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The Expected Value of an Advantage Blackjack player

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By

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Abstract

The Expected Value of an Advantage Blackjack Player

by

Kamron Paul Jensen, Master of Science
Utah State University, 2014

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The following paper takes an in-depth look at the gambling game Blackjack, also known as Twenty-One, and asks the question: If the game is beatable, how much can one expect to win playing Blackjack? This paper starts by explaining how the game is played and continues by telling of how Thorp (1962) discovered that the game can be beaten. It then goes into detail of how the game has changed over the past 50 years and what it takes to beat the game today. To find the expected value of a winning strategy, I create a computer program to run millions of simulations to output thousands of results. Conclusions are drawn as a result of the simulation analysis. Results show how to win more than $50 each hour of play.

Content

Introduction
More than 50 years has passed since Dr. Edward O. Thorp wrote the book that changed gambling forever. ***Beat the Dealer*** was published in 1962, a book on the game of Blackjack that proved once and for all that Blackjack was beatable. Almost overnight Thorpe overthrew the popularity of Craps and made Blackjack the number-one played casino game in America. His book sold over 700,000 copies. People skimmed through it and said to themselves, “Sounds easy enough to me,” before heading to the nearest casino.

The massive influx of players attempting to play with an advantage scared casinos into changing Blackjack to prevent the house’s edge from disappearing. Casinos introduced multi-deck games, some as large as eight decks in one game. This proved to be just a speed bump in the gambling world. In 1966 Thorp released a revised addition of *Beat the Dealer***,** which contained a new card counting system to overcome the multi-deck games. It is called the High-Low count. The game’s popularity continued to grow. Once more, casinos altered the game by decreasing the player’s choice of play with rules like “Double-down on 10s and 11s only,” ”No double-downs after splitting” or “Blackjack pays even money”. Each of these rules increased the casino’s advantage to the game and in some cases made it impossible for a player to beat the game. Patrons of casinos showed their displeasure by not playing and casinos faced decreases in revenue.

Few players attempting to use Thorp’s card counting methods were actually successful in beating the game, not because the system didn’t always work, but because few people actually applied the full system. The system allowed a small edge over casinos, and if used properly, would generate money over the long-run (tens-of-thousands of hands played). Unless used almost perfectly, the player could not overcome the house edge in the long-run. Realizing this, casinos relaxed their rules on the game and increased their security. Teaching pit bosses and surveillance personnel how to look for card counters. This began the cat and mouse game between casinos and card counters. Casinos offering an appealing game to its patrons while keeping an eye out for the card counting sharks looking for a big score.

From the day Thorp’s book hit the shelves stories have popped up in the media of card counters making big scores. Further books have been written and rewritten on the subject telling stories of card counters making multi-million dollar scores. Casinos becoming wise to the idea of card counting have pushed advantage players into forming teams and grouping together to uses strategy to make the card counting almost undetectable. Stories like that of the MIT Blackjack team, who supposedly took millions from casinos, or the church team based out of Seattle, Washington who claimed to have won 3.5 million in just a few short years. Others like Tommy Hyland have claimed to have “made a six-figure income each year since 1979 and sometimes more [by playing Blackjack].”

In this study, I test whether card counting is profitable. I will answer this question by discovering what a one-man card counting team's expected value would be for a years worth of play and compare this to my highest opportunity cost. Other questions I wish to answer include: What are some of the lesser known variables I can look for in the game of Blackjack that will increase my expected value?

How Blackjack is played

Anywhere from one to seven players can play the game of blackjack at a time. Unlike poker, the players are not playing against each other. In fact, they are playing against the dealer or "the house." Each player is dealt two cards as well as the dealer. However, one of the dealer's cards is dealt face down. This is known as the hole card. The player then sums the value of his two cards and makes a decision on how to play the hand by whether or not to draw another card. The object of the game is for the sum of the player's cards to be closer to 21 than the dealer's cards without going over. Whoever does this wins, and is awarded double the chips of their original wager.

When summing the cards, face cards are worth ten and numbered cards are worth the value of the number on the card (for example: a 7 is worth seven). The ace can have a value of either 11 or one, it is the player's choice and the values can change in the middle of play according to the desires of the player.

