Oct 27th, 12:00 AM

Designing Smooth Mixed-Geometry Canal Transition

D. C. Froehlich
Cary, dcfroehlich@aol.com

Recommended Citation
Froehlich D.C. (2022). "Designing Smooth Mixed-Geometry Canal Transition" in "9th IAHR International Symposium on Hydraulic Structures (9th ISHS)". *Proceedings of the 9th IAHR International Symposium on Hydraulic Structures – 9th ISHS*, 24-27 October 2022, IIT Roorkee, Roorkee, India. Palermo, Ahmad, Crookston, and Erpicum Editors. Utah State University, Logan, Utah, USA, 9 pages (DOI: 10.26077/3051-fc31) (ISBN 978-1-958416-07-5).

9th International Symposium on Hydraulic Structures Roorkee, India 24-27 October 2022
ISBN: 978-1-958416-07-5 DOI: https://doi.org/10.26077/3051-fc31

Designing Smooth Mixed-Geometry Canal Transition

D.C. Froehlich[1]

[1]*Consulting Engineer, Cary, North Carolina, USA*

E-mail: dcfroehlich@aol.com

Abstract: *When a canal's size or shape changes, usually over a short distance, a section of the channel, known as a transition structure, is needed to connect the waterway's two stretches. Fifth-degree parametric equations are developed to calculate the cross-section dimensions and bed centerline elevations (thus, the geometric surface coordinates) between the two ends of a warped transition structure in a water-supply canal. The parametric modeling approach provides a smooth representation of the mixed geometry that results from terminal sections having vastly different shapes. A generalized cross-section defined by four parameters enables a straightforward model of various forms ranging from trapezoids to semi-circles. This approach significantly simplifies the interpolation of surface coordinates between the terminal points of a transition structure. It also maintains a smoothness that helps avoid undesirable consequences of channel contractions and expansions. An example is presented that applies the parametric modeling approach to design a significant canal transition where the cross-section changes from a standard trapezoidal shape with rounded bottom vertices to a rectangular section in a steeper aqueduct that carries the flow across a broad valley.*

Keywords: *Canal, water supply, transition, parametrically smooth, mixed-geometry*

1. Introduction

An artificial canal supplying water for irrigation or public consumption may wind hundreds of kilometers from its source to its destination. Along the way, the flow rate delivered by the waterway usually decreases because of diversions, evaporation, and seepage losses, which reduces the size of the channel cross-section needed to convey the flow. The longitudinal slope might also vary, increasing or decreasing, accompanied by a change in cross-section shape. The channel cross-section also changes at flumes, siphons, and aqueducts, where contractions and expansions of the flow occur (Aisenbrey et al. 1978).

When the size or shape of a canal changes, usually over a short distance, a section of the channel, known as a transition structure, is needed to connect the waterway's two stretches. The sidewalls' geometrical shape (for example, conical frustum) and the physical appearance of the sidewalls (flared, splayed, straight, or streamlined) characterize the transition type. The most common types of transitions connecting channels having trapezoidal and rectangular cross-sections are the cylindrical quadrant, wedge, and semi-elliptical (Laycock 2007, page 154), which is like the so-called "warped" or twisted structure proposed by Hinds (1928) nearly a century ago. In Hinds' warped transition, a surface is generated between the end (terminal) sections (which usually have flat walls either sloped or vertical) by a straight line moving so that no two consecutive positions are in the same plane.

In comparatively small concrete-lined canals where energy losses at transitions are tolerable, the special formwork and skilled labor required to build elaborate structures are usually not justified economically. The most straightforward construction is suitable (Simmons 1964, page 1). However, for large canals on gentle slopes where energy losses at transitions must be kept small, the design typically tries to provide a gradual changeover that avoids turbulence caused by flow contractions and expansions and keeps the generation of surface waves to an acceptable level. Scobey (1933) notes that a cylindrical quadrant transition works well for contractions with 2.4 m/s or less velocities. However, when the velocity-head changes by 0.6 m or more, he finds that a warped structure is best. Ippen (1949) found that warped transitions produce a minor energy loss in expansions.

The procedure developed by Hinds (1928) for designing a warped transition relies on the hypothesis that the longitudinal water-surface profile consists of two reverse parabolas tangent to each other at the transition mid-length and merge tangentially with the profiles at the entry and exit sections. It also assumes a constant value of the energy head-loss coefficient for the entire length of the transition and a dimensioning of the intermediate cross-section that is subjective to a large extent. Detailed explanations of the strategy are given by Chow (1959, page 319), Vittal and Chiranjeevi (1983), and Mazumder (2020, pages 82-85 and 157-167). However, contemporary methods for evaluating canal transition hydraulic performance use computational techniques more advanced than those devised by Hinds (1928). These approaches involve the numerical solution of differential equations that describe the canal flow in one, two, or three dimensions. For this reason, the technique developed by Hinds (1928) for computing water-surface profiles through warped transitions now has little practical relevance.

