A two-fraction model for the transport of sand/gravel mixtures

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[1] A mixed-size sediment transport model using only two fractions, sand and gravel, offers practical advantages in terms of the required input and conceptual advantages by readily incorporating the nonlinear effects of sand content on transport rate. A two-fraction model is developed from flume and field transport observations. Sand and gravel transport rates are collapsed to a single transport function using a scaling parameter representing incipient motion for each fraction. The incipient motion function is consistent with established values in the limit of pure sand and gravel beds and shows a sharp change with increasing sand content over the transition from a clast- to matrix-supported bed. A two-fraction model captures an important and previously unmodeled nonlinear effect of sand content on gravel transport rate and suggests important implications for a number of fluvial processes, including the response of channels to sediment supply and the mechanism for abrupt sorting in gravel/sand transitions. *INDEX TERMS:* 1815 Hydrology: Erosion and sedimentation; 1824 Hydrology: Geomorphology (1625); 3022 Marine Geology and Geophysics: Marine sediments—processes and transport; *KEYWORDS:* sediment transport, sand and gravel, gravel bed rivers, transport models, fluvial geomorphology

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1. Introduction

[2] The wide range of grain sizes found in most gravel bed rivers poses a difficult and incompletely solved problem for the successful modeling of transport rate. The influence of grain size on transport rate can be described qualitatively as a competition between absolute and relative grain size effects. The absolute size effect causes the inherent mobility of sediment grains to decrease with increasing grain size. A given flow will transport finer sediment more rapidly than coarser sediment. When different sizes are placed in a mixture, the relative size effect tends to increase the transport rate of larger grains and decrease the transport rate of smaller grains. The magnitude of the relative size effect and, therefore, the transport rate of individual sizes within a mixture, will be sensitive to the composition of the mixture, which can change during transport and in response to variation in flow and sediment supply. A quantitative model for mixed-size transport must account for the distribution of grain sizes available for transport.

[3] Early transport models avoided some, but not all, of the difficulties associated with specifying size distribution by predicting the total transport rate as a function of a single representative grain size [e.g., *Meyer-Peter and Müller*, 1948; *Engelund and Hansen*, 1972]. This approach is relatively practical because the only sediment information required is the representative size, but it is unable to predict changes in grain size and it is also likely to underpredict the transport rate of the finer fractions, which may be much larger than that of the coarser fractions [*Leopold*, 1992; *Lisle*, 1995]. To account for size-dependent variations in transport, transport models can be formulated for many finely divided size fractions. This approach is able to capture variation in transport rate among different sizes, as well as interactions among different sizes. This detail comes at the expense of greater computational effort and, more critically, requires specification of the full size distribution of the bed sediment. The grain size distribution in a reach is generally not accurately known, is subject to variation during transport, and is sensitive to the history of flow and sediment supply. At present, these data constraints make many-fraction transport models useful primarily for simulation, rather than for predicting the transport at a particular location, for which specific grain size information is required [*Wilcock*, 2001b].

[4] This paper presents a transport model in which the bed size distribution is divided into two fractions, sand and gravel. The effort required to determine the proportion of sand and gravel in a reach is comparable to that required to determine a representative grain size (and much less than that required to determine the full size-distribution), so a two-fraction model retains much of the practicality of a single-fraction estimate, while permitting variation in bed grain size through changes in the relative proportion of sand and gravel. This provides a means of predicting the variation in the fines content of the bed, which may often be more variable than that of the coarse fraction, and whose passage, intrusion, or removal may be a specific environmental or engineering objective. A two-fraction transport relation also admits relatively large sand transport rates at low to moderate flows that transport little gravel.

[5] That a two-fraction approximation of widely sorted sediment might capture mixed-size transport dynamics of practical significance is suggested by a number of observations, including differences in the behavior of the sand and gravel, similarity of transport rates of different sizes within the two fractions, consistent variation of sand and gravel behavior with bed sand content f_s, and the fact that the fines

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content of a river bed tends to be more transient than the gravel/cobble framework of the bed. In this paper, we outline previous work that indicates why a two-fraction approach might be effective and why the particular model proposed here is consistent with (in fact, could be deduced from) previous observations.

[6] In addition to practical considerations, the binary nature of a two-fraction model has the particular advantage that it supports a simple description of the interaction between the two fractions. Because the proportions of sand and gravel sum to one, the effect on transport of variations in bed composition may be represented as a simple function of the proportion of either fraction. We find that sand and gravel transport rates depend on f_s not only as it specifies the amount of each fraction available for transport, but also through an additional nonlinear effect on the mobility of each fraction. With a two-fraction model, we can efficiently and directly ask: how does f_s affect gravel transport rates? Earlier work indicates that this effect is quite pronounced and not adequately accounted for in current transport models [*Jackson and Beschta*, 1984; *Ikeda and Iseya*, 1988; *Wilcock et al.*, 2001].

[7] The two-fraction model presented here was partially described by *Wilcock* [1998]. This treatment differs in several respects. First, it presents a complete model, including a function for transport rate as well as incipient motion. Second, it incorporates new data from flume experiments with four sediments specifically designed to define the two-fraction model in its most sensitive range. Third, it explicitly develops versions of the model referenced to the grain size of either the bed surface or subsurface.

2. Background of a Two-Fraction Approach

[8] Many have observed that sand can be preferentially transported at low flows that move little or no coarse material [e.g., *Jackson and Beschta*, 1982; *Ferguson et al.*, 1989]. The variation in sand and gravel transport as flow increases has been described in terms of phases, with Phase I representing transport of sand over an immobile coarse bed and higher phases involving an increasing amount of coarser grains in transport [*Emmett*, 1976; *Jackson and Beschta*, 1982; *Carling*, 1983]. The dominance of sand transport at lower flows, combined with the larger frequency of these flows, can produce annual sediment loads that contain far more sand than is present in the bed [*Leopold*, 1992; *Lisle*, 1995].

[9] Sand, when present in more than trace amounts, can form into well-sorted patches and stripes that can increase the sand transport rate by locally eliminating the hiding effect that tends to reduce the transport of finer fractions in mixed-size sediment [e.g., *Ferguson et al.*, 1989; *Wilcock*, 1993; *Seal and Paola*, 1995]. *Paola and Seal* [1995] invoked sand patches to model the preferential downstream transport of sand in a downstream fining model.

[10] There is evidence that the variation of transport rates among different sizes within the sand and gravel fractions is small relative to that between the sand and gravel. The limiting case, in which the transport rate of all sizes (scaled by their proportion in the bed) is identical, is the condition of equal mobility defined by *Parker et al.* [1982b]. Similar fractional transport rates are observed over a range of grain sizes in sediments with a unimodal grain size distribution [*Wilcock*, 1993] and for widely sorted gravel with only a small sand content [*Parker et al.*, 1982b]. Divergence from similarity tends to be largest for the very smallest fractions, suggesting the grouping we make in the two-fraction model. Sediments with the largest deviations from transport similarity tend to have bimodal size distributions, with a principal gravel mode and a secondary sand mode [*Kuhnle*, 1992; *Wilcock*, 1992]. Field evidence indicates that sand transport over a coarse gravel-cobble bed approaches equal mobility when averaged over a hydrograph [*Church et al.*, 1991; *Wathen et al.*, 1995].

[11] The relative influence of the mechanisms controlling sand and gravel transport change systematically with sand content f_s and the associated change from a clast-supported to matrix-supported gravel bed. Processes that vary with f_s include the relative size effects producing "hiding" of fine grains and "exposure" of coarse grains, the tendency of sand to be removed from the bed surface through vertical winnowing or to become sorted into relatively homogeneous surface patches, and the mechanisms for gravel entrainment and deposition.

[12] When f_s is small (less than roughly 10%), the bed consists of a framework of gravel clasts and the sand tends to become well hidden among the gravel interstices [*Parker et al.*, 1982a; *Wilcock et al.*, 2001]. If the sand grains are sufficiently small, they can settle through the bed surface layer and become unavailable for transport unless the gravel is entrained [*Diplas and Parker*, 1985]. There is not sufficient sand in transport to make persistent patches. Substantial sand transport requires disruption of the gravel framework, so that the sand and gravel tend to begin moving at comparable flows. A trace amount of sand on the bed surface should have little effect on the gravel transport.

[13] At larger f_s, the bed is still clast-supported, but the pore spaces become filled with sand. Pore filling becomes nearly complete in the range $10\% < f_s < 30\%$, depending on the relative size of the sand and the pores, which influences the tendency of sand to 'bridge' between gravel clasts and prevent deeper percolation. Moving sand may congregate into patches on the bed surface, reducing the "hiding" effect and increasing its relative ease of transport. The combined effect of pore filling and patch formation is that measurable amounts of sand transport occur at flows too low to transport gravel and sand transport can persist as long as there is an upstream supply. The effect of increased sand content on gravel transport is more complex. As the pore space between grains is filled with sand, the gravel may be partially or temporarily buried by sand, which will suppress the gravel transport rates. Once a gravel grain is entrained, however, it may move faster over the relatively smooth bed provided by the filled pores and it may move further because the number of available resting places has been reduced by sand filling of surface pores. Empirical information is needed to determine the relative importance of these competing effects.

