A Comparison of the Bootstrap Particle Filter and the Extended Kalman Filter and the Effects of Non-linear Measurements in Spacecraft Attitude Determination Using Light Curves

Arun J. Bernard and David K. Geller

Utah State University

The attitude of a space object can be obtained from looking at the photometric light curves of the object. This approach has traditionally been employed by astronomers for asteroids, and has recently been developed for application to satellites. Estimation algorithms such as particle filters or Kalman filters are employed to acquire the attitude information from the light curves through a process called light curve inversion. The use of a bootstrap particle filter and an extended Kalman filter in light curve inversion are examined. In many instances the extended Kalman filter is able to match the performance of the bootstrap particle filter. However, there are some cases where the extended Kalman filter diverges. The reasons leading to this filter divergence are investigated.

I. Introduction

The attitude of a space object can be obtained from looking at the photometric light curves of the object. Attitude is the spatial orientation of an object. As an object rotates or changes its attitude, there can be a change in the apparent magnitude, or brightness, of the object as perceived by an observer. These magnitudes can be measured using an Electro-optical sensor, or a telescope. The time histories of these magnitude measurements are called light curves. In analyzing the light curve of an object, it is possible to determine its attitude.

Using light curves to determine the attitude of an object has traditionally been employed by astronomers for asteroids [1], and the adaptation of this approach to satellites has recently been a research subject of great interest [2]. The application to man-made, Earth-orbiting objects has proved difficult due to factors such as symmetry of spacecraft, non-linear dynamics and measurements, and the fact that the satellite may be controlling its own attitude [3].

Due to these challenges extracting spacecraft attitude information from the light curve measurements, or performing light curve inversion, has typically been realized using unscented Kalman filters (UKF), and particle filters (PF) [4]. The UKF is a state estimation technique that is well suited for estimating systems where the dynamics and the measurements are non-linear. The filter operates by propagating the state using a number of points from the state covariance matrix called sigma points, and the states are updated using a gain weighted by the available measurements. The particle filter is another estimation technique that is suitable for non-linear systems. Typically, the PF employs a more "Monte Carlo" type approach to state estimation. A distribution of particles is created and then propagated forward in time. Then the particles that are likely to match the truth are kept while the other particles are thrown away. Linares et. al. in [4] and Holzinger et. al. in [3] claim that a particle filter may be better suited to perform light curve inversion than a UKF as the severe non-linearities can become non-Gaussian.

However, despite the effectiveness of these different approaches, they are both computationally expensive, and it is desirable to determine if there is another approach that could yield similar results, but in a more computationally efficient way. This work focuses on two techniques for light curve inversion, a bootstrap particle filter (BPF) and an extended Kalman filter (EKF). The BPF is also known as the sample-importance re-sampling filter. This filter operates like other particle filters, but instead of throwing away all of the particles, it re-samples them so that there remains the same number of particles, but they are all distributed in the region of greatest likelihood.

The extended Kalman filter is another Kalman filter that has been developed for use with nonlinear systems. The EKF operates by linearizing the dynamics and the measurements about the current estimate and uses the linearized models to estimate the truth. However, in the case where the linearized models don’t accurately represent the non-linearized dynamics and measurements, the EKF may break down. This paper serves to compare the results obtained using a highly effective, computationally expensive algorithm to a more efficient linearizing estimation technique to determine if the EKF may perform sufficiently well as to provide usable information on the attitude of the satellite.

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*Graduate student, Mechanical and Aerospace Engineering, Utah State University, 4130 Old Main Hill Logan, UT 84322-4130
† Associate Professor, Mechanical and Aerospace Engineering, Utah State University, 4130 Old Main Hill Logan, UT 84322-4130
In addition, the goal of this research is to further work started by the authors in [5]. In that paper, the authors used a BPF and EKF to obtain attitude estimates of a given spacecraft. This work is expanded to include the following contributions: an examination of the cases where the particle filter is able to estimate the attitude of the spacecraft while the EKF diverges, and an examination of the measurements that are causing this issue.

II. Scope

The satellite shape model, attitude, orbit, BRDF formulation, and one of the observatory locations are the same as used in the baseline case of [3]. The novel work includes the expansion of that work to two observatories, implementing an EKF along with a BPF, and an analysis of the effects of the non-linearities on the attitude determination of the satellite. This work does not include shape or reflection parameter estimation as the shape model with its reflection properties is assumed to be perfectly known.

