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The Reflection of Light from Periodic Conducting Interfaces

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THE REFLECTION OF LIGHT FROM PERIODIC CONDUCTING INTERFACES

by

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ABSTRACT

It is known that carbon-nanotube (CNT) forests, nanopillar arrays, and other high aspect-ratio conducting and semi-conducting nanostructures can have extremely low reflectance in a wide range of wavelengths, and that reflectance begins to rise at long wavelengths. The mechanism for this behavior is poorly understood. It has been shown that the reflectance of CNT forest varies with the morphology of the forest, which indicates that the interface of such a material may play a primary role in its reflectance and absorptance.

Simulations of the reflection of light from arrays of conducting nanorods using commercial finite-difference-time-domain software predict a similar increase in reflectance at long wavelengths, and the reflectance spectra have characteristics that hint at the presence of interference phenomena. In order to understand the role of interference in the properties of a broadband absorber with a nonuniform conducting interface, the reflection of light from a two dimensional square array of conducting patches is modeled with scalar waves using the Huygens-Fresnel principle. This model qualitatively explains the characteristic increase of reflectance with an increase in wavelength, and the observation that a decrease in CNT forest density increases the onset wavelength of reflectance.

However, a true theoretical understanding must be based on Maxwell’s equations which describe electromagnetic radiation precisely. As a first step, the reflection of light from a one dimensional periodic conducting interface is calculated based on electromagnetic scattering theory. Contrary to the prediction of the scalar wave model, this solution predicts a decrease in reflectance with an increase in wavelength, indicating that a nonuniform conducting interface may have a decreased role in reflectance at long wavelengths. Additionally, this solution fails to explain the extremely low reflectance of short wavelength light observed from many conducting and semi-conducting nanostructures. Further analysis and an extension of this scattering theory to a two dimensional interface is necessary to fully understand the properties of broadband absorbing nanostructures.
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1. INTRODUCTION

One of the most important problems in modern technology is the need to understand and control the propagation of light. Through the invention and development of transistors, the ability to control the movement of electrons has been mastered to make computers cheap, powerful, and small. But we are quickly approaching the limit of the miniaturization of electronics. Processors are being developed with architectures that are so small that the classical rules of circuits begin to break down as electrons jump through insulating gaps. The next technological revolution will be associated not with electrons, but with photons. With proper control over light, we could have the capability to carry quantum information from atom to atom to create quantum computers with unprecedented computational power and a quantum internet which uses quantum mechanics to achieve perfect security. In order to achieve this, it is necessary to be able to understand how light propagates and interacts with matter at subwavelength scales.

For many applications, the challenge is to control the propagation of light from one medium to another. One technology which relies on this control of light is the broadband absorber—a materials which can absorb light in a wide range of wavelengths. Engineering such a material is not an easy task, as it requires a mechanism of allowing incident light to propagate into the material, and a mechanism of absorbing the light once it is in the material. The accidental discovery of carbon-nanotubes (CNTs), which are rolls of graphene with radii on the order of 1 nanometer, provided a material with exactly these properties. It is known that forests of CNTs can exhibit an almost complete absorption of all incident light in a very wide wavelength range. [1,2] Other high aspect ratio nanostructures have also displayed broadband absorption with reflectances less than 1% in the visible range, including arrays of dual-diameter germanium nanopillars, [3] coatings of silicon nanotips, [4] silicon nanowires, [5] and metallic cones. [6] Even the cellular structures of some butterfly wings have reflectances lower than 1% in the visible range and remain dark after being coated in gold. [7] Such materials hint at the answer to the problem of the transmittance of light from medium to medium, but the mechanism for this remarkable behavior is still poorly understood. Because metals and semi-conductors are typically
reflective in planar form, the extremely low reflectances of these materials are a consequence of their structures.

