A GPS-Based Attitude Determination System for Small Satellites (SSC06-VII-1)

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Overview

- Carrier phase Differential GPS (CDGPS) Primer
  - Centimeter resolution baseline vectors
  - Fast convergence time
  - Real-time attitude determination and relative navigation
  - Developed by Dr. Mark Psiaki and Shan Mohiuddin

- Attitude Determination
  - Absolute and relative attitude

- Design Considerations
  - Antennas
  - Orbits
  - Missions
Why fly a GPS-based ADS on small sats?

- Lightweight

- Multipurpose functionality
  - Absolute and relative position information
  - Absolute and relative attitude information

- Relatively Inexpensive

- Very accurate

- No extensive calibration
Operational Concept

- Use two baseline vectors to compute attitude
- One baseline vector between spacecraft
- These vectors come from differential GPS carrier signal measurements (CDGPS)
- Objective of CDGPS baseline vectors
  - Provide extremely accurate baseline vectors (cm, mm)
  - Construct a highly accurate attitude estimate
Hardware Design

Overview
Motivation
Concept
Hardware

CDGPS
Carrier Phase
CP Ranging
CDGPS Concept
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Attitude
Concept
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Design
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CDGPS Primer
Why use carrier phase measurements?

- Modern GPS Receivers have ~1% measurement accuracy
  - 0.01\(\lambda\)

- Code solution (f=1.023 MHz, \(\lambda=293m\))
  - Best case accuracy of ~3m

- L1 Carrier signal (f=1.57542 GHz, \(\lambda=19cm\))
  - Best case accuracy of ~2mm

- Remember
  - There is substantial noise in the system
  - 3-10 meters
Carrier Phase Ranging

- **Known quantities (1):**
  - $\phi_A^j$

- **Unknown quantities (5):**
  - $\rho_A^j(x,y,z)$
  - $N_A^j$
  - $\delta_{RA}$

- **Range from GPS satellite $j$ to receiver $A$**
  \[ \rho_A^j = \lambda_{L1}(\phi_A^j + N_A^j) + f_{L1}\delta_{RA} \]

**Notation:**
- Superscripts denote GPS satellites
- Subscripts denote GPS receivers
CDGPS Concept

- Want a relative vector between points
  - Subtract absolute range solution vectors
    - Noisy
  - Double differenced carrier phase measurements
    - Carrier phase difference between two satellites (i,j) & between receiver A and B*
    - Noise cancels out
    - Phase offsets cancel out
    - Receivers are within 1km of each other
      - Transmission errors and satellite errors cancel
      - Ephemeris, satellite clock, ionospheric and tropospheric errors
    - Removes receiver errors
      - Oscillator drift, receiver clock

\[ \mathbf{\rho}_A - \mathbf{\rho}_B = \nabla \Delta \mathbf{\rho} \]

\[ \Phi_{RL}^{jj} = \mathbf{\rho}_A - \mathbf{\rho}_B \]

Search Spaces

- Double Difference: \[ \nabla \Delta \tilde{p}_A^j = \lambda_{L1} (\nabla \Delta \phi_A^j + \nabla \Delta N_A^j) \]
- Find the correct \( \nabla \Delta N_A^j \)
- Least-squares Ambiguity Decorrelation Adjustment (LAMBDA) method
  - Ambiguity is an integer through receiver design
- Residual Error
  \[ Z_A^j = \rho_A^j - P_A^j - c\delta_{RA} \]
Solution Convergence

(1 of 2)

- Line of sight vectors
  - If values are too similar, inconsistent integer ambiguities
  - Geometric Dilution of Precision (GDOP)
    - Difference of the ranging vectors from receiver A to multiple satellites should not close to 0
  - Rate of change

- Carrier to noise ratio
  - Affects measurement accuracy

- Multipath
  - < 1us transmission bounces (300m)
Solution Convergence

(2 of 2)

- Large residual errors
  - GDOP issue
  - Increases the search space

- Cycle slips
  - Receiver loses track briefly
  - Have to recompute solution

- Number of satellites
  - 5 is the minimum
  - More satellites reduce convergence time
On Orbit Considerations

- Mitigates many of the convergence concerns
- Low multipath environment
  - Ensure spacecraft doesn’t interfere with itself
- High line of sight dynamics
- High GPS satellite visibility
  - Relative little elevation mask
Results – Simulated

- Spirent GSS7700 GPS
  - $3\sigma$ error of 4mm
  - Converge within minutes

![Graph showing CDGPS error magnitude over time for different numbers of GPS satellites (5, 6, and 7). The x-axis represents time in seconds, and the y-axis represents CDGPS error magnitude in meters.]
Results – Measured

- Terrestrial Field Test
  - Cornell University GPS Autonomous Receiver (COUGAR)
  - GPS Patch Antennas (Synergy Systems AN-10SC)
Results – Measured

- 3σ Error of 4mm

![Graph showing CDGPS Error Magnitude over time](image)
Results Comparison

- Noise model matches
- Convergence time
  - Line of sight dynamics
  - Multipath

<table>
<thead>
<tr>
<th>Test</th>
<th>Mean Error (cm)</th>
<th>Std. Dev Error (cm)</th>
<th>Convergence time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEO Simulation</td>
<td>0.22</td>
<td>0.1</td>
<td>1-70</td>
</tr>
<tr>
<td>Terrestrial</td>
<td>0.19</td>
<td>0.12</td>
<td>150</td>
</tr>
</tbody>
</table>
Attitude Estimation
Attitude Estimation

- Find the direction cosine matrix (DCM) that rotates CDGPS vectors in ECEF to the body frame
- CDGPS vectors from two bodies

- Markley’s SVD Method
  - Stable
  - Robust
  - Scalable
Results – Simulated

- Monte Carlo simulation
  - 100,000 iterations
  - 25cm baseline distance, 3 vectors
  - 7.5 mm 3σ error

PDF as a histogram of 100,000 cases

Mean Error: 2.033 degrees
Std. Dev: 0.884 degrees
Design Considerations
Antenna Considerations

- Phase center
  - Not necessarily in the physical center
  - Changes with attitude
  - Place antennas in same orientation
    - Cancel in the double difference
    - Only occurs for fixed relative distances
- Visibility
  - Must ensure that each antenna has nearly identical field of view
- Grounding
  - Antenna radiation patterns can be negatively affected
How many antennas should I use?