In this investigation, the focus is not on the hydraulic calculations that go into designing a canal transition but on creating a layout that mirrors Hinds' (1928) idea of connecting the terminal sections with a warped structure. Without abrupt changes in geometry, such a shape keeps energy losses small for contracting and expanding flows. A parametric modeling procedure produces a smooth, gradually-varying warped transition of mixed geometry. The coordinates of the intermediate cross-sections are calculated by mathematical expressions that employ specified coefficients to create a three-dimensional representation or parameterization of the structure surfaces.

A fifth-degree polynomial equation is applied that provides a high measure of smoothness of the transition surfaces between the end sections. A similar fifth-degree polynomial equation interpolates the longitudinal variation of the channel centerline bed elevation. This approach dramatically simplifies the cross-section layout for warped transitions connecting channels with a wide range of conventional shapes. Although this investigation focuses on open channel transitions in water-supply canals, the same principle exists for waterways used for other purposes, such as diversion structures at dams (Hager et al. 2020, Chapter 7) and hydroelectric plant intake structures (Gemperline and Crane 1995).

2. Generalized Canal Cross-Section

Continuous excavating, trimming, and lining equipment is the most economical method for building large canals lined with erosion-resistant material. Widespread acceptance and use of canal cross-sections that conform to standard trapezoidal shapes enable manufacturers to build excavating and lining equipment more efficiently and lower replacement parts and service costs. For these reasons, the Bureau of Indian Standards (BIS 2004) recommends model cross-sections that are trapezoidal with rounded bottom vertices, emphasizing those lined with concrete. The BIS standard cross-sections set the bottom vertices' radii to the full-supply-depth (that is, the typical channel depth at the design discharge) for comparatively shallow excavation and one-half of the supply depth where deep cuts are needed.

In this analysis, the development of smooth, mixed geometry transitions is based on a generalization of the BIS standard cross-section shapes introduced by the writer (Froehlich 2008). The cross-section is defined by four parameters (see Fig. 1): b = the horizontal bottom width, h = the full-supply depth, m = the side-slope ratio (horizontal to vertical), and $\kappa = r \div h$, where r = the radius-of-curvature of the bottom vertices and $0 \le \kappa \le 1$. Cross-sections of various shapes ranging from trapezoids to semi-circles can be formed with different combinations of the four parameters, as illustrated in Table 1. With $b > 0$, setting κ to 1 and 0.5 gives the two BIS standard canal cross-sections. With $\kappa = 0$, a trapezoid with sharp bottom vertices is produced.

The flow area A for the generalized cross-section is calculated as

$$A = \left(b + \alpha h\right) h, \tag{1}$$

the wetted perimeter P (that is, the boundary length of the section that is in contact with the flow) is given by

$$P = b + \beta h, \tag{2}$$

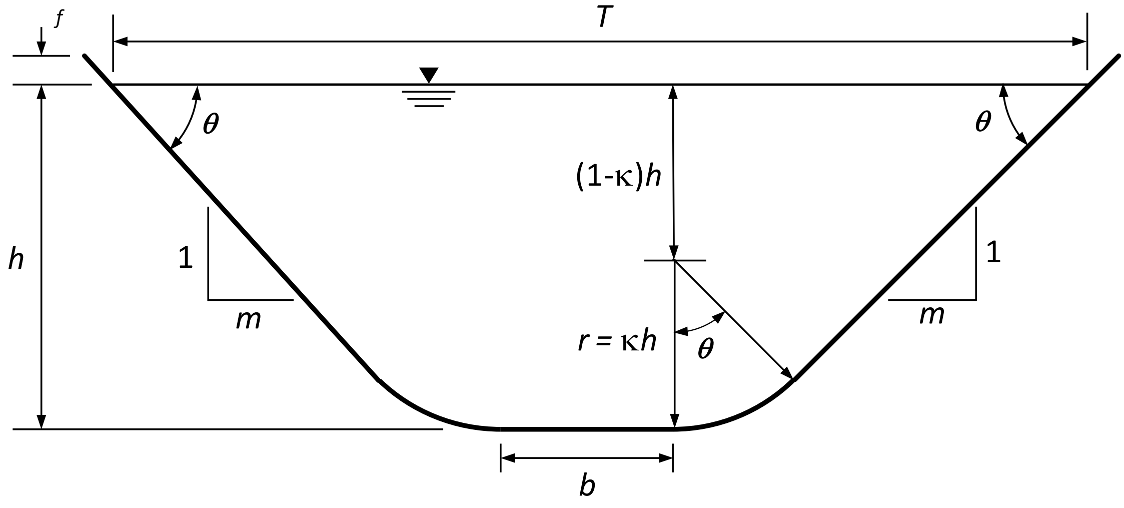
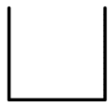
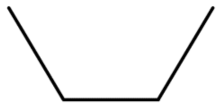
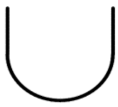


Figure 1. Generalized trapezoidal cross-section.

