A Linear Programming Decision Model for Elk and Deer Management on the Logan Peak Winter Range Area

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A LINEAR PROGRAMMING DECISION MODEL FOR ELK AND DEER MANAGEMENT
ON THE LOGAN PEAK WINTER RANGE AREA

by
Ronald P. Grove

A Plan B report submitted in partial fulfillment
of the requirements for the degree
of
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in
Forest Science
(Forest Management)

UTAH STATE UNIVERSITY
Logan, Utah
1973
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Ronald P. Grove
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Chapter I

INTRODUCTION

One consequence of our surging human population is the corresponding increased level of resource consumption. This occurrence renders it imperative that resource managers intensify their management of the world's natural resources. Failure to improve management techniques involving the use of these resources will result in premature depletion of non-renewable commodities and severe checks on the vigor of renewable resources.

Management is faced with the difficult task of searching for astute means of allocating natural resources. Because of the tremendous size and complexity of the natural world, the problem can be extremely complicated.

Mathematical techniques have proved useful in providing natural resource managers with increased competence in decision-making. This work illustrates the possibilities of a mathematical decision model for elk and mule deer management on the Cache Big Game Management Unit in northeastern Utah.

Problem

In large portions of the intermountain region the winter range of ungulates is the limiting factor in their production. On the Cache Big Game Management Unit in northeastern Utah, winter range is the critical factor in producing elk and mule deer. The Logan
Peak area, a subdivision of the Cache Big Game Management Unit, contains principal elk and mule deer winter ranges which are largely confined to the Wasatch face, but extend into the Logan River and Blacksmith Fork drainages. Since winter range is the principal factor limiting animal populations in this particular subdivision, game managers of the Cache National Forest and the Utah State Division of Wildlife Resources seek opportunities to improve winter range carrying capacity and to regulate hunting in this area so as to fully utilize but not deteriorate range quality.

Conceptually, management activities for the Logan Peak winter range area could be directed toward one of three possible alternatives: first, the manager may choose to manage the area primarily for elk, second, he may choose to manage the area chiefly for mule deer, or, third, he may choose to manage the area equally for both elk and mule deer. Both species are desired and current social, political, and economic factors require that the game manager select the third alternative. The current problem facing the manager is thus one of producing and regulating an optimum elk and mule deer population which is available for harvest by hunters and for sightseeing and other non-consumptive uses.

The game manager has several decisions to make in developing a management program for the Logan Peak Management Unit. For example, he must decide: (1) how much and what type of land management is required to produce adequate food and cover; (2) how many animals of each species and sex should or can be harvested; (3) what length, type, and time of hunting season is required to remove the desired
number of animals; and (4) how to allocate money and manpower between management of the two species: elk and deer.

Linear programming (LP) is an operations research technique that can be used to deal with a natural resource manager's problem of choosing between alternatives. This report illustrates its usefulness by developing a linear programming decision model for the Logan Peak unit described above.

Objectives

The purpose of this work is (1) to demonstrate the applicability of the linear programming technique to big game management problems and (2) to provide the foundation for a specific formulation of the Logan Peak winter range unit.

The process of developing linear programming models will be discussed and a time stage linear programming model will be constructed for analysis of elk and mule deer herd management.

Method of Procedure

The procedure to be followed in this paper will be to adapt the techniques developed by Davis (1967) to the Logan Peak Management Unit problem. This work develops a mathematical model relating controllable variables of land management and deer harvest as a linear programming problem for computer analysis. The principle difference between the two models is that winter range is a limiting factor on the Cache Big Game Management Unit, while in Davis's problem developed in the Southeast, there was no winter range limitation.
The model will depict its real world counterpart by describing and incorporating the significant variables and the biological and managerial aspects of the elk and deer management situation, identified and quantitatively expressed as linear equations.

Empirical data will be utilized as much as possible. However, where appropriate data are not available, estimates of numerical relationships will be obtained from the literature and knowledgeable experts. The experts will be qualified personnel from the U.S. Forest Service, the Utah State Division of Wildlife Resources, and the College of Natural Resources, Utah State University.
Chapter 2
LINEAR PROGRAMMING

One major application of linear programming is solving the manager's problem of allocating scarce resources among alternative management activities that are essential to accomplishing his predetermined goals. Richmond (1968) relates that the basic problem which can be solved by the linear programming technique is that of maximizing or minimizing a linear objective function which is subject to a set of linear constraints.

This mathematical technique may be applied to an immense variety of situations. It is applicable to practical problems of allocation in economics, government, military, and industrial operations, as well as to natural resource management.

According to Hiller and Lieberman:

Linear programming uses a mathematical model to describe the problem of concern. The adjective "linear" means that all the mathematical functions in this model are required to be linear functions. The word "programming" is essentially a synonym for planning. Thus, linear programming involves the planning of activities in order to obtain an "optimal" result, i.e., a result which reaches the specified goal best (according to the mathematical model) among all feasible alternatives. (Hiller and Lieberman, 1970, p. 127).

The Linear Programming Model

The mathematical model utilized in linear programming is developed around two components. One part consists of a linear function which is to be maximized or minimized. This equation is called the objective function. The second component consists of a group of
functions which represent restrictions or constraints relative to the objective function. The component is appropriately described by Davis as:

... a set of equations representing or describing a real world economic or biological activity, including the real world limitations on resources such as land, food, or labor. The variables in these equations are specified to be activities under control of the manager. (Davis, 1967, p. 668)

Spivey (1963) presents an elementary form of the linear programming model as:

Maximize: 

\[ Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n, \]

subject to

\[ a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \]
\[ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \]
\[ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \vdots \]
\[ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m \]

where \( Z \) is the objective function (the chosen over-all measure of effectiveness); \( a_{ij}, b_i, \) and \( c_j \) are known constants; and \( x_1, x_2, \ldots, x_n \) are the decision variables which represent the levels of \( n \) competing activities.

Since negative activity variables are undesired, non-negative restrictions are included in the model. These are written as \( x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0. \)

Limitations of Linear Programming

An essential requirement of the linear program model is linearity. A program is linear if the variables in the objective function and every constraint function appear only as linear forms. This form is an expression of the type \( y = a_1x_1 + a_2x_2 + \ldots + a_nx_n + b, \) where \( a_j \) and
b are constants. Hadley (1963, p. 5) comments that "Intuitively, linearity implies that products of the variables, such as $x_1x_2$, powers of variables, such as $x_3^2$, and combinations of variables, such as $a_1x_1 + \log x_2$, cannot be allowed."

In linear programming the activities must be additive. This means that if we use $h_1$ hours on a machine to produce product A, and $h_2$ hours on the machine to produce product B, then the total time used by the machine to produce both products is $h_1 + h_2$.

Proportionality is also a characteristic desired in linear programming. This property is illustrated by Hadley (1963 p. 5) as follows: ",.. (1) If it takes one hour to make a single item on a given machine, it takes ten hours to make ten parts; .. . (2) The total profit from selling a given number of units of a product is the unit profit times the number of units sold; .. ."

Another limitation of linear programming is that the variables can take on any values permitted by the constraints. This simply means that fractional values of the decision variables ($X_n$) are permitted.

A number of books are available which provide a comprehensive study of linear programming. A thorough development of the subject is presented in Hadley (1963) and an excellent introduction for the neophyte is given by Spivey (1963).