The model involves the simulation of a CubeSat placed into the orbit of the GEOdetic SATellite spacecraft. This is a low-Earth orbit (LEO). The initial orbital elements of this spacecraft at time of observation are $a = 7417$ km, $e = 1.06 \times 10^{-3}$, $i = 108$ deg, $\Omega = 177$ deg, $\omega = 80$ deg, and $M = 66$ deg. The spacecraft does not rotate, but maintains an inertially fixed attitude throughout the simulation. The principal observer location is the advanced Electro-optical System (AEOS) telescope in Maui, Hawaii (lat = 20.7081 deg, long. = $-156.257$ deg, and alt. = 3.058 km). The additional observatories are all placed in different locations relative to this principal observatory. For every case the measurements and dynamics are simulated as shown in the following sections.

III. Dynamics models

This section presents the dynamics governing the motion of the spacecraft that are being examined in this paper. As the spacecraft is non-rotating the only dynamics governing its motion are the orbital two-body orbital dynamics.

A. Orbital dynamics

The orbital motion of the spacecraft being examined in this paper is limited to two-body motion. The differential equations describing this motion are presented below.

$$\dot{\vec{r}} = \vec{v} \quad (1)$$

$$\ddot{\vec{r}} = -\frac{\mu}{|\vec{r}|^3} \vec{r} \quad (2)$$

In the above equations, $\dot{\vec{r}}$, $\vec{v}$, and $\mu$ are respectively the position and velocity of the spacecraft and the standard gravitational parameter of Earth.

The spacecraft is a non-rotating spacecraft and thus, the attitude doesn’t change with time. However, it is important to establish the attitude representation that is being employed. This paper uses a 3-2-1, or $Z\cdot Y\cdot X$, Euler angle attitude representation. In this paper, these Euler angles are represented by a vector, $\theta_{1\rightarrow\beta}$. The direction cosine matrix for the transformation from the inertial to the satellite body frame as presented in [6] is shown below.

$$A_{I\rightarrow\beta} =$$

$$\begin{bmatrix}
c\theta_2 c\theta_1 & c\theta_2 s\theta_1 & -s\theta_2 \\
-s\theta_3 s\theta_2 c\theta_1 - c\theta_3 s\theta_1 & s\theta_3 s\theta_2 s\theta_1 + c\theta_3 c\theta_1 & s\theta_3 c\theta_2 \\
c\theta_3 s\theta_2 c\theta_1 + s\theta_3 s\theta_1 & c\theta_3 s\theta_2 s\theta_1 - s\theta_3 c\theta_1 & c\theta_3 c\theta_2
\end{bmatrix} \quad (3)$$

In the above matrix, the $\theta$ values correspond to the components of the $\theta_{1\rightarrow\beta}$ vector and the $c$ and $s$ are the cosine and sine trigonometric functions.

IV. Measurement Model

The attitude of the spacecraft is being estimated by analyzing light curves which are the time history of the apparent brightness, or magnitude, of the object. The governing equation for developing light curves is the measurement of the apparent magnitude of the spacecraft:

$$M_A = -2.5 \log_{10} \left( \frac{I_A}{I_{sun}} \right) - 26.74 \quad (4)$$

Where $I_{sun}$ is the average illumination intensity of the sun at a given distance. The term, $I_A$ is the calculated photometric flux of the object at that time. The amount of flux that is being captured by a sensor is dependent upon the amount of light that is being reflected by the object. Thus, to be able to accurately determine the attitude of the spacecraft, it is necessary to implement an accurate shape and reflection model.

The approach used to represent the shape model in this paper is breaking up the shape into component facets under the assumption that the shape model is a convex shape. Each facet represents a face of the object. The different facets are defined using the area, $A$, and the normal vector $\vec{n}$ of the facet. Using this representation, it is then possible to find the reflected light from each of the facets and then sum them to obtain a value for the total reflection of the object.

The apparent magnitude measurement is dependent upon the reflection characteristics of the spacecraft which are a function of the location of the illumination source, the sun, the location of the observer, and the shape of the object. These vectors are illustrated in Figure[1] The other vectors shown in the figure are the facet normal vector, $\vec{n}$,
Fig. 1 Reflection geometry for a given facet of the spacecraft shape model

and the bisector between the observation vector and the sun vector, $\vec{h}$.