Possible Play Moves and an Example of play.

Once a bet has been placed the dealer deals two cards to the player and two cards to his/her self. One of his/her cards is face up. The player has up to six different decisions in any given hand. First is to play or to **surrender**. Few casinos offer surrender but it is beneficial to the player when used correctly. When a player surrenders, the hand is forfeited and only half of the original bet is lost. If the player chooses to play, his next decision is to double-down, split, hit or stand.

Double-down is when the player doubles his original bet and receives only one more draw card. If the player wins he receives four-times the original bet. The player may only choose to **split** if the two first cards of the hand match, for example, if the player had two 8s. The player would place another bet matching his original bet and split the two eights, creating two hands. The dealer would draw another card for the first of the two hands, if the new card matches the original card, in our case another eight, the player may re-split by repeating the process. When the player splits two cards into two hands he will continue to play the two hands as usual. To **hit** is to simply draw another card and **stand** is to end your turn. If the players hand sum is greater than 21 the player has "busted" and loses the hand and his wager. If the player is to make 21 on the first two cards, an ace and a 10, that is considered **Blackjack.** The player, as long as the dealer does not also have Blackjack, instantly wins and is usually awarded a bonus.

One last move a player can make is to buy insurance. **Insurance** is only offered when the dealer has an ace as her up card. The dealer will offer insurance to the players before they play their hands. When a player buys insurance the player is betting that the dealer has Blackjack, that his/her hole card is a ten. To buy insurance the player would place one-half of his original bet out in front of his original bet. The dealer would then look at his/her hole card and if it is in fact a ten, then the player would lose his/her original bet, but be paid 3:1 on the insurance. Essentially the player did not lose any money on the hand. However, if the dealer's hole card is not a ten the player loses the insurance bet and the play continues.

The dealer's play is fixed. The dealer must draw cards until the dealer's hand sum is equal to or greater than 17. This is called a dealer stand 17 rule. There is also a dealer hit 17 rule where if the dealer has a **soft 17**, meaning that one of the dealer's cards is an ace, then the dealer would hit the soft 17 until the dealer's sum was a hard 17 or greater. If the dealer reaches a sum greater than 21, the dealer has "busted" and all players remaining in the game win.

Not all Blackjack is Created Equal

Rules of the game of Blackjack can vary from casino to casino or even table to table within the casino. Some of these rules are governed by the state or country where the game is held, but usually it is the casino and its management that govern the rules. However, each of these rules have their own effect on the house advantage. For instance, a six deck game where the dealer stands on all 17's and the player may double-down after splitting, split up to four times, double-down on any two cards and Blackjack pays 3:2 a game that can be found just about anywhere, considering perfect Basic Strategy play (we will discuss Basic Strategy soon) the house advantage is about 0.42 percent --meaning for every $100 bet the player will lose approximately 42 cents, given long run play with hundreds of thousands of wagers.

Now take that same game and make Blackjack pay 6:5 and the house advantage jumps to 1.78 percent. Blackjack pays 6:5, is one of the worst game rules a person can play with. In any situations it makes the game unbeatable. I am about to get into more detail of how we know the size of the house advantage in a moment, but for now we simply need to understand that different rules create different expected values for both the casino and the player.

Optimal strategy for Blackjack

It is often argued amongst novice gamblers what the best play for Blackjack is. What does a player do when the dealer has a 10 and the player is holding 16? If the player hits, they bust. If the player stands the dealer gets 20. Or a soft 17 against a dealer's 9. It presents the same problem.

It is widely accepted that the typical Blackjack gambler who is trying to beat the game, but bases his game decisions on hunches, luck, or superstitions, is playing a game with about a four percent disadvantage. In other words, the expected value of the typical player is to lose $4 for every $100 he bets. However, in September of 1956, the **American Journal of Statistical Association** published a paper by Roger R. Baldwin, Wilbert E. Cantey, Herbert Maisel, and James P. McDermott called *The*

Optimal Strategy in Blackjack. Rather than following the hunches of your gut or superstitions, these four men brought forward the mathematical way to play every hand of Blackjack. They figured that both the player and the dealer had a finite number of possible hands, the player had only up to six possible decisions to make and the dealer's play being fixed, there had to be a optimal way to play the game. This paper provided the proof that the house advantage could be cut down to about 0.62 percent* In other words, rather than losing $4 for every $100 dollars bet a player would only be losing $0.62, considering what was called Reno/Tahoe rules in those days.

