Size and Book-to-Market Factors in Returns

Qian Gu
Utah State University

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Abstract
Fama and French (Multifactor Explanations of Asset Pricing Anomalies, The Journal of Finance, March 1996) showed that average returns on common stocks are related to firm characteristics like size, earnings/price, cash flow/price, book-to-market equity, past sales growth, long-term past return, and short-term past return. These factors are considered anomalies because they are not included in theoretical models like the CAPM. In replicating their analyses, I find that firm size and the book-to-market of equity explain a large portion of the average excess returns on common stocks.

1. Introduction
This study replicates Multifactor Explanations of Asset Pricing Anomalies, Fama and French (1996), from The Journal of Finance, Vol. 51, No. 1 (Mar., 1996), pp. 55-84. However in this paper, I am only concerned with two of the anomalies that they considered: firm size and book-to-market equity, I examine these anomalies with respect to the average excess returns of common stocks during the last decade.

Many research papers that show that common stock expected returns are related to factors that are not captured by the CAPM. For instance, Banz (1981) showed that small firms tend to yield returns greater than those predicted by the traditional CAPM. However, Brown, Kleidon and Marsh (1982) found that the size effect is linear in the logarithm of size, but reject the hypothesis that the ex-ante
excess return attributable to size is stable through time. Then Basu (1983) confirmed that the common stock of high E/P firms earns higher risk-adjusted returns than the common stock of low earnings/price (E/P) firms. This result remains significant even after controlling for differences in firm size. Also, Fama and French (1993) identified five common risk factors in the returns on stocks and bonds. Three of them are stock-market factors which include an overall market factor and factors related to firm size and book-to-market equity (BE/ME). Two of them are bond-market factors that relate to maturity and default risks. These kinds of factors provide evidence that CAPM often does a poor job in predicting stock returns. Factors describing the incidence when the actual results under a given set of assumptions are different from the expected results are called anomalies (Anomaly).

Over the last few decades, scholars performed several empirical and practical tests on the CAPM. Jagannathan and Wang (1996) have shown that the static CAPM is unable to explain the cross-section of average returns on stocks. However, it also possible that the CAPM could be influenced by continuous factors. Confirmed anomalies are the size effect, the earnings/price, the cash flow/price, the book-to-market equity (BE/ME) and the past sales growth. (Banz (1981), Basu (1983), Rosenberg, Reid, and Lanstein (1985), and Lakonishok, Shleifer and Vishny (1994).) One of these tests is the cross-section of expected stock returns. The most well-known test of the cross-section of average returns
was done by Fama and French (1992). They thought that the two easily measured variables, book-to-market equity (BE/ME) and size (ME), provide a simple and powerful characterization of the cross-section of average stock returns for the 1963-1990 period. Thus, they found a factor model that expands on the CAPM by adding size and value factors in addition to the market risk factor in CAPM. This model is called the Fama and French three factor model which considers the fact that value and small cap stocks outperform markets on a regular basis (Fama And French Three Factor Model).

In this essay, I discuss the data and the Fama French three factor model to examine the role of size and BE/ME, and how they combine to explain the cross-sectional variation in average excess returns of common stocks.

2. Methodology

Fama and French (1993) proposed a three-factor asset-pricing model that includes a market factor and risk factors related to size and BE/ME seems to capture the cross-section of average returns on U.S. stocks. In 1992, Fama and French showed that the simple relation between β and average return is weak in the last half century (1941-1990) of returns on NYSE stocks. Unlike the simple relation between β and average return, the univariate relations between average return and size, leverage, E/P, and BE/ME are strong. They demonstrated β does not seem to help explain the cross-section of average stock returns. Then in their
multivariate regression, the combination of firm size and BE/ME absorb the effect of leverage and E/P and provide a strong explanation on average stock returns. Although β did little in explaining the model, when added in the combined regression of firm size and BE/ME, the fitted degree of regression model was enhanced. Thus, Fama and French (1992) concluded that for the 1963-1990 period, the combination of size and BE/ME capture the cross-sectional variation in average stock returns associated with the size, E/P, BE/ME, and leverage.