A generic behavior of a broadband absorber is the fact that, at a long enough wavelength, the reflectance begins to rise significantly. If we could understand how to increase this onset wavelength of reflectance, then we would take an important step in the direction of controlling the transmission of light. For a CNT forest, the interface between air and the forest is the forest's top crust. The importance of the interface is clear from the fact that, if a CNT forest is pressed down, it can become quite reflective. [8] Because the forest grows from catalyst particles on the substrate, the crust represents the initial state of the forest's growth and is more entangled than the rest of the forest. The nanotubes also tend to clump together due to the van der Waals force, forming conducting patches and voids in the surface. [Fig. 1] It has been shown that, by introducing a random height-modulation that breaks up the top crust, the reflectance of a CNT forest can be decreased for even longer wavelengths. [9] Along with the general morphology of broadband absorbing nanostructures, this indicates that there may be some property of non-uniform conducting interfaces that prevents reflectance in a wide spectrum.

Using commercial finite difference time domain (FDTD) software, we simulated the reflectance of light from square arrays of metallic nanorods. Similar to the reflectance of light from broadband absorbing nanostructures, the reflectance was predicted to rise with wavelength. We found
that, by varying the spacing of the rods, the reflectance could be decreased by a factor of 5 for wavelengths ranging from 0.2 to 10 μm. For an array of nanorods 30 nm in diameter and 30 μm tall, when the spacing between the rod centers is 60 nm, the rods are touching. This situation results in a reflectance very similar to substrate reflectance, except at the very short wavelengths, where the reflectance approaches zero. [Fig. 2] As the spacing of the rods is increased to 130 nm, the reflectance drops to less than 20%, in accordance with the observation of CNT forests that a decrease in density can lead to an increase in the onset wavelength of reflectance. [9] As the spacing is increased further, the reflectance increases as more light reflects from the substrate, until the reflectance spectra resemble that of the substrate again. An important feature of this simulation is the fact that the reflectance spectra oscillate with wavelength. This is a common feature in systems that involve interference. For example, anti-reflection coatings are thin dielectric coatings which cause reflected light to interfere with itself, such that the majority of certain wavelengths of light is transmitted through the material. The reflectance spectra of such coatings also oscillate with wavelength. This is the primary observation which motivates our analysis of the reflectance of light from two dimensional interfaces within the framework of interference.

Here, I will investigate the reflection of light from interfaces of periodic conducting material. I will begin in section 2 by studying the reflection from arrays of conducting patches using the Huygens-Fresnel scalar wave approximation, in order to understand the general dependence of reflected power on wavelength and on the spacing of the patches. I will also introduce the tools for determining the diffraction from periodic structures using Fourier optics. In section 3, I will study both the reflection and transmission of light from periodic conducting gratings using electromagnetic scattering theory. To test the veracity of the theory, I will take limiting cases of long wavelength, wide aperture spacing, and parallel incidence. I will compare the diffraction patterns with those obtained from Fourier optics. I will discuss the effect on reflectance and transmittance of varying the size and spacing of conducting strips with an approximate solution to the scattering equations.
2. SCALAR DIFFRACTION

2.1. The Huygens-Fresnel Principle

A wave is essentially a spatial variation of amplitude that oscillates with time due to some elastic force. For example, an acoustic wave in air is a variation of pressure that oscillates in time due to the electrostatic forces between particles, and an electromagnetic wave is a variation of electric field strength that oscillates due to the electromagnetic force constraining the movement of the source charge. When waves from different sources overlap, their amplitudes sum together in the phenomenon known as interference. Constructive interference occurs when the amplitudes of two waves at a point in space sum together to make a larger amplitude, and destructive interference occurs when the amplitudes cancel out to make a smaller total amplitude. When an electromagnetic wave reflects from or passes through a nonuniform surface, the form of the wave changes significantly as each electron in the surface reacts by emitting its own wave in the phenomenon of diffraction. Because even the simplest incident wave is turned into an infinitude of waves when it interacts with a nonuniform surface, the prediction of the form of the reflected or transmitted wave is extremely complicated. The remarkable physicist Christiaan Huygens (1629–1695) found an intuitive way to understand the propagation of a wave of light. Huygens’ principle, as it is now called, claims that every point on a wavefront can be imagined to be the source of a spherical secondary wavelet with the same frequency. The wavefront of the propagating wave corresponds to the envelop of all the secondary waves. This idea, however, ignores most of the secondary wavelets, retaining only that portion common to the envelope. As a result, Huygens’ Principle cannot account for diffraction patterns. Augustin-Jean Fresnel (1788–1827) later improved this principle by claiming that these spherical secondary wavelets emanate from every unobstructed point on the wavefront, extending the application of the principle to the calculation of diffraction patterns by summing the waves from virtual point sources in the aperture. [Fig. 3] This principle ignores many of