*In Thorp's revised addition to Beat the Dealer, Thorp mentions Mr. Wilbert E. Cantey discovered an error in the mathematics of his work after it was published and that the Reno/Tahoe rules actually favored the player by 0.09 percent

Blackjack is a Dependent Variable Game

Casino gambling games can be classified into two groups: Independent Variable and Dependent Variable Games. Independent meaning the outcome of one game does not affect the out come of another. Games such as Roulette or craps are examples of Independent games, one role of the dice or spin of the roulette wheel does not affect the outcome of the next game on a fair pair of dice or Roulette wheel.

Blackjack is a Dependent Variable Game. The deck is not shuffled after every round of play. Once a card has been used and the play is over for that round, that card will not be used again until the cards are reshuffled and put back into play. Thus, the outcome of one round is dependent on what cards were used in the previous round.

Thorp's Experiment

When Thorp learned about The Optimal Strategy and realized that it was a dependent variable game, he figured it had to be beatable. The first thing he wanted to do was recreate the study he had read about by Baldwin, Cantey, Maisel and McDermott. To do so he used an IBM 704 super computer and wrote a Monte Carlo simulation using Fortran. He determined what move would generate the greatest expected value for the player in every possible Blackjack situation. One of the first things he discovered is that in some of the more liberal Blackjack games casinos were offering a losing game, meaning the player had and edge off the top. Dr. Thorp discovered that using Reno/Tahoe rules, the player had an advantage of 0.09 percent. Thorp's strategy was very similar to that of the original publication of The Optimal Strategy but different. Thorp called his new strategy the Basic Strategy. Even with 0.09 percent player advantage, Thorp knew there had to be more, a way to increase profits for the player. As a professional card counter I will have to know the Basic Strategy frontwards and backwards with no mistakes.

With the game being a dependent variable game, Thorp wanted to determine the significance of each card and how it affected the expected value to both the player and the casino. To do this he went back to his IBM 704 and once again created a

Monte Carlo simulation with a computer program that mimicked the game of Blackjack. Rather than using a 52-card deck, Thorp would select one of the cards to be missing from the deck. Using a 51 card deck, Thorp was able to measure the significance of the missing card. For example, Thorp knew that using the Basic Strategy with Reno/Tahoe rules the player had a 0.09% advantage over the casino but when he removed an Ace from then deck and ran the simulation, the casino gained an additional 0.58% advantage. On the contrary, if a five was removed from the deck and the ace placed back in, the player's advantage was increased by about 0.80%, making the game almost a full percent in the players favor. When two 5s were removed the advantage rose an astounding 1.69% in the players favor. Thorp did this for each card and determined what effect the removal of each and every card would have on the continually changing expected value in the game of Blackjack.

Effect of Card Removal*

Card	Effect
2	0.3875%
3	0.4610%
4	0.6185%
5	0.8018%
6	0.4553%
7	0.2937%
8	-0.0137%
9	-0.1997%
10	-0.4932%
A	-0.5816%

*Negative numbers indicate a decrease in the expected value for the player.

BASIC STRATEGY

BLACKJACK *Apprenticeship*

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INSURANCE INDEX=3

Pair Splitting

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
(A,A)	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
(T,T)	N	N	N	N	N	N	N	N	N	N
(9,9)	Y	Y	Y	Y	Y	N	Y	Y	N	N
(8,8)	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
(7,7)	Y	Y	Y	Y	Y	Y	N	N	N	N
(6,6)	Y	Y	Y	Y	Y	N	N	N	N	N
(5,5)	N	N	N	N	N	N	N	N	N	N
(4,4)	N	N	N	Y	Y	N	N	N	N	N
(3,3)	Y	Y	Y	Y	Y	Y	N	N	N	N
(2,2)	Y	Y	Y	Y	Y	Y	N	N	N	N

First Number/Letter is DAS game. Second is NDAS.
R=Reverse BS at that number or higher.

