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Stochastic Orbital Lifetime Analysis

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ABSTRACT: Given the dynamic environment in which spacecraft exist, a better methodology for performing orbital lifetime analyses over the current practice of point analyses was desired. The approach chosen was to utilize Monte Carlo based predictions, which provides the ability to gauge the probability of meeting mission lifetime goals, as well as identifying driving factors. The Monte Carlo analysis, called Orbital Lifetime Monte Carlo (OLMC), is based on the NASA Langley Research Center long term orbit propagator Orbital Lifetime. OLMC incorporates the ability to model variations in predictions of solar flux levels and timing of associated peaks, the variation in launch vehicle orbit insertion accuracy (altitude, velocity, and flight path angles), spacecraft ballistic coefficients, and launch delays. Desired repeatability, distribution smoothness and code runtime are considered for the purposes of establishing values for code specific parameters and number of Monte Carlo runs. Results demonstrate that solar flux predictions are the primary driver for variations in lifetime; of which, due to their variability, multiple prediction sets should be utilized to fully characterize the lifetime range of a spacecraft.

INTRODUCTION

Given the dynamic environment in which spacecraft exist, a better methodology for performing orbital lifetime analyses over the current practice of point analyses was desired. The avenue that was defined for this approach was to utilize Monte Carlo (MC) based probabilistic predictions. This capability is expected to provide the ability to more appropriately gauge the inherent risks of meeting nominal and/or minimum mission lifetimes as defined by science or operational requirements and limited by mission design. The tool created to address these requirements is Orbital Lifetime Monte Carlo (OLMC).

OLMC, a compiled executable written in Fortran, uses a demonstrated long term orbit propagator, Orbital Lifetime (OL) with a Monte Carlo wrapper. The variance of key physical parameters that affect orbit lifetime, such as launch vehicle injection errors, solar flux variation, and launch delays are modeled with probability distributions (normal and other types). The products of OLMC are lifetime predictions (months or years above a minimum altitude) for the number of user specified cases. Histograms can then be used to determine the probabilities of the spacecraft meeting the lifetime requirements.

In addition to an explanation of the OLMC code, this document provides information on the uncertainties that factor into modeling of orbit lifetimes. Results are provided that explore the adequate number of MC cases to run, along with recommended values for code specific parameters. Runtime statistics are reported along with a comparison of OLMC with Satellite Tool Kit/Professional (STK/Pro). Sensitivity to different solar flux profiles and the investigation of tool use for maximum lifetime requirements are also explored.

STOCHASTIC ORBITAL LIFETIME ANALYSIS

Orbital lifetime analyses are nominally performed to determine the likelihood of spacecraft meeting minimum and maximum lifetime goals and requirements. Minimum lifetime goals are driven by science or operational requirements. Low cost missions without propulsion systems are most susceptible to failure of meeting minimum lifetime goals. Post mission maximum lifetime requirements for Low Earth

Orbit spacecraft are defined by the governing agency in charge of the spacecraft.

Spacecraft lifetimes are generally driven by ballistic coefficients, orbital parameters, and atmospheric drag conditions. Ballistic coefficient is defined as $m/(C_D A)$, where m is the spacecraft mass, C_D is the drag coefficient, and A is the average projected area normal to the velocity vector. In general, the larger the ballistic coefficient, the longer the spacecraft will stay in Earth orbit. Orbital parameters of the spacecraft also factor into the lifetime. Spacecraft in lower orbits (300 – 500 km range) will reenter faster than spacecraft in higher orbits (1000 km range) mainly due to atmospheric density variations at different altitudes above the Earth. In addition, atmospheric density at a given altitude is directly affected by the solar activity. Measured in radio flux at F10.7 cm wavelength, the solar flux shows a cyclic pattern over an average 11 year cycle. Spacecraft launched close to the peak activity of a solar cycle will experience greater early orbit altitude degradation than those launched close to the minimum.

Historically, orbit lifetime assessments have relied on point design analyses with specific cases limited to small subsets of parameters manually varied prior to execution of each case. A stochastic approach, in which a MC routine is utilized, allows the user to simultaneously vary multiple parameters based on their probability of occurrence. Stochastic analyses allow the user to explore the trade space without excessive implementation time. A key benefit of this approach is a product that provides a clear, visual representation of the probability of meeting a given lifetime. Figure 1 provides an example histogram of prediction results and a quantification of the probability of achieving a five year lifetime.

OLMC

OL is a Fortran-based long term orbit propagator, originally developed at Massachusetts Institute of Technology Lincoln Laboratory, modified by the Rand Corporation, and obtained and further modified by NASA Langley Research Center. Environmental perturbations are used in OL to decay the spacecraft orbit. Only long term orbital variations are considered, since short term variations over a single orbit are assumed to average out[1]. The user has the ability to specify whether numerical integration extends over one or multiple spacecraft orbits through the code parameter called orbits per iteration. In the section entitled *Determination of Adequate Number of Monte Carlo Runs and Orbits per Iteration*, more information is presented on the relationship between this quantity and analysis results. OL utilizes two distinct atmospheric models: the U.S. Standard Atmosphere 1976 for altitudes below 90 km and the Jacchia 1970 atmospheric density model for altitudes between 90 km and 2,500 km. The code will execute if the user inputs an altitude above 2,500 km, however, since the Jacchia 1970 atmospheric density model is based on fit coefficients derived within the 90-2,500 km region, the validity of the density data will be suspect due to extrapolation beyond the intended range. The user can also specify a minimum stopping altitude in lieu of the default reentry altitude. This allows the user to explore "what if" scenarios, such as the duration a spacecraft remains above an altitude at which spacecraft controllability issues may be experienced. The perturbations used in OL are atmospheric drag, solar radiation pressure, rotating atmosphere, and gravitational effects due to the Earth's oblateness, the Sun, and the Moon[2]. Each of the perturbations can be toggled on or off for any of the analyses. The solar flux prediction profiles can be any text file properly formatted for use with OL. If necessary, the solar flux profile is repeated in OL over additional 11 year cycles after the last date defined in the file. More discussion on this facet of OL is provided in the *Solar Flux Predictions* section. Additional information regarding the operation of OL can be obtained from [1] and [2].

A MC routine, also written in Fortran, was implemented for use in conjunction with OL as illustrated in Figure 2. The MC routine sets up the key parameter sigma values used throughout each run and calls OL a specified number of times, up to 10,000. For each key parameter, a random number is generated and is used to determine the sigma value of that parameter. Key parameter values for all planned MC runs are generated at the start of the OLMC execution. For each execution of the OL kernel, the key parameter values are held constant. As with the number of orbits per iteration for OL, the number of MC cases that should be run must be determined to get 'reasonable' results. There is more discussion on the effects of varying the number of MC runs provided in the section entitled *Determination of Adequate Number of Monte Carlo Runs and Orbits per Iteration*.

Key parameters chosen as MC variables are the solar flux magnitude and timing of the peak, launch vehicle insertion altitude, velocity, and flight path angle, and launch delays. Normal distributions are used for all the key parameters except for launch delay. The launch delay is based on historical data and is set as a Weibull distribution. The user can also specify fixed delays by varying the launch date. A more detailed discussion of the key parameters and distributions used for simulation is included in the next section.

KEY PARAMETER DISCUSSION

Solar Flux Predictions

Measured F10.7 cm solar flux levels vary on a daily basis and can be difficult to predict. In general, the solar flux is cyclic over an 11-year period. However, the minimum peak-to-peak time recorded is 9 years and the maximum is 17 years. In addition, the solar flux cycles have been observed to follow an Even-Odd behavior, where the odd number cycles are larger than the preceding even number cycles[3], although cycle 23 did not conform to this pattern. Also, solar flux may be cyclic over an 80 to 90 year period, as suggested by Gleissberg[3], however, more recent cycles do not conform to this duration and suggest an even longer period variation. These factors, along with others, contribution to the uncertainty associated with the modeling and prediction of the solar flux.

Solar flux prediction sets currently utilized by OLMC are generated by the NASA Marshall Space Flight Center's Marshall Solar Activity Future Estimation (MSAFE) model[4] and Dr. Kenneth Schatten[5]. The former have been utilized in OL and the latter in STK/Pro. Both prediction methods supply nominal and +two-sigma solar flux predictions over approximately two cycles of solar activity. Figure 3 shows examples of solar flux predictions used for OLMC lifetime analyses and Table 1 gives a comparison of the data sets. As seen in the figure and table, the MSAFE and Schatten predictions do not agree in magnitude and timing and vary as a function of prediction date. This can have a significant impact on a spacecraft lifetime (see *Sensitivity to Solar Flux Profile* section for more detail).

The MSAFE is a 13-month smoothed solar flux and geomagnetic index estimation technique that utilizes a modified McNish-Lincoln linear regression method. Future activity is estimated based on a mean cycle and deviations derived from previous cycles, initialized based on maximum-to-maximum or minimum-to-minimum cycle values.

The methodology used to produce the data by Dr. Kenneth Schatten is a precursor method and is explained as the SODA (SOlar Dynamo Amplitude) index. "The SODA index is a composite index attempting to combine the changing toroidal and poloidal fields of the Sun. As these fields change with time, the combined SODA index allows us to monitor the 'buried magnetic flux' present in the Sun's ever-changing dynamo."[6]

Solar flux variations in OLMC are broken into two factors – solar activity magnitude and timing of the peak. Both can have significant effects on a spacecraft's lifetime. Regardless of which prediction set is used (MSAFE, Schatten, or others), the variations in solar activity magnitude about the nominal are represented by normal distributions determined by the supplied nominal and +2 sigma profiles. The timing of the peak is the variation of the date at which the maximum occurs relative to the predicted peak. The timing of the peak is also represented with a normal distribution with a user specified one-sigma value.