The value of using linear programming techniques for solving managerial problems of the type mentioned above is significant.

According to Davis:

The utility of linear programming arises because of its solution method. The values of the activity variables are found which maximize the value of the separate linear equation and which, at the same time, are consistent with the whole
set of production relationship equations specified on the same variables. If achieving the maximum value of the separate or objective equation corresponds in the real world to achieving "best" or optimum results, then LP analysis effectively finds an approximation to the best plan of management." (Davis, 1967, p. 668)

**Time Stage Linear Programming**

The procedure for dealing with optimization problems for allocating resources over a period of time is termed time stage linear programming. It differs from normal linear programming in that production relationships are linked over a specified time period and that management objectives are directed to maximizing total production throughout several time periods. In the problem considered in this work, there are dynamic ties between animals and forage production from one time interval to another.
Chapter 3

CONSTRUCTION OF THE MODEL

In order for the Logan Peak model to accurately portray its real world counterpart, it must formally describe and include all of the significant factors and affiliations relating to elk and deer production.

The aspects to be incorporated into the model include fertility, mortality, food requirements, breeding requirements, harvesting, browse production, and the quantity of land, money, and labor that is available to the resource manager.

These factors will be described as a series of linear equations which will depict the ecological and managerial aspects of the elk and deer management situation on the Logan Peak winter range area. Each significant relationship will be separately identified and expressed as a specific equation. The management objective will also be specified and combined with the series of linear equations described above to form the linear programming decision model.

Geographic Area of Study

The Logan Peak area (Figure 1) is located in northeastern Utah and lies entirely within Cache County. The boundaries of the region are established by the Utah Division of Wildlife Resources. On the north, it is bounded by Highway U.S. 89. The southern boundary is the Left Hand Fork of the Blacksmith Fork River and the main stem of the Blacksmith Fork River. The eastern boundary is Cowley Canyon and
Figure 1. The Logan Peak Game Management Unit.
Herd Hollow. The western limit of the area is the big game fence which extends between Logan Canyon and the main Blacksmith Fork River.

The winter range portion of the Logan Peak area (Figure 2) is the specific study unit for this paper. It comprises approximately 8,000 acres of Cache National Forest and privately owned range and forest land. It is bounded on the west by the big game fence, and on the south by State Highway 242 and Forest Route 055. The eastern limit is defined by the ridge top of the Wasatch face excluding Logan, Dry, and Profidence Canyons. The area extends into the Blacksmith Fork drainage approximately half way up along the face of the mountains, and on to Herd Hollow. The northern boundary extends about one quarter mile along the south side of State Highway 89 into Logan Canyon. There are also some isolated areas located near Logan Peak, Spring Hollow, and Card Canyon.

Three of the five major vegetational types defined for the Cache Big Game Management Unit by Hancock (1955) are found within the study area: juniper, mahogany, and sagebrush.

The northern portion of the area is currently inhabited by deer only. The southern part of the range is inhabited by both deer and elk. It is estimated that approximately 50 percent or 4,000 acres of the southern sector are occupied by both species.¹

Decision Variables

Decision variables, or management activities, are the variables

¹Information about the winter range and area inhabited by each species was obtained from Jon Gates, Conservation Officer, Utah State Division of Wildlife Resources, during an interview on September 26, 1972.
Figure 2. The winter range area of the Logan Peak Game Management Unit.
about which the production relationships are developed. They are referred to as activity variables and are defined and used in the model as follows:

<table>
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<th>Definition and Units of Measurement</th>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Area inhabited by deer only</td>
<td></td>
</tr>
<tr>
<td>$X_1$ : Number of acres of normal (unmanaged) land</td>
<td></td>
</tr>
<tr>
<td>$X_2$ : Number of acres treated to produce deer food</td>
<td></td>
</tr>
<tr>
<td>$X_3$ : Number of pounds of surplus (un-utilized) food</td>
<td></td>
</tr>
<tr>
<td>Area inhabited by deer and elk</td>
<td></td>
</tr>
<tr>
<td>$X_4$ : Number of acres of normal (unmanaged) land</td>
<td></td>
</tr>
<tr>
<td>$X_5$ : Number of acres treated to produce deer food</td>
<td></td>
</tr>
<tr>
<td>$X_6$ : Number of acres treated to produce elk food</td>
<td></td>
</tr>
<tr>
<td>$X_7$ : Number of pounds of surplus (un-utilized) food</td>
<td></td>
</tr>
<tr>
<td>Applicable to entire area</td>
<td></td>
</tr>
<tr>
<td>$X_8$ : Number of harvested buck deer</td>
<td></td>
</tr>
<tr>
<td>$X_9$ : Number of harvested doe deer</td>
<td></td>
</tr>
<tr>
<td>$X_{10}$ : Number of harvested fawn deer</td>
<td></td>
</tr>
<tr>
<td>$X_{11}$ : Number of remaining buck deer</td>
<td></td>
</tr>
<tr>
<td>$X_{12}$ : Number of remaining doe deer</td>
<td></td>
</tr>
<tr>
<td>$X_{13}$ : Number of remaining fawn deer</td>
<td></td>
</tr>
<tr>
<td>$X_{14}$ : Number of harvested bull elk</td>
<td></td>
</tr>
<tr>
<td>$X_{15}$ : Number of harvested cow elk</td>
<td></td>
</tr>
<tr>
<td>$X_{16}$ : Number of harvested calf elk</td>
<td></td>
</tr>
<tr>
<td>$X_{17}$ : Number of remaining bull elk</td>
<td></td>
</tr>
<tr>
<td>$X_{18}$ : Number of remaining cow elk</td>
<td></td>
</tr>
<tr>
<td>$X_{19}$ : Number of remaining calf elk</td>
<td></td>
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</table>
Objective Function

The decision maker's purpose is to select that course of action which will result in obtainment of specific management goals. In a linear programming model these management goals are expressed as mathematical statements termed objective functions. The most common objectives of managers in the economic community are maximization of profit or minimization of costs.

The game manager's problem is one of utilizing his limited financial and human resources to obtain an objective within certain limitations expressed as a system of constraints. That is, he must attempt to employ these resources in such a manner as to satisfy as completely as practicable man's desires relative to the hunting or aesthetic values of the wildlife resource, within the confines of the ecological and managerial aspects previously mentioned. In this circumstance the manager faces many difficult decisions, for the specific desires of individuals cannot be fulfilled by blind attention to the wants of the average public citizen.

For illustrative purposes, this work assumes that the game manager's goal is to maximize the total number of animals which could be harvested over the 20-year period. The model's objective function weights the harvested animals by the relative hunter cost of harvesting the different animals. An analysis by the Utah State Division of Wildlife Resources indicates an average cost of $7 per harvested deer. This

2 Information obtained from Dr. J. Juan Spillett, Utah State University, during an interview on October 25, 1972. He and other members of the Department of Wildlife Science feel that this method of determining costs is not realistic. The author concurs.
cost is based on money spent solely for deer hunting by resident hunters, above and beyond what they spend for other hunting and recreation. It includes variable expenditures such as gasoline, mileage rates, and ammunition, and does not include the cost of a hunting license, food, or a firearm. For this objective all deer have a relative value of $7 which implies that the hunter receives equal satisfaction from taking any buck, doe, or fawn. Ashcroft (1967), in his socio-economic study of the Cache Elk Herd, concludes that an average cost for harvesting elk is $25. This figure includes $15 for the price of an elk permit because it can be used only for this type of hunting. Otherwise the cost is based on the variable expenditures previously mentioned. All bulls, cows, and calves have a relative value of $25 which indicates equal hunter satisfaction from taking any animal of this species.