In 1993, Fama and French expanded their research in three ways: (a) they expand the set of returns from common stocks to U.S government and corporate bonds as well as stocks. (b) They also expand the set of variables to include bond returns, which help to explain stock returns, and vice versa. (c) They introduced a new approach which is a time-series regression to test asset-pricing models. In this paper, Fama and French illustrated that monthly returns on stocks and bonds are regressed on the returns to a market portfolios of stocks and mimicking portfolios for size, BE/ME, and term-structure risk factors in returns. According to this paper, they suggest that the return premiums $E(R_i) - R_f$ are associated with these factors: (a) their proxy for the market factor in stock returns is the excess market return $E(R_M) - R_f$. (b) They proposed measuring the size factor in each period as the differential return on small firms versus large firms, which is called SMB for small minus big. (c) Similarly, the other extra-market factor is typically measured as the return on firms with high BE/ME minus that on firms with low
BE/ME, or HML for high minus low. Therefore, the Fama and French three-factor model is

\[
E(R_i) - R_f = a_i + b_i[E(R_M) - R_f] + s_iE[SMB] + h_iE[HML] \quad (1)
\]

\(E(R_i)\) is the expected return on asset \(i\). \(E(R_M)\) is the expected return on the value-weight market portfolio. \(R_f\) is the risk-free interest rate of 1-month Treasury bills. The coefficients \(b_i, s_i\) and \(h_i\) are the \(\beta\)s of the stock on each of the three factors.

To create portfolios that track the size and BE/ME factors, Davis, Fama and French (2000) use a broad market index, the value-weighted return on all stocks traded on U.S. national exchanges (NYSE, AMEX and NASDAQ) to compute the excess return on the market portfolio relative to the risk-free rate. Then Fama and French (1993) decide to sort firms into three groups on BE/ME and only two on ME following the evidence in Fama and French (1992) that book-to-market equity has a stronger role in average stock returns than size. The splits are arbitrary, however, and we have not searched other alternatives. In June and each year \(t\) from 1963 to 1991, all NYSE stocks on CRSP are ranked on size (price times shares). The median NYSE size is then used to split NYSE, Amex, and (after 1972) NASDAQ stocks into two groups, small and big (S and B). They also break NYSE, Amex, and NASDAQ stocks into three BE/ME groups based on the breakpoints for the bottom 30\% (Low), middle 40\% (Medium), and top 30\% (High) of the ranked values of BE/ME for NYSE stocks. After that, they construct six
portfolios (S/L, S/M, S/H, B/L, B/M, B/H) from the intersections of the two ME and the three BE/ME groups. For example, the S/L is the value-weight return on the portfolio of stocks that are below the NYSE median in size and in the bottom 30% of BE/ME.

The size premium, SMB, is the average return on the three small portfolios minus the average return on the three big portfolios:

\[ \text{SMB} = \frac{1}{3} (S/L + S/M + S/H) - \frac{1}{3} (B/L + B/M + B/H). \]

Similarly, HML in each period is the average return on the two value portfolios (that is, with high BE/ME ratios) minus the average return on the two growth portfolios (low BE/ME ratios):

\[ \text{HML} = \frac{1}{2} (S/H + B/H) - \frac{1}{2} (S/L + B/L). \]

Monthly value-weighted returns on the six portfolios are calculated from July of year t to June of t + 1, and the portfolios are reformed in June of t + 1. We calculate returns beginning in July of year t to be sure that book equity for year t - 1 is known.

To test the three-factor model, Davis, Fama and French (2000) form nine portfolios in June each year as the intersections of independent sorts of stocks into three size groups (small, medium, and big; or S, M, B) and three BE/ME groups (high, medium, and low; or H, M, L). For example, S/H is the return of stocks that is in the smallest third of firms and the top third of BE/ME.