Table 1. Generalized cross-section shapes for various values of b, m, and κ.

Bottom width	Corner curvature	Side Slope (Horizontal/Vertical) $m = 0$	Side Slope (Horizontal/Vertical) $m = 1/\sqrt{3}$
$b > 0$	$\kappa = 0$		
	$\kappa = 1/2$		
	$\kappa = 1$		
$b = 0$	$\kappa = 0$	--[a]	
	$\kappa = 1/2$		
	$\kappa = 1$		

[a]Rectangular cross-section with $b = 0$ is not possible.

and the section topwidth T is found as

$$T = b + \delta h, \tag{3}$$

where the coefficients α, β, and δ are calculated as follows:

$$\alpha(\kappa, m) = \left[1 - 2\kappa(1-\kappa)\right]m + 2\kappa(1-\kappa)\sqrt{1+m^2} + \kappa^2 \arctan\frac{1}{m}, \tag{4}$$

$$\beta(\kappa, m) = 2\left[\kappa m + (1-\kappa)\sqrt{1+m^2} + \kappa \arctan\frac{1}{m}\right], \tag{5}$$

$$\delta(\kappa, m) = 2\left[(1-\kappa)m + \kappa\sqrt{1+m^2}\right]. \tag{6}$$

When $\kappa = 0$, the expressions for A, P, and T reduce to those for a trapezoidal section with sharp bottom vertices if $b > 0$, or a triangular section if $b = 0$.

Freeboard is an extra height (f, shown in Fig. 1) added to the full-supply depth to set the top elevation of the channel's lining. The freeboard aims to accommodate waves, a flood surcharge, or a flow surge caused by a faulty operation. It is also a safety margin that can absorb the effects of low construction tolerances or inaccurate estimates of the channel lining surface roughness or longitudinal bed slope, factors affecting the calculated full-supply depth (Liria Montañés 2006, pages 164-170).

The best hydraulic section provides the maximum flow-carrying capacity for a fixed area or the minimum cross-sectional area and perimeter to pass a given discharge. As it is a minimum area and minimum perimeter section, it provides the maximum hydraulic radius and, therefore, requires the least excavation and lining costs. The most efficient cross-section shape for carrying open-channel flows is a semi-circle and a regular half-hexagon when a sharp-cornered trapezoid (Froehlich 1994). A shape's hydraulic efficiency can be judged by comparing its non-dimensional hydraulic radius $R_* = R/\sqrt{A} = (A/P)/\sqrt{A} = \sqrt{A}/P$ to the value for by a semi-circle ($1/\sqrt{2\pi}$). For the generalized cross-section given by Eqs. *(1)* and *(2)*, $R_* = \sqrt{(b+\alpha h)h}/(b+\beta h)$.

3. Curve and Surface Continuity and Smoothness

Transition structures must efficiently connect the terminal cross-sections with continuous wall and bed surfaces to achieve the goals of small energy loss production and minimal generation of surface waves. Continuity is associated with the smoothness of the geometry and describes how two items come together. These items may be two curves that meet in some way or two portions of the same curve. The function derivatives define the concept of the smoothness of curves. A straight curve (a line) has a zero second derivative, and a nonlinear curve with a sharp kink has a high second derivative at the abrupt bend.

This analysis evaluates the smoothness of transition surfaces using a parametric equation that interpolates between the generalized terminal cross-sections, whose shapes are defined by the parameters b, h, m, and κ in Eqs. *(1)* through (6). Parametric continuity is determined by the n-th derivative of the defining expression, so-called C^n continuity. If a curve or surface is continuous at the n-th derivative, it is said to have an n-th degree of continuity (or degree of continuity n). A function n times continuously differentiable is n-th order parametric continuous, with smoothness increasing with n (Veltkamp 1992). A curve is said to possess a particular degree of continuity when the continuity is at least that degree for all points on the curve's interior. The same holds for surfaces.