Utilizing these cost figures and the managerial (activity) variables presented for this work, the objective function can be expressed mathematically as:

$$\text{Maximize the sum: } \sum_{t=1}^{20} 7x_8 + 7x_9 + 7x_{10} + 25x_{14} + 25x_{15} + 25x_{16}.$$
Constraints

This portion of the linear programming decision model comprises the set of equations which describe the ecological and economic aspects of deer and elk production.

Mortality and Fertility

All mortality is assumed to occur between the end of the hunting season and before fawns and calves are born in late spring or early summer of the following year. Natural mortality rates are used in the model; hunting mortality is excluded. The coefficients of mortality employed here for each species, sex, and age are: buck, 25%, doe, 20%, fawn, 40%, bull elk, 20%, cow, 20%, and calf, 34%.³

According to the Utah Division of Wildlife Resources (1972), average mule deer fawn production for the Cache Deer Herd over the 5-year period from 1967 through 1971 is approximately 80 fawns per 100 does. Kimball and Wolfe (1972) present a winter trend count and productivity estimate of approximately 50 calves per 100 cow elk on the same management unit.

Using these mortality and fertility estimates, and assuming that fawns and calves have a 50:50 sex ratio, the following equations indicate the number of animals of each species, by sex, available in

³Mortality data were obtained from Dr. Spillett during an interview on October 10, 1972. Although Kimball and Wolfe (1972) state that elk mortality for the Cache elk herd is: bulls 28%, cows 19%, and calves 35%, appropriate estimates for the study area are as stated above.
the spring of period (t) relative to the remaining animals after harvest in period (t-1):

\[
(Bucks)_t = 0.75X_{11,t-1} + 0.30X_{13,t-1} \\
(Does)_t = 0.80X_{12,t-1} + 0.30X_{13,t-1} \\
(Fawns)_t = 0.64X_{12,t-1} \\
(Bulls)_t = 0.80X_{17,t-1} + 0.33X_{19,t-1} \\
(Cows)_t = 0.80X_{18,t-1} + 0.33X_{19,t-1} \\
(Calves)_t = 0.40X_{18,t-1}
\]

Equations (3) and (6) are derived by applying both mortality and fertility elements to the remaining doe and cows of period (t-1). For example, \((Fawns)_t = 0.80(0.80X_{12,t-1}) = 0.64X_{12,t-1}\).

**Herd Identity**

Davis (1967, p. 660) expressed the relationship of animals present at the beginning and end of a year, reporting, "The number of animals at the beginning of a year must add up to the sum of animals harvested or left to carry over at the end of the same year."

By applying equation (1) for the number of bucks at the beginning of the year, the appropriate equation can be expressed as \(0.75X_{11,t-1} + 0.30X_{13,t-1} = X_8,t + X_{11,t}\). By rearranging terms and writing the equation in standard form, it becomes:

\[-0.75X_{11,t-1} - 0.30X_{13,t-1} + X_8,t + X_{11,t} = 0 \tag{7}\]

The same procedure is used to derive equations for the other animals as follows:

\[
\begin{align*}
(Does) &\quad -0.80X_{12,t-1} - 0.30X_{13,t-1} + X_9,t + X_{12,t} = 0 \tag{8} \\
(Fawns) &\quad -0.64X_{12,t-1} + X_{10,t} + X_{13,t} = 0 \tag{9}
\end{align*}
\]
(Bulls) \[-0.80X_{17,t-1} - 0.33X_{19,t-1} + X_{14,t} + X_{17,t} = 0 \] (10)

(Cows) \[-0.80X_{18,t-1} - 0.33X_{19,t-1} + X_{15,t} + X_{18,t} = 0 \] (11)

(Calves) \[-0.40X_{18,t-1} + X_{16,t} + X_{19,t} = 0 \] (12)

**Food Production and Consumption**

All of the food produced by the vegetation in a year is either eaten by the animals or left as surplus \((X_7)\) and \((X_7)\). Food production rates are: An acre of normal (unmanaged) land \((X_1)\) and \((X_4)\) produces approximately 1600 lbs. of food (grasses, forbes, and browse) per year. An acre treated to produce deer food \((X_2)\) and \((X_5)\) produces 1700 lbs. during the first two years and steadily decreases in production at a rate of about 6 lbs. per acre per year for 16 years, and then loses about 4 lbs. per acre per year for the remaining 4 years. An acre of land treated to produce elk food \((X_6)\) produces the same amounts of food per acre per year over the same time periods as an acre treated to produce deer food. However, the ratio of browse to grasses and forbes is substantially increased for deer food production and the ratio of grasses to forbes and browse is greatly increased when land is treated to produce elk food.\(^4\)

There is a paucity of information regarding vegetational succession patterns that might occur on land treated to produce deer and elk food. It is generally assumed, however, that after about the first

\(^4\)Data regarding food production were obtained from Frank Gunnell, biologist, Cache National Forest, during an interview on September 28, 1972. The author intuitively disagrees with the very slight improvement in food production resulting from appropriate land treatment. However, since no other data is available, the amounts are used with no attempt at justification.
two years the amount of food produced would decline in the manner just described, and that these areas would probably revert to normal land in about 20 years. Since no accurate information is available at this time, the figures mentioned above will be used in this model.

The winter food supply is consumed only by remaining animals of each species. Food consumption rates are based on a 120-day range utilization period and have been determined for each animal as follows: bucks, 680 lbs., does, 540 lbs., fawns, 300 lbs., bulls, 1810 lbs., cows, 1540 lbs., and calves, 910 lbs. These quantities were derived by relating appropriate body weights of each animal to the average weight and food consumption for a deer and an elk.

These two food production and consumption equations needed for this decision model are developed by incorporating this information as follows:

Total area (Area inhabited by deer only plus area inhabited by both deer and elk):

\[ 1600X_{1,t} \] 
\[ + 1700X_{2,t} + 1694X_{2,t-1} + 1688X_{2,t-2} + 1682X_{2,t-3} + 1676X_{2,t-4} + 1670X_{2,t-5} + 1664X_{2,t-6} + 1658X_{2,t-7} + 1652X_{2,t-8} + 1646X_{2,t-9} + 1640X_{2,t-10} + 1634X_{2,t-11} + 1628X_{2,t-12} + 1622X_{2,t-13} + 1616X_{2,t-14} + 1610X_{2,t-15} + 1604X_{2,t-16} + 1608X_{2,t-17} + 1600X_{2,t-18} + 1600X_{2,t-19} \] 

This phenomenon was discussed with Dr. John Malechek, Department of Range Science, Utah State University, on October 5, 1972.