For each of these nine portfolios, Davis, Fama and French (2000) estimate
Equation (1) as a first-pass regression over the 816 months between 1929 and 1997 by using the regression model

\[ R_i - R_f = a_i + b_i(R_M - R_f) + s_iSMB + h_iHML + e_i \] (2)

For the post-formation returns on nine portfolios in June each year as the intersections of independent sorts of stocks into three size groups and three BE/ME groups.

The three-factor model in (1) seems to capture much of the cross-sectional variation in average stock returns. In Fama and French (1993), they show that size and BE/ME proxy for sensitivity to risk factors that capture strong common variation in stock returns and help explain the cross-section of average returns.

The evidence presented in Fama and French (1995) shows that size and BE/ME are related to profitability. Not surprisingly, firms with high BE/ME is associated with persistently low earnings on assets. Then controlling for book-to-market equity, small firms tend to have lower earnings on assets than big firms, however, is largely due to the 1980s. Prior to 1980, given BE/ME, small firms are only slightly less profitable than big firms. But for small stocks, the recession of 1981 and 1982 turns into a prolonged earnings depression. In other words, small firms do not participate in the economic boom of the middle and late 1980s. Though there is no exact reason for the small stock depression of the 1980s, it does imply that a size effect exists in fundamentals that might cause a size-related risk factor in returns.
In the long-term, Fama and French (1992) provide an economic foundation for the empirical relations between average stock return and size, and average return and book-to-market-equity. However, in a rational market, short-term variation in profitability should have little effect on stock price and BE/ME. BE/ME should be associated with long-term differences in profitability, which is illustrated in Fama and French (1995). Firms with high BE/ME (a low stock price relative to book value) tend to be persistently distressed. They have low ratios of earnings to book equity, and the low earnings persist for at least five years before and five years after BE/ME is measured. Conversely, low BE/ME (a high stock price relative to book value) is associated with sustained strong profitability.

In the application of Fama French three-factor model, we utilize theoretical assumptions based on finite theory. There are several basic assumptions: (a) a large number of investors exist. (b) All investors schedule their own portfolios in the same security holding period. (c) The scope of investment is restricted to assets traded in the open financial market. (d) Securities transaction cost (including commission and service fee) and taxes do not exist. (e) Investors have the same expectation to mean, variation, and standard deviation of security returns. (f) All investors have the same perspective on the evaluation of security and economy. Fama French three-factor model is a polynomial regression model, thus its statistical assumptions are just the same as general multivariate regression model.
According to the arbitrage pricing model, if these are the relevant factors, excess returns should be fully explained by risk premiums due to these factor loadings. In other words, if these factors fully explain asset returns, the intercept of the equation should be zero. To specify macroeconomic factors as proportions of related systematic risks, we utilize firm characteristics to proxy for exposure to systematic risk on an empirical level. The factors chosen are variables based on past evidence that seem to predict average returns well and thus, might be capturing risk premiums. These two firm characteristic variables are chosen because in the long-term observation, using firm size and BE/ME to model expected average stock returns is identical to which the CAPM predicted. Fama and French proved the model through empirical test: although SMB and HML is not the significant proxy of related risk factors, they can substitute unknown and more fundamental variables yet. For instance, Fama and French pointed out that firms with high BE/ME are easier to be deep in financial crisis, and that small firms are more sensitive to the change of business conditions. Therefore, these variables can capture sensitivity to risk factors in the macroeconomy.

Empirical approaches similar to the Fama French three factor model adopt some proxy to describe sources of risk outside the market. This leads to a problem that none of these factors could indicate to hedge a significant source of uncertainty. Black (1993) demonstrated when researchers doing data snooping, they might find that this patterns are purely accidental. Black thought risk premium
to factors like firm size has justified contradictory when first observed. However, Fama and French pointed out that size and BE/ME have predicted average stock returns in various time period over the global market, thus mitigating potential effects on data snooping.