Figure 3. Huygens-Fresnel principle for single aperture diffraction.
the effects of the aperture on the form of the wave because it does not consider the interaction of the wave with the charge carriers in the material. It instead assumes that the light just outside of the aperture is completely stopped by the material, and that the light just inside the aperture continues to propagate unobstructed. However, the method still allows for a qualitative understanding of diffraction.

2.2. Diffraction through a One Dimensional Aperture

The classic problem in diffraction is the transmission of light through a one dimensional aperture. According to the Huygens-Fresnel principle, each source in the aperture emits a wave with amplitude $U_i$, which oscillates sinusoidally with frequency $\omega$ and wavelength $\lambda$ as it propagates towards the screen. This wave represents the electric or magnetic field of the light emitted from each source. Because the wave emitted by each source is assumed to be spherical, its strength decreases as $1/r$ from an initial strength $U_0$. The point on the diffraction screen has coordinates $(X, Z)$, and the source has coordinates $(x_i, 0)$.

$$U_i(X, Z) = \frac{1}{r} U_0 e^{i(\omega t - kr_i)} \quad \text{(2.1)}$$

$$r_i = \sqrt{Z^2 + (X - x_i)^2} \quad \text{(2.2)}$$

where the wavenumber $k = \frac{2\pi}{\lambda}$. The strength of the wave at $(X, Z)$ is just the sum of all $U_i(X, Z)$. In the limit that there are infinitely many sources, this sum turns into an integral over the aperture, and the initial strength becomes a field strength per unit length $\gamma$.

$$U(X, Z) = e^{i\omega t} \int_{-a/2}^{a/2} \frac{\gamma}{r} e^{-ikr} dx \quad \text{(2.3)}$$

$$r = \sqrt{Z^2 + (X - x)^2} \quad \text{(2.4)}$$

Since $U(X, Z)$ represents the electric or magnetic field strength of an electromagnetic wave, the energy intensity is proportional to the squared magnitude of the complex amplitude $|U(X, Z)|^2$, which decreases as $1/r^2$. The energy flux is a function of the angle of incidence of the field. For example, if the screen is very close to the aperture, then the virtual sources near one edge will have a greater contribution to the diffraction pattern at that point than the virtual sources on the
other edge, a distance $a$ away. To convert the energy intensity to an energy flux, a factor of $\cos(\theta) = \frac{Z}{r}$ is included in the integral.

$$|U(X,Z)|^2 = \left[ e^{i\omega t} \int_{-a/2}^{a/2} \frac{a}{r} e^{-ikr} \left( \frac{Z}{r} \right) dx \right] \times \left[ e^{-i\omega t} \int_{-a/2}^{a/2} \frac{a}{r} e^{ikr} dx \right] \quad (2.5)$$

The four parameters that determine the diffraction pattern are the field strength density $\gamma$, the aperture width $a$, the wavelength $\lambda$, and the distance between the aperture and the source, $Z$. Fig. 4 shows a series of diffraction patterns, measured along the $\hat{x}$ direction on the screen, with varying aperture dimension. Because the different diffraction patterns depend on the relative phases of the fields at the screen, similar patterns can also be generated by varying the distance to the screen.