To aid in formulating the equations for the reflection model the satellite to sun, $\vec{s}$, and the satellite to observer, $\vec{v}_r$, vectors need to be expressed in the body frame of the spacecraft. This is done by transforming these vectors from the inertial frame.

$$\vec{v}_r^B = A_{I \rightarrow B} \vec{v}_r^I$$  \hspace{1cm} (5)

$$\vec{s}^B = A_{I \rightarrow B} \vec{s}^I$$  \hspace{1cm} (6)

Note that the superscripts on a vector refer to the frame in which a vector is expressed. The facets used to describe the shape model of the spacecraft can be represented by an area, $A_i$, and an orientation using the facet normal vector, $\hat{n}_i$.

It is assumed that the reflection of the light is equal over all wavelengths. Under this assumption, the photometric flux intensity becomes

$$I_A = \frac{1}{\vec{v}_r^B \vec{v}_r} \bar{\psi}_{sun} (\bar{\delta}) \sum_{i=1}^{N_f} A_{i,vis} \rho_i \left( \vec{v}_r^B, \vec{s}_B^B, p_i \right)$$  \hspace{1cm} (7)

where $p_i$ is used to represent the physical reflection properties of the given facet. The term $A_{i,vis}$ in the above equation is the projected area of each facet that is visible to the observer. It is given by

$$A_{i,vis} = A_i \langle \hat{n}_i \cdot \hat{v}_r \rangle$$  \hspace{1cm} (8)

where $\langle \cdot \rangle$ is the nonnegative operator defined as

$$\langle x \rangle = \begin{cases} 
  x & x \geq 0 \\
  0 & x < 0
\end{cases}$$  \hspace{1cm} (9)

Thus, when the different facets are properly aligned with the observer vector, the reflection from the facet will be viewed. The following section discusses the method to quantifying the reflection of the different facets.

A. Bi-directional reflectance distribution function

A bi-directional reflectance distribution function (BRDF) is a model that is used to describe the reflection from a surface given the direction of the impinging light and the direction of the observer viewing the reflected light. There are two main types of BRDF formulations, those based on empirical data, and those based on the physical properties of the reflecting object. This paper utilizes the BRDF model introduced by Cook et. al. [7]. This is a BRDF that is based on the physical properties of the reflecting object. Out of BRDF models utilized by the aerospace community, this one was shown to best match actual light curve data Ceniceros et al. [8]. Equations for the formulation of this reflectance function are presented below.

The reflectance of an object is found using a weighted combination of specular and diffuse reflection terms. Diffuse being the light that is scattered due to a rough surface, and the specular contribution being the mirror-like reflection component.

$$\rho_i = \xi_i R_d + (1 - \xi_i) R_s$$  \hspace{1cm} (10)

In this equation, $\xi$ is a weight that determines how much of the reflection is due to the specular component, and how much is due to the diffuse reflection component. The diffuse reflection term is a Lambertian model. This means that the diffusive light term reflects equally in all directions. This diffuse reflection term is found using

$$R_d = \frac{a_i (\hat{n}_i \cdot \hat{v}_r)}{\pi}$$  \hspace{1cm} (11)

where $a_i$ is the diffuse albedo term. The specular reflection is found using the following equations.

$$R_s = \frac{F \, D}{4 \, (\hat{n}_i \cdot \hat{s}_B^B) \, (\hat{n}_i \cdot \hat{v}_r)}$$  \hspace{1cm} (12)

Where $F$ is the value of the Fresnel equation found using Equations 13–16.
The geometric attenuation factor vector $\vec{F}$ to represent the actual state that is being estimated. The distribution of particles that are weighted with the measurements are presented in the following sections.

V. Estimation Algorithms

Extracting the attitude information from the light curves is accomplished through a process called light curve inversion. Two estimation techniques are employed in this paper to perform this light curve inversion: a bootstrap particle filter (BPF), and an extended Kalman filter. The operation and formulation of these different estimation methods is are presented in the following sections.

A. Bootstrap particle filter

A particle filter is an estimation method that is used to recursively estimate the state variables by creating a distribution of particles that are weighted with the measurements to represent the actual state that is being estimated. The Bootstrap particle filter (BPF) was developed by Gordon et. al. in [9]. It is also known as the sample importance re-sample filter. This filter first samples the given distribution to place the particles such that they represent the current probability density function. Then the particles are propagated forward in time, and when a measurement is available the filter assigns weights and determines the particles of highest likelihood. Then after this, it uses what is called a re-sampling algorithm. This involves the reassigning of particle values based on those particles that have the largest weights so that the same number of particles is kept at each update phase of the filter.