As referenced in the *OLMC* section, one facet of OLMC and its use of solar flux profiles is that after the solar flux file end date, the data is repeated as necessary. This can lead to discontinuities in the solar flux data used for a simulation. Solar flux predictions generally cover up to two periods of the solar cycle, which is approximately 22 years. After the last date in the solar flux file is reached, the solar flux is repeated over an assumed 11 year period. This 11 year period starts at the end of the file and finds the date 11 years prior to the end date to produce one "cycle", which is repeated as necessary. As a cautionary note, this assumption can lead to discontinuities in the repeated solar flux profile, as illustrated in Figure 4.

Launch Vehicle Dispersions

Launch vehicle dispersions represent the accuracy with which the launch vehicle is capable of placing a spacecraft into its target orbit. Launch vehicle Payload Planner's Guides typically specify the launch injection errors for both injection and non-injection apse. These errors can be used to determine the one-sigma value for given orbital parameters. In OLMC, the launch vehicle dispersions are accounted for by injection apse, velocity, and flight path angle. These are functionally equivalent to specifying injection and non-injection apse through orbital mechanics (Figure 5). For the solid upper stage launch vehicles that were the focus of this paper, launch dispersions are represented by a normal distribution in OLMC and the one-sigma values are set to reflect expected performance of the launch vehicle. For example, the Payload User's Guide for Pegasus states a three-sigma non-insertion apse error of ±90 km (no Hydrazine Auxiliary Propulsion System), which corresponds to an injection velocity of 7.63±0.0083 km/s (±one-sigma) for a 500 km circular orbit. The reader should be aware that discussions with launch vehicle providers about specific target orbits can lead to realistic reductions in the insertion and non-insertion apse errors. Launch vehicle guidance scheme, target altitude, and spacecraft mass can all have an effect on these quantities.

Launch Delays

Launch delays are typically programmatic in nature. Consideration of this factor for determining orbital lifetime is complicated by two primary issues: availability of applicable data (project scope, funding constraints, selected launch vehicle, etc.) and non-standardized methodology of characterizing delays (early or on-time launches, launch date definition, completeness of data set, etc.). Although it was originally envisioned that launch delays would be normally distributed, proprietary data obtained for the purpose of these analyses lead to the conclusion that Weibull distributions based on mission classes were more representative.

In place of having historical data, a fixed launch delay (non-MC parameter) can be modeled by varying the launch date. For example, the user can determine what the estimated lifetime would be for a spacecraft that launches one, two, or three years later than originally planned. This may or may not extend the lifetime but allows the user to test potential mitigation strategies.

Ballistic Coefficient

Variations of the ballistic coefficient allow for accommodation of uncertainties associated with projected area, drag coefficient, and mass. Of specific concern are design contingencies that may impact projected area and mass, variations in operational timelines, and drag coefficient uncertainties (which are typically in the range of 2.0-2.2). The ballistic coefficient can be varied by use of a normal distribution and is fixed over each MC case run.

RESULTS

Comparison of Tools

To check the validity of OLMC, fixed (non-MC) runs were performed to compare discrete results with those from STK/Pro. Test cases and results are shown in Table 2. The launch date for each case was April 1, 2000, and the solar flux file was an MSAFE solar flux prediction from 1997. Test cases 1-5 focused on varying the ballistic coefficient of the spacecraft with a fixed orbit. Test cases 6 & 7 covered elliptical orbits. Test cases 8 & 9 varied the inclination of the orbit. Test case 10 utilizes nominal solar flux.

All STK/Pro results were within 7% of OLMC except case 5. For case 5, STK/Pro version 6.2 was within 2% of OLMC, but STK/Pro version 7.0 reported an extremely long lifetime. The reason for this discrepancy is unknown.

Determination of Adequate Number of Monte Carlo Runs and Orbits per Iteration

For any defined scenario (solar flux profile, launch date, launch vehicle injection uncertainty, etc.), it is necessary for the user to determine the code settings that will produce a distribution of lifetime results that are ‘reasonable’ and repeatable. Reasonable refers to the realism of the distribution of the lifetime data. The driving parameter controlling the shape of the distribution is the predicted solar flux profile. Assuming that this profile is smooth and continuous, a nominal lifetime distribution should loosely mimic the shape of the solar flux curve. Other effects, such as launch delays, small variations in projected area, etc., are secondary drivers that have a tendency to ‘flatten out’ the lifetime distribution, decreasing the sharpness of any peak and spreading the data over a wider range of lifetimes. A repeatable distribution is one that can be numerically duplicated with a second and third run without significant variation in the lifetime distribution. It should be noted that the term ‘significant variation’ is fairly subjective. One wishes to establish that the mean (or median) lifetime and the probability of meeting a desired lifetime are repeatable within acceptable limits. Higher altitude orbits that span multiple solar cycles may exhibit multiple corresponding lifetime groupings (peaks), but each peak should still be smoothly distributed (see *Long Term Estimation* section).

Absent of other considerations, ensuring repeatability would consist of clamping down on the number of orbits per iteration in OLMC and increasing the number of MC runs to an exceedingly high number. However, the real world impact to this approach is a large increase in the runtime of the code. For practical purposes, two user controllable parameters with which the user can determine the ‘sweet-spot’ trade between runtime and repeatability are the number of MC runs conducted and the number of orbits per iteration used within the OL code when computing lifetime for each MC run.

Three representative Earth orbits were selected to investigate the sensitivity of results to varying these parameters, and to use as guidelines when establishing an OLMC run under different conditions. The chosen orbits were a 300 km circular, 525 km circular, and 300x700 km altitude elliptical orbit. The ballistic coefficient used for the analyses was ~40 kg/m^2. Pegasus launch vehicle injection errors, in cooperation with Orbital Sciences Corporation and NASA Kennedy Space Center, were defined for injection and non-injection apses based on the specifics of the test cases performed.

The total number of MC runs was varied for the setting of one orbit per iteration to determine the resultant effect on spacecraft lifetime distribution. Results for each of the chosen orbits are shown in Figures 6-8 as a histogram of the MC run data. Tables 3-5 show the mean and median of the data referenced to a chosen data set. The reference data set, plotted with a thicker line in each histogram, represents the best data set run for each orbit (most MC runs). Visual inspection of the histogram plot can and should be used to qualitatively review the adequacy of the chosen number of MC runs. Distributions should be smooth and largely free of localized minima or maxima if properly binned for the histogram. Quantitatively, if an adequate number of MC runs has been selected the median and mean should show little percentage change in subsequent runs.

This situation is illustrated in Figure 6 for the 300 km orbit, where the profiles approach the distribution of the 2,000 MC run case as the number of MC runs is increased. Table 3 indicates that even though the profiles become very rough with fewer MC runs, the median and mean stay relatively constant at less than 5% error from the reference data set. This error is indicative that such a low orbit is being dominated by high atmospheric density, and the lifetime is tightly grouped around about 0.7 months.

Results for the 525 km orbit are shown in Figure 7. At this altitude, the mean and median are much more sensitive to the variation in MC runs than for the 300 km orbit. At 500 MC runs and less, the median and mean both show increased percentage difference from the reference data. The repeated runs at 1,000 MC cases show a 5% (3.4 month, in this case) variation in median from each other, which indicates repeatability is questionable. The user would be recommended to run with at least 1000 MC runs, and preferably more if analysis time permits.

Results for the 300x700 km orbit are shown in Figure 8. Again, as the number of MC runs is increased, the distribution smoothes out and approaches the reference data set. There is noticeable jaggedness in the 100 and 500 MC run cases, although the mean and median are not seen to shift (Table 5). This effect is attributed to the low perigee driving the fairly tight clustering of lifetime values.

The lifetime distribution sensitivity to the number of orbits per iteration within the OL kernel code was also investigated. Unless otherwise specified, all results are for 1,000 MC run cases. Results are shown in Figures 9-11 and Tables 6-8.

The 300 km circular orbit shows a high sensitivity to the number of orbits per iteration, as might be expected (Figure 9). The high drag, due to a low starting altitude, means that more frequent iterations are necessary to ensure accuracy. Significant shifts in the mean and median are seen at runs above 10 orbits per iteration. It is also interesting to note that at 100 orbits per iteration the jagged, unrealistic lifetime distribution was repeatable and insensitive to the number of MC runs (both 1,000 and 2,000 runs were performed). This implies some artificial binning of lifetimes due to an averaging of too many orbits between iterations. This case clearly illustrates why repeatability alone is not an adequate measure of the correct setting of runs and orbits per iteration. One must also consider the actual lifetime distribution and its realism.

Results for the 525 km orbit are shown in Figure 10. The higher starting altitude makes this case much less sensitive to the value of orbits per iteration chosen. No significant shift in mean and median are seen out to 200 orbits per iteration, with only one data set (20 orbits per iteration) showing a change in median greater than 2% from the reference data set.

The 300x700 km orbit results (Figure 11, Table 8) displayed more sensitivity to orbits per iteration than 525 km, but less sensitivity than the 300 km circular orbit. Mean and median shifts were noticeable in both the histogram and the statistical data above 10 orbits per iteration.