When the wavelength is much smaller than the aperture width, the path lengths of the sources can differ by multiple wavelengths at any point on the screen. From Fig. 4 (a) and (b), this results in many oscillations inside the shadow of the aperture. The diffraction region in the screen has approximately the same dimensions as the aperture (100 μm, and 50 μm, respectively) because the light does not spread out appreciably when the wavelength is smaller than the

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**Figure 4.** Diffraction patterns with varying aperture width. $\lambda = 500$ nm, $Z = 1$ mm. The aperture width is (a) 100 μm (b) 50 μm (c) 20 μm (d) 0.2 μm. The red curve in (d) is from a point source with field amplitude $a\gamma$. 
aperture dimension. In the limit that the wavelength approaches 0 or when the screen is very close to the aperture, the diffraction profile becomes a shadow of the aperture. As the aperture dimension decreases, the oscillation peaks spread out and higher order peaks appear. [Fig. 4(c)] Because we ultimately want to understand and control the reflectance of light at long wavelengths, we are mostly interested in the case when the wavelength is larger than the aperture width. If the width of the aperture is less than one half of a wavelength, then no two sources will ever be able to interfere destructively. Thus, there will be no minima in the intensity distribution as depicted in the blue curve of Fig. 4 (d). In this limit the distribution resembles that of a point source.

The total power from the light transmitted through the aperture can be calculated as the integral of the diffraction pattern over the entire screen. When the screen is very close to the aperture, [Fig. 5] the diffraction pattern assumes the form of a square wave, and the power is proportional to the width of the distribution times its height. However, because the field strength is proportional to $1/r^2$, the intensity diverges as the screen approaches the sources. This demonstrates the limitation of applying the Huygens-Fresnel principle to describe energy flux, as the secondary waves cannot generate fields with finite energy. However, integrating the diffraction patterns may still be a useful way to compare relative energy flux with constant aperture width and screen distance as wavelength is varied. For example, in Fig. 4 (d), even though the field strength of the point source is equal to the field strength per unit length of the aperture times the aperture width, the area under the curve of the point source is greater. This is because the fields from the virtual sources in the aperture are not completely in-phase at the screen, resulting is
some destructive interference. Figure 6 shows the power from a one dimensional aperture of constant width as a function of wavelength. At short wavelengths, the destructive interference decreases the power while, at long wavelengths, the power approaches a maximum value as the sources become more in-phase. This result suggests that the total energy of the light reflected from or transmitted through a periodic structure could be wavelength dependent.

2.3. Diffraction from Arrays of Point Sources

This idea can be extended to qualitatively understand the light reflected from an array of conducting patches. The light reflected from a perfect conductor with a small surface area may be approximately similar in form to the light transmitted through an aperture. Under this approximation, the reflected power from arrays of conductors could be compared by using the Huygens-Fresnel principle to calculate the diffraction patterns and integrating over a screen. Because the diffraction patterns generated by long-wavelength light are similar to the patterns generated by point sources, we can begin by considering the diffraction from a square lattice of point source. 5×5 arrays of 1 μm square apertures and point sources both generate diffraction patterns that are square lattices whose lattice spacings are inversely proportional to the spacings of the sources. [Fig. 7]

Figure 7. Diffraction patterns from 5x5 arrays of point sources and aperture sources with varying lattice constant. λ = 500 nm, Z = 50 μm. (a) point sources, a = 5 μm (b) apertures, a = 5 μm (c) point sources, a = 10 μm (d) apertures, a = 10 μm where a is the lattice constant.

