A New Adaptive Decision Maker Framework for the Exploration in an Unknown Region Using Multiple Mobile Sensors

Mohammad Shekarami, Todd K. Moon, and Jacob H. Gunther
Information Dynamics Laboratory, ECE Dept., Utah State University

Abstract

- Objective: Configuration of multiple mobile sensors to explore an unknown region, where the exploration trades off between two desiderata:
  - To continue taking data in a region known to be interesting with the intent of refining the measurements versus
  - Taking data in unobserved areas to attempt to discover new interesting regions
- Proposed Framework:
  - Prediction stage: Predicts the behavior of phenomenon of interest (PoI) at unseen locations
  - Decision maker stage: Deciding on the next trajectory of sensors

Background on Gaussian Processes (GPs)

- Gaussian process (GP) modeling
  - Assumption: The PoI can be evaluated via an unknown function \( f(\cdot) \)
  - GP is a non-parametric model
  - Core idea: Any observation is an outcome of a Gaussian random variable
  - All of these random variables are jointly Gaussian
- Gaussian Process Regression (GPR)
  - Suppose we have access to a set of \( N \) noisy observations \( y = f(U) + \varepsilon \sim \mathcal{N}(0, \Sigma_U) \)
  - Goal: Prediction of \( f \) evaluated at some other input data set \( U_o = [u_1, \ldots, u_M]^T \)
    \[ f(U_o|U) = \mathcal{N}(\mu, \Sigma) \]
    where
    \[ \mu_U = \mu_U(U) = K(U, U) (K(U, U) + \sigma^2 I)^{-1} y \]
    \[ \Sigma = K(U_o, U) - K(U_o, U) K(U, U) (K(U, U) + \sigma^2 I)^{-1} K(U, U_o) \]

Proposed Framework: (1), Prediction (GPR) Stage

- \( S = \{ y^{(1)}, y^{(2)}, \ldots, y^{(M)} \} \): A set of all feasible trajectories of the sensors through the region
- \( U_{km} = [u^{(1)}_{km}, u^{(2)}_{km}, \ldots, u^{(M)}_{km}] \): The location of measurements along the trajectory \( s^{(k)} \)
- Defining the kernel function
  \[ K(u, u') = \exp \left[ -\frac{1}{\sigma^2} \| u - u' \|^2 \right] \]
  - \( T_{km} \): the mth time the satellite visits the region under study
- Applying the GPR model, we obtain a prediction of the PoI throughout the region and the corresponding amount of uncertainty
  - \( \hat{y}(U_o) \): The estimate of the PoI for the pth location along the trajectory \( s^{(k)} \)
  - \( \hat{\Sigma}(U_o) \): The associated measure of variance for the pth location along the trajectory \( s^{(k)} \)

Proposed Framework: (2), Decision Maker Stage

- Optimization problem
  \[ \hat{k} = \arg \max_k \quad \hat{J}(U_{km}) + \lambda \hat{\Sigma}(U_{km}) \]
  - \( \hat{k} \) represents the kth trajectory from the dictionary of feasible trajectories
  - \( \lambda \) is a tuning parameter that emphasizes on the desire to explore
  - \( \hat{J}(U_{km}) = \frac{1}{2} \sum_{j=1}^{M} \hat{y}(u_{kj}) \) and \( \hat{\Sigma}(U_{km}) = \frac{1}{2} \sum_{j=1}^{M} \hat{\Sigma}(u_{kj}) \)
  - \( \lambda \) may depend on time
- For the satellite constellation problem, \( \lambda \) can be related to the required \( \Delta V \)-budget for the orbit transfer and the available thrust on the satellites

Simulation Results

- An example showing the satellites orbits and the region of under study
- From left to right: Initial measurement, measure of variance, and reconstruction of PoI
- Results using the initial and the first two determined trajectories for case (1) i.e., when \( \lambda = 0.25 \)
- Results after 13 determined trajectories for case (1) i.e., when \( \lambda = 0.25 \)
- Results after 13 determined trajectories for case (2) i.e., when \( \lambda = 2 \)
- Results after 13 determined trajectories for case (3)
- Results after 13 determined trajectories for case (4)

Future Work

- Sensor array configuration via the feasible trajectories and orbit assignment via \( \Delta V \)-budget
  - Assumption: There is a dictionary of possible trajectories generated via Keplerian orbital elements
  - Compute the required \( \Delta V \) for the orbit transfer for all the satellites in the constellation
  - Find the one which requires minimum \( \Delta V \) to transfer to the target orbit
  - Example: The decision making on the orbits here is only based on the exploration desire

Acknowledgments

This work is supported by the Grant NASA NNX13 AD 39A.