OLMC Runtime Estimates

Runtime values for the test cases performed are summarized in Figures 12-14. As expected, runtime increases significantly as the number of orbits per iteration are reduced, or the number of MC runs is increased. Notice that the runtime is extremely sensitive to the number of orbits per iteration (plotted on a log scale), while runtime increases linearly with the number of MC runs. Runtime is also seen to be highly dependent upon the orbit selected and the solar flux predictions selected for the run.

Sensitivity to Solar Flux Profile

As shown in Figure 3 and Table 1, solar flux predictions can vary significantly, which underscores the intrinsic volatility of solar flux predictions not only between distinct models but also predictions made at different times by the same source. For illustrative purposes, Figure 15 displays results of a spacecraft's lifetime using Schatten predictions from March 2004 (0304Schatten) and July 2005 (0705Schatten), and a Marshall Space Flight Center prediction from July 2005 (0705MSAFE). It is clear from the figure that the histogram peak shifts significantly to the left for 0705MSAFE compared to 0705Schatten, resulting in a lower probability of meeting a specified lifetime requirement. The histogram peak moves in a similar fashion from 0705Schatten to 0304Schatten, highlighting the impact of volatility of the predictions from a given source over time. Consequently, given the sensitivity of lifetime predictions to solar flux profiles coupled with their intrinsic variability, it is recommended that the user consider more than one profile when assessing orbit lifetimes.

Long Term Estimations

Although not originally a driver in the creation of OLMC, the long term estimations of lifetime became of interest upon reviewing some of the solar flux variation sensitivities (refer to Figure 15). In these sensitivities, the lifetime distribution adopted the behavior of having multiple peaks, about which the lifetimes are more frequently clustered. While this behavior was originally a bit of a mystery, a comparison of the solar flux profile to the lifetime distribution, coupled with the fact that the orbit spans multiple solar cycles, illustrates that the lifetime peaks are clustered around the repetitive solar cycle peaks. This highlights the dependence of orbital lifetime on the integral of solar flux (see Figure 16), where, as the integral increases rapidly, more and more lifetime cases are 'captured'. When solar flux is at a minimum (the flatter portion of the integral curve), fewer lifetime cases are expected to terminate. The histogram for these analyses is shown in Figure 17.

CONCLUSIONS

A Monte Carlo approach was implemented that utilized demonstrated orbit lifetime analysis software for the purpose of providing probabilistic estimates of spacecraft lifetime. Code verification and test case demonstration for multiple representative orbits were conducted. This effort resulted in the following significant findings:

- For a given target orbit, solar flux predictions are the primary driver for variations in lifetime. Given their intrinsic variability, caution should be exercised when utilizing single predictions. It is recommended that multiple prediction sets be used to fully characterize the lifetime variations of a spacecraft with respect to solar flux.
- Solid upper stage launch vehicle performance for a Monte Carlo simulation is probably best modeled as normal distributions around injection altitude (or 3-D position, if desired), injection velocity, and flight path angle. Liquid systems may exhibit a different behavior.
- Weibull distributions were determined to be a better fit for launch delays when compared to normal distributions. Launches are rarely, if ever, early which intuitively supports a skewed distribution, such as a Weibull.
- Normal distributions may not be the best solution to characterize the variation of solar flux predictions, since they exhibit multi-dimensional dependence (phase, amplitude, frequency, prediction source, prediction date).
- Lower altitude orbits display a high sensitivity to the orbits per iteration parameter, and less sensitivity to the total number of Monte Carlo runs.
- For elliptical orbits, the perigee value, and not the semi-major axis, should drive selection of the orbits per iteration parameter and the number of Monte Carlo runs.
- For any scenario, visual inspection of the lifetime histogram and computation of mean and median variation should be done at multiple settings for orbits per iteration and number of Monte Carlo runs to determine acceptable settings for desired 'reasonable' distribution and repeatability. To minimize the total runtime, the maximum acceptable value for orbits per iteration and the minimum acceptable number of Monte Carlo runs should be used.

FUTURE WORK

As one of the largest contributors to the uncertainty in lifetime analyses, the focus of the future work will be on the solar flux predictions. A new prediction method, developed by NCAR (National Center for Atmospheric Research), will be examined with the goal of integrating its products into OLMC. Another significant development area will be the mathematical characterization of multi-dimensional solar flux predictions to better capture their inherent uncertainties.

Methodology to address post-mission disposal requirements as defined in NASA Safety Standard (NSS) 1740.14, Guidelines and Assessment Procedures for Limiting Orbital Debris, will be investigated. It is proposed that the generation of OLMC-based probabilistic lifetime distributions to address post-mission disposal requirements will provide additional insight when compared to the current NASA standard Debris Assessment Software with its assumption of a constant 130 solar flux unit.

ACKNOWLEDGMENTS

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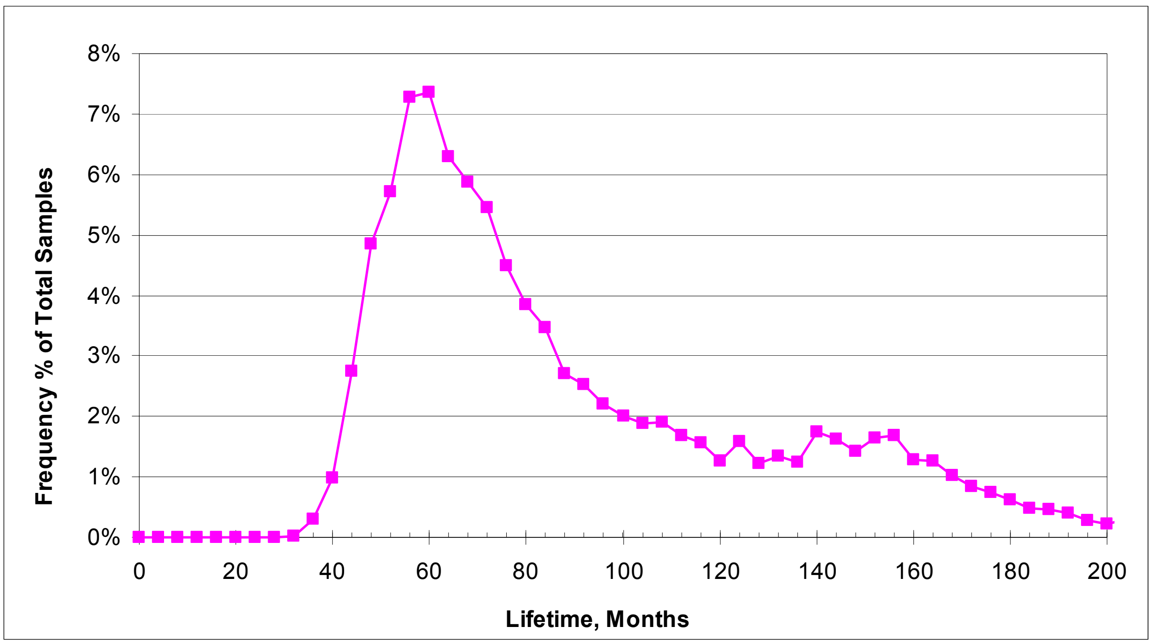
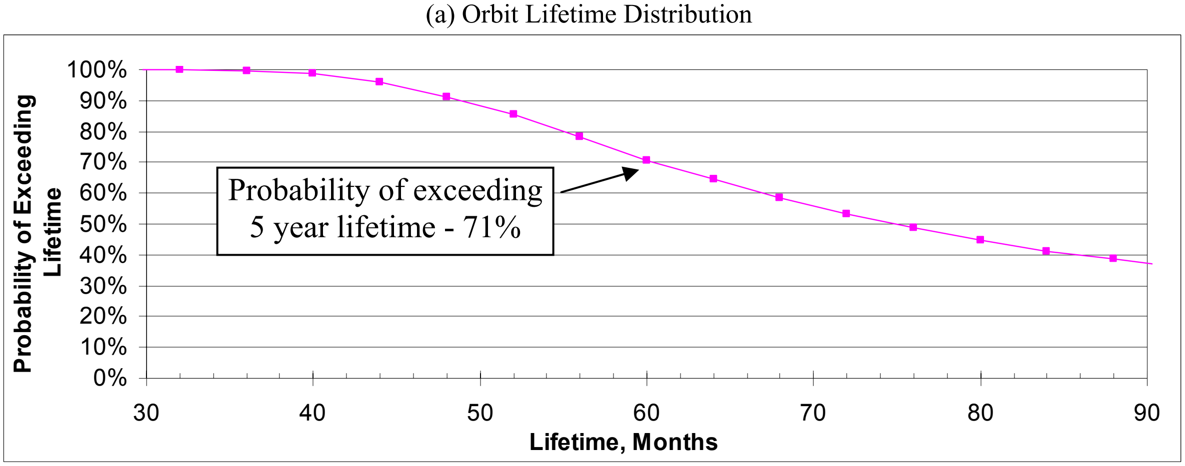