[14] As fines content increases further, the proportion of gravel grains in direct contact is reduced and approaches zero as the fines content exceeds approximately 50% and the bed is fully matrix supported. Patch development is extensive and sand transport rates will be similar to those for a bed of uniform sand. Gravel transport will commence at smaller flows than those producing transport of unisize

Table 1. Sediment Grain Size

	Sand/Gravel Boundary, mm	Subsurface D50, mm	Subsurface D90, mm	Subsurface fs	Subsurface Ds, mm	Subsurface Dg, mm	Surface D50, mm	Surface D90, mm	Surface Fs	Surface Dg, mm
Laboratory										
J06	2.00	12.2	38	0.06	1.0	13.4	17	39	0.001	16
J14	2.00	9.8	37	0.15	1.0	13.4	14	39	0.013	13.5
J21	2.00	8.4	36	0.21	1.0	13.4	8.5	34	0.072	10
J27	2.00	6.7	33	0.27	1.0	13.4	6.4	31	0.20	9.4
Bed of many colors	2.00	5.3	31	0.34	0.5	13.4	2.6	21	0.48	8.5
Field										
East Fork River, Wyoming	2.00	1.2	28	0.59	0.6	12.0	–	–	–	–
Goodwin Creek, Mississippi	2.00	8.3	30	0.34	0.6	16.0	12.0	34	0.25	17.5
Jacoby Creek, California	2.00	14	81	0.22	1.0	24.0	27	104	0.12	34.0
Oak Creek, Oregon	2.38	20	65	0.15	1.2	26.0	53	98	0.036	53.0

gravel, because the gravel clasts can be entrained by removal and undercutting of the surrounding sand and, once in motion, the gravel will be able to keep moving over the relatively smooth sand bed.

[15] The differences between sand and gravel behavior, and the shift in this behavior with f_s, suggest that a transport model describing the transport of the two fractions should include an explicit dependence on f_s. That is, f_s not only controls the amount of sand and gravel available for transport, but also influences the inherent mobility of the two different fractions. Previous work suggesting that the distinct behavior of sand and gravel merited independent transport computations for each fraction did not account for possible between-fraction interaction [*Bagnold*, 1980; *Kuhnle*, 1992]. We find in our experimental work that the effect of f_s on transport is strong and not adequately accounted for in existing models, regardless of the number of fractions used to model the transport [*Wilcock et al.*, 2001]. Beyond any practical efficiency of using a simple two-fraction model, the ability to directly incorporate the effect of f_s on transport rate is an important advantage of the two-fraction approach.

3. Data

[16] The two-fraction transport model is developed from both laboratory and field observations. We place somewhat more emphasis on the laboratory data in developing the model because the laboratory experiments provide a controlled test of the effect of f_s on transport rate. We were also able to estimate the grain shear stress with greater certainty in the laboratory and surface grain size distribution was measured with greater accuracy and frequency compared to the single measurements available for each field case.

[17] The laboratory experiments used five sediments prepared by adding different amounts of sand to the same gravel mixture. The gravel ranged in size from 2.0 mm to 64 mm, the sand from 0.5 mm to 2.0 mm (Table 1). The sand content f_s of the mixtures varied from 0.062 to 0.343; four of the sediments were named according to the target sand content such that the sediment name and actual sand content were J06 (6.2%), J14 (14.9%), J21 (20.6%), and J27 (27%). The fifth sediment (BOMC, 34.3% sand) was the initial mixture from which the other sediments were developed [*Wilcock and McArdell*, 1993]. About one-half of the sand in BOMC is in the range 0.21 mm to 0.5 mm, making its sand size approximately half that of the other sediments. Runs were initiated from a homogenized planar sediment bed 8–10 cm thick. Water discharge was varied over nine or ten runs with each sediment and flow depth was held within a narrow range, thereby providing controlled measurement on the variation of transport rate with flow strength for different f_s. The flow conditions were chosen to produce a wide range in transport rate (including conditions of incipient motion) for both sand and gravel. Both water and sediment were recirculated and transport rates, flow, and bed surface composition were measured after the system reached equilibrium (defined as a stable mean transport rate and grain size). Further detail on the experimental conditions and reference to the archived data may be found in *Wilcock et al.* [2001].

[18] Transport data from four gravel bed rivers is also used in developing the two-fraction model. In three of the field cases, East Fork River, Wyoming [*Emmett*, 1980; *Emmett et al.*, 1980, 1985], Oak Creek, Oregon [*Milhous*, 1973], and Goodwin Creek, Mississippi [*Kuhnle*, 1992], the transport of the entire stream was sampled using either a slot trap extending across the entire river width or a weir through which all of the transport was directed. The grain size distributions in these streams vary widely (Table 1) and sand content f_s varies between 0.15 and 0.59. In the fourth field case, Jacoby Creek [*Lisle*, 1986, 1989], transport was measured using repeated traverses with a hand-held sampler and $f_s = 0.22$, which is close to the transition from a framework- to matrix-supported bed. For all four field sites, the transport data were grouped into narrow ranges of transport rate (the range of transport in each group was nearly always within 10%) and the geometric mean of the grouped data was carried forward in the model development.

[19] Bed surface armoring (expressed as the ratio of surface to subsurface D_{50}) was relatively well developed for Oak Creek (2.65) and Jacoby Creek (1.93) and less well developed for Goodwin Creek (1.45), J06 (1.37) and J14 (1.43). Separate surface and subsurface grain sizes are not available for East Fork River, for which the reported bed grain size is a composite of 232 grab samples of approximately 5 cm depth [*Emmett*, 1980]. In all cases except BOMC and East Fork River, the sand content of the bed surface was smaller than that of the subsurface. The proportional difference between surface and subsurface sand content is largest for those sediments with the least sand (J06, J14, J21, Oak Creek, Jacoby Creek), indicating that the substrate is able to accommodate a larger proportion of the available sand. The proportional difference in sand content

also tends to be larger for the lab sediments. For J06 and J14, the bed surface was nearly devoid of sand (Table 1). We expect more thorough winnowing of sand is possible during the development of equilibrium transport conditions in the flume. In the relatively homogeneous transport field in a recirculating sediment flume, vertical winnowing of sand into the subsurface may proceed to a maximum extent permitted by the available storage in the subsurface, whereas in the field, persistent upstream supply of sand from, for example, channel margins or pools, is likely to prevent the bed from reaching a steady state vertical sorting.

4. Determining Grain Stress

[20] Development of a transport model using transport observations from different sediments and channels requires a consistent basis for scaling the flow. The bed shear stress τ is used here, requiring a means of determining the portion of the total boundary stress acting on the sediment grains. Determining grain stress is relatively easy in the laboratory. The bed was essentially planar in nearly all runs, such that the entire shear stress acting on the bed can be considered to be skin friction. The only correction needed is a minor adjustment to account for the difference in roughness between the flume bed and the smooth flume walls. This correction is made using the method of *Vanoni and Brooks* [1957], as modified by *Chiew and Parker* [1994]. The resulting shear stress was consistently 8% smaller than that calculated using the flow depth, and 24% larger than that calculated using the flow hydraulic radius (flow depth was ca. 10 cm and channel width was 60 cm).

[21] In the four field cases, the total boundary shear stress includes forces acting on bed forms, vegetation, and channel banks. On Oak Creek, *Milhous* [1973] estimated that approximately half the roughness was due to form drag. On Goodwin Creek, *Kuhnle and Bowie* [1992] report that large, steep gravel dunes form at some flows and that nonuniform flow during the passage of a flood wave influences the observed water slope and therefore τ. East Fork River includes alternate bars, sand dunes, and a bend in the approach to the study section [*Leopold and Emmett*, 1997]. Drag losses on Jacoby Creek are associated with gravel bars, bends, woody debris, and rough banks, although the sampling reach was relatively straight and unobstructed [*Lisle*, 1986]. To provide a consistent flow scaling among these different cases, skin friction is estimated using the Einstein/Keulegan relation

$$\frac{U}{\sqrt{gh'S}} = 2.5 \ln\left(\frac{11h'}{k_s}\right) \quad (1)$$

where U is mean velocity, g the acceleration of gravity, S is slope, k_s is roughness, h' is the portion of the flow depth attributed to grain roughness ($h' = h$ in the absence of other sources of drag) and grain stress is calculated as $\tau = \rho g h' S$. Using specified values of U, S, and k_s, (1) is solved for h'. The roughness k_s is typically specified as a multiple of the coarser grain sizes on the bed surface. A range of k_s estimates has been reported, varying from $2D_{65}$ [*Engelund*, 1967] to $2D_{90}$ [*Kamphuis*, 1974] to $3D_{90}$ [*Whiting and Dietrich*, 1990; *Wilcock*, 1996]. For some of the streams considered here, setting k_s to multiples of D_{90} larger than 1.5 produces estimates of τ that are larger than the total stress. To provide consistency with the laboratory data, we set $k_s = 0.84\ D_{90}$, a value back calculated from the sidewall-corrected values of τ in the flume experiments. For the field sediments, 0.84 D_{90} corresponds to a multiple of 1.9 to 3.1 times D_{65}, which falls within the reported range of k_s.