Figure 8. Diffracted power from arrays of point sources. a = 1 μm. (a) 2x2 arrays. (b) 3x3 arrays.
Figure 8 shows the general behavior of the power calculated from the diffraction patterns of arrays of point sources. In general, the power is low for short wavelengths, where the fields on the screen destructively interfere. As the wavelength increases, the power oscillates until the wavelength is greater than the lattice constant and the destructive interference diminishes, at which point the power stops oscillating and increases monotonically. This behavior does not vary significantly with the number of sources. The number of sources only changes the minimum and maximum values of the power. It is interesting to note that, with a special choice of unit, the interference causes the power to oscillate around the value equal to the simple sum of sources, and when the interference diminishes, the maximum value is equal to the sum of sources squared. This can be seen in Fig. 8(a), as the power from four sources oscillates around 4, and has a maximum value near 16. A similar result for nine sources is shown in Fig. 8(b). The maximum value of $N^2$ originates from the fact that the intensity is proportional to the electric field squared. If all the sources are in-phase, then the total field will be proportional to $N$, and the intensity to $N^2$. Strong interference in the short wavelength region leads to fast oscillations and near complete cancellation of the interference terms, resulting in an intensity proportional to $N$.

2.4. Diffraction from Arrays of Apertures

The calculation of the interference from an array of aperture sources is similar to the calculation for a single one dimensional aperture described in Sec. 2.2, with a few modifications. First, the electric field emanating from each aperture is an integral of all point sources within the two dimensional area. Second, we assume that the dimensions of the total aperture array are much smaller than the distance to the screen, such that the fields from the virtual point sources at the screen have essentially the same strength, proportional to $\frac{1}{R}$, though their phases still differ as a function of position on the screen. Based on this assumption, the Poynting vector can be assumed to point radially outward wherever it is calculated and is perpendicular to a hemispherical surface when computing irradiance. From Fig. 4(d), as the wavelength exceeds the aperture dimension, the central lobe of the diffraction pattern spreads out, eventually approaching the distribution of a point source. Therefore, it would be expected that the
diffraction pattern from a square array of apertures would resemble that from an array of point sources. Indeed, Figure 9 shows the similarity between the two kinds of sources.

For an array of square apertures, an increase in the lattice constant $\alpha$ decreases the slope of the rising power, keeping it low for a longer range of wavelengths, as shown in Figure 10. This is like the result from an array of point sources, except that the oscillations at shorter wavelengths are not present. We have already seen from Figure 8 that, in the short wavelength region, power oscillates around a value proportional to the number of sources $N$ and asymptotically approaches $N^2$ in the long wavelengths. We have also seen that, as the number of sources is increased, the amplitude of the oscillations decreases relative to the peak irradiance. In the limit that $N$ approaches infinity, as in the aperture calculation, the oscillations disappear.

Figure 9. Diffraction patterns from 5x5 arrays of point sources and apertures. $a = 20$, $\lambda = 1 \mu m$, $Z = 4 mm$. (a) 5x5 100x100 nm apertures (b) 5x5 point sources.

Figure 10. Diffracted power from 5x5 arrays of 1x1 $\mu m$ apertures. (a) $a = 2 \mu m$ (b) $a = 4 \mu m$. 
2.5. Fourier Diffraction

Though the point sources and apertures in the preceding sections were evenly spaced, the sources and diffraction patterns were not truly periodic. The diffraction of light from periodic surfaces can be calculated using Fourier optics. Consider the reflection of light from a one-dimensional periodic diffraction grating. [Fig. 11] Periodicity here refers to a type of translational symmetry, such that the structure is identical under certain discrete transformations. In this case, the structure has a discrete symmetry under translations of a distance $L$ along the $x$-axis.

The most natural expression for periodic functions is the sine wave. The sinusoidal functions are complete, meaning that any function $U(x)$ can be expressed as a linear combination of sinusoids or complex exponentials with different wavenumbers.