Key:
- Y — Yes, Split the pair.
- N — No, Don't split the pair.

Soft Totals

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
(A,10)	S	S	S	S	S	S	S	S	S	S
(A,9)	S	S	S	S	S	S	S	S	S	S
(A,8)	S	S	S	S	Ds	S	S	S	S	S
(A,7)	Ds	Ds	Ds	Ds	Ds	S	S	H	H	H
(A,6)	H	D	D	D	D	H	H	H	H	H
(A,5)	H	H	D	D	D	H	H	H	H	H
(A,4)	H	H	D	D	D	H	H	H	H	H
(A,3)	H	H	H	D	D	H	H	H	H	H
(A,2)	H	H	H	D	D	H	H	H	H	H

Key:
- H — Hit
- S — Stand
- D — Double if allowed; If not, hit.
- Ds — Double if allowed; If not, stand.

Hard Totals

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
17	S	S	S	S	S	S	S	S	S	S
16	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
12	H	H	S	S	S	H	H	H	H	H
11	D	D	D	D	D	D	D	D	D	D
10	D	D	D	D	D	D	D	D	H	H
9	H	D	D	D	D	H	H	H	H	H
8	H	H	H	H	H	H	H	H	H	H
7	H	H	H	H	H	H	H	H	H	H

Key:
- H — Hit
- S — Stand
- D — Double if allowed; If not, hit.

Hard Totals- Surrender

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
17										
16								Sur	Sur	Sur
15									Sur	
14										

Key: H Hit

Colin Jones BlackjackApprenticship.com

The Basic Strategy chart may vary from one game to another but this chart is accrete for the majority of Blackjack games offered in the U.S.

Once Thorp realized what effect the removal of each card had on the game, he knew there had to be a way to tell when the deck was in favor of the player and when the deck was in favor of the casino. Thorp came up with what is called the Ten Count System, a complicated card counting system that keeps track of the 10 cards as a ratio to the other cards. With there being 16 tens in a deck and 36 others, as Thorp calls anything other than a ten, Thorp would then divide the others by the tens, giving the player a ratio. For instance, a fresh deck with all the cards would have 16 tens and 36 others, 36/16 = 2.25 there are 2.25 others in the deck for every ten. A player would watch the cards as they are dealt and keep a running count of each card as they are played, counting back wards from both 16 and 36 at the same time. (That is why I said the Ten Count System is complex. Try doing this at Las Vegas casino speed.) When it comes time for the player to make any kind of a playing decision, the player would then divide the current number of other by the current number of tens and base his decision on what ration the cards were at.

Ratio	Players Advantage
3.0	-2.20%
2.25	-0.20%
2.0	1.00%
1.75	2.00%
1.63	3.00%
1.5	4.00%
1.35	5.00%
1.25	6.00%

The greatest thing the Ten Count System did was allow the player to know when to bet more. For instance, if a player knows the cards are in advantage of the house, that player should bet as little as possible or nothing at all. However, if the player knows the cards are in the player's favor, that player would want to bet a significant amount. Now this does not guarantee the player is going to win, it just means that given the current situation the player is going to win more often then lose when the ratio is in the player's advantage. Under the law of large numbers, this means that the player is going to eventually beat the game of Blackjack.

The High-Low Count

Almost from the day Thorp's book hit the shelves people began redesigning his counting system. As a result, dozens of counting systems have stemmed from the original Ten Count. Many of these counting systems were necessary, because the first action casinos did to prevent card counting was to add more decks to the game. Could you imagine counting down a six-deck game using the Ten Count system? One would have to start at 216/96 and start counting backwards.

The High-Low was first introduced in 1963 by Harvey Dubner and since then has become the industry standard for card counters. It is simple, yet effective. It has been the preferred card counting system for many card counting professionals and teams such as the M.I.T, the Hyland, and the Church. The High-Low count simply assigns a value to each card, the player then keeps a running count of what cards have been dealt. Low cards, 2 though 6 are given the value of +1 and high cards such as 10s and aces are given the value of -1. 7 through 9 are counted as zero. A newly shuffled deck would start at a count of zero and as each card is dealt the player would keep a running count of what cards have been dealt, adding plus-one for small cards and subtracting one for big cards. This offered a much more simple way to know who has the advantage.