(a) Orbit Lifetime Distribution



(b) Probability of Exceeding a Given Orbit Lifetime

Figure 1. Example of Histogram and Lifetime Probability

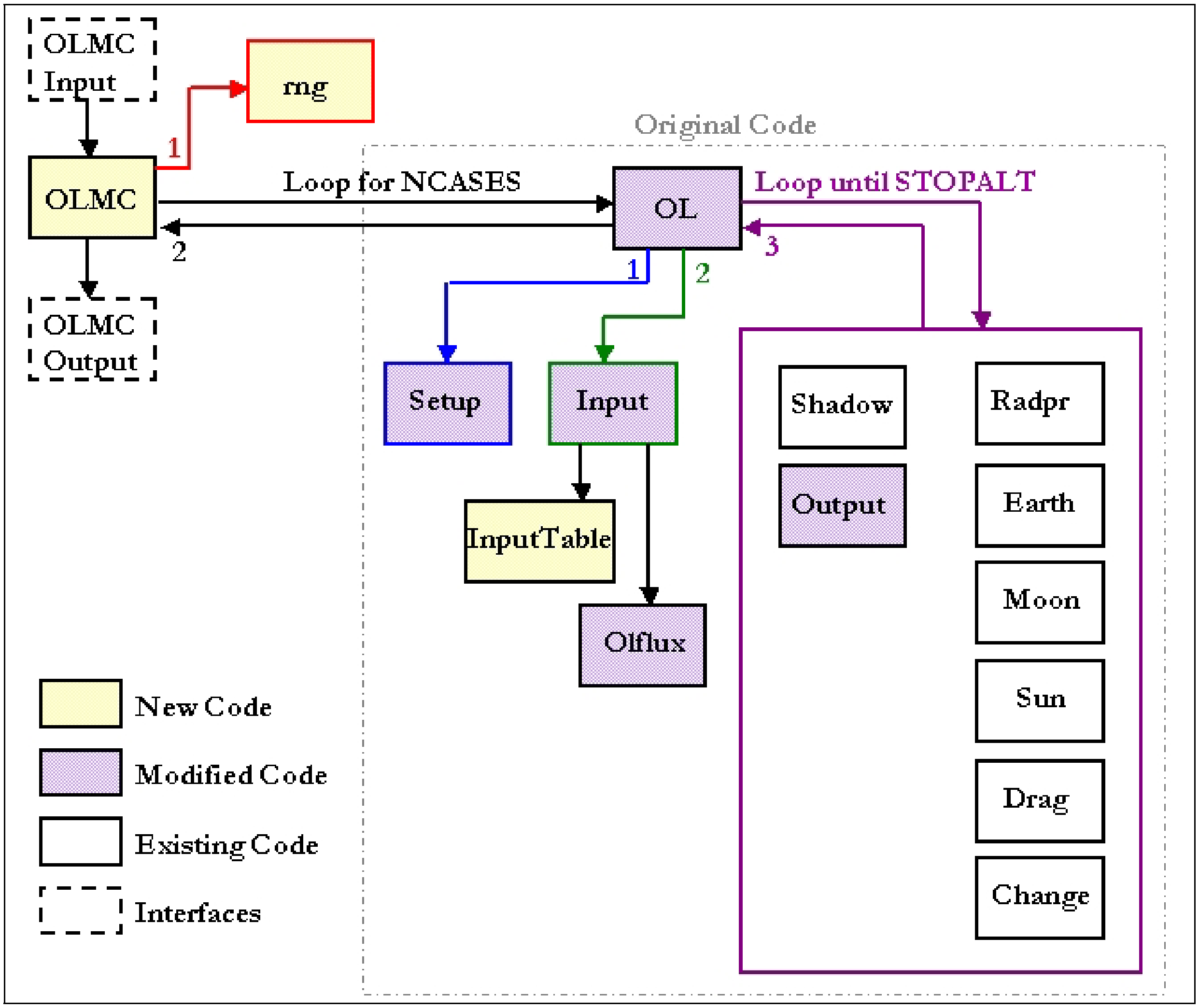


Figure 2. OLMC Flow Chart

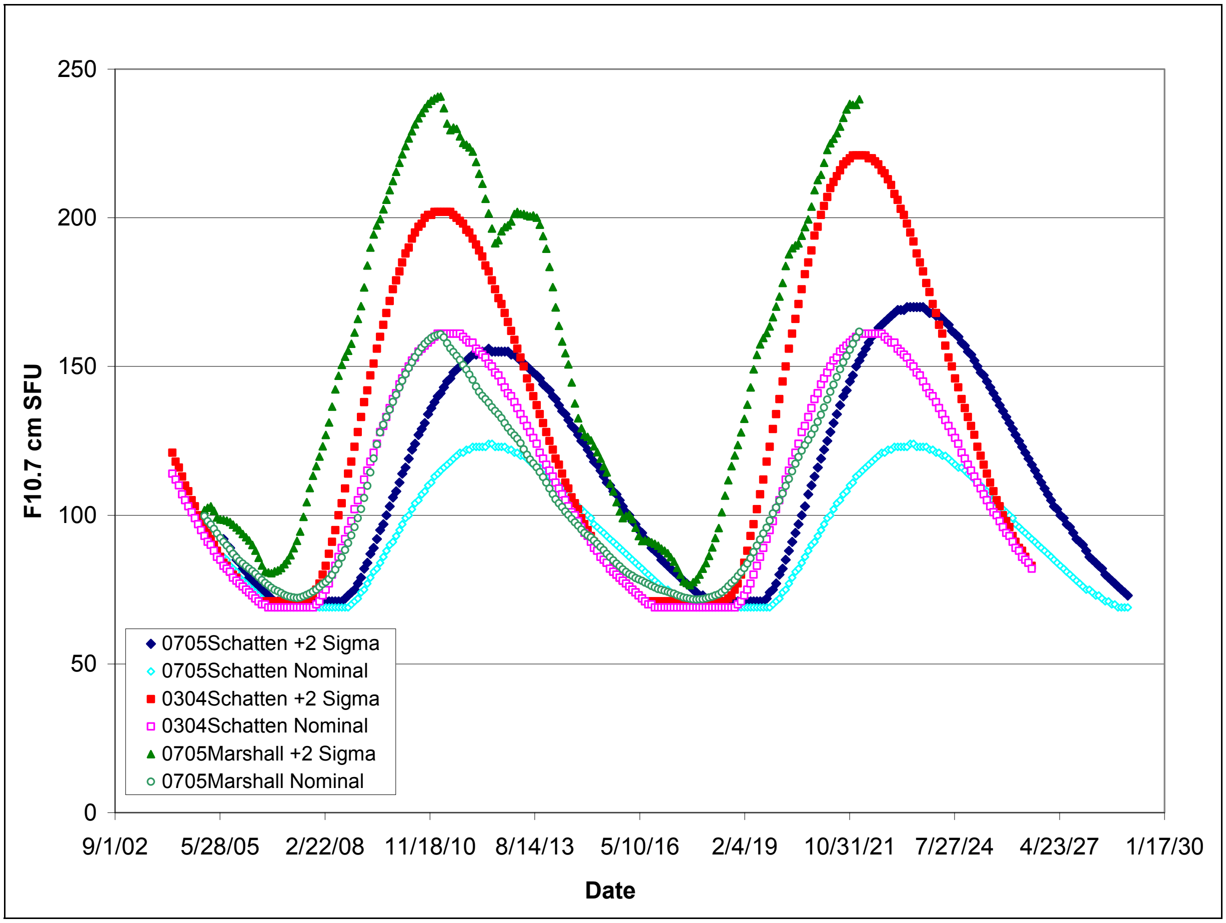


Figure 3. Solar Flux Predictions

Table 1. Solar Flux Predictions Peak Values

Solar Flux Data	Nominal	+2 Sigma	Cycle 24 Peak
0705Schatten	124 SFU	156 SFU	June 2012
0304Schatten	161 SFU	202 SFU	Feb-May 2011
0705MSAFE	161 SFU	241 SFU	March 2011

SFU – Solar Flux Unit, 10^{-22} W/(m^2 Hz)