C^1 continuity means the parametric equation associated with the curve or surface exhibits first derivative continuity. It follows that C^1 continuity implies C^0 continuity; if a curve or surface is C^1, it is also C^0 continuous. Similarly, C^2 continuity implies C^1 continuity. More precisely, C^0 continuity means that a curve or surface is continuous (that is, it has no gaps) but may exhibit kinks or sharp bends. C^1 continuity means that the parametric equation associated with the curve possesses first derivative continuity – in addition to C^0 continuity – so that the tangents are identical. Likewise, C^2 continuity implies parametric second derivative continuity and being C^1 continuous. The curvature is said to be continuous if its entities are C^2.

While it may be evident that a curve would require C^1 continuity to appear smooth, higher geometric continuity levels are required for pleasing aesthetics (Barnhill 1985, Boehm 1988). For example, a reflective surface, such as an automobile's body, does not appear smooth unless it has C^2 continuity. C^2 means that the curvature is continuous across two connected curves. Again, all previous conditions must be fulfilled for C^2 to be possible. Although the reflective property of light from the surface of a concrete-lined transition is not a concern, the reflection of surface waves from its sidewalls is worrying. To avoid the generation of cross-waves and runup along the sidewalls, enforcing C^2 continuity is a prudent design condition.

4. Transition Parameterization

Parametric modeling, also known as constraint modeling, automatically merges two structures into a single unit governed by the union of all the constraints in a coordinated way. A parametric equation defines a group of entities as functions of one or more independent variables called parameters. Parametric equations are commonly used to express the coordinates of the points that make up a geometric object, such as a curve or surface. The equations are collectively called a parametric representation or parameterization of the object. In this analysis, the item analyzed is a canal transition surface formed from the cross-section properties and the longitudinal bed profile as they vary between the ends of the structure.

4.1. Cross Sections

Considering a transition of length L, constraints are imposed at the terminal sections on the values of the generalized cross-section factors b, h, m, and κ along with their first and second derivatives with respect to channel distance x (as measured along the channel centerline.) A general profile shape function $F(\xi)$, where $\xi = x/L$ is defined so that

$$F(0) = F'(0) = F''(0) = F'(1) = F''(1) = 0 \text{ and } F(1) = 1, \tag{7}$$

where $\xi = 0$ is at the upstream end of the transition and $\xi = 1$ at the downstream end. The following fifth-degree parametric polynomial equation meets these requirements, as can easily be verified:

$$F(\xi) = 10\xi^3 - 15\xi^4 + 6\xi^5;\ 0 \le \xi \le 1. \tag{8}$$

The function is fifth-order parametric continuous. Moreover, because the function is C^2 continuous, if $F''(\xi_0) = 0$ and $F'''(\xi_0) \ne 0$ for some ξ_0 – that is, if $F''(\xi) < 0$ on one side of ξ_0 and $F''(\xi) > 0$ on the other side), F has an inflection point at ξ_0. At an inflection point, the curvature is continuous, and the surface smoothness is assured. Because these conditions are satisfied at $\xi = 0$ and $\xi = 1$, there exist inflection points at the terminal cross-sections. Consequently, the transition surface curvature is in line with the upstream and downstream channels, which is a critical property because any discontinuities in curvature at the end sections, or if $F''(0) \ne 0$ or $F''(1) \ne 0$, can lead to flow separation and sudden steep increases in the local fluid pressure field.

On the plane $\xi = 0$ (that is, the upstream end of the transition), the four cross-section parameters are b_0, h_0, κ_0, and m_0, and on the plane $\xi = 1$ (the downstream end), they are b_L, h_L, κ_L, and m_L. The cross-section parameters between the terminal sections $(0 \le \xi \le 1)$ interpolated with the same properties attributed to $F(\xi)$ are found as follows:

$$\begin{aligned} b(\xi) &= (b_L - b_0)F(\xi) + b_0 \\ h(\xi) &= (h_L - h_0)F(\xi) + h_0 \\ \kappa(\xi) &= (\kappa_L - \kappa_0)F(\xi) + \kappa_0 \\ m(\xi) &= (m_L - m_0)F(\xi) + m_0 \end{aligned} \tag{9}$$

The parametric equations give points on the warped transition surface between the two end sections of the structure. Transition surface schematics in Table 2 illustrate three combinations of entrance and exit sections.

