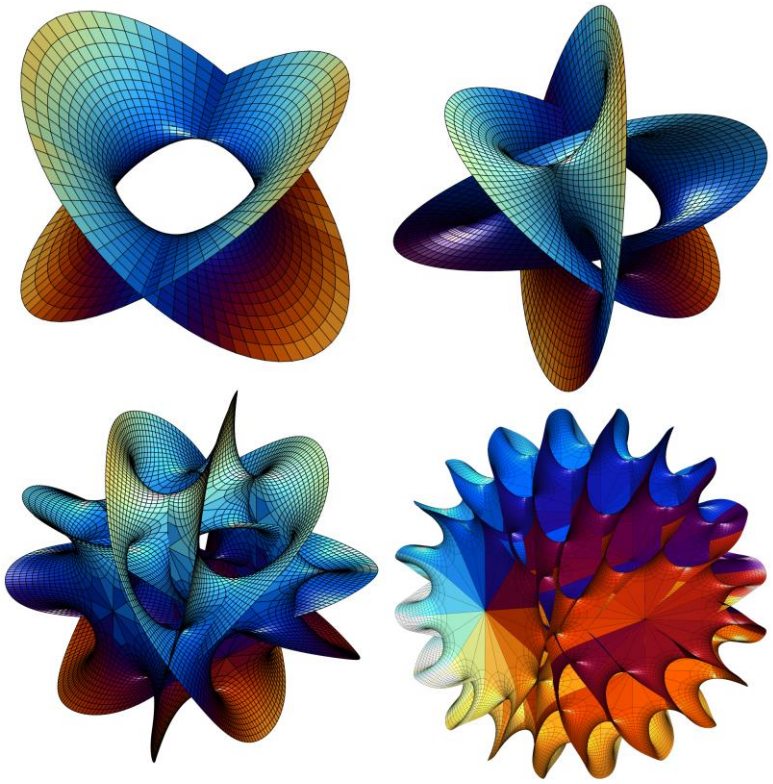
Differential Equations and Geometry of Calabi-Yau Manifolds



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Student Research Symposium

Utah State University

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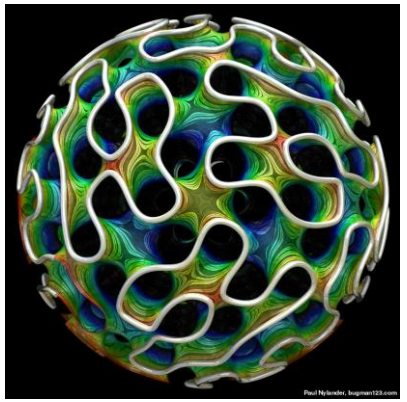
Research Snapshot

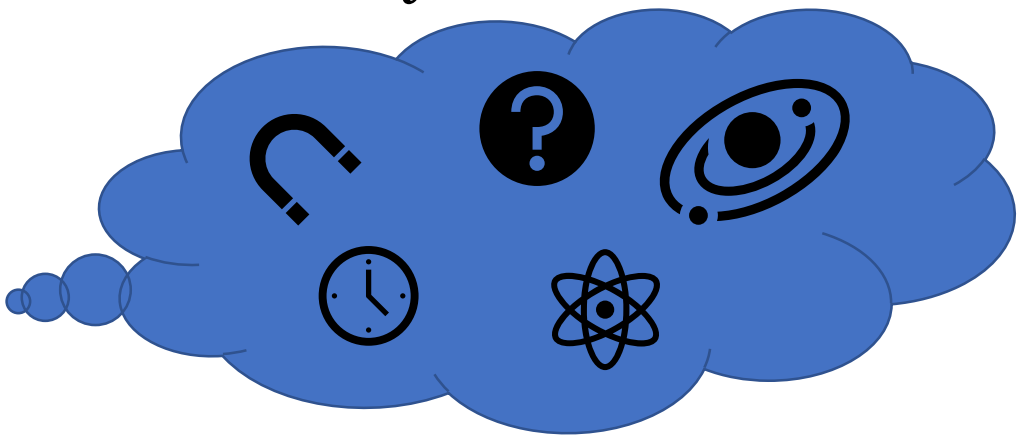
A Calabi-Yau manifold



- I'm a PhD student working under Dr. Andreas Malmendier.
- My research is in Algebraic Geometry.
- We are creating new families of Calabi-Yau 3-folds from special known families that are fibered by elliptic K3 surfaces by doing operations to the differential equations that govern the variation of Hodge structure.
- This approach is new to the field. We will be able to construct new families of these special geometries that have not been seen before.