$$U(x) = \sum_{n=-\infty}^{\infty} c_n e^{i k_n x} \quad (2.6)$$

For a structure with a single periodicity $L$, like the problem at hand, the possible wavenumbers can only be those such that the transformation $x' = x + nL$, where $n$ is an integer, leaves the value of the complex exponential unchanged. Hence, the wavenumbers are of the form

$$k_n = k_0 + \frac{2n\pi}{L}. \quad (2.7)$$

This is a form of Floquet's theorem, [10] which is the fundamental assumption that will be used to solve for the wave reflected from periodic structures. Because we are dealing with the current problem in terms of scalar fields, the best we can do to determine the reflected field is to assume the form of the field on the structure and use Floquet's theorem to determine how it evolves as it propagates away from the surface. A reasonable guess for the setup would be that the reflective surfaces reflect all of the incident light, and the openings between the surfaces transmit...
all of the incident light. Thus, the wave on the surface looks like a periodic square wave. [Fig. 12] We can write this square wave as a Fourier series with periodicity $L$. Let $\beta_n$ be the spatial wavenumber along the $x$ direction for each mode.

\[ U(x) = \sum_{n=-\infty}^{\infty} c_n e^{-i\beta_n x} \]  

\[ c_n = \frac{1}{L} \int_{-L/2}^{L/2} U(x) e^{i\beta_n x} \, dx = \frac{L - W}{L} \text{sinc} \left( \frac{\beta_n (L - W)}{2} \right) \]  

\[ \beta_n = \frac{2n\pi}{L} \]  

We now need to determine how the wave evolves as it propagates away from the surface. We know that the total wavenumber of the light $k$ must be a constant. We can use this and the wavenumber in the $x$ direction for each mode, $\beta_n$, to determine the wave number in the $z$ direction for each mode, $q_n$.

\[ U(x) = \sum_{n=-\infty}^{\infty} c_n e^{-i\beta_n x + i q_n z} \]

\[ q_n = \sqrt{k^2 - \beta_n^2} \]  

Figure 13. Diffraction patterns from a grating using Fourier optics. Spacing width is 500 nm, conductor width 500 nm, wavelength 500 nm and screen distance (a) 0 μm, (b) 2 μm.
Figure 13 shows the evolution of a diffraction pattern as the distance to the screen is increased. The wavenumber in the \( z \) direction has an important effect on modes with large wavenumber in the \( x \) direction. When \( |\beta_n| > |k| \), \( q_n \) becomes imaginary, and such modes do not propagate.

3. PERIODIC ELECTROMAGNETIC SCATTERING

Scalar diffraction theory is an excellent tool for understanding the generation of diffraction patterns and can help to give an intuitive sense of the mechanisms involved in the reflectance of light from periodic gratings, but its intrinsic assumptions make it unsuitable for a rigorous calculation of reflected power. For example, the calculations with scalar diffraction have assumed that the profile of the field on the grating is a square wave. This is a reasonable first pass approximation, as we know that the electric field should go to zero at the boundaries of a conducting aperture, but the actual field on the grating cannot be so simple. More importantly is the fact that, according to scalar diffraction theory, the strength of the field on the grating is independent of wavelength. By considering the boundary conditions of the electromagnetic field in the aperture, we can obtain a more physical understanding of the reflection of light from a periodic grating.

3.1. The Form of the Scattered Magnetic Field

We are interested in the form of the electromagnetic wave reflected from infinitely thin strips of perfectly conducting tape that are periodic in the \( \hat{x} \) direction. [Fig. 14] We will deal only with the transverse magnetic field, such that the incident and reflected fields only have components in the \( \hat{y} \) direction. Scattering theory relies on the fact that electromagnetism is a linear theory, which means that any field, no matter how complicate, can be dealt with in terms of its constituent fields, one at a time. The simplest components are the incident plane wave \( H_{yi} \) and the reflected plane wave \( H_{yr} \), which are functions of the angle of incidence. Each wave oscillates with the same frequency \( \omega \), so it is sufficient to set \( t = 0 \) and consider only the spatial distributions.
\[ H_{yi} = A_0 e^{+i q_0 z - i \beta_0 x} \]  
(3.1)

\[ H_{yr} = A_0 e^{-i q_0 z - i \beta_0 x} \]  
(3.2)

\[ \beta_0 = k \sin \theta_i \]  
(3.3)

\[ q_0 = k \cos \theta_i \]  
(3.4)