It also works for multi-deck games. A player simply keeps what is called the running count. As each card is dealt the player adds or subtracts accordingly, but when it comes time for the player to make a playing decision, the player must make what is called a true count conversation**.** The player would take the running count and divide it by the number of decks left in the shoe. The player does this by estimating how many decks have been placed in the discard tray and subtracting the number of decks in the discard tray from the number of decks the game was started with. For example, a player is playing a six-deck game and has a running count of +12, the player than estimates the number of decks in the discard tray. Let's say there are three decks in the discard tray. Take six (The number of decks started with), subtract three (the number in the discard tray) and you end up with three decks left in the shoe. Twelve divided by three is four. The true count is four. The true count conversation is a simple way to convert any running count into a ratio that the player can use to make their playing decisions.

Deviations

Almost as important as knowing when the advantage is for the player, the High-Low Count provides additional information – knowing when to deviate from the Basic Strategy. Again, all playing decisions should have one goal in mind and that is to maximize expected value. As the true count fluctuates in and out of the player's advantage, the Basic Strategy no longer becomes the best way to maximize expected value. Now it is very demanding to know when to deviate from the Basic Strategy and may seem impossible to memorize every possible hands deviation, so below are twenty-five deviation moves that will generate about 90% of the possible expected value that can be generated from deviations.

STAND 17 T20

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INSTRUCTIONS:

MEMORIZE THE DEVIATION #'S IN RED

A NEGATIVE (-) AFTER A "0" MEANS YOU DEVIATE AT THE COUNT AND BELOW.

A POSITIVE (+) AFTER A "0" MEANS YOU DEVIATE AT THE NUMBER AND ABOVE.

INSURANCE = 3

Pair Splitting

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
(A,A)	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
(T,T)	N	N	6	5	4	N	N	N	N	N
(9,9)	Y	Y	Y	Y	Y	N	Y	Y	N	N
(8,8)	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
(7,7)	Y	Y	Y	Y	Y	Y	N	N	N	N
(6,6)	Y/N	Y	Y	Y	Y	N	N	N	N	N
(5,5)	N	N	N	N	N	N	N	N	N	N
(4,4)	N	N	N	Y/N	Y/N	N	N	N	N	N
(3,3)	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N
(2,2)	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N

First Letter is DAS game. Second is NDAS.

Key:

Y	Yes, Split the pair.
N	No, Don't split the pair.

Soft Totals

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
(A,10)	S	S	S	S	S	S	S	S	S	S
(A,9)	S	S	S	S	S	S	S	S	S	S
(A,8)	S	S	3	1	1	S	S	S	S	S
(A,7)	Ds	Ds	Ds	Ds	Ds	S	S	H	H	H
(A,6)	1	D	D	D	D	H	H	H	H	H
(A,5)	H	H	D	D	D	H	H	H	H	H
(A,4)	H	H	D	D	D	H	H	H	H	H
(A,3)	H	H	H	D	D	H	H	H	H	H
(A,2)	H	H	H	D	D	H	H	H	H	H

Key:

H	Hit
S	Stand
D	Double if allowed; If not, hit.
Ds	Double if allowed; If not, stand.

Hard Totals

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
17	S	S	S	S	S	S	S	S	S	S
16	S	S	S	S	S	H	H	4	0+	H
15	S	S	S	S	S	H	H	H	3	H
14	S	S	S	S	S	H	H	H	H	H
13	-1	S	S	S	S	H	H	H	H	H
12	3	2	0-	S	S	H	H	H	H	H
11	D	D	D	D	D	D	D	D	D	1
10	D	D	D	D	D	D	D	D	4	3
9	1	D	D	D	D	3	H	H	H	H
8	H	H	H	H	2	H	H	H	H	H
7	H	H	H	H	H	H	H	H	H	H

Key:

H	Hit
S	Stand
D	Double if allowed; If not, hit.