3. Data

**Table I**


<table>
<thead>
<tr>
<th>Size</th>
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<th>Low</th>
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<td>1.16</td>
<td>1.36</td>
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<td>3.90</td>
<td>4.56</td>
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**Table II**


<table>
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<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Standard Error</th>
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<td>Mkt-Rf</td>
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<td>SMB</td>
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<tr>
<td>HML</td>
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<tr>
<td>Rf</td>
<td>0.12</td>
<td>0.15</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Number of observations: 132
Correlation between SMB and HML: 0.15
The data comes from Kenneth French's web site at Dartmouth. There are 132 observations and 29 variables in the 25 Portfolios Formed on Size and Book-to-Market (5 x 5). The 25 size-BE/ME portfolios are constructed as the intersections of 5 portfolios formed on size and 5 portfolios formed on the ratio of BE/ME.

Market equity (size) is stock price times shares outstanding. Price and shares outstanding are both from CRSP.

According to Kenneth R. French’s website, book equity is constructed from Compustat data. BE is the book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. Depending on availability, redemption, liquidation, or par value (in that order) were used to estimate the book value of preferred stock. Stockholders' equity is the value reported by Compustat. (Davis, Fama, & French, February 2000)

The book-to-market ratio (BE/ME) used to form portfolios in June of year t is book equity for the fiscal year ending in calendar year t-1, divided by market equity at the end of December of t-1.

As stated above, the Fama-French factors are constructed using the 6 value-weight portfolios formed on size and book-to-market (S/L, S/M, S/H, B/L, B/M, B/H).

\[
SMB = \frac{1}{3} (S/L + S/M + S/H) - \frac{1}{3} (B/L + B/M + B/H).
\]
HML = 1/2 (S/H + B/H) - 1/2 (S/L + B/L).

\[ R_M - R_f, \] the average excess return on the market, value-weighted return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month t.

\[ R_M \] (market return) is the return on the value-weighted portfolio of the stocks in the six size-BE/ME portfolios, plus the negative-BE stocks excluded from the portfolios. \[ R_f \] (risk-free rate) is the one-month treasury bill rate. (Fama & French, February 1993)

Each year t from 2003 to 2013 NYSE quintile breakpoints for size, measured at the end of June, are used to allocate NYSE, AMEX and NASDAQ stocks to five size quintiles. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. Similarly, NYSE quintile breakpoints for BE/ME are used to allocate NYSE, AMEX and NASDAQ stocks to five book-to-market equity quintiles. The portfolios for July of year t to June of t+1 include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for December of t-1 and June of t, and (positive) book equity data for t-1.

4. Results

**Table III**


In June of each year t from 2003 to 2013, all NYSE stocks on CRSP are ranked
on size (price times shares). The median NYSE is then used to split NYSE, AMEX and NASDAQ stocks into two groups, small and big (S and B) based on whether their June market capitalization, ME (stock price times shares outstanding), is below or above the median for NYSE stocks on CRSP. Stocks are allocated in an independent sort into three BE/ME groups on the breakpoints for the bottom 30% (Low), middle 40% (medium), and top 30% (High) of the ranked values of BE/ME for NYSE stocks in our samples. BE, as the COMPUSTAT book value of stockholders’ equity, plus balance-sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. The BE/ME ratio used to form portfolios in June of year t is then book common equity for the fiscal year ending in calendar year t-1, divided by market equity at the end of December of t-1. We do not use negative-BE firms when calculating the breakpoints for BE/ME or when forming the size-BE/ME portfolios. Also, only firms with ordinary common equity (as classified by CRSP) are included in the tests. This means that ADRs, REITs, and units of beneficial interest are excluded.