[22] Unfortunately, the range of k_s reported in the literature is sufficiently large to give a range of τ estimates of approximately 50%. Because the dependence of transport on τ is nonlinear, uncertainty in τ remains an important, persistent, and sometimes neglected problem in developing or applying any sediment transport model. In discussing the transport model developed here, we will indicate the most likely consequences of this uncertainty, which arise in comparisons between field and flume data and in the grain stress estimates for Jacoby Creek and Goodwin Creek. On Jacoby Creek, grain stress was more than 80% of the total stress, a value that is probably too large. On Goodwin Creek, the grain stress proportion of the total shear stress decreases with increasing discharge, which contradicts the expected trend in which bed form roughness becomes progressively drowned with increasing flow [e.g., *Parker and Peterson*, 1980].

5. Choice of Surface or Subsurface Grain Size

[23] A transport model can be defined relative to the grain size of either the bed surface or the bed subsurface. The surface size distribution is most relevant because it defines the population of grains immediately available for transport. The relation between the subsurface and the transport is mediated by the composition of the bed surface, which may be sorted and coarser than the subsurface and is contingent on the history of flow and sediment supply. The bed surface grain size can be measured relatively easily using point counts at low flows, but is difficult or impossible to measure during active transport. Only a single low-flow surface size distribution is available for each field case (in the case of East Fork River, the grain size is for the upper 5 cm of the bed). Use of a single surface size distribution in developing an empirical transport model requires the assumption that the low flow surface grain size persists at larger sampled flows. Although some have proposed that coarse surface layers vanish at large flows [e.g., *Andrews and Parker*, 1987], calculations using a correctly formulated surface-based transport model indicate a persistent armor layer [*Wilcock and DeTemple*, 2001], a trend supported by laboratory observations [*Wilcock et al.*, 2001] and limited field evidence [*Andrews and Erman*, 1986]. The subsurface size distribution is relatively stable in that it can change only when appreciable scour or aggradation occurs. The subsurface grain size is measurable in the field, although this requires special methods for sampling in the wetted parts of the channel and it can be difficult to obtain a large enough sample to be representative of all grain sizes [*Church et al.*, 1987].

[24] Because the transport depends directly on the surface size distribution, the two-fraction model is developed here in terms of the surface grain size. A surface-based model most directly represents the physical process and also permits calculation of transient conditions when coupled with an appropriate sediment mass conservation algorithm [e.g., *Parker*, 1990; *Parker et al.*, 2000; *Wilcock*, 2001a]. An alternative model based on the subsurface size distribution is also presented. Although any relation between the transport rate and subsurface size distribution must implic-

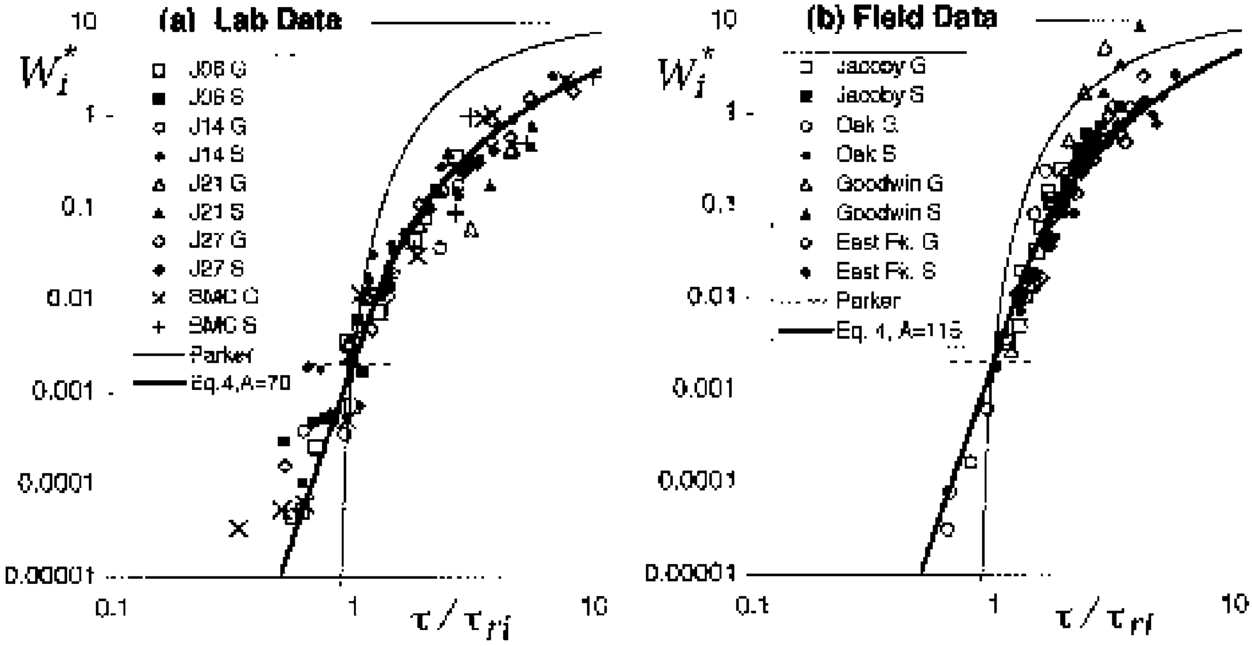

Figure 1. Dimensionless transport rate W_i* as a function of bed shear stress τ scaled by reference shear stress τ_{ri}. (a) Laboratory data. (b) Field data. "G" indicates gravel transport rates; "S" indicates sand transport rates. The reference transport rate W_i* = 0.002 is indicated with a dotted line.

itly represent sorting processes that are contingent on factors not included in the model (such as the history of flow and sediment supply), the subsurface size distribution may be specified in some applications and a subsurface-based relation may be subsequently shown to provide useful approximations over longer time and space scales.

6. Model Development

[25] The two-fraction transport model is developed using a similarity collapse of the transport rate of the sand and gravel fractions. The objective is to define appropriate values of a similarity parameter such that all fractional transport rates collapse about a common trend in suitable nondimensional space (see *Ashida and Michiue* [1972] and *Parker et al.* [1982b] for application to a many fraction model). The complete model consists of the transport relation and a relation to predict the similarity parameter for each fraction.

[26] The form of the similarity collapse is

$$W_i^* = f(\tau/\tau_{ri}) \tag{2}$$

where τ_{ri} (the similarity parameter) is a reference value of τ,

$$W_i^* = \frac{(s-1)gq_{bi}}{F_i u_*^3} \tag{3}$$

s is the ratio of sediment to water density, g is gravity, q_{bi} is volumetric transport rate per unit width of size i, u_* is shear velocity ($u_* = [\tau/\rho]^{0.5}$), F_i is proportion of fraction i on the bed surface and the subscript i represents either the sand (s) or gravel (g) fraction.

[27] The reference shear stress τ_{ri} is defined as the value of τ at which W_i^* is equal to a small reference value W_r^* = 0.002 [*Parker et al.*, 1982b; *Wilcock*, 1988]. Values of τ_{ri} were determined by eye on plots of scaled fractional transport rate q_{bi}/F_i against τ. Greatest weight in choosing τ_{ri} was given to transport observations in the vicinity of W_r^*, thereby preserving an interpretation of τ_{ri} as a surrogate for the critical shear stress for incipient motion τ_{ci}. In cases requiring extrapolation, or where the transport data were more scattered, increased weight was given to the overall trend of the fractional transport data, in which case a relation similar to the final transport curve was used to aid the choice of τ_{ri}. Fitting was done by eye because the procedure is multipart, involves strongly nonlinear functions, and optimized solutions for τ_{ri} tend to be strongly influenced by outliers and are, to the eye, inferior. Fitting by eye also facilitates assignment of different weights to individual data points according to the scatter in the individual trends and the need to extrapolate them to W_r^*. In general, the range of τ_{ri} that provides a conceivable fit to the transport data is much smaller than ±10% [*Wilcock*, 1988]. The choice of τ_{ri} values can be evaluated in Figure 1, which shows all values of W_i^* plotted as a function of τ/τ_{ri} . In any case, subjectivity in the choice of τ_{ri} is highly constrained. Values of τ_{ri} must not only provide a good collapse of W_i^* versus τ/τ_{ri} via (2), but must also must be predicted as a function of sediment properties. Adjustment of τ_{ri} to improve the fit of one relation directly influences the fit of the other.