4.2. Bed Centerline Profile

The centerline bed profile of a lined canal is usually composed of stretches with constant slopes connected by vertical curves. Consequently, the transition centerline bed elevation (z_b) is parameterized slightly differently than cross-sections to allow non-zero values of the longitudinal bed slope to be specified at the terminal sections. The following fifth-degree polynomial is used to interpolate z_b between the entrance and exit sections at $\xi = 0$ and 1:

$$z_b(\xi) = z_{b0} + z'_{b0}\xi + (10k_1 - 4k_2)\xi^3 - (15k_1 - 7k_2)\xi^4 + (6k_1 - 3k_2)\xi^5 \tag{10}$$

where $z'_b = dz_b/d\xi = L \times dz_b/dx$, and the coefficients k_1 and k_2 are

$$k_1 = z_{bL} - z_{b0} - z'_{b0} \quad \text{and} \quad k_2 = z'_{bL} - z'_{b0} \tag{11}$$

The conditions $z_b(0) = z_{b0}$, $z_b(1) = z_{bL}$, $z'_b(0) = z'_{b0}$, $z'_b(1) = z'_{bf}$, and $z''_b(0) = z''_b(1) = 0$ as a function of ξ are satisfied and are easily verified.

Sharp concave (that is, the tangent line to the longitudinal profile lies below the bed in the vicinity of a point) and convex (the tangent line lies above the bed) vertical curves create undesirable conditions usually avoided in a water-supply canal. Concave curves should have an amply long radius-of-curvature to control the dynamic pressures exerted on the floor by the centrifugal force resulting from the flow direction change. Convex curves should be flat enough to maintain positive pressures and prevent the flow's tendency to separate or spring free from the channel's floor, as described by the U.S. Bureau of Reclamation (USBR 1987, page 384). The radius-of-curvature of the bed centerline profile is

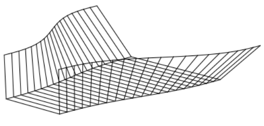
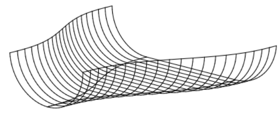
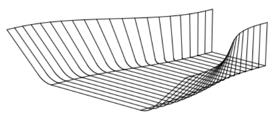
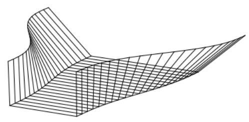
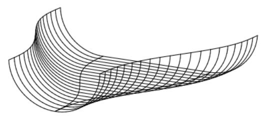
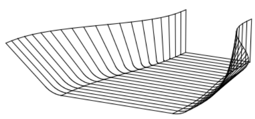
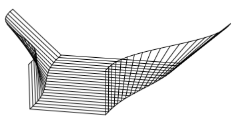
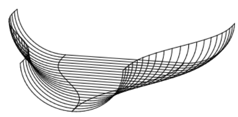
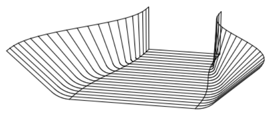
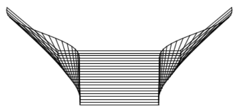
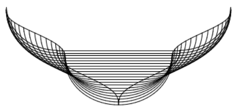
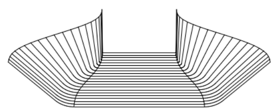
$$r_c = \frac{\left[1 + (dz_b/dx)^2\right]^{3/2}}{d^2 z_b/dx^2} \tag{12}$$

where from Eq. *(10)*,

$$\frac{dz_b}{dx} = \frac{dz_b}{d\xi}\frac{d\xi}{dx} = \left[z'_{b0} + 3(10k_1 - 4k_2)\xi^2 - 4(15k_1 - 7k_2)\xi^3 + 5(6k_1 - 3k_2)\xi^4\right]\frac{1}{L} \tag{13}$$

and

Table 2. Transition surface schematics for three combinations of entrance and exit sections.

Rotation angle about the vertical axis (deg)	Terminal Section Parameters		
	$b_0 = 20$ m, $m_0 = 1, \kappa_0 = 0$ $b_L = 15$ m, $m_L = 0, \kappa_L = 0$	$b_0 = 20$ m, $m_0 = 0, \kappa_0 = 1$ $b_L = 0$ m, $m_L = 0, \kappa_L = 1$	$b_0 = 15$ m, $m_0 = 0, \kappa_0 = 0$ $b_L = 20$ m, $m_L = 1, \kappa_L = 1/2$
45			
30			
15			
0			

[a]All views have an elevation angle (that is, the angle made with a plane normal to the vertical axis) of 30 degrees. The azimuth angle defines the horizontal rotation of the view about the vertical axis.

$$\frac{d^2 z_b}{dx^2} = \frac{d^2 z_b}{d\xi^2}\frac{d^2\xi}{dx^2} = \left[6\left(10k_1 - 4k_2\right)\xi - 12\left(15k_1 - 7k_2\right)\xi^2 + 20\left(6k_1 - 3k_2\right)\xi^3\right]\frac{1}{L^2} \qquad (14)$$

A positive value of r_c indicates a concave vertical curve, and a negative value one that is convex. Concave profiles present no difficulty provided the curvature radius is not smaller than $10 \times y_1$, where y_1 = the flow depth at the start of the curve. However, a larger radius is better.