Average weight and consumption data were obtained from Frank Gunnell on September 28, 1972; weight for each animal obtained from Dr. Spillett on October 10, 1972.
treated for deer food production in area inhabited by deer only the past 19 years) $+1600X_4,t$ (food from normal land in area inhabited by both species) $+1700X_5,t+1700X_5,t-1+1694X_5,t-2+1688X_5,t-3+1682X_5,t-4+1676X_5,t-5+1670X_5,t-6+1664X_5,t-7+1658X_5,t-8+1652X_5,t-9+1646X_5,t-10+1640X_5,t-11+1634X_5,t-12+1628X_5,t-13+1622X_5,t-14+1616X_5,t-15+1612X_5,t-16+1608X_5,t-17+1604X_5,t-18+1600X_5,t-19$ (food from areas treated for deer food production in area inhabited by both species)

$+1700X_6,t+1700X_6,t-1+1694X_6,t-2+1688X_6,t-3+1682X_6,t-4+1676X_6,t-5+1670X_6,t-6+1664X_6,t-7+1658X_6,t-8+1652X_6,t-9+1646X_6,t-10+1640X_6,t-11+1634X_6,t-12+1628X_6,t-13+1622X_6,t-14+1616X_6,t-15+1612X_6,t-16+1608X_6,t-17+1604X_6,t-18+1600X_6,t-19$ (food from acres treated for elk food production in area inhabited by both species the past 19 years)

$-1810X_{17},t-1540X_{18},t$ (food consumed by remaining animals) $-X_3,t-X_7,t$ (surplus food in both areas) $= 0$ (13)

Area inhabited by deer and elk:

$1600X_4,t$ (food produced from normal land in area inhabited by both species for the current year) $+1700X_5,t+1700X_5,t-1+1694X_5,t-2+1688X_5,t-3+1682X_5,t-4+1676X_5,t-5+1670X_5,t-6+1664X_5,t-7+1658X_5,t-8+1652X_5,t-9+1646X_5,t-10+1640X_5,t-11+1634X_5,t-12+1628X_5,t-13+1622X_5,t-14+1616X_5,t-15+1612X_5,t-16+1608X_5,t-17+1604X_5,t-18+1600X_5,t-19$ (food from land treated for deer food production in area inhabited by both species the past 19 years) $+1700X_6,t+1700X_6,t-1+1694X_6,t-2+1688X_6,t-3+1682X_6,t-4+1676X_6,t-5+1670X_6,t-6+1664X_6,t-7+1658X_6,t-8+1652X_6,t-9+1646X_6,t-10+1640X_6,t-11+1634X_6,t-12+1628X_6,t-13+1622X_6,t-14+1616X_6,t-15+1612X_6,t-16+1608X_6,t-17+1604X_6,t-18+1600X_6,t-19$ (food from land treated to produce elk food in area inhabited by both species over the past 19 years) $-1810X_{17},t-1540X_{18},t$
Equation (14) states that food produced in the area inhabited by both deer and elk which is not eaten by elk or left as surplus is eaten by deer.

A simple line diagram (Figure 3), illustrates how the model deals with habitational patterns and maintains consistency.

Breeding Requirements

Accurate information regarding the number of does one buck will mate and the number of cows one bull will mate is not available. Darling (1964) and Follis (1972) report that breeding usually occurs at night and little is known about actual copulation. However, informed personnel of the Utah State Division of Wildlife Resources and the Department of Wildlife Science at Utah State University believe that one buck will mate with approximately 6 does and one bull will mate with about 8 cows.

Integrating this information with the requirement that at least these ratios of bucks to does and bulls to cows must be maintained in the remaining herds to support capacity breeding, the appropriate equations are:

\[
\begin{align*}
\text{(Deer)} & \quad 6X_{11},_t - X_{12},_t \geq 0 \quad (15) \\
\text{(Elk)} & \quad 8X_{17},_t - X_{18},_t \geq 0 \quad (16)
\end{align*}
\]

Land Identity

The approximate total acreage of the Logan Peak winter range area, as previously noted, is 8,000 acres. There are five land
### Area Inhabited by Deer Only (4,000 Acres)

**Appropriate Decision Variables**

- $X_1$: No. Acres Unmanaged Land
- $X_2$: No. Acres Treated for Deer Food
- $X_3$: No. Lbs. Surplus Food

**Appropriate Constraint Equations**

- Food Production and Consumption (13)
- Food Production and Consumption (14)
- Total Land Identity (17)
- Land Identity Both Species (18)

### Area Inhabited by Deer and Elk (4,000 Acres)

- $X_4$: No. Acres Unmanaged Land
- $X_5$: No. Acres Treated for Deer Food
- $X_6$: No. Acres Treated for Elk Food
- $X_7$: No. Lbs. Surplus Food

---

**Figure 3.** Line diagram illustrating how the model deals with habitational patterns and maintains consistency.
classes \((X_1, X_2, X_4, X_5, X_6)\) on the whole tract of land. The sum of land acreage in all of these classes must always equal 8,000. This condition is expressed mathematically as:

\[
X_1 + X_2 + X_4 + X_5 + X_6 = 8,000
\]  \hspace{1cm} (17)

That is, the number of acres of normal (unmanaged) land in the area inhabited by deer only \((X_1)\); plus the number of acres treated to produce deer food in the same area \((X_2)\); plus the number of acres of normal land in the area inhabited by both deer and elk \((X_4)\); plus the number of acres treated to produce deer food in this area \((X_5)\); plus the number of acres treated to produce elk food in the same area \((X_6)\) equals the total land area of 8,000 acres.

In order to distinguish between the total land area and that portion of the winter range which elk inhabit (deer dwell on the entire tract), it is necessary to include another constraint equation in the model. There are three classes of land \((X_4, X_5, X_6)\) in this part of the total area. The sum of these classes of land must equal 4,000 acres. This identity is written as:

\[
X_4 + X_5 + X_6 = 4,000
\]  \hspace{1cm} (18)

**Monetary Limitations**

Treatment of land to produce food for deer and elk requires, of course, an outlay of funds. Monetary expenditures are also required to provide a harvest of the animals. These expenses are related to law enforcement, manning checking stations, etc.

Costs required to treat an acre of land to produce deer food are estimated to be about $313. An acre of land treated to produce
food for elk costs approximately $56. These figures are based upon records compiled by personnel of the Cache National Forest.⁷

The substantial difference in costs for the two treatments is attributed to the manner in which they are conducted. Treatment of land to produce deer food involves approximately 6 man-days of hand labor required for thinning stands of juniper trees and planting bitterbrush seedlings. Also, cost of the seedlings is estimated at $150 per acre. Treatment to produce elk food requires only one man-day for thinning juniper trees and the area is seeded by fixed-wing aircraft.

Harvest management costs are estimated to be about $5 per harvested deer and $10 per harvested elk. These amounts are based on estimated expenditures of the Utah State Division of Wildlife Resources for this type of work, and checking station records.⁸ Several times as many deer are harvested as are elk. Hence the difference in estimated harvested costs.

An accurate estimate of funds allotted for these operations is not available. Therefore, an arbitrary amount of $15,000 per year is assumed available for this problem. Before actual implementation, the appropriate amount would need to be established.