7. Transport Function

[28] The laboratory transport data are plotted in part (a) of Figure 1 and field data in part (b). Sand and gravel transport are combined on each plot. Somewhat greater scatter is evident in the lab data, which may attributed to the fact that the lab data are for individual runs, whereas

the field data are grouped samples. Some scatter in the lab data also results from the fact that the duration of transport sampling was limited to avoid interfering with the flow/bed/transport interaction in the sediment recirculating flume.

[29] The transport data collapse, with scatter, about a common trend for both sand and gravel. Transport rates for the field data are somewhat larger than for the lab transport. Larger values of transport for field measurements may be attributed to spatial variability in grain size and bed topography and, therefore, in bed mobility [*Lisle et al.*, 2000] and in τ [*Brownlie*, 1981; *Paola et al.*, 1999]. Because transport is a strong nonlinear function of τ, total transport rate will be greater for a channel with spatially variable τ than for a channel with spatially uniform τ and the same mean shear stress. In his empirical transport model, *Brownlie* [1981] used a correction factor to account for the difference between field and lab transport rates. *Paola et al.* [1999] developed a basis for estimating the influence on transport rate of spatial variation in topography. The trend fitted to the field data is a factor of 1.64 larger than that for the laboratory data (Figure 1), an increase in transport rate that falls within the range of three calculated for a braided sand channel by Paola et al.

[30] Other factors may contribute to a difference between field and laboratory transport rates. One is unmeasured variation in the bed surface size distribution. A single low-flow measurement of bed grain size is available for each field case, whereas the surface size distribution was measured for each run in the laboratory experiments. Another factor is error in the estimate of τ and its variation with discharge for the field case. This may be particularly important in the case of Goodwin Creek, for which transport rates appear to increase more rapidly with τ/τ_{ri} than the remaining cases. Recalling that the estimated grain stress on Goodwin Creek decreases as a proportion of total boundary shear stress as discharge increases (a trend opposite to that expected if bed form drag washes out at higher flows and suggesting that true grain stress may become larger than estimated using (1) as discharge increases), the trend for Goodwin Ck on Figure 1 may be too steep and was not allowed to strongly influence the fit of the transport function.

[31] The transport function has the form

$$W_i^* = \begin{cases} 0.002\,\phi^{7.5} & \text{for } \phi < \phi' \\ A\left(1 - \frac{\chi}{\phi^{0.25}}\right)^{4.5} & \text{for } \phi \geq \phi' \end{cases} \quad (4)$$

where $\phi = \tau/\tau_{ri}$, A is a fitted parameter, and ϕ' and χ are chosen to match the value and slope of the two parts of the function. This function is similar to some previous transport functions for gravel bed rivers. In particular, the surface-based transport model of *Parker* [1990] uses a function similar to (4) to define W_i^* for $\phi < 1$ and $\phi > 1.59$ with a third exponential function providing a smooth fit between the two. In that case, the exponent on the power law is 14.2 and, in the function for large ϕ, $A = 11.2$, and the exponent on ϕ is one. The Parker function is plotted for comparison on Figure 1. The lack of fit between the Parker function and the data (including Oak Creek, to which the Parker function was originally fit) may be attributed to a number of factors, including the grouping of sizes into two fractions and the calculation of grain stress via (1), whereas the total boundary stress was used by *Parker* [1990; see also *Parker et al.*, 1982b]. Rather than using a third function to smoothly match the value and slope of the two limbs of (4), the model presented here matches the two parts at ϕ', which is calculated as the value of ϕ at which the slope of the two functions match and χ is specified to match the value of W_i^* at ϕ'. For the laboratory data, $A = 70$, giving $\chi = 0.908$ and $\phi' = 1.19$. For the field data, $A = 115$, giving $\chi = 0.923$ and $\phi' = 1.27$.

8. Incipient Motion Function

[32] Completion of the two-fraction model requires a basis for predicting τ_{ri}. The form of this submodel can be largely deduced from prior knowledge about incipient motion. Because this argument speaks to the generality of a two-fraction approach, and also illuminates an essential dependence on f_s that is captured by a two-fraction model and is absent from other transport models, we begin with a general discussion before presenting the fitted model.

[33] To account for the effect of fraction size, τ_{ri} is predicted in terms of the Shields Number τ_{ri}^*

$$\tau_{ri}^* = \frac{\tau_{ri}}{(s-1)\rho g D_i} \quad (5)$$

Expected values of τ_{ri}^* can be defined in the limit of $f_s = 0$ and $f_s = 1$. For the gravel, τ_{rg}^* at $f_s = 0$ must take a standard value for incipient motion of well-sorted, unimodal gravel. As f_s approaches 1, τ_{rg}^* should decrease to a small constant value because the influence of surrounding grains on the motion of a gravel clast becomes small relative to the influence of the weight of the grain and the drag force acting it, which are directly represented in τ_{rg}^*. A limiting value of approximately 0.01 has been suggested for grains that are large relative to their surroundings [*Fenton and Abbott*, 1977; *Andrews*, 1983]. For the sand, τ_{rs}^* at $f_s = 1$ must take a standard value for incipient motion of well-sorted sand. As f_s approaches zero, we expect that trace amounts of sand will be largely hidden among the pores of the gravel grains and that entrainment of the sand will require entrainment of the gravel. This suggests that $\tau_{rs} \approx \tau_{rg}$ as f_s approaches zero or, using (5),

$$(\tau_{rs}^*)_0 = \alpha(\tau_{rg}^*)_0(D_g/D_s) \quad (6)$$

where $(\tau_{rs}^*)_0$ and $(\tau_{rg}^*)_0$ are the values of τ_{rs}^* and τ_{rg}^* at $f_s = 0$ and α is an order one constant. If the sand and gravel begin moving at the same value of τ, $\alpha = 1$.

[34] This general argument indicates that τ_{ri}^* for both gravel and sand is larger at $f_s = 0$ than at $f_s = 1$. We postulate that the decrease in τ_{ri}^* with f_s will be associated with the transition from a grain- to a matrix-supported bed and with the associated shift in entrainment mechanisms discussed above. If so, the shift from "gravel-like" to "sand-like" entrainment should be largely complete by $f_s \approx 0.3$ [*Wilcock*, 1998]. In general, we expect that vertical winnowing will cause F_s to be smaller than f_s, with the difference increasing with smaller f_s.

[35] Values of τ_{ri} used to scale τ in Figure 1 are presented in Table 2 and the corresponding Shields numbers are

Table 2. Reference Shear Stress

		Transport Scaled By	
	τ_{rg}, Pa	Surface τ_{rs}, Pa	Subsurface τ_{rs}, Pa
J06	8.8	8	17
J14	7	6	9
J21	3.5	2.9	4
J27	2.1	1.7	1.7
BOMC	1.7	0.6	0.6
East Fork River	1.9	1.4	1.4
Goodwin Creek	2.8	2	2
Jacoby Creek	13	9	11
Oak Creek	12	7.5	10

presented as a function of F_s in Figure 2. The data can be fitted with an exponential model

$$\tau_{ri}^* = (\tau_{ri}^*)_1 + \left[(\tau_{ri}^*)_0 - (\tau_{ri}^*)_1\right] e^{-14F_s} \quad (7)$$

For the gravel, the limiting value $(\tau_{rg}^*)_0$ at $F_s = 0$ is 0.035 and, for the sand, the limiting value $(\tau_{rs}^*)_1$ at $F_s = 1$ is 0.065, both of which fall within the standard range of values for clean, well-sorted sediment. At $F_s = 1$, $(\tau_{rg}^*)_1 = 0.011$, close to the indicated limit for very large grains on a fine-grained bed. At $F_s = 0$, $(\tau_{rs}^*)_0$ is not fitted, but calculated as a function of $(\tau_{rg}^*)_0$ and D_g/D_s using (6) with $\alpha = 1$, indicating that $\tau_{rs} = \tau_{rg}$.

[36] For the gravel, the laboratory data match the curve well and Goodwin Ck and East Fork River also fall near the curve, but Oak Ck and Jacoby Ck deviate on either side of the trend. The transition from gravel-like behavior to sand-like behavior is essentially complete by $F_s \approx 0.2$ (Figure 2a). Because D_g/D_s varies among different sediments, predicted values of τ_{rs}^* do not fall along a common trend and τ_{rs}^* is represented in Figure 2b as a family of curves for D_g/D_s = 10, 20, 35, and 50, which span the range of D_g/D_s for the sediments. A direct comparison of the predicted and observed τ_{rs}^* is given in the inset of Figure 2b. The laboratory values tend to be well predicted by the fitted trend, whereas the field data, particularly for Oak Creek and Jacoby Creek, are poorly predicted.