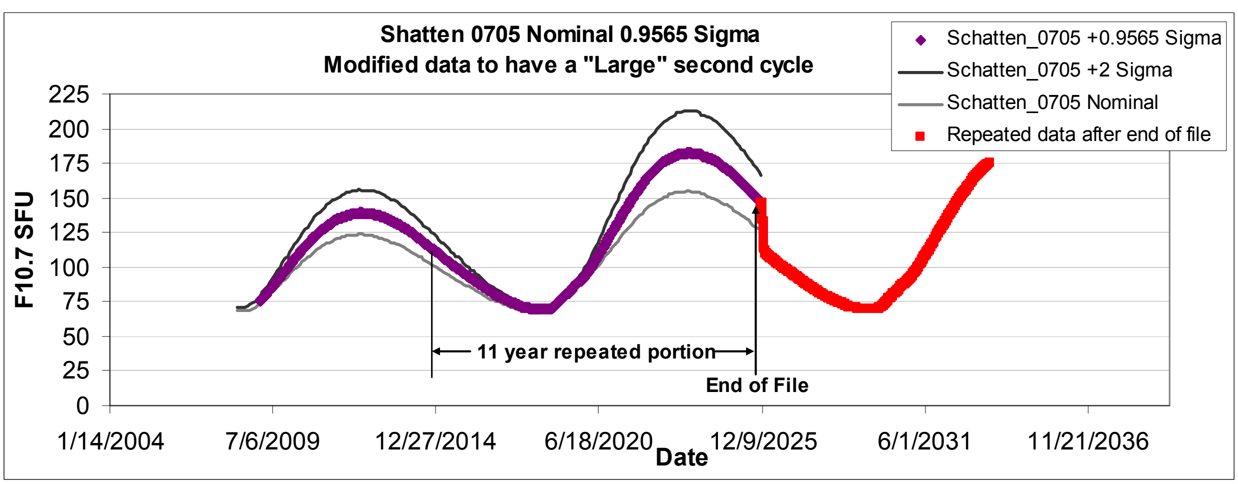


Figure 4. Solar Flux File Discontinuity Example

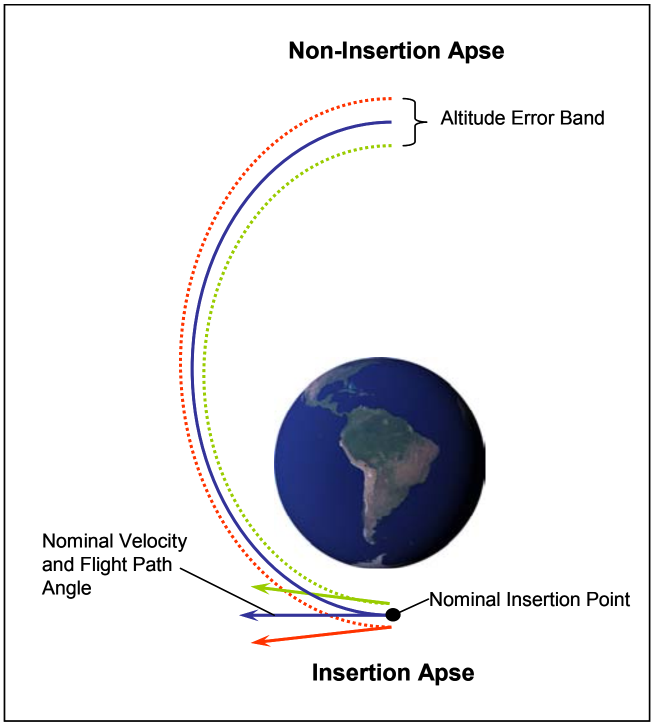


Figure 5. Relation of Injection Apse Velocity and Flight Path Angle Variation to Non-injection Apse Altitude Error (Not to Scale)

Table 2. Results of Tool Comparison

Case Number	1	2	3	4	5	6	7	8	9	10
Mass (kg)	1000	2300	230	23	23000	2300	2300	2300	2300	2300
Drag area (m^2)	10	10	10	10	10	10	10	10	10	10
Sun area (m^2)	10	10	10	10	10	10	10	10	10	10
Drag Coefficient	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3
Reflection Coefficient	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Nominal or +2 sigma	+2	+2	+2	+2	+2	+2	+2	+2	+2	Nom
Apogee (km)	500	500	500	500	500	500	1000	500	500	500
Perigee (km)	500	500	500	500	500	350	350	500	500	500
Inclination (deg)	28.5	28.5	28.5	28.5	28.5	28.5	28.5	51.6	90	28.5
Argument of Perigee (deg)	0	0	0	0	0	0	0	0	0	0
Long. of Asc. Node (deg)	0	0	0	0	0	0	0	0	0	0
Results (d – day, y – year)										
STK/Pro version 7.0	**355 d**	**2.6 y**	**75 d**	**7 d**	**132 y**	**223 d**	**4.5 y**	**2.7 y**	**2.6 y**	**9.9 y**
STK/Pro version 6.2	**343 d**	**2.4 y**	**72 d**	**7 d**	**40.1 y**	**217 d**	**4.0 y**	**2.5 y**	**2.4 y**	**9.8 y**
OLMC	**343 d**	**2.5 y**	**71 d**	**6.8 d**	**40.9 y**	**218 d**	**4.2 y**	**2.6 y**	**2.5 y**	**9.3 y**

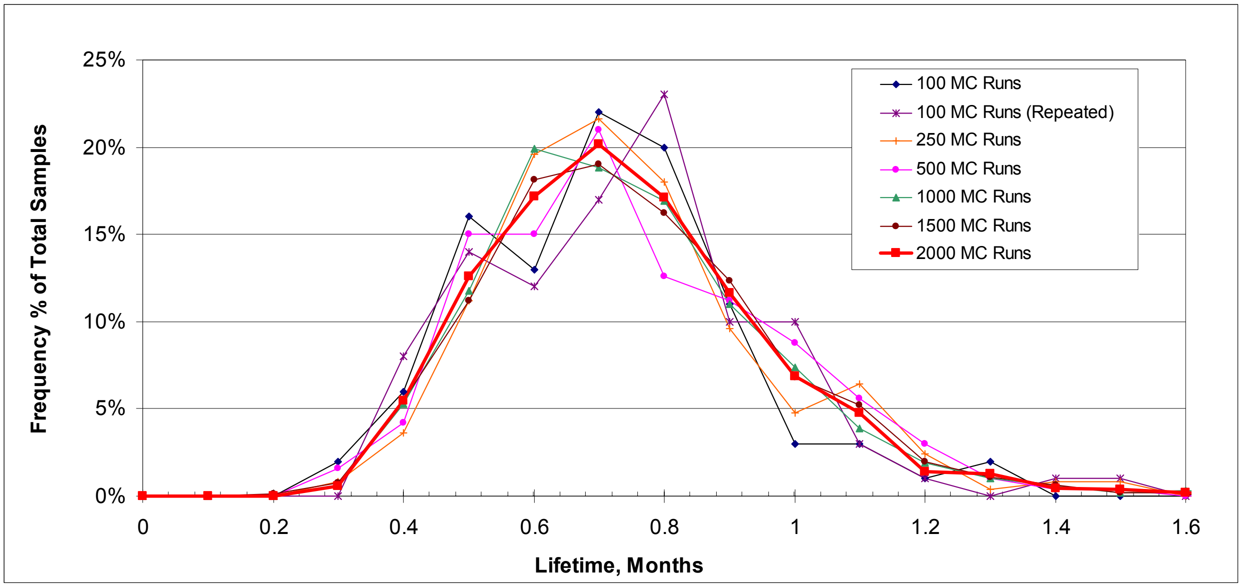


Figure 6. Lifetime Distribution for 300 Km Circular Orbit Sensitivity to Number of MC Runs

Table 3. Statistics for 300 Km Circular Orbit Runs (1 Orbit per Iteration)

Case	Mean	Median
	Months	Months
100 MC Runs	0.668	0.639
100 MC Runs (Repeated)	0.691	0.685
250 MC Runs	0.699	0.673
500 MC Runs	0.701	0.662
1000 MC Runs	0.692	0.666
1500 MC Runs	0.702	0.669
2000 MC Runs	**0.691**	**0.670**

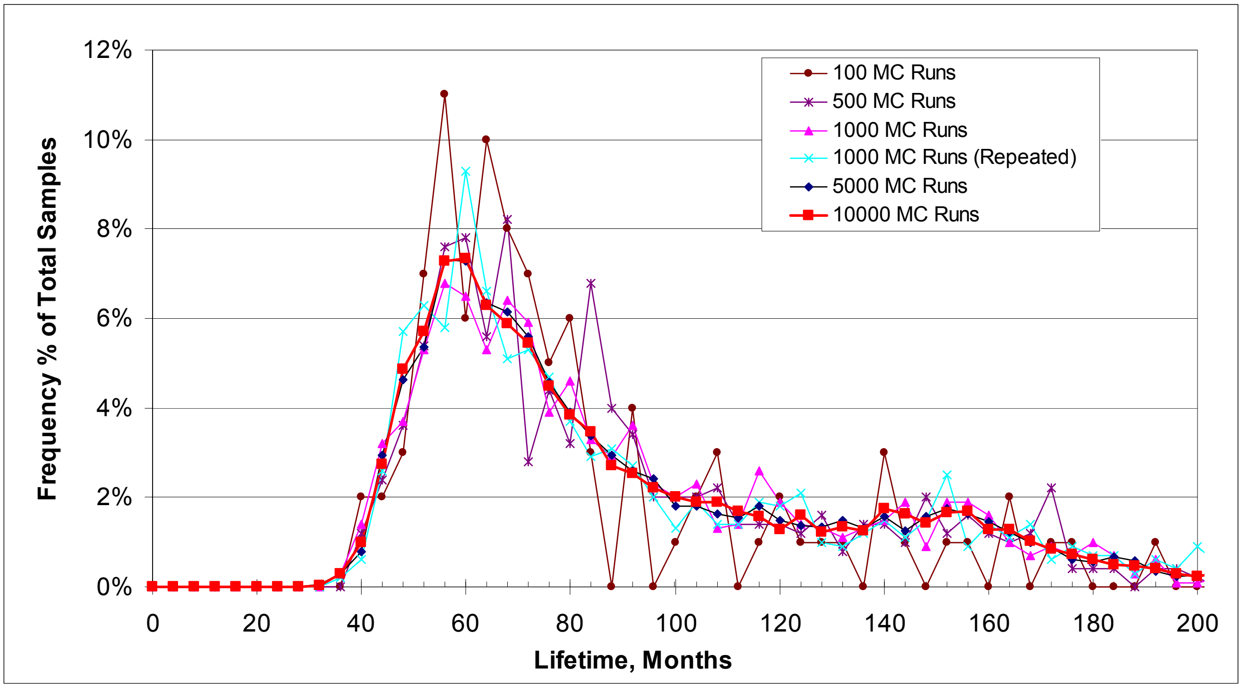


Figure 7. Lifetime Distribution for 525 Km Circular Orbit Sensitivity to Number of MC Runs

Table 4. Statistics for 525 Km Circular Orbit Runs (1 Orbit per Iteration)

Case	Mean	Median
	Months	Months
100 MC Runs	83.9	68.5
500 MC Runs	89.6	77.3
1000 MC Runs	91.8	77.4
1000 MC Runs (Repeated)	91.0	74.0
5000 MC Runs	91.1	74.5
10000 MC Runs	**91.1**	**74.6**