Using this information, the monetary constraint equation is:

\[
\begin{align*}
\text{Money: } 313x_2, t + 313x_5, t + 56x_6, t + 5x_8, t + 5x_9, t + 5x_{10}, t + 10x_{14}, t + 10x_{15}, t + 10x_{16}, t & \leq 15,000 \\
(19)
\end{align*}
\]

⁷Data obtained from Frank Gunnell during a visit of September 28, 1972.

⁸Discussed with Dr. Spillett on October 24, 1972.
Labor Limitations

Labor constraints must also be identified for performing the work required for land treatments and for harvesting the animals.

Information relative to man days of labor required for the specified tasks was obtained from the same sources which provided the monetary cost data.  

The estimated number of man-days required to perform each activity is:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Labor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 acre treated to produce deer food ($X_2$) &amp; ($X_5$)</td>
<td>7 man-days</td>
</tr>
<tr>
<td>1 acre treated to produce elk food ($X_6$)</td>
<td>3 man-days</td>
</tr>
<tr>
<td>1 harvested buck ($X_8$)</td>
<td>0.1 man-days</td>
</tr>
<tr>
<td>1 harvested doe ($X_9$)</td>
<td>0.1 man-days</td>
</tr>
<tr>
<td>1 harvested fawn ($X_{10}$)</td>
<td>0.1 man-days</td>
</tr>
<tr>
<td>1 harvested bull ($X_{14}$)</td>
<td>0.4 man-days</td>
</tr>
<tr>
<td>1 harvested cow ($X_{15}$)</td>
<td>0.4 man-days</td>
</tr>
<tr>
<td>1 harvested calf ($X_{16}$)</td>
<td>0.4 man-days</td>
</tr>
</tbody>
</table>

The number of man-days allotted for the work is not specified in budget allocations. Therefore, this problem arbitrarily assumes that 300 man-days of professional labor are available per year for the 8,000 acre tract.

The appropriate man-power equation is, therefore:

\[
\text{Labor: } 7X_2 + 7X_5 + 3X_6 + 0.1X_8 + 0.1X_9 + 0.1X_{10} + 0.4X_{14} + 0.4X_{15} + 0.4X_{16} \leq 300 \tag{20}
\]

---

9 Discussed with Frank Gunnell on September 28, 1972 and with Dr. Spillett on October 24, 1972.
The linear programming decision model of deer and elk production through the specified period of time is now fully developed. It is composed of the series of constraint equations (7) through (20), and the objective function. The model matrix contains 19 decision (activity) variables, 14 constraint equations, and 1 objective function. The appropriate matrix is illustrated in Figures 4 and 5.

The initial herd size and structure for each species can be numerically injected into the model in the first time period. This is accomplished by setting the numbers of deer and elk that are harvested and remaining in period 1 equal to some initial estimate of herd size and composition.

The herd size and structure used for this problem are:

**Deer**
- (Bucks) : 60
- (Does) : 120
- (Fawns) : 95

**Elk**
- (Bulls) : 20
- (Cows) : 158
- (Calves) : 97

---

10 The number of deer and a desirable ratio of bucks : does : fawns provided by John Kimball, Jr., Utah State Division of Wildlife Resources, on September 11, 1972. Similar data for elk provided by Dr. Spillett, on September 13, 1972.
### Period 1. Activity Variables

<table>
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<tr>
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<td>Bucks (A)**</td>
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<td>Does</td>
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**Note:**

- **Total matrix size for 20-year period problem = 281 x 300**
- **Sub-matrix designation used in Figure 5.**

---

**Figure 4.** Linear programming matrix* for 10-year period of deer and elk management problem.
<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
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<td>D** C B</td>
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<td>G** F E D C B</td>
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** Sub-matrix of coefficients from Figure 4.

Figure 5. Matrix formulation for entire 20-year period linear programming analysis of deer and elk problem.
Chapter 4

DISCUSSION

The model generated in this report is a representation of the specified real world situation. Reality is incorporated into the system through the use of significant coefficients for the ecological variables. Material for deriving these coefficients was obtained from informed persons and a review of the available literature. The accuracy of the model is, however, limited by the paucity of information germane to the problem.

The Objective Function

It is important to point out that the game manager has distinct responsibilities in construction of the mathematical decision model. He must state explicitly precisely what the management objectives are. Since the objective function of the model is most important to any efficient analysis of game resource management, the game manager must carefully select the appropriate values to be used in the function. It is also his responsibility to determine the significance of figures used as coefficients in all of the constraint equations.

As previously stated, there are 19 decision variables and 14 constraint equations in this model. Any group of values for the variables in the constraint equations which satisfies all of the requirements specified in the model, constitutes a feasible solution to the problem. The number of variables and equations in this model
will generate an infinite number of solutions. Davis (1967) comments on this situation, stating that the "... linear programming techniques are needed to find the best of all possible solutions."

In his endeavor to maximize benefits received from the wildlife resource, the game manager needs to consider a diversity of qualitative and quantitative aspects. These circumstances provide predicaments for him as he attempts to meet the conglomeration of demands of his clientele. The different desires may vary from hunting aims--such as obtaining trophy specimens, the thrill of a successful stalk, and improving hunting skill--to related satisfactions of nature study; physical exertion; aesthetic interests; and obtaining food. Because of the paramount importance of providing clients with a maximum of satisfaction, the game manager must alter his objectives to meet changing situations and wants of his clientele.

According to White (1965), the manager's primary objective should be accomplished:

(1) at the least cost for management in terms of funds and manpower,
(2) with the least disruption of the ecological complex,
(3) with the least interference with man's utilization of other related land resources, or
(4) with the greatest benefit to suppliers of goods and services catering to primary beneficiaries. (White, 1965, p. 73)

These management concepts cannot, however, be literally incorporated into the mathematical model. Only one item at a time can be maximized or minimized as an objective function. Furthermore, monetary, manpower and other constraints must be explicitly quantified to their limitations.
The objective function specified in this problem embodies present estimated expenditures of resident hunters for taking an average deer and elk. A more realistic function could be developed, however, if relative values for each sex and age class of animals in each species were known. This information would permit one to construct an objective function which would relate the relative worth of individual deer and elk in both harvested animals and the remaining herds.

Any one of a number of objective functions could be inserted into the model to coincide with the specific aim of management. For example, an objective might be to maximize hunter satisfaction over the 20-year period. For this objective relative values could be assigned to each animal according to some predetermined scale to indicate the satisfaction received by the hunter in taking an animal of a particular species, size, and sex. Relative values based on a scale of 1 through 10, in ascending order of value, could be assigned so that a bull elk had a relative value of 10; a buck deer 8; a cow elk 6; a doe deer and calf elk 5; and a fawn 2. This objective stated explicitly is:

\[
\text{Maximiz} \sum_{t=1}^{20} (8x_8 + 5x_9 + 2x_{10} + 10x_{14} + 6x_{15} + 5x_{16})t
\]

Another objective could be to maximize revenue returned to the Utah State Division of Wildlife Resources over the 20-year period. Obviously maximum revenue would be obtained by selling all licenses to non-resident hunters, but this policy would not be feasible. It could be stipulated that licenses would be divided equally between resident and non-resident hunters; and that elk permits would cost $100 and deer permits $50 for non-residents, and resident licenses
would cost $20 for elk and $10 for deer. Appropriate studies should be conducted to determine the number of licenses which could be sold relative to hunter success. This relationship could then be used to establish the number of licenses sold as a function of the number of animals harvested. Finally, revenue from license sales per harvested animal can be established to obtain the weights for this objective function. By averaging these costs and assuming equal satisfaction for taking any animal of a species, all deer would have a relative value of $30 and the value for elk would be $60. The appropriate equation for this objective function is:

$$ \text{Maximize the sum: } \sum_{t=1}^{20} \left( 30X_8 + 30X_9 + 30X_{10} + 60X_{14} + 60X_{15} + 16X_{16} \right) t. $$