[37] The same exponent ($-14F_s$) is used in (7) for both sand and gravel. This ensures that the predicted trend of τ_{rs}/τ_{rg} decreases monotonically with sand content, a useful consideration for modeling downstream sorting (R. Fergu-

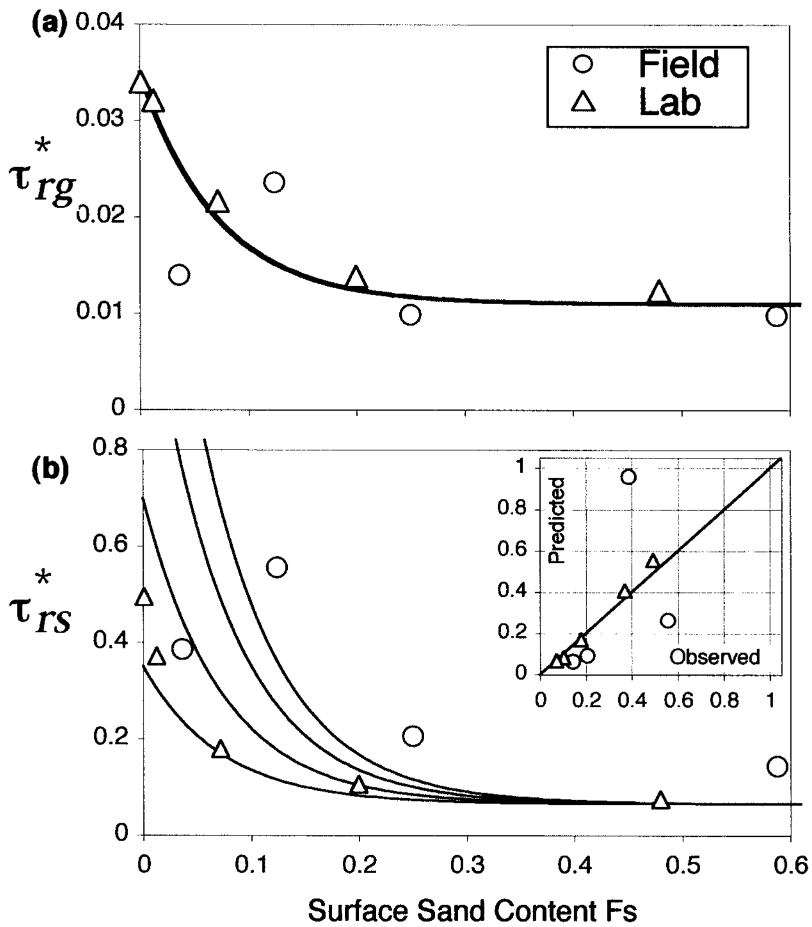

Figure 2. Dimensionless reference shear stress τ_{ri}^* as a function of surface sand content F_s. (a) Gravel; (b) sand. Trend in Figure 2a is equation (7) using limiting values of τ_{rg}^* from Table 3. Family of curves in Figure 2b is from equations (6) and (7) using limiting values of τ_{rs}^* from Table 3 and D_g/D_s = 10, 20, 35, and 50. Inset in Figure 2b shows predicted versus observed τ_{rs}^*.

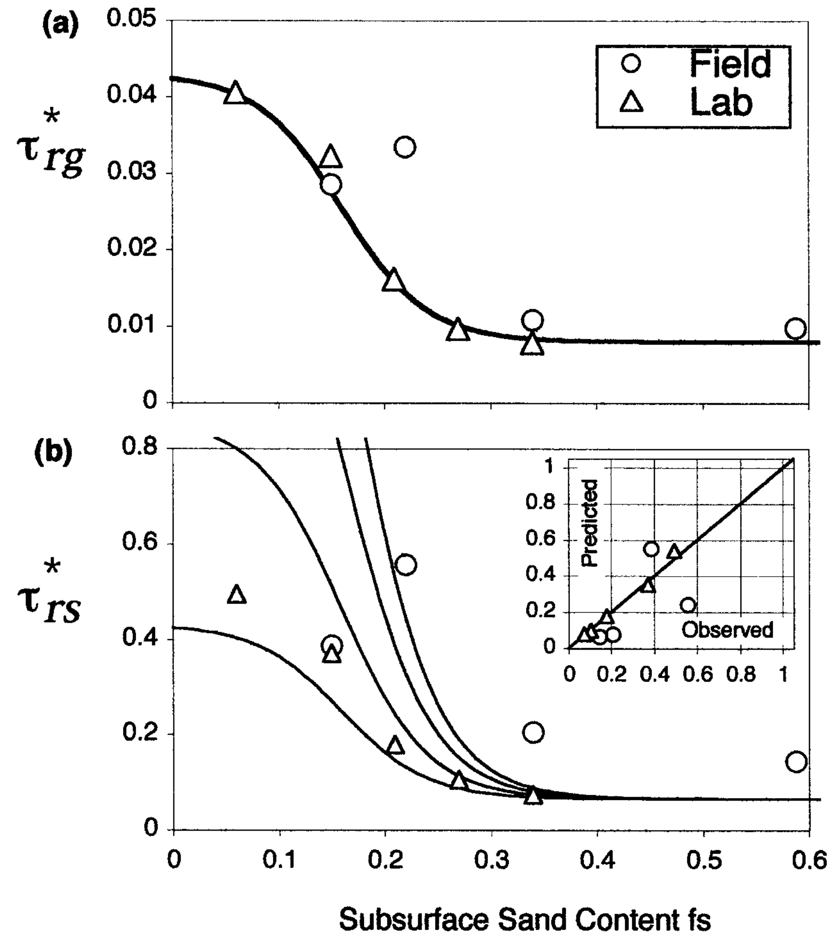

Figure 3. Dimensionless reference shear stress τ_{ri}^* as a function of subsurface sand content f_s. (a) Gravel; (b) sand. Trend in Figure 3a is equation (8) using limiting values of τ_{rg}^* from Table 3. Family of curves in Figure 3b is from equations (6) and (8) using limiting values of τ_{rs}^* from Table 3 and D_g/D_s = 10, 20, 35, and 50. Inset in Figure 3b shows predicted versus observed τ_{rs}^*.

son, personal communication). That a common model can be defined for both sand and gravel also supports the conclusion that the observed variation in τ_{rs} and τ_{rg} is associated with a fundamental change in the sediment bed, namely the transition from a clast-supported to matrix-supported structure, producing a consistent response in the transport of both sand and gravel fractions.

[38] The explicit dependence of τ_{ri}^* on F_s is an important feature of the two-fraction model. Because of the strong, nonlinear dependence of q_{bi} on τ_{ri} (e.g., as shown in (4) and Figure 1), small changes in τ_{ri} will correspond to very large changes in q_{bi}. *Wilcock et al.* [2001] isolate the effect of F_s on transport and demonstrate that increases in F_s can increase gravel transport rates by orders of magnitude. Existing transport formulas do not contain a dependence on F_s (or other parameter representing size fraction proportion), except as it determines the proportion of a fraction available for transport on the bed surface. The trends shown in Figure 2, as well as the general argument given here for the form of these trends, clearly indicate that τ_{ri} and, therefore q_{bi} should vary nonlinearly with F_s. The binary nature of a two-fraction model provides a natural means of capturing this dependence: because F_s and F_g sum to one, the variation in both τ_{rs} and τ_{rg} can be captured as a function of F_s alone.

[39] The model fit is better for τ_{rg} than for τ_{rs}. There are several possible reasons. First, D_s may have a larger proportional variation than D_g. For example, an influx of sediment from land disturbance might reasonably produce a factor of two variation in D_s, whereas such a change in D_g would require an unusually large addition of bed material. Grain size enters the model directly through (6) such that uncertainty in D_s produces proportional uncertainty in $(\tau_{rs}^*)_0$. A second problem is the sensitivity of W_s^* and τ_{rs} to variations in surface sand content. Because F_s is typically much closer to zero than F_g, relatively small changes in sediment supply can have a proportionally greater effect on W_s^*, and therefore τ_{rs}, than on W_g^* and τ_{rg},. Variation in the extent of vertical winnowing can cause F_s to differ from f_s not only between field and flume, but also in the field from one stream to the next, or one time to the next, depending on sand supply.

9. Alternative Incipient Motion Function

[40] Values of τ_{ri}^* are plotted in Figure 3 as a function of subsurface sand content f_s. For consistency, τ_{ri}^* is calculated using the subsurface value of D_g (Table 1; in the absence of contrary evidence, D_s is assumed the same for either case).