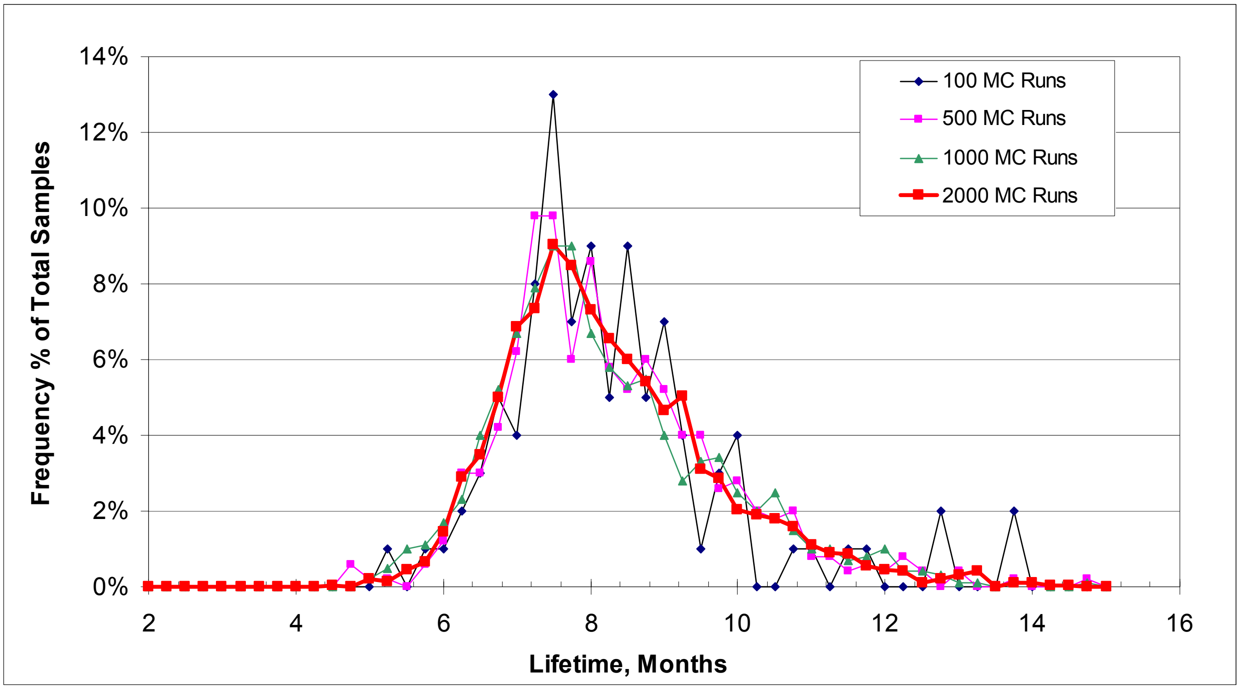


Figure 8. Lifetime Distribution for 300x700 Km Orbit Sensitivity to Number of MC Runs

Table 5. Statistics for 300x700 Km Orbit Runs (1 Orbit per Iteration)

Case	Mean	Median
	Months	Months
100 MC Runs	8.17	7.86
500 MC Runs	8.16	7.88
1000 MC Runs	8.11	7.79
2000 MC Runs	**8.14**	**7.88**

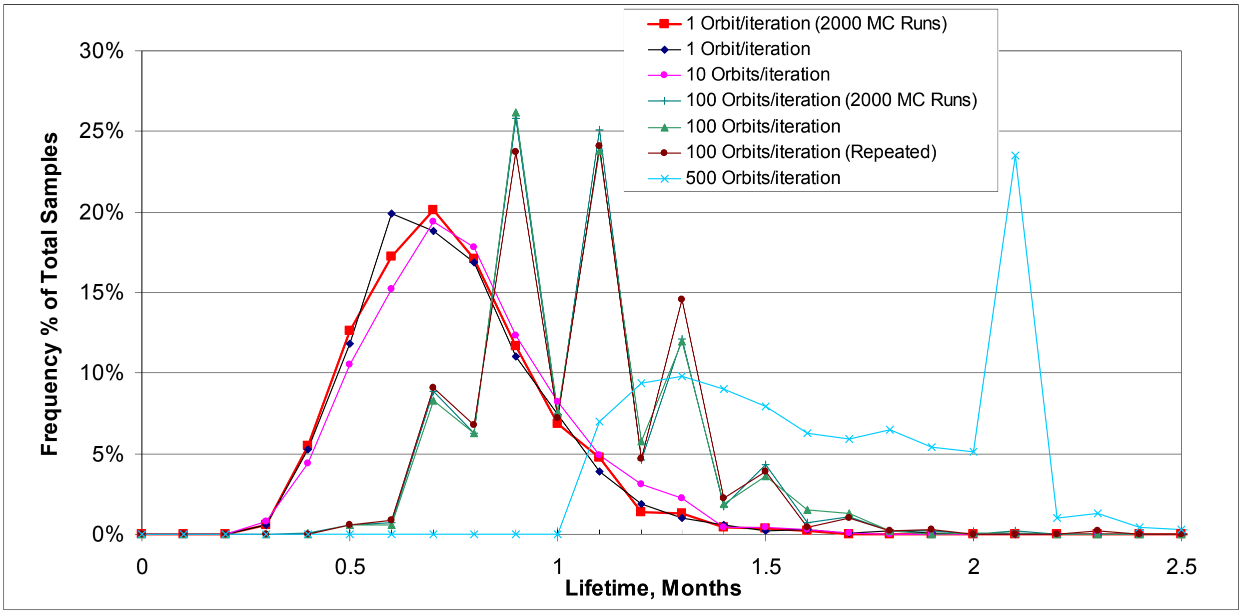


Figure 9. Lifetime Distribution for 300 Km Circular Orbit Sensitivity to Number of Orbits per Iteration

Table 6. Statistics for 300 Km Circular Orbit Runs (1000 MC Runs)

Case	Mean	Median
	Months	Months
1 Orbits/iteration (2000 MC Runs)	**0.69**	**0.67**
1 Orbit/iteration	0.69	0.67
10 Orbits/iteration	0.72	0.69
100 Orbits/iteration	1.01	1.01
100 Orbits/iteration (Repeated)	0.99	1.01
100 Orbits/iteration (2000 MC Runs)	0.97	1.01
500 Orbits/iteration	1.62	1.61

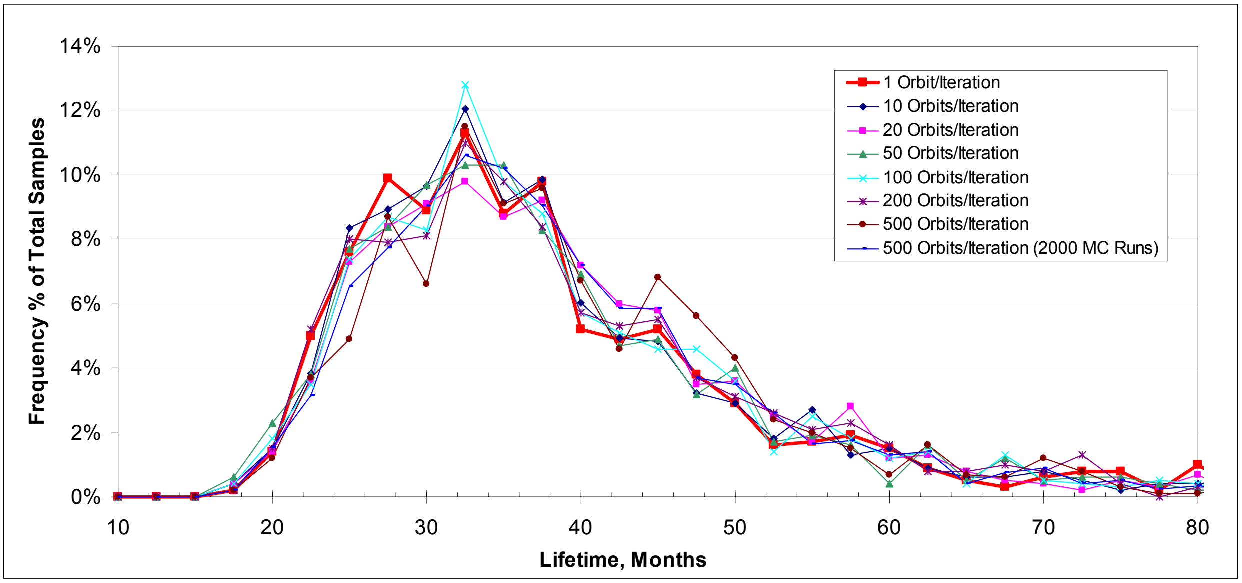


Figure 10. Lifetime Distribution for 525 Km Circular Orbit Sensitivity to Number of Orbits per Iteration

Table 7. Statistics for 525 Km Circular Orbit Runs (1000 MC Runs)

Case	Mean	Median
	Months	Months
1 Orbit/Iteration	**38.88**	**33.94**
10 Orbits/Iteration	38.47	33.91
20 Orbits/Iteration	39.51	35.26
50 Orbits/Iteration	38.92	34.01
100 Orbits/Iteration	38.68	34.27
200 Orbits/Iteration	38.84	34.29
500 Orbits/Iteration	40.86	36.21
500 Orbits/Iteration (2000 MC Runs)	39.80	35.29