5. Example Application

The design of a smooth transition connecting a concrete-lined water-supply canal on a gentle slope to an aqueduct with a steeper incline that crosses a broad valley illustrates the geometric surfaces' parametric representation. Only development of the geometric layout is presented. Procedures for computing water-surface profiles in contracting and expanding transitions can be found elsewhere (see, for example, Swamee and Chahar 2015). Standard software that numerically solves differential equations that describe flow in open channels (in one, two, or three dimensions) can be used for such calculations.

The canal cross-section is typical of large waterways in India (BIS 2004), with rounded bottom vertices having a curvature radius equal to the full-supply water depth $h_0 = 4.5$ m with a design discharge $Q = 100$ m^3/s. The canal has a gentle longitudinal slope $dz_{b0}/dx = -1/15500$ m/m, a horizontal bottom width $b_0 = 14.6$ m, a side-slope ratio $m_0 = 2$ (horizontal to vertical), and a freeboard $f = 0.75$ m. The entrance section wall height is then $h_{w0} = 4.5 + 0.75 = 5.25$ m.

Table 3. Example transition structure terminal section parameters.

Section Parameters	Terminal Section Parameter Value	
	Entrance (canal)	Exit (aqueduct)
Bottom width b (m)	14.6	10.0
Lined bank height h (m)	5.25	6.0
Side slope m (horizontal:vertical)	2	0
Bottom vertex curvature ration κ	1	0
Channel bottom elevation z_b (m)	7.5	6.6
Full-supply water depth h_n (m)	4.5	5.25
Longitudinal channel slope dz_b/dx (m/m)	1/15500	1/5000

The longitudinal slope of the channel increases to dz_{bL}/dx = -1 / 5000 m/m in the aqueduct, which has a sharp-cornered rectangular cross-section with b_L = 10.0 m, h_L = 5.25 m, h_{wL} = 5.25 + 0.75 = 6.0 m, m_L = 0, and κ_L = 0. The bed elevation drops 0.9 m through the transition (z_{b0} = 7.5 m and z_{bL} = 6.6 m), which is 50 meters in length (L = 50 m), giving $z'_{b0} = L \times dz_{b0}/dx$ = 50 × (– 1/15500) = – 0.003223 m/m, and $z'_{bL} = L \times dz_{bL}/dx$ = 50 × (– 1/5000) = – 0.01 m/m. Dimensions of the canal and the aqueduct are summarized in Table 3.

The cross-section measures b, h, m, and κ given by Eq. *(9)*, the centerline bed elevation z_b obtained from Eq. *(10)*, the first and second derivatives of z_b with respect to the channel distance x, and the radius-of-curvature of the bed centerline r_c from Eq. (12), are listed at five-meter spacings in Table 4. For the calculation of z_b from Eq. *(10)*, the coefficients k_1 = 6.6 – 7.5 – (– 0.00323) = – 0.897 and k_2 = – 0.01 – (– 0.00323) = – 0.00677. The minimum radius-of-curvature of the bed r_{c_min} = 481.2 m at ξ = 0.789 (x = 39.5 m), which was found by a binary search, and is concave. A three-dimensional plot of the smooth transition surface is presented in Fig. 2.

6. Summary and Conclusions

Fifth-degree parametric equations are developed to calculate the cross-section dimensions and bed centerline elevations (thus, the geometric surface coordinates) between the two ends of a warped transition in a water-supply canal. The parametric modeling approach provides a smooth representation of the mixed geometry that results from terminal sections having vastly different shapes. Additionally, modifications to the transition structure parameters during the design process can be easily implemented and evaluated using this technique.

Table 4. Example transition structure cross-section and bed centerline parameters

Transition distance x (m)	$\xi = x/L$ (--)	$F(\xi)$ (--)	Cross-section Shape Parameters				Bed Centerline Profile			
			b (m)	h (m)	m (m/m)	κ (--)	z_b (m)	dz_b/dx (m/m)	d^2z_b/dx^2 (m/m^2)	r_c[a,b] (m)
0	0.0	0.000	14.600	4.500	2.000	1.000	7.500	-6.667×10^{-5}	0	∞
5	0.1	0.009	14.561	4.506	1.983	0.991	7.492	-0.0044	-0.00155	-647
10	0.2	0.058	14.334	4.543	1.884	0.942	7.448	-0.0138	-0.00206	-485
15	0.3	0.163	13.850	4.622	1.674	0.837	7.353	-0.0237	-0.00180	-555
20	0.4	0.317	13.140	4.738	1.365	0.683	7.215	-0.0310	-0.00103	-969
25	0.5	0.500	12.300	4.875	1.000	0.500	7.051	-0.0336	0.00000	-2.50×10^5
30	0.6	0.683	11.460	5.012	0.635	0.317	6.887	-0.0310	0.00103	977
35	0.7	0.837	10.750	5.128	0.3226	0.163	6.749	-0.0238	0.00180	557
40	0.8	0.942	10.266	5.207	0.116	0.058	6.654	-0.0139	0.00206	487
45	0.9	0.991	10.039	5.244	0.017	0.009	6.609	-0.0045	0.00154	648
50	1.0	1.000	10.000	5.250	0.000	0.000	6.600	-0.0002	0	∞

[a]Negative values of r_c show convex curvature; positive values are concave.
[b]The minimum r_c = 481.2 m at ξ = 0.789.