Although the values of τ_{ri} used were determined for surface-based transport rates (Figure 1), presentation in terms of f_s allows a useful interpretation in terms of the overall sand content of the bed. (In actuality, the same values of τ_{rg} are found whether transport is scaled by the surface or subsurface grain size, although the values of τ_{rs} differ in the two cases.) In contrast to the exponential decrease indicated in (7) and Figure 2, the trend in Figure 3 can be described by a logistic function

$$\tau^*_{ri} = (\tau^*_{ri})_0 - \frac{(\tau^*_{ri})_0 - (\tau^*_{ri})_1}{1 + \exp[-25(f_s - 0.16)]} \tag{8}$$

The fit in Figure 3b uses the same value of $(\tau^*_{rs})_1$ as in (7) and Figure 2b, and the value of $(\tau^*_{rs})_0$ is again determined by (6) using $\alpha = 1$. The gravel values are slightly different, $(\tau^*_{rg})_0 = 0.043$ and $(\tau^*_{rg})_1 = 0.008$, reflecting the difference in D_g between surface and subsurface (Table 1).

[41] For the gravel, all sediments match the fitted trend well, with the exception of Jacoby Creek. For the sand, the laboratory data are again well predicted by the curve (Figure 3b) and the fit for the field data is weaker, although the scatter is reduced relative to Figure 2b (residuals between observed and predicted are reduced by a factor of almost three).

[42] Figure 3 indicates that the transport has "gravel-like" behavior over a range of small f_s and "sand-like" behavior for f_s grater than approximately 0.25. Equation (8) indicates that 90% of the decrease in τ^*_{ri} occurs between f_s = 0.06 and f_s = 0.26. This supports the idea that shift from gravel behavior to sand behavior occurs over the transition from a relatively clean clast-supported bed to the onset of a matrix-supported bed.

[43] The proportional shift in sand content between F_s and f_s is greatest for the sediments with the least sand (especially Oak Creek and for J06 and J14, for which vertical winnowing of sand into the bed subsurface over the course of a flume run leaves only a trace of sand on the bed surface; Table 1). Comparing Figures 2 and 3, the effect of using F_s versus f_s is to translate the shift in τ^*_{ri} toward the left.

10. Subsurface Two-Fraction Model

[44] In application, a transport problem may be specified in terms of the subsurface, rather than surface, grain size, so a two-fraction model based on subsurface grain size is presented here. The relation between subsurface grain size and transport rate is mediated by any sorting between surface and subsurface and, because this sorting cannot be explicitly included in a transport model (it is appropriately predicted by combining a transport model with a sorting algorithm based on sediment mass-conservation and is contingent on the history of flow and sediment supply), a subsurface-based model cannot be general [*Parker*, 1990; *Wilcock*, 2001a]. In developing a subsurface-based model, we find that the most important sorting effect is due to vertical winnowing of fine sediment for $f_s < 0.1$, which causes $F_s \ll f_s$. At smaller transport rates measured with J06 and J14, for example, the sand fraction of the transport was vanishingly small for the simple reason that there was almost no sand present on the bed surface. In a surface-based formulation, small values of q_{bi} and F_s tend to balance when calculating W_i^* and a common trend can be fitted using plausible values of τ_{rs}. In a subsurface model, tiny values of q_{bi} are scaled by a constant f_s, producing very small values of W_i^* which require very large values of τ_{rs} in the similarity collapse. This effect is more artificial than real, reflecting the action of a vertical sorting processes that is contingent on flow and sediment supply history and cannot be explicitly included in the transport model.

[45] We find that the same transport function (4) can be fitted to the subsurface-scaled transport rates and, for the gravel, the same values of τ_{rg} suffice for both surface and subsurface-based models, a result of the fact that differences between F_g and f_g are sufficiently small that their effect on W_g^* is negligible relative to its large range. As a result, (8) defines τ^*_{rg} with the same limiting values as the surface-based model: $(\tau^*_{rg})_0 = 0.043$ and $(\tau^*_{rg})_1 = 0.008$. Because of the winnowing effect, however, values of τ_{rs} are much larger in the subsurface-based model for small f_s (Table 2). For the subsurface model, τ^*_{rs} can be found from (8) using $(\tau^*_{rs})_1 = 0.045$ and $(\tau^*_{rs})_0$ found from (6) using $\alpha = 1.8$. This latter result indicates formally that τ_{rs} is almost twice as large as τ_{rg} as f_s approaches 0. Although this reflects the influence of factors not included in the transport model, it is not entirely unreasonable. For small f_s, the sand may percolate into the substrate and may be largely unavailable for transport until substantial gravel transport rates are achieved, indicating that $\tau_{rg} < \tau_{rs}$ and $\alpha > 1$, although the interwoven effects of vertical sorting, flow and sediment history, and transport dynamics make any simple interpretation or quantitative prediction difficult.

11. Application

[46] To calculate the transport rate, τ must be specified along with F_s, D_s, and D_g for the bed surface. Transport rate is found using (7) with (5) to calculate τ_{ri} and using (4) with (3) to calculate q_{bi}. Values of $(\tau^*_{ri})_0$ and $(\tau^*_{ri})_1$ are given in Table 3 or, for $(\tau^*_{rs})_0$, found from (6). If subsurface f_s, D_s, and D_g are available, τ_{ri} can be alternatively calculated using (8) with (5), requiring different values of $(\tau^*_{rg})_0$ and $(\tau^*_{rg})_1$ given in Table 3. This alternative is attractive because $\alpha = 1$ in the fitted model, assuring that $\tau_{rs} \leq \tau_{rg}$, as observed in all four field cases, because τ_{rs} is predicted with more accuracy (Figure 3b versus Figure 2b), and because the shift from gravel to sand behavior occurs over a range of f_s associated with the expected framework/matrix threshold. Values of τ_{ri} can be predicted more accurately for gravel than sand. We note that τ_{rs}/τ_{rg} is equal to 0.63 to 0.74 for all four field cases, suggesting that τ_{rs} may be estimated as a simple fraction of τ_{rg}, although further field data are needed to evaluate this alternative.

[47] If only subsurface grain size information is available (e.g. from bulk samples), then f_s must be used to scale W_i^* in (3) and τ_{ri} must be found from (8) using the subsurface parameters in Table 3. In this case, transport predictions will be more uncertain because the mediating effects of surface sorting are not included in the model and the prediction of sand transport for $f_s < 0.1$ is likely to be highly uncertain. The factor $\alpha = 1.8$ is likely to overestimate τ_{rs} for field cases, unless an absence of sand supply permits the same degree of vertical winnowing that is achieved at steady state transport in a recirculating flume. We note that τ_{rs}/τ_{rg} is equal to about 0.7 to 0.85 for all four field cases, suggesting again that it may be possible to estimate τ_{rs} as a constant fraction of τ_{rg}.

Table 3. Incipient Motion Parameters

Transport Model	Incipient Motion Equation	$(\tau^*_{rg})_0$	$(\tau^*_{rg})_1$	α (equation (6))	$(\tau^*_{rs})_1$
Surface	(7)	0.035	0.011	1	0.065
Surface	(8)	0.043	0.008	1	0.065
Subsurface	(8)	0.043	0.008	1.8	0.045

[48] A two-fraction transport model offers potential practical advantages when applied to the prediction of transport rates at specific locations. Areas with similar fines content may be mapped and combined in a weighted average to give the proportion of sand in the reach [*Wilcock et al.*, 1996]. Combined with size analyses in individual facies (giving D_s and D_g), this approach offers the potential to develop a reach-averaged measure of grain size with reasonable effort. The effort required to determine grain size may be further reduced if a few measurements of small sand and gravel transport rates are made. In this case, *Wilcock* [2001b] demonstrates that measurement of D_s and D_g is not necessary (and an increase in prediction accuracy is gained) if τ_{ri} is determined from transport observations. If changes in bed size distribution are due primarily to the introduction or removal of fines from a reach, remapping of fines content may provide a relatively efficient means of monitoring bed changes compared to the effort required to characterize the full grain size distribution of the reach.

12. Discussion

12.1. Definition of the Sand/Gravel Threshold

[49] The threshold between fine and coarse sediment used here is 2 mm (with the exception of Oak Creek, for which 2.38 mm was the nearest class boundary). This size threshold is consistent and well established, but somewhat arbitrary. Because the conceptual basis of a two-fraction model rests on the different transport dynamics of grains forming the framework and matrix of a gravel bed river, it would be advantageous and consistent to define the size threshold based on the likely behavior of the two fractions, at the expense of a simple, constant grain size. For example, we have used a boundary at 8 mm on a coarse gravel/cobble river with a distinct fine mode in the sand and pea gravel range [*Wilcock et al.*, 1996]. No specific size boundary, whether 2 mm or 8 mm, will work for all size distributions, so the choice must be made on a case-by-case basis. Distinction between framework and matrix sizes is relatively straightforward for size distributions with a principal gravel mode, a secondary sand mode, and a small amount of sediment in the "grain size gap" (typically in the range 2 mm to 8 mm). The distinction is potentially ambiguous, but also less important, for gravel beds with no sand mode, which can be interpreted as openwork gravels with an absence of matrix.