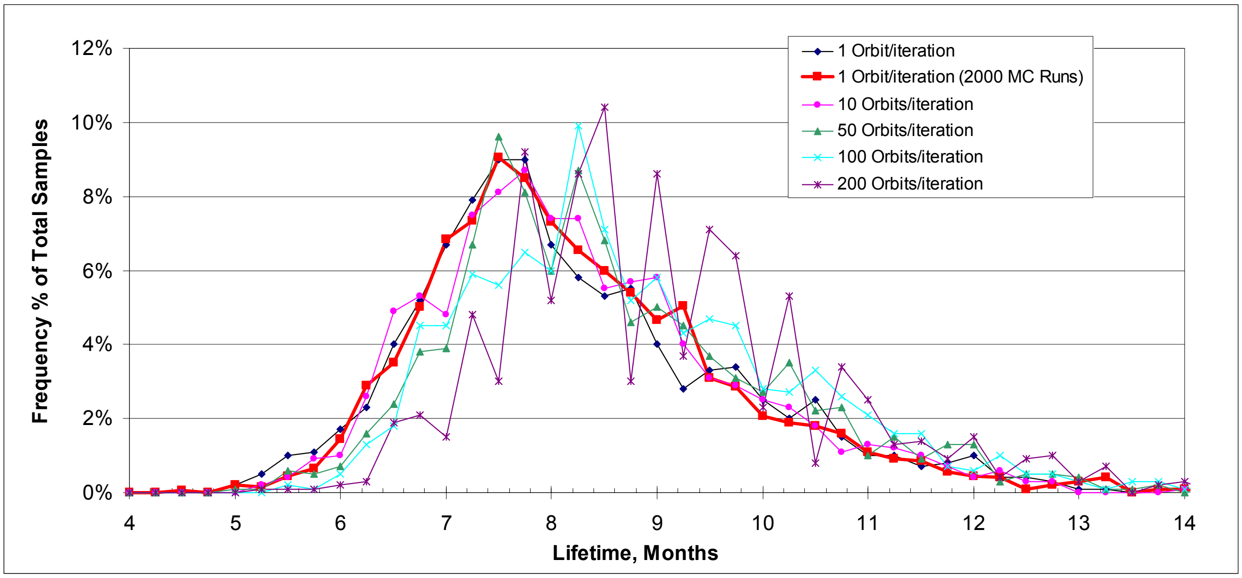


Figure 11. Lifetime Distribution for 300x700 Km Orbit Sensitivity to Number of Orbits per Iteration

Table 8. Statistics for 300x700 Km Orbit Runs (1000 MC Runs)

Case	**Mean**	**Median**
	Months	Months
1 Orbit/iteration	8.11	7.79
1 Orbit/iteration (2000 MC Runs)	**8.14**	**7.88**
10 Orbits/iteration	8.16	7.95
50 Orbits/iteration	8.48	8.17
100 Orbits/iteration	8.65	8.43
200 Orbits/iteration	8.92	8.67

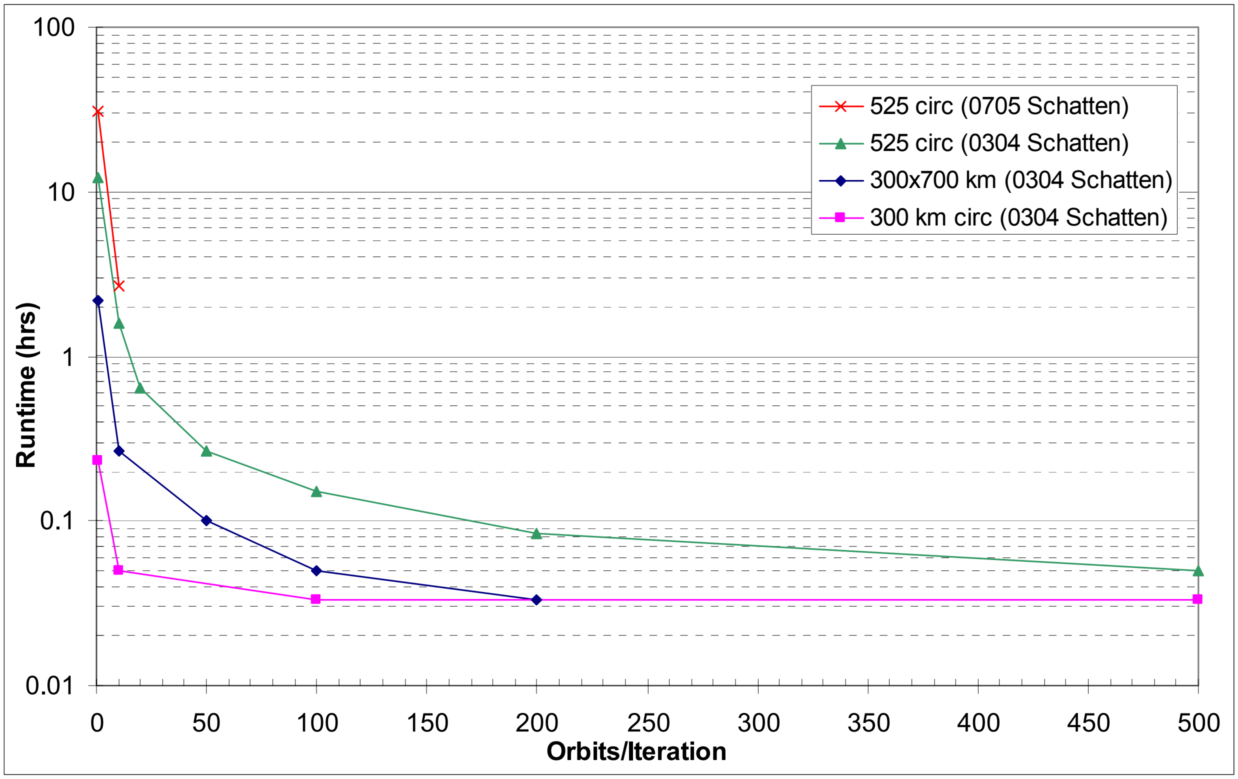


Figure 12. Code Execution Time Sensitivity to Orbits per Iteration, All for 1000 Monte Carlo Runs (3.6 GHz Pentium 4 with 4 GB RAM)

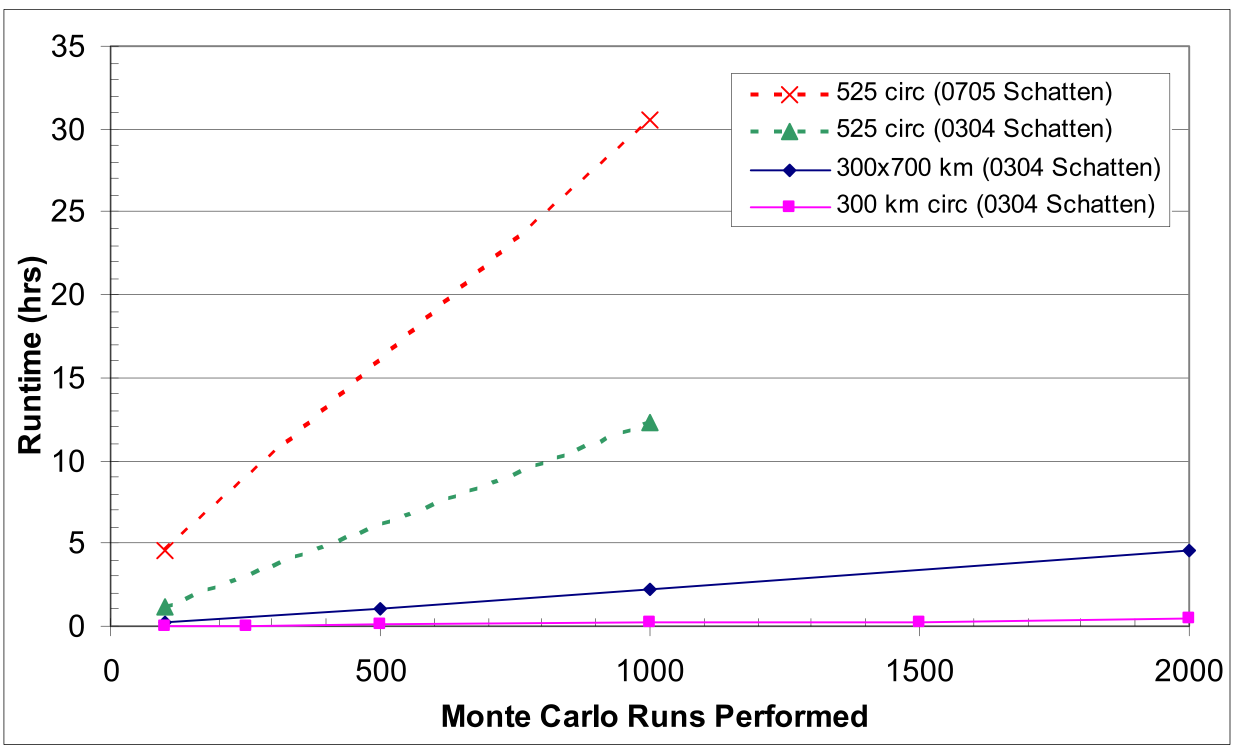


Figure 13. Code Execution Time Sensitivity To Number Of MC Runs, All at 1 Orbit per Iteration (3.6 GHz Pentium 4 with 4 GB RAM)

