key parameter to explain decrease of efficiency with increasing head.

![Figure 7. Flow over PK weir and Nappe Interference](image1.jpg)

The impact of curvilinear outlet key profiles on discharge efficiency of PK weir may be described as follows: The lower surface of an aerated nappe flowing over a sharp crested weir for a single upstream head determines the form of the ogee profile. The ogee spillway having a S-shaped crest profile which provides a high discharge coefficient without causing cavitation. Furthermore, across most of the outlet key, the curvilinear profiles have higher slopes than the linear profiles, allowing for quick stream escape. As a result, at identical high flow circumstances, the limits of nappe interference for CL-PKW were seen to have shifted less towards the downstream than for L-PKW, as shown in Figure 7.
4. Conclusion

In this paper two experimental models with linear slope (L-PKW) and curvilinear slope (CL-PKW) of outlet key were examined to see how the form of the outlet key slope affects the discharge capacity of piano key weir. The water surface profile for both the models were also analysed at the outlet and inlet keys of the piano key weir separately. The conclusions based on the results of this experimental study is as follows:

- The coefficient of discharge for the CL-PKW is higher than the L-PKW in the given range of data. For low head the coefficient of discharge is quite higher for CL-PKW as compared to L-PKW. The maximum relative increase in coefficient of discharge was observed to be 18.15% for $0.17 < H/P < 0.58$.
- Both the model shows similar trend with increase in head, the coefficient of discharge decreases because of the nappe interference at the outlet key and the lateral crest submergence.
- Nappe interference were observed more downstream side on L-PKW in comparison to CL-PKW.
- Water surface profile shows that for the same discharge, the hydraulic head ($H$) is less in the case of CL-PKW, that’s why the coefficient of discharge is more for CL-PKW.

The findings of a single PK weir model with oggee shape outlet keys are presented in this study. To enhance and reinforce the findings of this paper, more tests and modifications on the curvilinear shape of PK weir can be performed. Further additional research can be done on the curvilinear shape PK weir with different design head to weir height ratio to enhance the discharge capacity of the weir. More experimental study will be helpful to design an optimal shape of these weirs.

5. REFERENCES