Hard Totals- Surrender

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
17										
16							4	-1	Sur	Sur
15								2	0-	2

HIT 17 T21

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INSURANCE = 3

Pair Splitting

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
(A,A)	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
(T,T)	N	N	6	5	4	N	N	N	N	N
(9,9)	Y	Y	Y	Y	Y	N	Y	Y	N	N
(8,8)	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
(7,7)	Y	Y	Y	Y	Y	Y	N	N	N	N
(6,6)	Y/N	Y	Y	Y	Y	N	N	N	N	N
(5,5)	N	N	N	N	N	N	N	N	N	N
(4,4)	N	N	N	Y/N	Y/N	N	N	N	N	N
(3,3)	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N
(2,2)	Y/N	Y/N	Y	Y	Y	Y	N	N	N	N

First Number/Letter is DAS game. Second is NDAS.

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Y	Yes, Split the pair.
N	No, Don't split the pair.

Soft Totals

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
(A,10)	S	S	S	S	S	S	S	S	S	S
(A,9)	S	S	S	S	S	S	S	S	S	S
(A,8)	S	S	3	1	0-	S	S	S	S	S
(A,7)	Ds	Ds	Ds	Ds	Ds	S	S	H	H	H
(A,6)	1	D	D	D	D	H	H	H	H	H
(A,5)	H	H	D	D	D	H	H	H	H	H
(A,4)	H	H	D	D	D	H	H	H	H	H
(A,3)	H	H	H	D	D	H	H	H	H	H
(A,2)	H	H	H	D	D	H	H	H	H	H

Key:

H	Hit
S	Stand
D	Double if allowed; If not, hit.
Ds	Double if allowed; If not, stand.

Hard Totals

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
17	S	S	S	S	S	S	S	S	S	S
16	S	S	S	S	S	H	H	4	0+	3
15	S	S	S	S	S	H	H	H	4	5
14	S	S	S	S	S	H	H	H	H	H
13	-1	S	S	S	S	H	H	H	H	H
12	3	2	0-	S	S	H	H	H	H	H
11	D	D	D	D	D	D	D	D	D	D
10	D	D	D	D	D	D	D	D	4	3
9	1	D	D	D	D	3	H	H	H	H
8	H	H	H	H	2	H	H	H	H	H
7	H	H	H	H	H	H	H	H	H	H

Key:

H	Hit
S	Stand
D	Double if allowed; If not, hit.

Hard Totals- Surrender

	Dealer's Upcard									
	2	3	4	5	6	7	8	9	T	A
17										Sur
16							4	-1	Sur	Sur
15								2	0-	-1

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My Experiment

Now that we know how card counting works and the theory of one beating the game of Blackjack, what I want to know is; if the theory is correct, how much can one expect to win? To do so I wanted to follow Thorp's example by creating my own Blackjack simulation program to measure the expected value. In researching how card counting works, I found several different House Edge Calculators online. What these consist of are programs where the user goes through the different rules of Blackjack in a checklist fashion. Once the rules have been collected, the program runs hundreds of thousands of simulations then averages the results and determines how great the house's advantage is.

I wanted to take it further and create a program that would tell me what my expected value would be given the amount of money I want to bet and the rules of the game I plan to play with. Using object oriented style of programing I created classes to act as each of the different functions of the game of Blackjack. I wrote this program in C++ and wanted it to mimic every aspect of the game of Blackjack. A user would enter the rules of the game, each rule would be turned off or on by using true or false statement, which then activated or deactivated different classes with in the code. The Bank Roll amount, (how much money the user is planning to play with) is a class as well and allows the program to keep track of the wins and losses. The bet spread (How much the user plans to bet at each level of the true count) is a setting the user predetermines and is an Int that is called upon to act as a betting unit. I built a class for the shoe, the shoe is what contains the decks of cards. Just as 52 cards in a deck I created 16 tens, 4 aces, 4 twos, 4 threes and so forth. I used a random shuffler to randomize the order of these numbers, acting as the deck being shuffled. In addition, the shoe may contain more than one deck, simulating a multi-deck game and the random shuffler shuffles the multiple decks together. Again, the user predetermines the number of decks at the beginning of the program.