The manager may also choose to maximize revenue to the local economy over the 20-year period. For this objective it might be determined that an elk hunter spends $40 while hunting bulls, $30 for cows, and $20 for hunting calves throughout the season. Similar values for deer could be $30 for bucks, and $20 each for does and fawns. These values would be a function of hunter preference and time spent in taking a specific animal. Quantitatively this objective function is:

$$ \text{Maximize the sum: } \sum_{t=1}^{20} \left( 30X_8 + 20X_9 + 20X_{10} + 40X_{14} + 30X_{15} + 20X_{16} \right) t. $$

Resident deer and elk herds have significant value for sightseers. An additional objective, therefore, could be to maximize the leave herd animals over the 20-year period. In this circumstance some harvest would still be permitted but the objective function would consider only the remaining animals. Visual value for each animal to sightseers could be determined from proper surveys. Appropriate values
might be such that a bull elk had a relative value of 10; a buck deer, fawn, and calf 8; and a doe deer and cow elk 6. This objective function is quantitatively expressed as:

Maximize the sum: \[ \sum_{t=1}^{20} \left( 8X_{11} + 6X_{12} + 8X_{13} + 10X_{17} + 6X_{18} + 8X_{19} \right)_t. \]

Another goal of the manager might be to minimize the cost of treating land for food production for deer and elk, in order to sustain a predetermined number of both animals on the area. For this objective the game manager or modeler would need to specify the number of animals desired and maintain consistency with the other variables in the model to develop the objective function. With these data and information presented in this paper, the objective function for this goal would be written:

Minimize the sum: \[ \sum_{t=1}^{20} \left( 313X_2 + 313X_5 + 56X_6 \right)_t. \]
Given: \[ \sum_{t=1}^{20} X_n = \text{a constant.} \]

**Area Competition of Deer and Elk**

Two different animal habitation patterns are defined in the decision model. That is, one half of the winter range contains deer only, and the other half has both deer and elk residing on it.

It would be unrealistic to assume that deer residing on the area containing deer only would not venture into the area containing both deer and elk. The model assumes that these creatures may move from one sector of the range to the other. This condition is postulated in the food production equations and in the equations expressing the land identity.
An intensive survey of the area might possibly reveal a third situation in which only elk inhabit a particular segment of the tract. If so, more refined equations could be developed to portray this added restriction.

Food Production

The specific land treatments for producing elk and deer food are assumed to be equally successful in any portion of the range. Furthermore, the same amount of biomass is produced as a result of either treatment. Likewise, normal or unmanaged land is assumed to produce the same number of pounds of food in either of the two areas. The author questions these assumptions and does not attempt to justify them. There are no studies currently available from which accurate information may be obtained regarding the specific quantities of food that would be produced from year to year as a result of such treatments.

The model assumes that food production in a given year is independent of the amount consumed during the previous year. It is also assumed that available sustenance can be obtained under all climatic conditions.

Information obtained as a result of range transects conducted by personnel of the Cache National Forest shows current plant production of the tract to be as follows: 11

<table>
<thead>
<tr>
<th>Vegetation Type</th>
<th>Pounds of Food per Acre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grasses</td>
<td>400</td>
</tr>
<tr>
<td>Forbs</td>
<td>600</td>
</tr>
<tr>
<td>Browse</td>
<td>600</td>
</tr>
</tbody>
</table>

Total 1600

11Discussed with Frank Gunnell on September 28, 1972.
Game biologists of the Cache National Forest believe that the land is presently producing its maximum amount of vegetation. Land treatments would, therefore, result in only a slight change in total biomass produced, but the composition of vegetation would be greatly altered. The author suggests that if the different foods and food habits of the two species could be formulated into the model, precision would be improved considerably.

Treatment of an acre of land to produce deer food would bring about an increase in food of about 100 pounds per year for the first 2 years after treatment. The vegetational composition resulting from the work would be as follows:

<table>
<thead>
<tr>
<th>Vegetation Type</th>
<th>Pounds of Food per Acre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grasses</td>
<td>300</td>
</tr>
<tr>
<td>Forbs</td>
<td>300</td>
</tr>
<tr>
<td>Browse</td>
<td>1100</td>
</tr>
<tr>
<td></td>
<td><strong>Total 1700</strong></td>
</tr>
</tbody>
</table>

Modification of the land for producing deer food would involve thinning of present juniper trees by cutting them with a power saw or an axe, hand planting antelope bitterbrush seedlings at a rate of 3,000 per acre, and hand sowing approximately 2 pounds per acre of big sagebrush, black sagebrush, and fourwing saltbush (Table 1). Monetary and labor costs for performing this work are stated in the appropriate constraint equations.

---

12 Estimated dollar values and man-day requirements in Tables 1 and 2 obtained from Cache National Forest records on September 28, 1972.
Table 1. Monetary and man-day requirements for treating an acre of land to produce deer food.

<table>
<thead>
<tr>
<th>Activity and Materials</th>
<th>Monetary Cost per Acre</th>
<th>Man-days Required per Acre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand thinning juniper trees</td>
<td>$26</td>
<td>1</td>
</tr>
<tr>
<td>Planting bitterbrush seedlings</td>
<td>130</td>
<td>5</td>
</tr>
<tr>
<td>Hand sowing seed</td>
<td>26</td>
<td>1</td>
</tr>
<tr>
<td>Antelope bitterbrush seedlings</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>Big sagebrush seed</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Black sagebrush seed</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Fourwing saltbush seed</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$313</strong></td>
<td><strong>7</strong></td>
</tr>
</tbody>
</table>

Treating an acre of land in order to produce food for elk would require altering the vegetation to produce forage in the following amounts:

<table>
<thead>
<tr>
<th>Vegetation Type</th>
<th>Pounds of Food per Acre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grasses</td>
<td>1100</td>
</tr>
<tr>
<td>Forbs</td>
<td>300</td>
</tr>
<tr>
<td>Browse</td>
<td>300</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1700</strong></td>
</tr>
</tbody>
</table>

Activities required to accomplish this work would again involve thinning of juniper trees, and seeding of desired plant species. Unlike the deer food improvement work, seeding in this treatment would be accomplished with a fixed-wing aircraft. An acre would be aerial seeded with about 2 pounds each of rambler alfalfa, yellow sweetclover, mountain brome grass, and Great Basin wildrye grass (Table 2). Mone-
tary and labor costs for this job are also contained in the specified constraint equations.

Table 2. Monetary and man-day requirements for treating an acre of land to produce elk food.