The final portfolios are the six intersections of the two ME and the three BE/ME groups (S/L, S/M, S/H, B/L, B/M, and B/H). Monthly value-weighted stock returns for the six portfolios are calculated from July of year t to June of year t + 1, and the portfolios are reformed in June of year t + 1. We calculate returns beginning in July of year t to be sure that book equity for year t - 1 is known. SMB is the difference between the average of the return on the three small-stock portfolios (S/L, S/M, and S/H) and the average of the returns on the three big-stock portfolios (B/L, B/M and B/H). HML is the difference, each month, between the average of returns on the two high-BE/ME portfolios (S/H and B/H), and the average of the return on the two low-BE/ME portfolios (S/L and B/L). The 25 size-BE/ME portfolios are formed much like the six size-BE/ME portfolios. We construct 25 portfolios from the intersections of the size and BE/ME quintiles and calculate value-weighted monthly returns on the portfolios from July of t to Jun of t+1. For stocks, we use excess returns on 25 portfolios for January 2003 to December 2013, formed on size and BE/ME, as dependent variables in the time-series regressions.

Our proxy for the market factor in stock returns is the excess market return, $R_M - R_f$. $R_M$ is the value-weight return on all NYSE, AMEX and NASQAD stocks in the six size-BE/ME portfolios, plus the negative-BE stocks exclude from the portfolios. $R_f$ is the one-month Treasury bill rate observed at the beginning of the month (from CRSP).
### Book-to-Market Equity (BE/ME) Quintiles

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<th>Size</th>
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<th>4</th>
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</table>
| Panel B: Regressions: \( R_i - R_f = a_i + b_i(R_m - R_f) + s_iSMB + h_iHML + e_i \)

<p>| | | | | | | | | | | |</p>
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<tr>
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<td>0.95</td>
</tr>
<tr>
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<tr>
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<td>0.78</td>
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<td>-0.93</td>
<td>-1.73</td>
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| b   |     |     |     |     |      |     |     |     |     |      |
| t(b) |     |     |     |     |      |     |     |     |     |      |
| Small | 1.15 | 1.04 | 0.96 | 0.85 | 0.97 | 24.14 | 31.35 | 36.80 | 29.39 | 28.84 |
| 2   | 1.07 | 1.00 | 0.95 | 0.97 | 1.04 | 35.08 | 38.51 | 35.69 | 33.99 | 29.94 |
| 3   | 1.10 | 1.04 | 1.01 | 1.07 | 0.96 | 35.83 | 35.81 | 29.91 | 28.28 | 23.02 |
| 4   | 1.05 | 1.10 | 1.19 | 1.03 | 1.10 | 36.39 | 35.48 | 29.40 | 26.19 | 26.79 |
| Big | 0.99 | 0.89 | 1.03 | 0.99 | 1.09 | 51.72 | 34.09 | 33.36 | 29.91 | 23.02 |

| s   |     |     |     |     |      |     |     |     |     |      |
| t(s) |     |     |     |     |      |     |     |     |     |      |
| Small | 1.17 | 1.02 | 0.95 | 1.03 | 1.05 | 13.52 | 16.97 | 19.90 | 19.38 | 17.10 |
| 2   | 1.01 | 0.89 | 0.93 | 0.86 | 0.96 | 18.18 | 18.80 | 19.07 | 16.36 | 15.06 |
| 3   | 0.68 | 0.66 | 0.54 | 0.42 | 0.65 | 12.08 | 12.30 | 8.76 | 6.14 | 8.50 |
| 4   | 0.45 | 0.36 | 0.33 | 0.38 | 0.14 | 8.57 | 6.37 | 4.47 | 5.34 | 1.86 |
| Big | -0.17 | -0.17 | -0.22 | -0.25 | -0.03 | -5.00 | -3.59 | -3.93 | -5.17 | -0.30 |