The BPF is especially suited to nonlinear state estimation problems. In [3] Holzinger et. al. suggest that the BPF is very well suited for the light curve inversion problem as the measurements and dynamics are exceedingly nonlinear and non-Gaussian in nature.

This following paragraphs outline the formulation of the BPF for use in the light curve inversion problem. The majority of the algorithm is taken as presented in [3] with the re-sampling algorithm implemented as shown in [9].

The state that is being estimated is the vector containing the $3 - 2 - 1$ Euler angles.

$$x = [\theta_{1-2-3}]$$

The spacecraft is non-rotating so the state is propagated as

$$x_{k+1} = x_k$$

where the subscript $k$ denotes the current step. This model also assumes that there is no process noise. The measurement model is

$$z_k = h_k(x_k, k) + w_k = Mx_k + w_k$$

The first step to using the BPF is creating a distribution of particles about the initial estimate of the state, $\hat{x}_0$, using the initial state covariance. This is done by sampling $x^i_0$ from the distribution $N(\hat{x}_0, \sigma^2_0)$ where $\sigma_0$ is the initial 1σ uncertainty in $\hat{x}_0$. As mentioned previously, $k$ refers to the time step of the filter, and the superscript $i$ in this case refers to the $i^{th}$ particle. When a measurement is available, the residual of the actual measurement subtracting the measurement based on the current value of $\hat{x}$ is found and used to weight the different particles as shown in the equation below.

$$w^i_k = z_k - h_k(x^i_k, k)$$

Using this weight based off the measurement residual the likelihood of this measurement matching the truth
state is found using the probability density function of the measurement given the state \( p\left(z_k|\xi_k^i\right) \) using
\[
p\left(z_k|\xi_k^i\right) = e^{-\frac{1}{2}w_k^iR_k^{-1}w_k^i}
\]
where \( R_k \) is the covariance matrix of the measurement. The above equation creates weights \( w_k^i \), which are then normalized using
\[
W_k^i = \frac{w_k^i}{\sum_{i=1}^{N} w_k^i}
\]
where \( N \) is the number of particles. With these normalized weights and the particle states, the particles can be re-sampled following the method presented in [9].

B. Extended Kalman filter

The extended Kalman filter (EKF) is an estimation technique that is widely used for nonlinear state estimation. The EKF operates by linearizing the state dynamics and measurements about the current estimate. Then using measurements that are weighted based on their accuracy, a gain is calculated that updates the state and covariance to better match the truth. The EKF is less expensive, computationally, than the particle filter, but the filter may cease to function when the linear dynamics and measurements approximations don’t closely match the nonlinear dynamics and measurements. One key purpose of this work is to determine under what situations the EKF might be able to match the performance of the BPF.

The state dynamics are
\[
\dot{x} = Fx
\]
where as with the particle filter
\[
x = [\theta_l \rightarrow \beta]
\]
and as the spacecraft is not rotating at all, but maintaining an inertially fixed attitude
\[
F = [0]_{3 \times 3}
\]
The state transition matrix for the system is
\[
\Phi = I_{3 \times 3}
\]
The initial covariance matrix is
\[
P = \begin{bmatrix}
\sigma_{\theta}^2 & 0 & 0 \\
0 & \sigma_{\theta}^2 & 0 \\
0 & 0 & \sigma_{\theta}^2
\end{bmatrix}
\]
where \( \theta \) is the 1\( \sigma \) uncertainty of the initial state estimate. There are two parts to the Kalman filter, the propagation and the update stages. These are discussed in detail in the following sections.

1. Propagation

This section discusses the propagation of the state estimate forward from time \( t_k \) to time \( t_{k+1} \). As the RSO is not rotating, this is
\[
\hat{x}_{k+1} = \hat{x}_k
\]
Here the decoration \( \hat{\cdot} \) refers to the best estimate at time \( t_k \). The state covariance matrix is propagated using
\[
P_{k+1} = \Phi_k P_k \Phi_k^T
\]
For this non-rotating case this reduces to being
\[
P_{k+1} = P_k
\]
The state is propagated at every time step and then updated when a measurement is available.

2. Update

When a measurement, \( z_k \) is available at time \( t_k \) the state estimate is updated using the following equations.