Using a pointer, numbers are assigned to be either the player's cards or the dealer's cards. The program than follows all of the possible Blackjack plays. First, the dealer checks its up card. If it is an ace, then the dealer offers insurance to the player. The player only buys insurance if the true count is three or greater, using the High-Low count and following the deviation strategy. If the dealer's up card is a 10, the dealer peeks to see if the second card is an ace. If it is, the hand is over and the player loses unless the player also has 21. If the player has 21 but the dealer does not, the hand is over and the player is given 1.5 times the current bet (I didn't bother putting in a rule where Blackjack pays anything less than 3:2 because is simply bad for the player). The current bet would be set to zero and the simulation would start all over, without shuffling the shoe.

If neither the player nor the dealer have 21 the play continues. The player counts each card as they are dealt with the High-Low system. Before each decision the player makes, the player ask its self what is the true count? Then the player sums the total value of the card it has. Using a series of if statements the program runs down the list of possible deviation moves followed by the Basic Strategy moves. Eventually landing on the correct play decision, the program cares out the correct move and then repeats the process. Asking itself, "What is the true count? What is the player's hand sum? What is the correct move?" until the player's hand receives a code for stand. Standing is the end of the player's hand. Then it is the dealer's turn. The dealer follows the same rules for every hand. It sums the total of the cards. If the value is less than 17 it hits until it has a value is greater than or equal to 17. If the user, when determining the rules of the game at the beginning of the program selected "Dealer hits soft 17," then whenever the dealer's hand total equals 17, the dealer checks if it has and ace in it's hand. If the dealer does have an ace in the hand, the value of the ace becomes a value of one and the dealer hits. The process starts over in summing the dealer's hand.

Once both the player and the dealer have returned a stand, the hands are over and the process of determining the winner begins. Because the player is allowed to split, the player may often have multiple hands and bets out. The program goes through each of the player's hands one at a time and determines the winner. Just as in the real game, the hands closest to 21 without going over are the winner and the payouts are awarded accordingly. Once the bets and payout have been collected, the whole process starts over. The shoe is never shuffled until it the pointer determining whose cards are whose reaches another pointer, a set point at which the shoe is to be shuffled at the end of the current round. The program continues to run until the set amount of simulation, predetermined by the user is reached. The results of each simulation are exported to a CSV file and can be uploaded into excel, which then the user can take the average and have the expected value.

Results

Intuitively, I figured as an advantage player, the more hands one plays the greater the expected value. Simple things like playing at tables that have few to no other players would increase the number of hands per hour played. I figured I should be able to play about 100 rounds per hour and so in my simulation I measured the expected value as per 100 rounds.

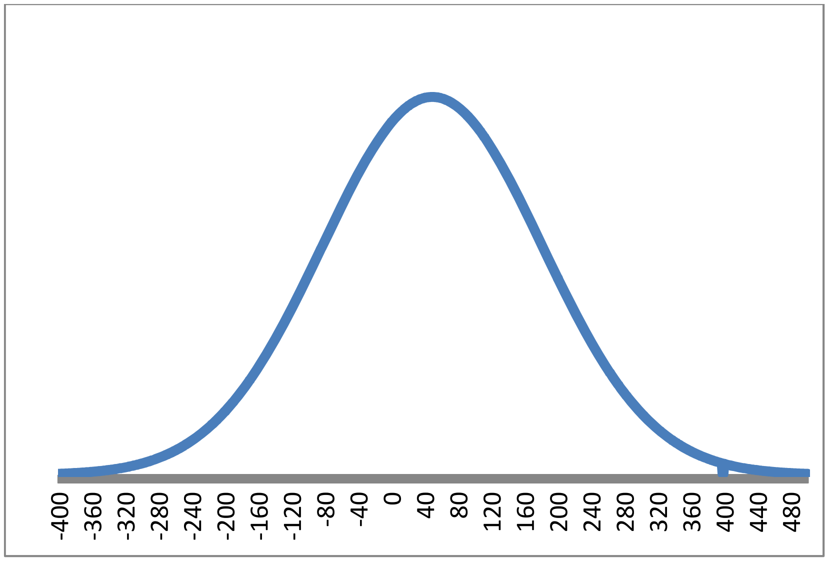
The game I would like to talk in detail is a six-deck, dealer stands on all 17's, player can double after splitting and slit up to four times, also player can surrender. It is a game that is very common in Las Vegas, especially in the High-Limit rooms. According to the House Edge Calculators online, this game is supposed to have a