- Minimum
  - 3 for 2 baseline vectors (can compute 3)

- Generalization
  - \( N \text{ antennas} = \frac{N(N-1)}{2} \text{ possible independent vectors} \)
  - Adds fault tolerance
  - Reduces noise in attitude estimate

Simulation Parameters
- 7.5mm 3\( \sigma \) error
- 25cm baseline distance
- 10k iterations

![Graph showing angular error magnitude vs. number of baseline vectors.](image)
How far apart do I need my antennas?

- The close the antennas, the greater percentage error is with respect to baseline distance
- Want
  - To find the closest distance that meets your pointing requirements

Simulation Parameters
- 7.5mm 3σ error
- Varied baseline distance
- 10k iterations

Good separation distance
- 20-25 cm
- 1-2 degree pointing accuracy
- Lower limit: 15cm
What are my orbital constraints?

- Altitude
  - LEO altitude is significantly smaller than GPS altitude
What are my orbital constraints?

- Independent of altitude in LEO
- Independent of inclination in LEO
  - GPS orbital planes are at 55 degrees
  - Is there poor visibility at high inclinations?
- Simulation

![Graph showing number of generated SVs vs time](image)

**Statistics:**
- Mean: 7.6282
- STD: 2.2760
What are my pointing constraints?

- Visibility is highly dependent on spacecraft attitude
  - Best case: Zenith pointing
  - Worst case: Nadir pointing
Satellite visibility over pointing scheme

Number of Generated SVs

Visible GPS Satellites
GPS Satellites Required for CDGPS

Mean: 5.9796
STD: 5.0395
Conclusions

- Benefits of CDGPS
  - High performance to cost ratio
    - Low cost implementation
    - Cm level accuracy
  - Adaptable, modular package
    - Offers complete coverage using existing technology (GPS)
  - Independence of design
    - Only need three antennas (or more)
    - Modest pointing constraints
    - Calibration free

- Provides accuracy sufficient for most small satellite missions
Future Work

- Incorporation of an attitude estimation filter
  - Further reduces error

- Flying these algorithms
  - Cornell University Satellite (CUSat) Project
Questions
Backup Slides
Double Differencing Carrier Phase

- **Single Difference**
  \[ \Delta(\hat{\phi})_{AB}^j = (\hat{\phi})_{A}^j - (\hat{\phi})_{B}^j \]
  - Carrier phase difference of the same satellite (j) between receiver A and B
  \[ \Delta \rho_{AB}^j = \lambda_{L1} (\Delta \phi_{AB}^j - \Delta N_{AB\rho}^j) \]
  - Receivers are within 1km of each other
    - Transmission errors and satellite errors cancel
    - Ephemeris, satellite clock, ionospheric and tropospheric errors

- **Double Difference**
  - A differenced single difference
    \[ \nabla \Delta(\hat{\phi})_{AB}^j = \Delta(\hat{\phi})_{iAB}^i - \Delta(\hat{\phi})_{AB}^j \]
  - Carrier phase difference of two satellite (i,j) between receiver A and B
    \[ \nabla \Delta \rho_{AB}^{ij} = \lambda_{L1} (\nabla \Delta \phi_{AB}^{ij} - \nabla \Delta N_{AB\rho}^{ij}) \]
  - Removes receiver errors
    - Oscillator drift, receiver clock
The Integer Ambiguity

- Tracking a GPS satellite \((j)\)
  - Lock onto satellite with initial \(N_A^j\)
  - \(N_A^j\) stays constant while tracking satellite \(j\)
  - Removes a variable in subsequent time steps
- There are more phase offsets to the system
  - Also constant when tracking a satellite
- L1 replica phase offset \((\eta_A^j)\)
  - Same for each channel in a receiver
- Epoch phase offset \((\Gamma_e^j)\)
  - Each GPS satellite has a phase offset at epoch time
  - Same offset for each GPS satellite
    - To within the accuracy of the GPS satellite clock

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Phase Ambiguities

- Combine the integer ambiguity with phase offsets
  \[ N_A^j = N_{AP}^j - (\eta_A^j + \Gamma_A^j) \]

- Single differencing can be used to obtain a relative solution, but the single differenced phase ambiguity is difficult to obtain a solution

- Double differencing can remove these phase offsets and guarantee that \( \nabla \Delta N_A^j \) is an integer
Phase Ambiguities

- Who cares if the ambiguities are actually integers?
  - Convergence time for real-valued ambiguities: minutes
  - Convergence time for integer ambiguities: seconds
SVD Attitude Estimation

- Davenport’s Attitude Matrix
  \[ B = \sum_{i=1}^{N} a_i \cdot B_{CBF} \cdot R_i \cdot ECEF \cdot R_i^T \]
  - Scale influenced by length of baseline vectors

- Perform SVD
  \[ B = U \Sigma V^T \]

- Find the DCM
  \[ ECEF^T Q_{CBF}^T = U \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det(U) \det(V) \end{bmatrix} V^T \]