| h   |     |     |     |     |      |     |     |     |     |      |
| t(h) |     |     |     |     |      |     |     |     |     |      |
| Small | -0.26 | -0.02 | 0.19 | 0.44 | 0.77 | -3.26 | -0.41 | 4.35 | 8.98 | 13.42 |
| 2   | -0.34 | -0.06 | 0.25 | 0.37 | 0.92 | -6.52 | -1.42 | 5.59 | 7.64 | 15.67 |
| 3   | -0.39 | -0.04 | 0.11 | 0.28 | 0.63 | -7.44 | -0.90 | 1.85 | 4.40 | 8.93 |
| 4   | -0.36 | -0.10 | 0.05 | 0.25 | 0.79 | -7.30 | -1.81 | 0.72 | 3.73 | 11.41 |
| Big | -0.38 | 0.09 | 0.21 | 0.46 | 0.57 | -11.63 | 2.12 | 3.95 | 10.16 | 6.17 |

<table>
<thead>
<tr>
<th>R square</th>
<th>s (e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>0.91</td>
</tr>
<tr>
<td>2</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>0.94</td>
</tr>
<tr>
<td>Big</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Table I shows the summary of average excess returns on the 25 Fama-French (1993) size-BE/ME portfolios of value-weighted NYSE, AMEX, and NASD stocks. This table implies small firms tend to have higher average returns than big firms and high BE/ME stocks tend to have higher average returns than low BE/ME stocks. However, the numbers are not strictly increasing or decreasing monotonically. This is probably because using independent size and BE/ME quintile of NYSE stocks to construct portfolios results in that the highest BE/ME quintile is likely fall towards the smallest stocks. Moreover, most of AMEX and NASDAQ stocks are small, and they tend to have lower BE/ME than NYSE stocks controlling similar size. In other words, NYSE small stocks are more likely to have low stock prices with big firms than that of small AMEX and NASDAQ stocks.

Table II states the summary statistics for simple Fama-French 3 factors. The mean of the market premium, \( R_M - R_f \), is 0.77% per month and it is 0.37 standard errors from zero for the whole 132 months sample period. Thus, there is a very strong market premium in returns. In contrast to the market premium, there are relatively small value premium in SMB and HML. The size effect in Table II is not obvious. The average SMB return for January 2003 to December 2013 is 0.35% per month (t-statistic = 0.20). The BE/ME effect in Table II is also puny. The average HML return is only 0.16% per month (t-statistic = 0.20), that is small in both practical and statistical terms. The relatively weak size effect and fragile BE/ME effect might due to the 6 proportions of SMB and HML portfolios are
measured with value-weighted returns to some extent.

SMB should be neutral with respect book-to-market effect, and it focuses on the
different return behaviors of small and big stocks. Similarly, HML should be neutral
with respect of the size factors, and it concentrates on the different return
behaviors of high and low BE/ME stocks. To justify their relations, we come out
the correlation between the monthly returns for SMB and HML is only 0.15 during
January 2003 to December 2013.

Table III shows the estimates of the three-factor time-series regression (2). In
practical terms, the intercepts of the regressions are almost close to zero and
generally statistically insignificant, with t-statistics below 2 except for two
unexplained returns. One is a large negative return for the smallest firms and
lowest BE/ME quintile in the portfolio. The other is a large positive return in the
highest BE/ME but the 3rd largest firm size for the portfolio. Table III illustrates that
for the risk factors, size premium SMB and value premium HML, actually capture
common variation in average excess stock returns missed by the CAPM β. The
size proxy on SMB for risks is especially important in returns in relatively small
firms. The smaller the firm is, the larger the estimated β of SMB. Controlling for
size, the value proxy on HML increase monotonically from the lowest BE/ME to
the highest BE/ME portfolios. The ts for the SMB βs for the small firms are all
greater than 15.06. The ts on the value factor HML βs for the largest BE/ME
stocks are all greater than 6.17. The large t-statistics on the size and value
loadings show that these factors contribute significantly to explanatory power. Moreover, the large R-square are all in excess of 0.84, show that the explanatory power of the three-factor portfolios is always high. Thus, as in Fama and French (1993), the regression βs for the three risk factors (bi, si, hi) capture most of the strong spread in the average excess returns on 25 portfolios formed on size and BE/ME reported in Table III.