What's the deal with all this abstraction?

- Calabi-Yau 3-folds are 6D!
- Mathematics that was once very abstract now plays a fundamental role in the lives and opportunities we have today.



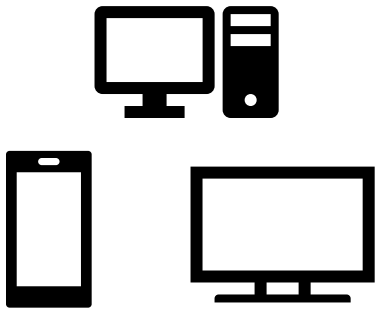
General Relativity

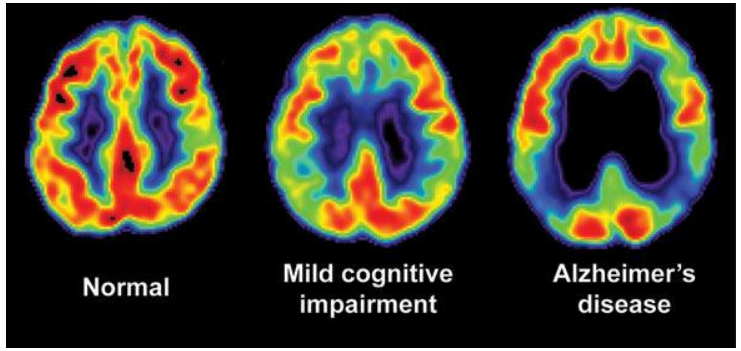
- The study of abstract manifolds in Differential Geometry was developed in mid 1800s. In the early 1910s, this became the language that Einstein used in his theory of General Relativity – describing Gravity as the curvature of spacetime.
- The effects of General Relativity must be accounted for to allow satellite systems like GPS to work correctly! [4]

A GPS satellite

Quantum Mechanics

- The mathematics of Quantum Mechanics was invented to explain experimental results that were completely unexpected.
- This lead to a branch of physics known as Quantum Field Theory.
- Predictions of these theories power all modern electronics.
- The PET scan, a modern medical test used for detection of cancer, heart problems, brain disorders, and more, was developed from predictions of Quantum Field Theory. [6]

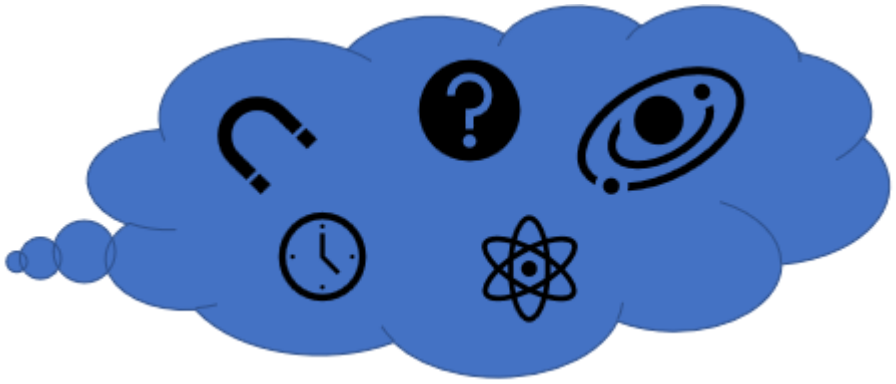




Results of a PET scan

The results of abstraction

- It is impossible to know what will come from abstract mathematical theories!
- These once abstract theories have fundamentally changed and even saved lives.



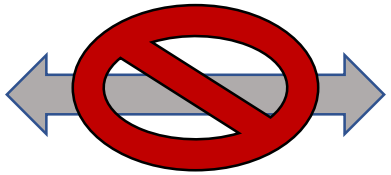
What is Calabi-Yau and why care?

- Calabi-Yau manifolds are special geometric objects with many rich and interesting structures.
- "Families of Calabi-Yau" – we mean a parameterized collection of such objects.
- Calabi-Yau 3-folds have become known in popular culture for their appearance in String Theory.
- Geometric properties of these objects translate into physical properties of particles and spacetime.