First the predicted measurement for the current best estimate of the state is predicted
\[
\hat{z}_k = h_k (\hat{x}_k, t_k)
\]
Then the measurement partial vector is found
\[
H = \left( \frac{\partial h}{\partial x} \right)_{\hat{x}_k} = \begin{bmatrix} \frac{\partial M_k}{\partial \theta_l \rightarrow \beta} \end{bmatrix}_{1 \times 3}
\]
For this work of this paper these partial derivatives were computed numerically using a central differencing scheme.

With the measurement partial vector, the residual covariance matrix can be computed
\[
\mathbf{g}_k = H_k P_k H_k^T + R_k
\]
As with the particle filter, \( R_k \) is the covariance matrix of the measurement (\( R_k \)). Next, the Kalman gain is calculated
\[
K_k = P_k H_k^T \mathbf{g}_k^{-1}
\]
With the Kalman gain, it is the state and covariance estimates can be updated.
\[
\hat{x}^+ = \hat{x} - K_k (z_k - \hat{z}_k)
\]
\[
P^+ = (I - K_k H_k) P_k (I - K_k H_k)^T + K_k R_k K_k^T
\]

VI. Results

This section presents the results obtained in performing attitude estimation on a non-rotating spacecraft with the shape model and reflection parameters perfectly known. These parameters are presented in Table 1 below.
Table 1  Shape and reflection model parameters of the spacecraft

<table>
<thead>
<tr>
<th>Face</th>
<th>A(m)</th>
<th>ξ</th>
<th>a</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Z</td>
<td>0.01</td>
<td>0.5</td>
<td>0.1</td>
<td>0.15</td>
</tr>
<tr>
<td>+Y</td>
<td>0.01</td>
<td>0.5</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>+X</td>
<td>0.01</td>
<td>0.5</td>
<td>0.40</td>
<td>0.15</td>
</tr>
<tr>
<td>−X</td>
<td>0.01</td>
<td>0.5</td>
<td>0.60</td>
<td>0.15</td>
</tr>
<tr>
<td>−Y</td>
<td>0.01</td>
<td>0.5</td>
<td>0.70</td>
<td>0.15</td>
</tr>
<tr>
<td>−Z</td>
<td>0.01</td>
<td>0.5</td>
<td>0.95</td>
<td>0.15</td>
</tr>
</tbody>
</table>

All observations have a common epoch of December 17, 2009 at 4:47:15 universal time. Unless otherwise specified all the results are based off of observations simulated for 350 seconds with measurements taken every 5 seconds. The measurement noise for the apparent magnitude measurements was set 0.3 1σ mag.

The initial attitude of the spacecraft is $\theta_1 = 247.8$ deg, $\theta_2 = 0$ deg, and $\theta_3 = 133.1$ deg. All of the other spacecraft parameters are presented in section [2] The initial uncertainty in the attitude estimate is 3 deg 1σ. For all results involving the particle filter, 5000 particles were used.

Figures 2 and 3 are adapted from the work of [5]. Figure 2 presents the BPF and EKF state estimate error and 3σ bounds for a single observatory. As can be seen from the figure, the covariance doesn’t really decrease over time. It is almost as large at the end of the observations as it is as the start. However, it can be seen that for this case, the EKF very closely matches the BPF results.

Figure 3 again presents the BPF and EKF state estimate error and 3σ bounds, but this time using two different observatories. The first observatory was located at AEOS ($lat = 20.7081$ deg, $long. = -156.257$ deg, and $alt. = 3.058$ km), and the second observatory was located at $lat = 23$ deg, $long. = -150$ deg, and $alt. = 3.058$ km. In this figure, it can be seen that the attitude error and 3σ bounds are better with the two observatories. Again, the EKF results closely match the BPF results.

In [5] it was shown that different results could be obtained using different locations of the second observatory site. To better understand how the location of the observatory affects the accuracy of the attitude estimate a number of trials were run and contour plots were made showing the final attitude estimation error as a function of location. Figures 4 and 5 show the contours for a single observatory and using a second observatory respectively. For the two-site contours the first observatory was located at AEOS, and the other observatory was moved to different locations. In these figures, the red line shows the ground track path of the satellite as it passes.