12.2. Three-Fraction Model

[50] A two-fraction transport model is an approximation that cannot capture the details of transport rate for all grain sizes in a mixture. In particular, we expect that the coarsest gravel fractions in widely sorted sediments will typically begin moving at larger flows than fractions in the pea gravel range. *Parker et al.* [1982b] demonstrated small but systematic deviations from fractional transport similarity using three size fractions for Oak Creek. The two-fraction model presented here is most similar to that for the finer two fractions of a three-fraction model. The very largest gravel fractions in the laboratory data examined here also tend to begin moving at larger flows and have a smaller scaled transport rate than the finer gravel fractions. Because these coarsest fractions tend to be a small component of the total gravel transport, they have little influence on the fitted model for gravel transport or on the prediction of total gravel transport rate. The two-fraction model is an approximation, intended to give the total transport rate and not the transport of individual sizes within the two fractions. It should not be used in cases, such as prediction of armoring, for which differences among gravel transport rates are important.

12.3. Channel Adjustment

[51] The sharp decrease in τ^*_{ri} with f_s suggests that the response of a gravel bed river to additions of fine sediment may include an important self-regulating component. If additions of fines (e.g., from logging, fire, land development, or reservoir flushing) cause f_s to increase, particularly in the approximate range $0.06 < f_s < 0.26$, the model predicts a sharp decrease in τ^*_{ri} and, therefore, a large increase in transport capacity. This suggests that adjustments of channel geometry and hydraulics in response to sand inputs may be smaller than predicted by other models. An increase in transport capacity also suggests that the period required to evacuate the excess sediment supply may be shorter than anticipated. Any reduction or acceleration of channel adjustment is not without environmental cost, however, because it involves an increase in f_s which is typically associated with negative impacts on the stream ecosystem.

12.4. Gravel/Sand Transition

[52] The trend of decreasing τ^*_{ri} has interesting implications for the formation and maintenance of sharp gravel-sand transitions reported in many rivers. At these transitions, grain size can decrease by an order of magnitude or more over distances as small as a few 100 m [*Yatsu*, 1955]. *Smith and Ferguson* [1995] report that the gravel-sand transition is often located where the transport capacity of the river decreases relative to the imposed load as a result of a change in river slope, a backwater, or lateral input of fine-grained sediment. Because the gravel/sand transition can occur over a shorter distance than would be implied by the hydraulic transition, its abruptness suggests a corresponding discontinuity in the sediment transport. If fraction mobility is assumed to vary smoothly with grain size, one explanation for a sharp size transition would be a gap in the size distribution of the sediment supply near the sand/gravel boundary [*Smith and Ferguson*, 1995; *Parker*, 1996]. The τ^*_{ri} trends in Figures 2 and 3 suggest an alternative explanation: a small increase in f_s (typically observed immediately upstream of the transition [*Smith and Ferguson*, 1995]) can produce a decrease in τ^*_{ri} for both sand and gravel, although the decrease in τ^*_{rs} is proportionately larger (Figure 4). The resulting enhanced transportability of the sand will accelerate hydraulic sorting at the transition, such that sand, but

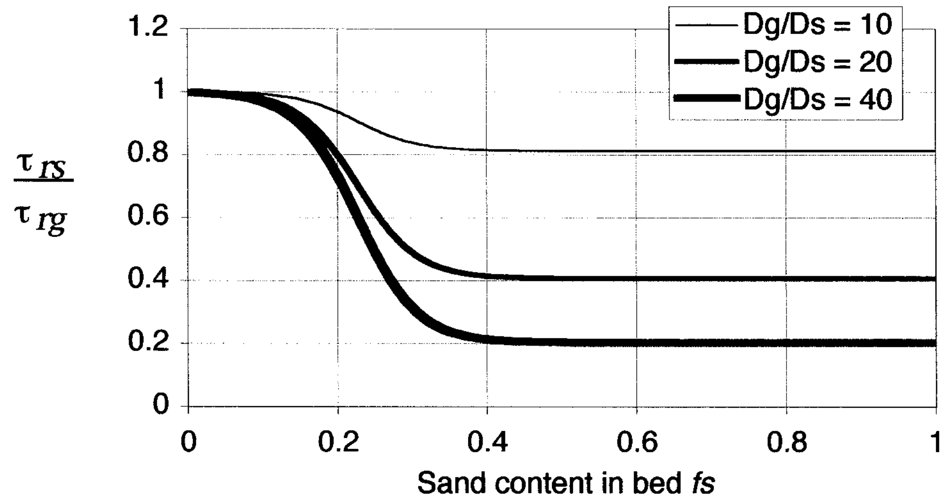

Figure 4. Ratio of τ_{rs} to τ_{rg} as a function of f_s for the case of surface-based transport rates, with τ_{rs} and τ_{rg} predicted using subsurface grain size.

not gravel, is able to proceed into the lower energy environment downstream.

13. Conclusions

[53] A model for mixed sand and gravel transport is proposed in which the different grain sizes are grouped into two fractions: sand and gravel. This approach offers practical advantages in terms of the required input data and conceptual advantages by readily accommodating the nonlinear effects of sand content on the transport rate of either fraction. The model is consistent with previous observations of the behavior of sand and gravel transport and, in fact, its general form can be entirely deduced from these earlier observations. The specific form of the model is determined from flume and field transport observations which demonstrate that sand and gravel transport rates can be collapsed to a single transport function in suitable nondimensional space. The scaling parameter used for this collapse represents incipient motion for the sand and gravel fractions and is shown to be consistent with well-known values in the limit of pure sand and gravel beds.

[54] The incipient motion function shows a sharp decrease over the range in sand content corresponding to the shift from a framework- to matrix-supported bed, giving a consistent conceptual basis for the empirical model. The form of this function efficiently captures the effect of sand content on gravel transport rate and suggests important implications for a number of fluvial processes, including the response of channels to sediment inputs and the mechanism for abrupt sorting in gravel/sand transitions.

[55] The two-fraction model is developed using the grain size of the bed surface, which is the grain population directly contributing to the transport and which can be readily measured using point counts. Because only subsurface grain size may be available in some applications, a subsurface-based version of the model is also presented. In this case, model fidelity is reduced because transport is influenced by bed sorting processes that cannot be included in the model. The same transport function applies in either case. The incipient motion estimate varies between the two cases, reflecting the effect of vertical sand winnowing, which causes the sand content of the surface and subsurface to differ.

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References

Andrews, E. D., Entrainment of gravel from naturally sorted riverbed material, *Geol. Soc. Am. Bull.*, *94*, 1225–1231, 1983.
Andrews, E. D., and D. C. Erman, Persistence in the size distribution of surficial bed material during an extreme snowmelt flood, *Water Resour. Res.*, *22*, 191–197, 1986.
Andrews, E. D., and G. Parker, Formation of a coarse surface layer as the response to gravel mobility, in *Sediment Transport in Gravel-Bed Rivers*, edited by C. R. Thorne, J. C. Bathurst, and R. D. Hey, pp. 269–325, John Wiley, New York, 1987.
Ashida, K., and M. Michiue, Study on hydraulic resistance and bedload transport rate in alluvial streams, *Trans. Jpn. Soc. Civ. Eng.*, *206*, 59–69, 1972.
Bagnold, R. A., An empirical correlation of bedload transport rates in dunes and natural rivers, *Proc. R. Soc. London, Ser. A*, *372*, 453–473, 1980.
Brownlie, W. R., Prediction of flow depth and sediment discharge in open channels, *Rep. KH-R-43A*, W. M. Keck Lab. of Hydraul. and Water Resour., Calif. Inst. of Technol., Pasadena, 1981.
Carling, P. A., Threshold of coarse sediment transport in broad and narrow natural streams, *Earth Surf. Processes Landforms*, *8*, 1–18, 1983.
Chiew, Y.-M., and G. Parker, Incipient sediment motion on non-horizontal slopes, *J. Hydraul. Res.*, *32*(5), 649–660, 1994.
Church, M. A., D. G. McLean, and J. F. Wolcott, River bed gravels: Sampling and analysis, in *Sediment Transport in Gravel-Bed Rivers*, edited by C. R. Thorne, J. C. Bathurst, and R. D. Hey, pp. 43–88, John Wiley, New York, 1987.
Church, M., J. F. Wolcott, and W. K. Fletcher, A test of equal mobility in fluvial sediment transport: Behavior of the sand fraction, *Water Res. Res.*, *27*(11), 2941–2951, 1991.
Diplas, P., and G. Parker, Pollution of gravel spawning grounds due to fine sediment, *Proj. Rep. 240*, St. Anthony Falls Lab., Univ. of Minn., Minneapolis, 1985.
Emmett, W. W., Bedload transport in two large, gravel-bed rivers, Idaho and Washington, *Proceedings of the Third Federal Inter-Agency Sedimentation Conference, Denver, Colorado*, pp. 4-101–4-114, Sediment. Comm., Water Resour. Council, Washington, D. C., 1976.