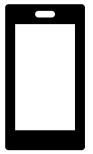
A family of Calabi-Yau manifolds

Calabi-Yau Manifolds & String Theory

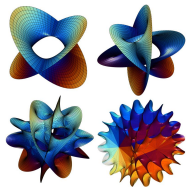
General Relativity

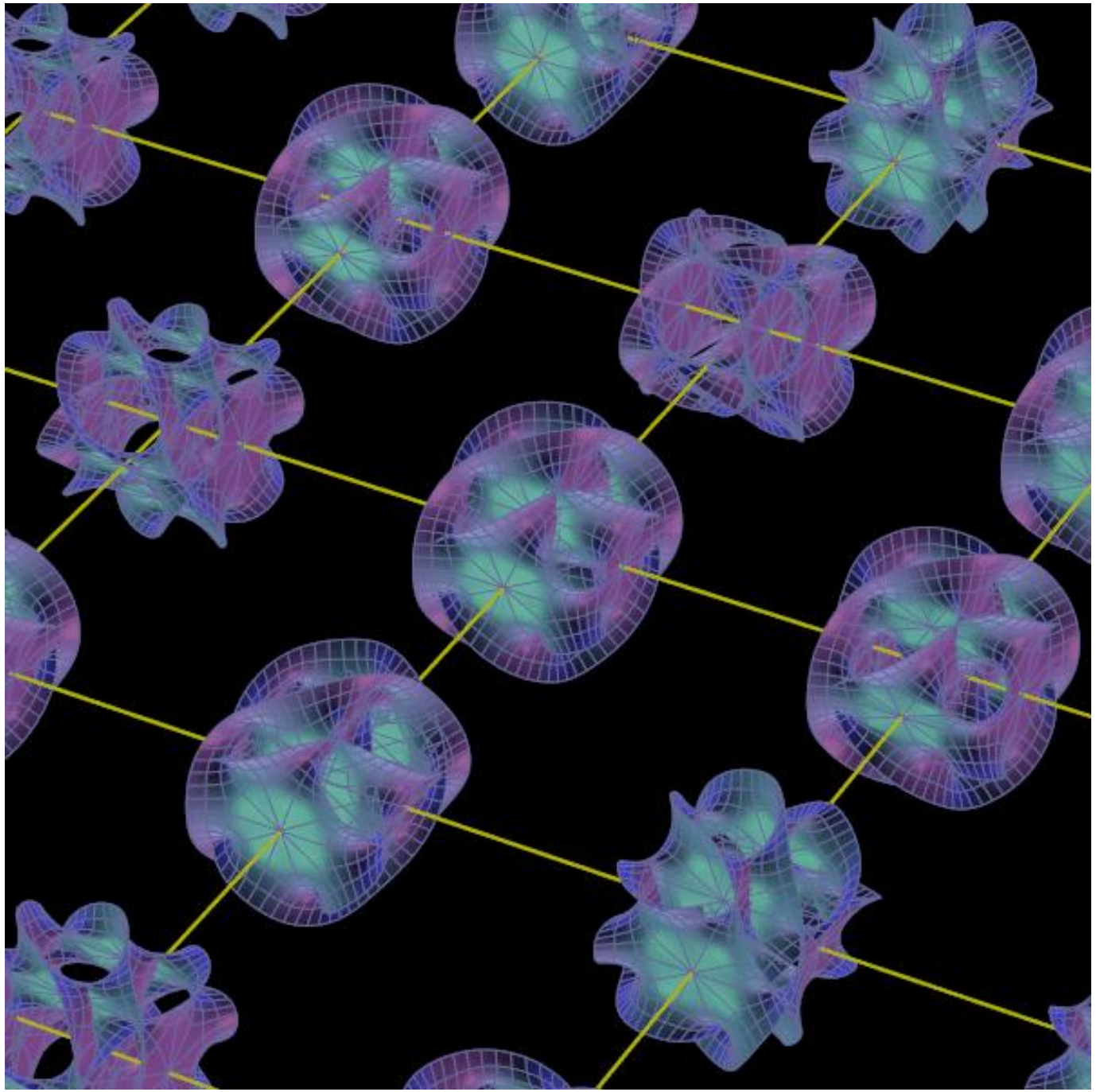


Quantum
Field Theory



String Theory
10 D

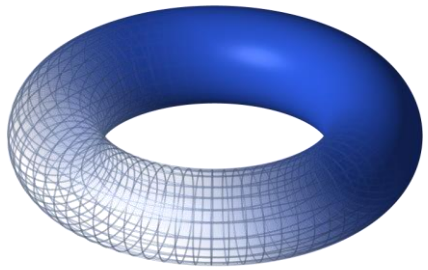




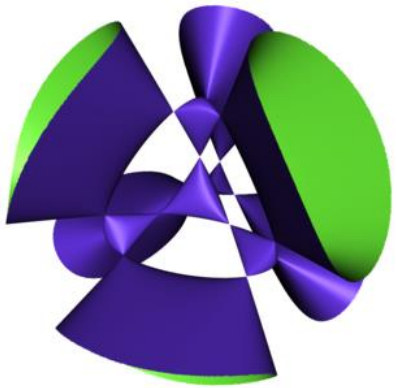
Calabi-Yau Manifolds in 10 dimensional Spacetime

What is the Big Picture?

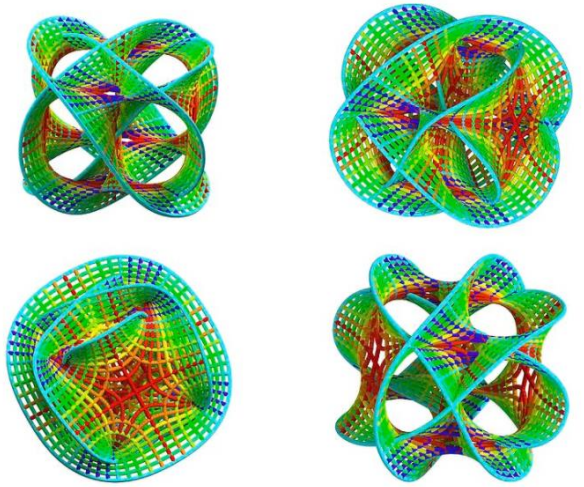
- The classification of Calabi-Yau manifolds is completely known in 2D and 4D.
- Very little is known about the classification for 6D.
- The work we are doing will add to the classification by creating families of Calabi-Yau 3-folds that have not been seen before.



2D



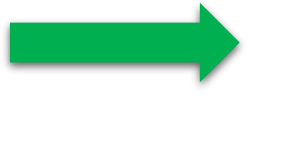
4D

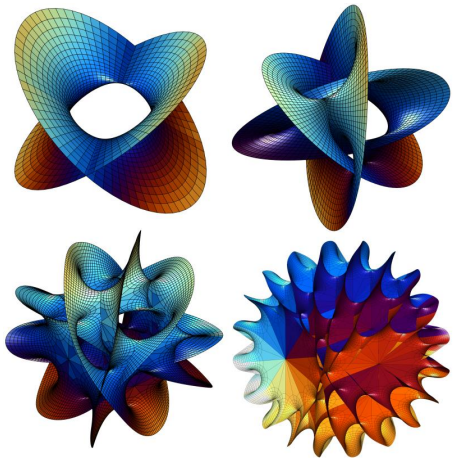


6D?

What is the Big Picture?

- Starting from 14 special families of Calabi-Yau 3-folds, we will create a new methodology using tools from algebra, calculus, differential equations, and geometry to create new differential equations associated to new families.
- The resulting equations are quite complicated – we are using many aspects of the symbolic computation engine Maple to solve (built in-house!)





"Part of our new process"

What will this do - and where next?

- Expand the classification of Calabi-Yau 3-folds by discovering new families
- Introduce new tools and perspectives (analytical vs. algebraic) for studying families of Calabi-Yau manifolds
- Two directions to go:

1. More complicated 3-folds with less symmetry
2. Higher dimensional Calabi-Yau's (8D, 10D, etc.) with similar properties

It is impossible to know what will come from abstract mathematical theories!

Many useful and practical applications come from mathematics that was at one time considered too abstract to have any applications.

THANK YOU!

Sources

- [1] http://aleph.se/andart2/math/calabi-yau-and-hansons-surfaces/
- [2,7] http://bugman123.com/MinimalSurfaces/index.html
- [4] https://www.novatel.com/an-introduction-to-gnss/chapter-3-satellite-systems/gps/
- [4] http://www.astronomy.ohio-state.edu/~pogge/Ast162/Unit5/gps.html \
- [5] http://historyofneuroscience.umwblogs.org/technology/pet-scan/
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- [11] https://www.maplesoft.com/