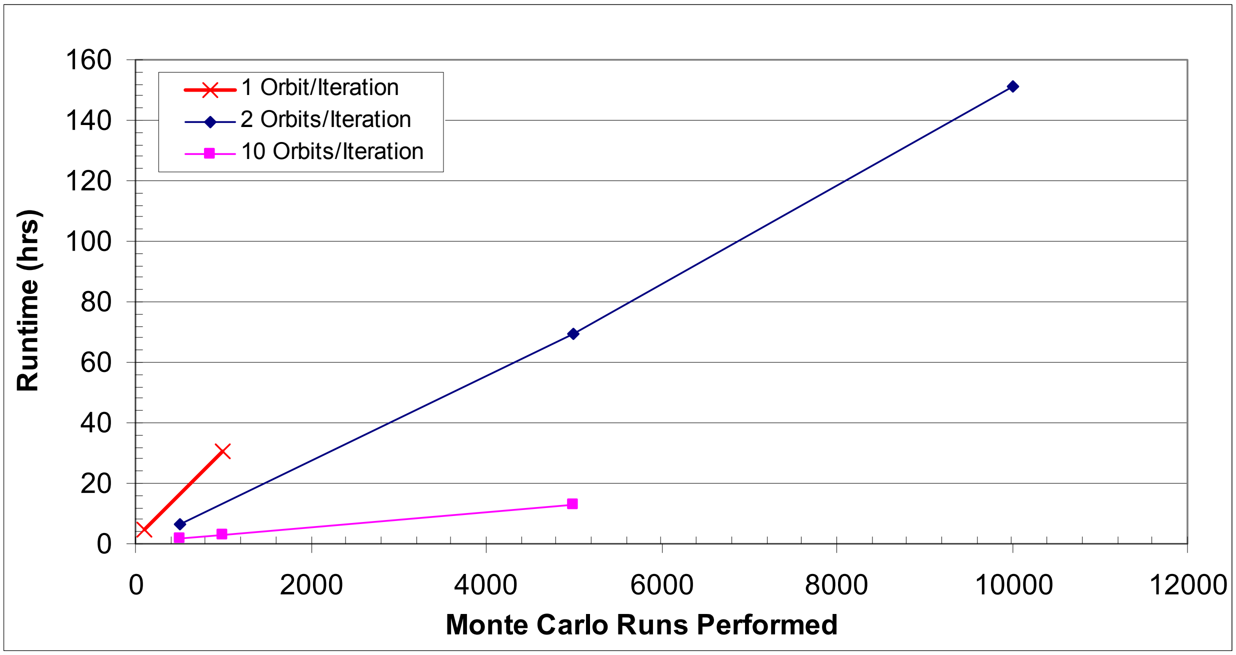


Figure 14. Code Execution Time Sensitivity To Number Of MC Runs, 525 Km Orbit Using 0705 Schatten Solar Flux Predictions
(3.6 GHz Pentium 4 with 4 GB RAM)

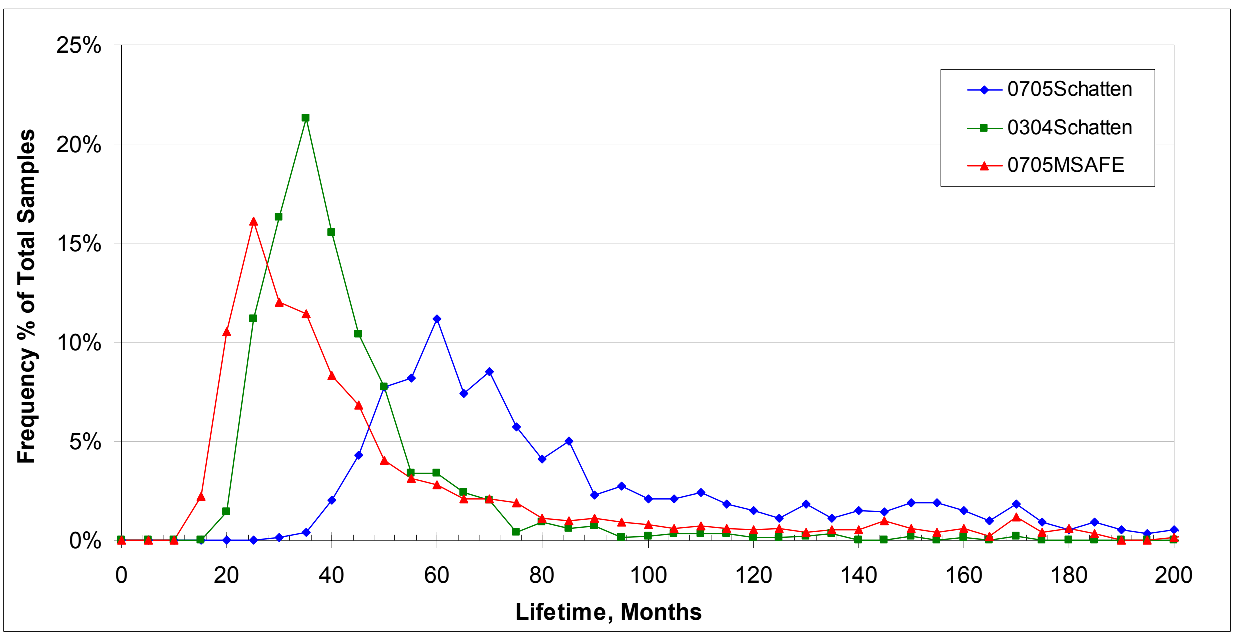


Figure 15. 525 Km Orbit Sensitivity to Solar Flux Profile

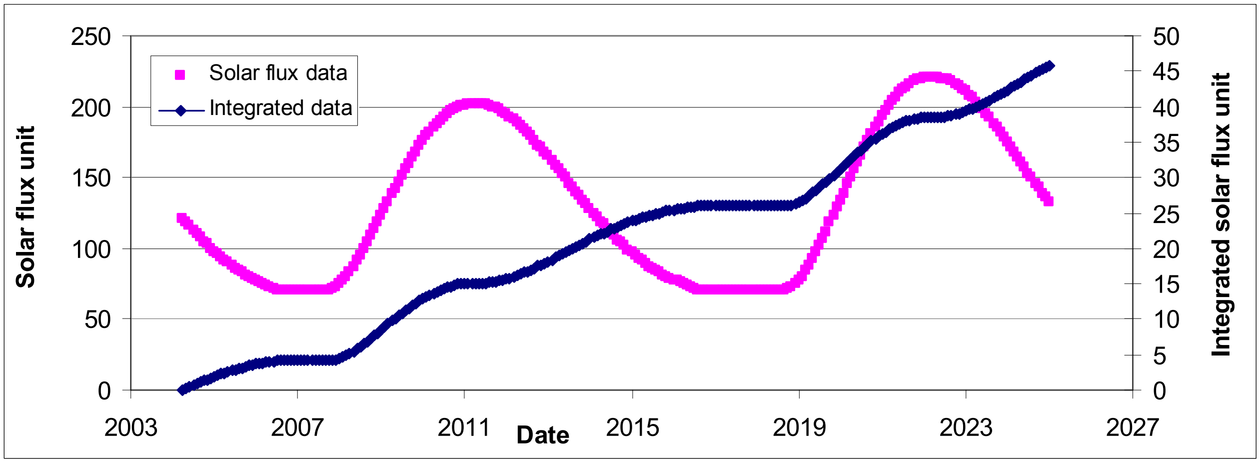


Figure 16. Integration of Solar Flux Profile

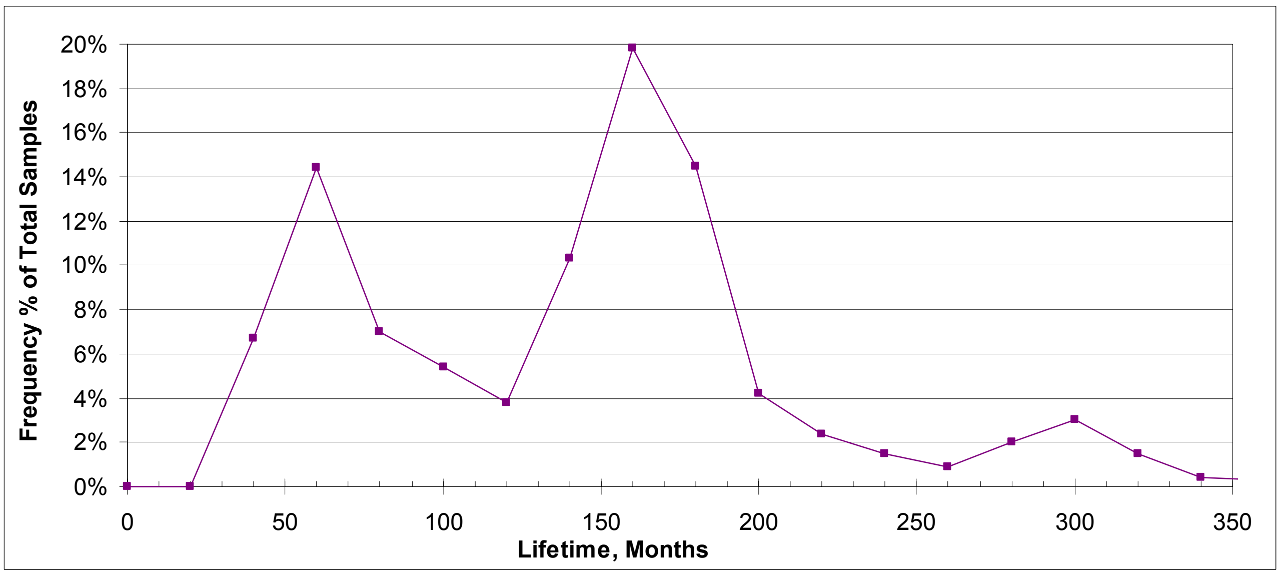


Figure 17. Long Term Results