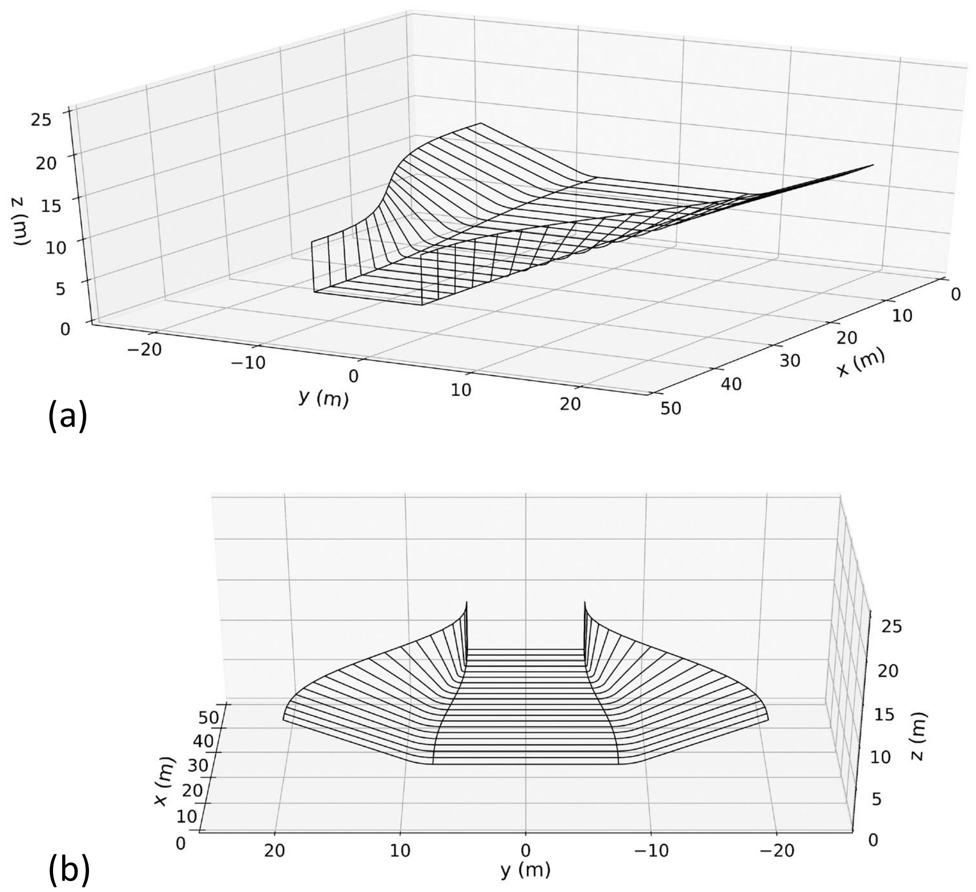


Figure 2. A three-dimensional representation of the smooth transition surface: (a) The view from downstream (aqueduct) end with a 45° rotation about the vertical axis, and (b) the view from the upstream (canal) end with no rotation about the vertical axis. Both views have an elevation angle (that is, the angle made with a plane normal to the vertical axis) of 30°.

The generalized cross-section shape introduced by the writer (Froehlich 2008) for canal design significantly simplifies the interpolation of sections between the terminal points of a transition structure and its geometric surface representation. The standard shape (see Fig. 1) is defined by four parameters (b, h, m, and κ), which enables a straightforward representation of cross-sections having various forms ranging from trapezoids to semi-circles (see Table 1). With $b > 0$, setting κ to 1 and 0.5 gives the two standard cross-sections used for major canals in India (BIS 2004).

Higher-degree polynomial formulas could be used for the parameterization. For example, Wilson (2005) uses a seventh-degree expression to develop smooth profiles of nozzles and transition ducts. However, in this analysis, the fifth-degree equation is sufficient to deliver a high level of surface smoothness that is esthetically pleasing (see Fig. 2). Additionally, the parametrization guarantees that inflection points exist along the warped sides at the terminal cross-sections. Consequently, the transition surface curvature is in line with the upstream and downstream channels, which avoids discontinuities in curvature at the end sections that can lead to flow separation, sudden steep increases in the local fluid pressure field, and generation of cross-waves and runup along the sidewalls.