Emmett, W. W., A field calibration of the sediment-trapping characteristics of the Helley-Smith bedload sampler, *U.S. Geol. Surv. Prof. Pap.*, *1139*, 1980.

Emmett, W. W., R. H. Myrick, and R. H. Meade, Field data describing the movement and storage of sediment in the East Fork River, Wyoming, part 1, River hydraulics and sediment transport, 1979, *U.S. Geol. Surv. Open File Rep.* 80-1189, 1980.

Emmett, W. W., R. H. Myrick, and H. A. Martinson, Hydraulic and sediment-transport data, East Fork River, Wyoming, 1978, *U.S. Geol. Surv. Open File Rep., 85-486*, 1985.

Engelund, F., Closure to hydraulic resistance of alluvial streams, *J. Hydraul. Div. Am. Soc. Civil Eng.*, *93*(HY4), 287–296, 1967.

Engelund, F., and E. Hansen, *A Monograph on Sediment Transport in Alluvial Streams*, Teknisk Vorlag, Copenhagen, 1972.

Fenton, J. D., and J. E. Abbott, Initial movement of grains on a stream bed: The effect of relative protrusion, *Proc. R. Soc. London, Ser. A*, *352*, 523–537, 1977.

Ferguson, R. I., K. L. Prestegaard, and P. J. Ashworth, Influence of sand on hydraulics and gravel transport in a braided gravel bed river, *Water Resour. Res.*, *25*, 635–643, 1989.

Ikeda, H., and F. Iseya, Experimental study of heterogeneous sediment transport, *Pap. 12*, Environ. Res. Cent., Univ. of Tsukuba, Tsukuba, Japan, 1988.

Jackson, W. L., and R. L. Beschta, A model of two-phase bedload transport in an Oregon coast range stream, *Earth Surf. Processes Landforms*, *9*, 517–527, 1982.

Jackson, W. L., and R. L. Beschta, Influences of increased sand delivery on the morphology of sand and gravel channels, *Water Resour. Bull.*, *20*(4), 527–533, 1984.

Kamphuis, J. W., Determination of sand roughness for fixed beds, *J. Hydraul. Res.*, *12*(2), 193–203, 1974.

Kuhnle, R. A., Fractional transport rates of bedload on Goodwin Creek, in *Dynamics of Gravel-Bed Rivers*, edited by P. Billi et al., pp. 141–155, John Wiley, New York, 1992.

Kuhnle, R. A., and A. J. Bowie, Loop rating curves from Goodwin Creek, paper presented at National Conferenece on Hydraulic Engineering, Am, Soc. of Civil Eng., Baltimore, Md., 1992.

Leopold, L. B., Sediment size that determines channel morphology, in *Dynamics of Gravel-Bed Rivers*, edited by P. Billi et al., pp. 297–307, John Wiley, New York, 1992.

Leopold, L. B., and W. W. Emmett, Bedload and river hydraulics—Inferences from the East Fork River, Wyoming, *U.S. Geol. Surv. Prof. Pap., 1583*, 52 pp., 1997.

Lisle, T. E., Stabilization of a gravel channel by large streamside obstructions and bedrock bends, Jacoby Creek, northwestern California, *Geol. Soc. Am. Bull.*, *97*, 999–1011, 1986.

Lisle, T. E., Sediment transport and resulting deposition in spawning gravels, north coastal California, *Water Resour. Res.*, *25*(6), 1303–1319, 1989.

Lisle, T. E., Particle size variations between bed load and bed material in natural gravel bed channels, *Water Resour. Res.*, *31*(4), 1107–1118, 1995.

Lisle, T. E., J. M. Nelson, J. Pitlick, M. A. Madej, and B. L. Barkett, Variability of bed mobility in natural gravel-bed channels and adjustments to sediment load at the local and reach scales, *Water Resour. Res.*, *36*(12), 3743–3756, 2000.

Meyer-Peter, E., and R. Müller, Formulation for bed load transport, in *Proceedings of the 2nd Congress*, pp. 39–64, Int. Assoc. Hydr. Res., Stockholm, Sweden, 1948.

Milhous, R. T., Sediment transport in a gravel-bottomed stream, Ph.D. thesis, Oreg. State Univ., Corvallis, 1973.

Paola, C., and R. Seal, Grain size patchiness as a cause of selective deposition and downstream fining, *Water Res. Res.*, *31*(5), 1395–1407, 1995.

Paola, C., G. Parker, D. C. Mohrig, and K. X. Whipple, The influence of transport fluctuations on spatially averaged topography on a sandy, braided fluvial plane, in *Numerical Experiments in Stratigraphy*, *Spec. Publ. SEPM Soc. Sediment. Geol.*, *62*, 211–218, 1999.

Parker, G., Surface-based bedload transport relation for gravel rivers, *J. Hydraul. Res.*, *28*(4), 417–436, 1990.

Parker, G., Gravel-bed channel instability, in *Issues and Directions in Hydraulics*, edited by Y. Nakata and R. Ettema, pp. 115–133, A. A. Balkema, Brookfield, Vt., 1996.

Parker, G., and A. W. Peterson, Bar resistance of gravel-bed streams, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *106*(HY10), 1559–1575, 1980.

Parker, G., S. Dhamotharan, and S. Stefan, Model experiments on mobile, paved gravel bed streams, *Water Resour. Res.*, *18*(5), 1395–1408, 1982a.

Parker, G., P. C. Klingeman, and D. L. McLean, Bedload and size distribution in paved gravel-bed streams, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *108*(HY4), 544–571, 1982b.

Parker, G., C. Paola, and S. Leclair, Probabilistic Exner sediment continuity equation for mixtures with no active layer, *J. Hydraul. Eng.*, *126*(11), 818–826, 2000.

Seal, R., and C. Paola, Observations of downstream fining on the North Fork Toutle River near Mount St. Helens, Washington, *Water Resour. Res.*, *31*(5), 1409–1419, 1995.

Smith, G. H. S., and R. I. Ferguson, The gravel-sand transition along river channels, *J. Sediment. Res.*, *A65*(2), 423–430, 1995.

Vanoni, V. A., and N. H. Brooks, Laboratory studies of the roughness and suspended load of alluvial streams, *Sediment. Lab. Rep. E68*, Calif. Inst. Technol., Pasadena, 1957.

Wathen, S. J., R. I. Ferguson, T. B. Hoey, and A. Werrity, Unequal mobility of gravel and sand in weakly bimodal river sediments, *Water Resour. Res.*, *31*(8), 2087–2096, 1995.

Whiting, P. J., and W. E. Dietrich, Boundary shear stress and roughness over mobile alluvial beds, *J. Hydraul. Eng.*, *116*(12), 1495–1511, 1990.

Wilcock, P. R., Methods for estimating the critical shear stress of individual fractions in mixed-size sediment, *Water Resour. Res.*, *24*(7), 1127–1135, 1988.

Wilcock, P. R., Experimental investigation of the effect of mixture properties on transport dynamics, in *Dynamics of Gravel-Bed Rivers*, edited by P. Billi et al., pp. 109–139, John Wiley, New York, 1992.

Wilcock, P. R., The critical shear stress of natural sediments, *J. Hydraul. Eng.*, *119*(4), 491–505, 1993.

Wilcock, P. R., Estimating local bed shear stress from velocity observations, *Water Resour. Res.*, *32*(11), 3361–3366, 1996.

Wilcock, P. R., Two-fraction model of initial sediment motion in gravel-bed rivers, *Science*, *280*, 410–412, 1998.

Wilcock, P. R., The flow, the bed, and the transport: Interaction in flume and field, in *Fifth Gravel-Bed Rivers Workshop*, edited by P. Mosley, pp. 183–209, N. Z. Hydrol. Soc., Christchurch, 2001a.

Wilcock, P. R., Toward a practical method for estimating sediment transport rates in gravel-bed rivers, *Earth Surf. Processes Landforms*, *26*, 1395–1408, 2001b.

Wilcock, P. R., and B. T. DeTemple, Armor layers in flumes and streams, *Geol. Soc. Am. Program Abstr.*, *33*, abstract T24-3, 2001.

Wilcock, P. R., and B. W. McArdell, Surface-based fractional transport rates: Mobilization thresholds and partial transport of a sand-gravel sediment, *Water Resour. Res.*, *29*(4), 1297–1312, 1993.

Wilcock, P. R., G. M. Kondolf, W. V. G. Matthews, and A. F. Barta, Specification of sediment maintenance flows for a large gravel-bed river, *Water Resour. Res.*, *32*(9), 2911–2921, 1996.

Wilcock, P. R., S. T. Kenworthy, and J. C. Crowe, Experimental study of the transport of mixed sand and gravel, *Water Resour. Res.*, *37*(12), 3349–3358, 2001.

Yatsu, E., On the longitudinal profile of the graded river, *Eos Trans. AGU*, *36*, 655–663, 1955.

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