The yellow "hot spots" on these contour figures are flags that correspond to cases where the estimation error at the final time exceeded the 3σ bounds from the covariance matrix. As can be seen from the figures, the single-site case doesn’t exhibit very many of these "hot spots" while the two-site case exhibits a locus of spots where the EKF error exceeds the 3σ bounds. As the covariance from the single observatory does not shrink as much as with the two observation sites, it is more likely that there will be less flags than the two-site case. This is consistent with what is shown in the figures.

These hot spots were furthered studied to understand under what conditions they occur. Figure 6 shows the BPF (left) and EKF (right) results of the two-site case when a hot spot occurred when the second observatory was at $lat = 20$ deg, $long. = -150$ deg, and $alt. = 3.058$ km. Here there is a very noticeable difference between the performance of the BPF and the EKF. In this case, the estimation state error of the EKF is much larger than what was previously seen. The errors in the BPF can be attributed to the impoverishment problem that occurs when too few particles are used. However, despite this, the BPF estimation error is still much lower than that of the EKF.

As mentioned in section V.B the EKF doesn’t perform well when the linearized dynamics or measurement models cease to accurately approximate the nonlinear models. As the attitude dynamics are constant for this work, the measurements were examined to see if the nonlinearities in the measurements were related to the EKF divergence.

Figure 7 shows the magnitude measurements of both observatories for different values of the angles, $\theta_1 \rightarrow \theta_3$, at three different times. The AEOS observatory measurements are on the left of the figures, and the second observatory measurements are on the right. These figures were made by holding two of the angles at their nominal values and then varying the labeled angle over ±9 deg. As can be seen from looking at the figures 7a and 7b the measurements are fairly linear and could be approximated using a linearized model. However Fig. 7c shows that as the time continued forward the measurements became exceedingly nonlinear. This may be the factor that causes the EKF to diverge.

In investigating why these measurements were becoming nonlinear, each of the terms of the EKF formulation...
were evaluated and it was found that the measurement partial/geometry vector was increasing at the time where the filter started to diverge. This implies that at that time, the magnitude measurements are very sensitive to changes in attitude. Figure 8 shows the change in the measurement partial vector over time for the first observatory (Fig. 8a), and the second observatory at the location of the hot spot (Fig. 8b).

In investigating why this measurement is so sensitive at these times, the dot product of the shape’s facet unit normal vectors, \( \hat{n}_i \) with the observer vector, \( \hat{v}_r \) was plotted over time in Fig. 9. As can be seen from the figure, at the time where the filter starts to diverge, the facet pointing in the \(+Z\) direction of the body frame is aligned directly with the vector to observatory 2. Furthermore, the \(+Z\) facet has the smallest weight for diffuse reflection (see Table 1). The locus of hot spots in Fig 5 generally correspond to locations where one of the spacecraft’s facets aligns with the observer vector. Thus, there may be a correlation between this phenomenon and the divergence of the EKF.

VII. Conclusion

The attitude of a spacecraft can be determined by processing the change with time of apparent magnitude measurements or light curves. Performing this estimation is accomplished using the process called light curve inversion. In this paper, the ability of two methods, the bootstrap particle filter, and the extended Kalman filter were examined. Some of the key assumptions and simplifications of this work include a non-rotating spacecraft, and equal light reflection over all wavelengths. Under these constraints it was found that in many cases the EKF was able to match the BPF for attitude estimation with the exception of a few cases where the EKF diverged due to the severe non-linearities present in the attitude measurements. These non-linearities may be related to the fact that during the observation period one of the spacecraft facets directly aligned with the observer vector. However, all the cases examined shared the \(+Z\) facet being the one that aligned with the observer vector. Further work is required to determine if this behavior will also be exhibited when other spacecraft facets are the ones that align with the observer vector.

Other future work includes analyzing the measurements and dynamics for a rotating spacecraft. It is possible that further non-linearities resulting from the rotational dynamics may render the EKF not suitable for this application. In addition, an analysis similar to that presented in this paper could also be performed for three observatories viewing the spacecraft.

References

Fig. 3  Particle filter and Kalman filter attitude estimation from two observation sites


Fig. 4  Single site contour plots
Fig. 5  Two-site error contour plots: only position of second observatory changed
Fig. 6  Particle filter and Kalman filter attitude estimation from two observation sites with second observatory located at hot spot
Fig. 7  Magnitude vs $\theta$ at three different times
Fig. 8  Measurement partial vectors vs time for both observatories

Fig. 9  Projection of shape facet normals into observatory vector