The F-test of Gibbons, Ross and Shanken (1989) rejects the zero-intercepts hypothesis very strongly with a p-value of zero to at least 14 places after the decimal point for the 132 months sample period. In other words, all independent variables for the three-factor regression are jointly significant. Two of the intercepts in the regressions are different from zero on a statistical basis with t-statistics more than 2. To some extent, as Fama and French (2000) stated, the relative t-statistics for the regression slope and average premiums for the three risk factors (the average value of $R_M - R_f$, SMB, HML) are large. This is not because the difference between the predictions of the three-factor model and average excess returns are large, but rather because the regressions absorb so much return variance.

Like the tests Fama and French (1993, 1996), Table III demonstrate the three-factor model with what it is, how the model operates, and the potential issues of the model. However, the three-factor model does provide a reasonable approximation for the excess returns on 25 portfolios formed on size and BE/ME.
Thus, it is a viable model to test the risk factors for expected stock returns against with the $\beta$ of CAPM. Moreover, as we argued above, the three-factor model indicates a more accurate description of average stock returns that the CAPM model.

5. Conclusion

By replicating Multifactor Explanations of Asset Pricing Anomalies, Fama and French (1996), we only provide two common risk factors in average returns associated with size and BE/ME based on an empirical test of the Fama French three-factor model (1993). The three factor model is a good model which explains the anomalies better than the CAPM due to rational asset pricing theory. As Fama and French (1993) demonstrated, we show that size and BE/ME indeed proxy for sensitivity to common risk factors in stock returns. We examine the intercepts for the excess market return from three-factor regressions are close to zero and the large t-statistics on the mimicking loadings for size and BE/ME show that these factors contribute significantly to explanatory power. Thus, a market factor and our proxies for the risk factors related to size and BE/ME could help explain the cross-section of average returns. For stocks, no matter what else is in the time-series regression, 25 portfolios formed on size and BE/ME capture strong common variation in stock returns.

Fama and French (1992) observed the long-term economic foundation for the
empirical relations between average stock returns related to size as well as BE/ME. However in a rational market, the average excess stock returns would have little effect on BE/ME. The lower BE/ME, which means higher stock price relative to book value, is always associated with a higher excess returns. This is adverse to our examined tendency for BE/ME related to average returns. Moreover, we only construct portfolios on a common stock basis but do not consider bonds and other securities. This means we did an incomplete description of expected average stock returns.

Finally, there leaves one important open question in our work. In Fama and French (1993), we argue that our common stock returns are consistent with multifactor of asset pricing anomalies. Except for firm size and BE/ME, there are some patterns apparently that are not explained by the CAPM. For instance, earnings/price, cash flow/price, past sales growth, long-term and short-term past returns. These kinds of variables are captured by market factors but not include by our works. Also there could be other variables that are even not acquired by an overall market factor which might have some effect on average stock returns. We know we cannot make a perfect or even correct explanation on what we concerned in this paper, but multiple scholars continue doing research and study this kind of issue to make the results more viable.
Appendix for R Code

```r
mean(25_Portfolios_5x5.vwr$SMALL.LoBM)

Portvwr$Y1 = Portvwr$SMALLLoBM - Portvwr$RF
myreg <- lm(Y1 ~ MktmRF + SMB + HML, data = Portvwr)
summary(myreg)

Portvwr$Y2 = Portvwr$ME1BM2 - Portvwr$RF
myreg2 <- lm(Y2 ~ MktmRF + SMB + HML, data = Portvwr)
summary(myreg2)

......

Portvwr$Y25 = Portvwr$BIGHiBM - Portvwr$RF
myreg25 <- lm(Y25 ~ MktmRF + SMB + HML, data = Portvwr)
summary(myreg25)
```

References


427-465.