<table>
<thead>
<tr>
<th>Activity and Materials</th>
<th>Monetary Cost per Acre</th>
<th>Man-days Required per Acre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand thinning juniper trees</td>
<td>$26</td>
<td>1</td>
</tr>
<tr>
<td>Aircraft and Seed</td>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>(One contract for all pertinent requirements)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$56</td>
<td>3</td>
</tr>
</tbody>
</table>

The resource manager could stratify the entire winter range area into desired management units of any size for habitat regulation and control. If he chose land units small enough, he might find all five vegetational types identified within the Cache Big Game Management Unit. These were defined by Hancock (1955) as conifer, juniper, mahagany, aspen, and sagebrush. Explicit land treatments for producing deer and elk food could then be applied to each vegetational type. This intensive management practice may result in a variety of food yields per acre.

Another alternative the resource manager might consider is the possibility of initiating a feeding program similar to that which is conducted at the Hardware Ranch. Although Murie (1957) and Taylor
(1956) comment that emergency winter feeding is not generally encouraged by informed game managers, feeding elk could be considered if, in the judgement of the game manager, the trade-offs between this activity and treating land to produce food for the elk supported the practice. The linear programming model could be formulated to estimate these trade-offs.

Food Requirements

Food requirements for each animal are also treated as constants in this model. This stipulation does not allow for changes in vigor of the animals, and each is always required to consume the same amount of food over the 20-year period. If the number of animals exceeds the food supply, the model necessitates their removal by harvest.

Deer and elk populations are controlled through harvest removals which are authorized by the Utah State Board of Big Game Control and administered by the Utah State Division of Wildlife Resources. The model assumes that a "harvested" animal will actually be harvested. Although this may be unrealistic, removal of animals too numerous for the food supply will help maintain a desired level of vigor.

Food is consumed only by the animals remaining after harvest. Legitimate consumption rates per animal have been determined from the relationship of live body weight to approximate energy requirements.

Mean dressed weights for each animal were obtained from records compiled at appropriate checking stations. These amounts were then

13 Data for dressed weights obtained from Dr. Spillett on October 24, 1972.
converted to live body weights (Table 3), by adding one third to the total dressed weight of each animal.

Table 3. Dressed and live weights for individual animals in each species.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Dressed Weight</th>
<th>Live Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>140</td>
<td>187</td>
</tr>
<tr>
<td>Doe</td>
<td>100</td>
<td>133</td>
</tr>
<tr>
<td>Fawn</td>
<td>45</td>
<td>60</td>
</tr>
<tr>
<td>Bull</td>
<td>425</td>
<td>567</td>
</tr>
<tr>
<td>Cow</td>
<td>350</td>
<td>467</td>
</tr>
<tr>
<td>Calf</td>
<td>175</td>
<td>233</td>
</tr>
</tbody>
</table>

An average daily food requirement was calculated for each animal based on maintenance energy requirements. Procedures for determining these requirements were adapted from Wilson (1971).

According to Wilson:

An average daily food requirement for moose was determined based on Kleiber's interspecies mean for calculating adult maintenance energy requirements. Energy requirements were calculated from the formula:

\[
\text{Kilocalories} = a \times b (W_{\text{kg}}^{0.75})
\]

where:

\[
W_{\text{kg}}^{0.75} = \text{metabolic size of animals (body weight in kilograms raised to power 0.75)}
\]

\[b = 70, \text{a constant—the kilocalories required per unit of metabolic size for resting metabolism} \]

\[a = 3, \text{the factor to convert the "resting" metabolic requirement to that for maintenance (activity, reproduction and thermoregulation)} (\text{Wilson, 1971, p.15}) \]
Using this formula, appropriate energy requirements (Table 4) were derived in terms of mean body weights and kcal. required.

Table 4. Mean body weights and daily caloric requirements for deer and elk.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Mean Body Weights</th>
<th>Kcal. Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>187</td>
<td>5,869</td>
</tr>
<tr>
<td>Doe</td>
<td>133</td>
<td>4,546</td>
</tr>
<tr>
<td>Fawn</td>
<td>60</td>
<td>2,503</td>
</tr>
<tr>
<td>Bull</td>
<td>567</td>
<td>13,487</td>
</tr>
<tr>
<td>Cow</td>
<td>467</td>
<td>11,660</td>
</tr>
<tr>
<td>Calf</td>
<td>233</td>
<td>6,922</td>
</tr>
</tbody>
</table>

An average deer weighs about 135 pounds and eats approximately 4.5 pounds of food per day. Similar values for elk are 430 pounds of body weight and 12 pounds of food consumed per day.  

These average body weights were converted to required kcals. for each species. Daily food consumption for each animal was then calculated by constructing a proportion which equated the ratio of daily food consumption to required kcals. of an average deer and an average elk to each individual in the proper species.

14 Values obtained from Frank Gunnell on September 28, 1972.
For example, the daily food requirements for a buck were calculated as follows:

\[
\frac{4.5 \text{ pounds of food (average deer)}}{4,597 \text{ (kcals. per average deer)}} = \frac{X \text{ pounds of food}}{5,869 \text{ (kcals. for buck)}}
\]

Solving this proportion yields a daily food requirement for a buck of 5.7 pounds.

Daily food requirements of each animal were converted to annual requirements by multiplying each value by 120 (Table 5); the number of days use on the winter range.

Table 5. Daily and annual food consumption rates of deer and elk on the Logan Peak winter range area.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Daily Food Requirements</th>
<th>Annual Food Requirements*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>5.7 pounds</td>
<td>680 pounds</td>
</tr>
<tr>
<td>Doe</td>
<td>4.5 &quot;</td>
<td>540 &quot;</td>
</tr>
<tr>
<td>Fawn</td>
<td>2.5 &quot;</td>
<td>300 &quot;</td>
</tr>
<tr>
<td>Bull</td>
<td>15.1 &quot;</td>
<td>1810 &quot;</td>
</tr>
<tr>
<td>Cow</td>
<td>12.8 &quot;</td>
<td>1540 &quot;</td>
</tr>
<tr>
<td>Calf</td>
<td>7.6 &quot;</td>
<td>910 &quot;</td>
</tr>
</tbody>
</table>

*Annual food requirements rounded to nearest 5 pounds.

The food consumption elements of the problem, like the food production factors, provide the resource manager an opportunity to innovate. The food components of this model are simply defined as the plant biomass which is consumed by the animals. The game manager
may choose to define food factors more precisely. He might, for example, wish to deal with specific plant classifications such as grasses, forbs, or shrubs. Or, he may elect to determine the amount of specified nutrients that could be produced for food consumption on both the natural and the treated areas.

Both animals are selective in their consumption of the three plant classifications. Taylor (1956) states that mule deer usually desire to eat shrubs but they do eat grasses and forbs. By the same token, Murie (1957) notes that elk usually prefer grasses but will also eat forbs and shrubs. If empirical data were available from the Logan Peak area to accurately account for the amounts of each of the types of vegetation that were utilized, more refined equations could be developed to represent the real-world conditions. Land management practices could thus be appropriately designated for the particular plant classes applicable to each of the ungulates.

Nutritional requirements of the animals are extremely important in the management of big game species. The game manager is always interested in these factors and may choose to specify game food in terms of its nutritional ingredients. For instance, he may seek to achieve some results in terms of, say: protein, carbohydrates, fats, minerals, and vitamins. With the required information, the game manager could construct elaborate food production and consumption constraints in terms of these constituents.