0.42 percent house advantage. I entered the below bet spread with a $10,000 bankroll, large enough that is should not go broke in the course of 100 rounds. The bankroll is reset at the end of every simulation, and I chose to run 10,000 simulations of 100 rounds.

True Count	Bet
-2 or less	5
-1	5
0	5
+1	10
+2	20
+3	30
+4	40
+5	50
+6 or greater	60

My idea behind this bet spread was to bet very little in negative and zero counts and progressively increase my wager as the count became more favorable. The program outputted the 10,000 simulations of 100 rounds. I loaded them into an Excel spreadsheet and took the average result of simulations. This is my expected value for 100 rounds. I found an increase in the bankroll of $49. In other words, a perfect card counter who wagers this amount of money at each level of true count is expected to win $49 every 100 rounds.

In addition to the expected value I also measured the type of variance in wins and losses a player may expect. While the arithmetic mean was $49 the standard deviation was over $132. This means it would extremely common for player to win over $300 one hour and than to loose it all back the next hour or for a player to be on a losing streak for several hours. With a standard deviation being so large there will be great variance in the wins and losses.



The graph above illustrates the expected value of wins and losses given a normal distribution. A mean of $49 and a standard deviation of $132 per 100 rounds of play.

Let us take a look at a game with different rule. This time, take away surrender and double after splitting but keep the same bet spread. We get a expected value $40. with a standard deviation of $108. It is still a beatable game, but not as good as other games. This is a demonstration of how rules can affect the expected value of a game. Taking away two favorable rules for the player amounted in over an 18 percent decrease in expected value.

This is the expected value for two different games, with games having dozens of variations in the rules and infinite combinations of bet spreads there are literally endless possibilities for what the expected value will be for a card counter. However, this was a bet spread I felt I could afford and feel comfortable playing with.

In addition to finding the expected value, I wanted to see if there are other factors that I would help generate greater expected value holding the rules of the games constant. I decided to look into the penetration of the cut card. When playing a shoe game the cut card is a card placed in the deck and that tells the dealer when it is time to shuffle the deck. The placement of the cut card is really up to the dealer. While the rules of the table are the same, each dealer is going to place the card differently. This is important because if the dealer places the cut card halfway through the deck, only half of the cards are going to be dealt before the shoe is reshuffled. However, if the cut card is placed 85 percent of the way to the end of the shoe, 85 percent of the cards will be dealt before the shoe is shuffled. More cards being dealt is more information for the player and results in higher probability of higher counts.

While I did not create a function in my program to adjust the penetration point (The point where the cut card sits), I did go into my code and adjust the penetration point manually. I set it once for 65 percent, 75 percent, and 85 percent. I ran the same 10,000 simulations of 100 rounds with the same rules and as my first experiment. My results are below.

Penetration	Expected Value
65%	$43
75%	$49
85%	$54

As you can see, as penetration increases so does expected value for a card counter. There are so many variables to the game of Blackjack it would almost be impossible to measure the effects of each one without the aid of project-oriented program. And that is why I simply love this program I have created. I could fill volumes of books talking about each rule of the game and its effect on expected value. Perhaps there are more variables to the game, like penetration, that are not rules but still have their effect on the game. Perhaps I will discover them in time.

Conclusion

This study runs thousands of simulations of the expected return on a particular bet spread in the game of Blackjack also known as 21. Results show given the before mentioned bet spread and particular rules of the game a player can expect to win over $50 per 100 rounds of play. The results also show that the player can expect large swings in winning and loosing streaks, given the standard deviation of $132. Further study has shown that the penetration point of shoe also affects the results. As more cards are dealt the expected value of the game for and advantage player will increase.

References

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