An example is presented that applies the parametric modeling approach to design a significant canal transition where the cross-section changes from a standard trapezoidal shape with rounded bottom vertices to a rectangular section in a steeper aqueduct that carries the flow across a broad valley. The bed centerline elevations are also calculated to ensure a smooth changeover and that the vertical curvature is not excessive. The example shows that the parametric approach dramatically simplifies the geometric design of warped transitions connecting channels having a wide range of conventional shapes.

Because the focus of this analysis is creating a smooth mixed-geometry layout that embodies Hinds' (1928) idea of connecting the terminal sections with a warped transition structure, it is a designer's task to evaluate energy losses using (most likely) numerical models of flow in the open channel (possibly a three-dimensional numerical hydrodynamic model). A small-scale physical model could also be constructed and used to evaluate the design. The ability of warped transition structures to minimize energy losses is documented by Hinds (1928), Scobey (1933), and Ippen (1949).

7. REFERENCES

Aisenbrey, A.J. Jr., Hayes, R.B., Warren, H.J., Winsett, D.L., and Young, R.B. (1978). *Design of Small Canal Structures*. U.S. Bureau of Reclamation, Denver, Colorado.

Barnhill, R. (1985). "Surface in computer-aided geometric design: a survey with new results." *Computer-Aided Geometric Design*, 2(1), 1-17.

Boehm, W. (1988). "Visual continuity." *Computer-Aided Design*, 20(6), 307-311.

BIS (2004). "Criteria for design of lined canals and guidelines for selection of type of lining (1^{st} revision)." *IS:10430:2000*, New Delhi, India.

Chow, V.T. (1959). *Open-channel Hydraulics*, McGraw-Hill, New York, New York.

Froehlich, D.C. (1994). "Width-and-depth constrained best trapezoidal cross-section." Journal of Irrigation and Drainage Engineering, 120(4), 828-835.

Froehlich, D.C. (2008). "Most hydraulically efficient standard lined canal sections." *Journal of Irrigation and Drainage Engineering,* 134(4), 462-470.

Gemperline, G. and Crane, C. (1995). "Hydraulic design." Chapter 3 in *Guidelines for Design of Intakes for Hydroelectric Plants*, by the Committee on Hydropower Intakes of the Energy Division, American Society of *Civil* Engineers, New York, 16-111.

Hager, W.H., Schleiss, A.J., Boes, R.M., and Pfister, M. (2020). *Hydraulic Engineering of Dams*. CRC Press, London, United Kingdom.

Hinds, J. (1928). "The hydraulic design of flume and siphon transitions." *Transactions of the American Society of Civil Engineers*, 92(1), 1433-1459.

Ippen, A.T. (1949). "Channel transitions and controls." *Engineering Hydraulics*, Proceedings of the Fourth Hydraulics Conference Iowa Institute of Hydraulic Research, 12-15 June 1949, 496-588.

Laycock, A. (2007). *Irrigation Systems: Design, Planning, and Construction*. CAB International, Wallingford, Oxfordshire, United Kingdom.

Liria Montañés, J. (2006). *Hydraulic Canals: Design, Construction, Regulation, and Maintenance*. Taylor & Francis, London, United Kingdom.

Mazumder, S.K. (2020). *Flow Transition Design in Hydraulic Structures*. CRC Press, Boca Raton, Florida.

Scobey, F.C. (1933). "The flow of waters in flumes." *Technical Bulletin 393*, USDA, Washington, DC.

Simmons, W.P. Jr. (1964). "Hydraulic design of transitions for small canals." *Engineering Monograph No. 33*, U.S. Bureau of Reclamation, Denver, Colorado.

Swamee, P. K., and Chahar, B. R. (2015). *Design of Canals*. Springer India, New Delhi, India.

USBR (1987). *Design of Small Dams (3^{rd} Edition)*. Denver, Colorado.

Veltkamp, R.C. (1992). "Survey of continuities of curves and surfaces." *Report CS-R9202*, Computer Science/Department of Interactive Systems, Centrum voor Wiskunde en Informatica, Amsterdam, Netherlands.

Vittal, N., and Chiranjeevi, V.V. (1983). "Open channel transitions: Rational method of design." *Journal of Hydraulic Engineering*, 109(1), 99-115.

Wilson, J.F. (2005). "Smooth mixed geometry profiles of nozzles and transition ducts." *Journal of Mechanical Design*, 127(6), 1211-1213