Breeding and Mortality

In this simplified account of the problem, natality and mortality
are treated as being constant from year to year. That is, they are regarded as being independent of herd size or density.

Some factors related to reproduction in both species of animals are: (1) nutrient intake, (2) total population density, (3) age of parent doe and cow, and (4) energy demands of the pregnant female. These elements appear to determine the number of embryos produced by a female of each species.

Reproduction in both species decreases and the rate of males to females in fawns and calves changes when the animals' diets are low in nutrition. Taylor (1956) and Murie (1957) indicate that reproduction in deer and elk, respectively, varies inversely with population density. This fact is correlated with the per capita food consumption; i.e., an increasing population density results in a decreasing supply of available food.

The number of fawns and calves born in the respective species is related to the age of the mothers. Biologists agree that the number of young born to very young and very old females is, on the average, lower than the average number born by the female population as a whole. This points out the importance of the female age structure of the herds, which is affected by the intensity of harvest.

The composition and amount of the diet of females is an important factor regarding their ability to meet the increased energy demands during pregnancy. If energy demands exceed the supply that is available from food and stored body reserves, a weakened condition results and fewer live fawns and calves are born. Severe winter conditions which constrain the mobility of the animals effectively reduces the
available food supply. When this happens, the animals must rely on the reservoir of energy that is stored within the body.

Natural mortality is defined as all mortality that is not related to hunting. It includes deaths occurring from old age, disease, sickness, predators, starvation, accidents, and natural disasters. Natural mortality is related to the factors discussed for reproduction.

Natural mortality is a complex phenomenon and the interaction of density dependent and density independent effects is not clearly understood or described in the available literature.

If empirical data were available for the animals in this study area, density dependent and density independent natality and mortality could be treated explicitly. Constraint equations could be refined to reflect these relationships and the model would thus be more realistic.

Another relationship which needs additional study is the specified breeding requirements. While empirical evidence obtained from the Forest Service and the Utah Division of Wildlife Resources indicates that, in general, 1 buck deer will mate with 6 does, and 1 bull elk will mate with 8 cows, there is no assurance that this is the case for the particular study area, or that this is a constant occurrence.

It appears realistic, too, to hypothesize that breeding capabilities of the animals may be related to factors which are similar to those already presented in the discussion of natality and natural mortality. Thus, breeding activities probably fluctuate over time.

All of the literature examined in search of breeding habits for elk proclaims that while rare instances of yearling calves breeding
do occur, the young do not normally breed until after about 2 years of age. Mule deer fawns do not usually breed either, and Hickman (1971) reveals that the reproductive capacity of fawns is considered insignificant by most biologists. He also believes that they do not mate until they are about 1 1/2 to 2 years old.

This work assumes that the breeding requirements are constant, that the breeding ratios are as stated in the model, and that any female of either species could breed after it is one year old.

It is currently impossible for mathematical models of the natural world to comprise total information of the real world condition. A paucity of suitable empirical data is a restrictive factor in developing game management models. The game manager should, therefore, be cognizant of this fact and realize that the model approximates reality through incorporation of available data concerning known factors related to the problem. It is vitally important, too, for the decision maker to understand that the accuracy of the model is dependent upon the precision of its inputs.

Formulating managerial problems in terms of decision oriented mathematical models is a deviation from traditional cost-benefit investigations. The mental gymnastics required of constructing models which are acceptable abstractions of reality are rewarded with priceless enlightenment.

The linear programming technique is an operations research method that can be of considerable value to resource managers. It is an excellent tool, to be sure, but it cannot replace the human element in decision making. In the final analysis, the decision maker must determine what are or are not equitable trade-offs among feasible alternatives.
Chapter 5
SUMMARY AND CONCLUSIONS

Summary

This paper discusses a mathematical decision model as an aid for solving a current game management problem of producing an optimum number of elk and mule deer to be harvested from the Logan Peak winter range area. A mathematical model for the deer and elk herd management situation is constructed as a time stage linear programming problem for computer analysis.

A study by Davis (1967) provides the framework about which the model is constructed. A series of equations are developed to depict herd identities, reproduction, mortality, food production and consumption, breeding requirements, land identity, monetary limitations, and labor constraints which represent the biological and managerial aspects of the management position. The management objective is also explicitly stated as the objective function in the model.

The elk and deer herd management problem is similar in many respects to the general economic problem of allocating limited resources for land management. Construction of the mathematical model permits the decision maker to express elements of the problem and their affiliations in an orderly and quantitative fashion. The linear programming model indicates its adaptability as a solution technique for the specified management problem.
Conclusions

The elk and mule deer management problem can be adapted to solution by the linear programming technique. The real world situation can be accurately characterized by describing the significant features bearing on the problem as a set of appropriate mathematical statements. The framework is provided herein from which an appropriate linear programming decision model can be developed.

The game manager has certain responsibilities in development of an efficient mathematical model. He must declare the management goals to be specified as the model objective function. He must also attest to the reasonableness of the model as an abstraction of reality and be convinced that the particular coefficients used in the model are both significant and precise.

Correct execution of the model development process compels the decision maker or modeler to precisely quantify his knowledge relative to the particular problem. Thus, mathematical models provide insights regarding gaps in knowledge and understanding into the decision problem. Use of the decision model can, therefore, provide the resource manager with improved knowledge and skill in his decisionmaking.
REFERENCES


White, W. M. 1965. The Economics of Sport Fisheries Management. Canadian Fisheries Reports. The Department of Fisheries of Canada. Publication No. 4:73-78.

## Common and Scientific Names of Animals and Plants Mentioned

<table>
<thead>
<tr>
<th>Common Name</th>
<th>Scientific Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elk</td>
<td><em>Cervus canadensis canadensis</em></td>
</tr>
<tr>
<td>Mule deer</td>
<td><em>Odocoileus hemionus hemionus</em></td>
</tr>
<tr>
<td>Alfalfa, Rambler</td>
<td><em>Medicago sativa</em></td>
</tr>
<tr>
<td>Bitterbrush, antelope</td>
<td><em>Purshia tridentata</em></td>
</tr>
<tr>
<td>Brome, mountain</td>
<td><em>Bromus carinatus</em></td>
</tr>
<tr>
<td>Sagebrush, big</td>
<td><em>Artemisia tridentata tridentata</em></td>
</tr>
<tr>
<td>Sagebrush, black</td>
<td><em>Artemisia arbuscula nova</em></td>
</tr>
<tr>
<td>Saltbush, fourwing</td>
<td><em>Atriplex canescens</em></td>
</tr>
<tr>
<td>Sweetclover, yellow</td>
<td><em>Melilotus alba</em></td>
</tr>
<tr>
<td>Wildrye, Great Basin</td>
<td><em>Elymus cincereus</em></td>
</tr>
<tr>
<td>Aspen type</td>
<td><em>Populus tremuloides</em></td>
</tr>
</tbody>
</table>
| Conifer type                 | **Comprising:** *Pseudotsuga mensiesii*  
|                              | *Pinus contorta*  
|                              | *Picea engelmanni*  
|                              | *Abies lasiocarpa* |
| Juniper type                 | *Juniperus spp* |
| Mahogany type                | *Ceroocarpus montanus*  
|                              | *Ceroocarpus ledifolius* |
| Sagebrush type               | *Artemisia tridentata* |