The incident wave in the aperture region between conducting strips will generate two scattered fields: \( H_{ys1} \), which propagates upward and combines with \( H_r \) to form the total reflected wave, and \( H_{ys2} \), which propagates downward and constitutes the transmitted wave. The scattered waves are solved for in terms of the boundary conditions of the problem. The choice here is somewhat arbitrary, as we could just as well have chosen an incident and transmitted wave to start with and then defined the scattered waves. Either way, the scattered waves would have to satisfy the boundary conditions, and the total reflected and transmitted waves would be the same. As we will see, the choice of having an incident and reflected wave in region 1 simply gives a convenient expression for the boundary conditions.

Via Floquet’s theorem, we can write the scattered fields, which are generated by a periodic structure, as a sum of plane waves with wavevectors that satisfy the periodic boundary conditions.

\[ H_{ys1} = \sum_{n=-\infty}^{\infty} B_n e^{-i q_n z - i \beta_n x} \]  
(3.5)

\[ H_{ys2} = \sum_{n=-\infty}^{\infty} C_n e^{+i q_n z - i \beta_n x} \]  
(3.6)

\[ \beta_n = \beta_0 + \frac{2n \pi}{L} \]  
(3.7)

\[ q_n = \sqrt{k^2 - \beta_n^2} \]  
(3.8)

The scattered fields are generated by the electric field in the aperture at \( z = 0 \), whose \( x \) component we will denote as \( f(x) \). The aperture field is itself generated by the incident wave, which is the driving force. These scattered waves contain all the necessary information about the reflected and transmitted waves. For example, when \( C_0 = A_0 \) and all other \( C_n = 0 \), \( H_{ys2} \) will be
identical to the incident wave, and the entire wave will be transmitted. In order to satisfy the continuity of the magnetic field at the boundary, we would expect $B_0 = -A_0$, so that the scattered wave in region 1 would cancel the reflected plane wave. We can use the Maxwell-Ampere law in a medium with no current to solve for the scattered fields in terms of the electric field in the aperture.

$$\nabla \times \vec{H} = \varepsilon \frac{\partial}{\partial t} \vec{E}$$

(3.9)

$$\left( \nabla \times \vec{H} \right)_x = \varepsilon \frac{\partial}{\partial t} f(x)$$

(3.10)

Here, $\varepsilon$ is the permittivity of the dielectric that the grating is embedded in, and $f(x)$ is assumed to oscillate with frequency $\omega$. Using the fact that all space harmonics are orthogonal, we can solve for the coefficients $B_n$ and $C_n$.

$$B_n = \frac{1}{L} \int_{-L/2}^{L/2} \frac{\omega \varepsilon}{q_n} f(x) e^{i\beta_n x} dx$$

(3.11)

$$C_n = -\frac{1}{L} \int_{-L/2}^{L/2} \frac{\omega \varepsilon}{q_n} f(x) e^{i\beta_n x} dx$$

(3.12)

This relation agrees with intuition. First, we expect the strength of the scattered magnetic field to be somehow proportional to the strength of the field in the aperture. Second, the scattered wave is antisymmetric. That is, the scattered waves have the same magnitude and opposite signs. This can be explained in terms of the magnetic field generated by a current in a wire. Such a current generates a circular magnetic field around the wire, so that the fields above and below the wire are pointing in opposite directions and have opposite signs. In this case, the current is replaced by a time-changing electric field.

3.2. The Form of the Scattered Power

In general, the time-averaged energy flux density of an electromagnetic wave is given according to the Poynting vector $\vec{S}$ as [11]

$$\langle \vec{S} \rangle_t = \frac{1}{2} \vec{E} \times \vec{H}^*$$

(3.13)
where $\vec{H}^*$ denotes the complex conjugate of the magnetic field. For the case of a periodic structure, we are interested in the spatial average of this vector over the length of one period $L$.

$$\langle \vec{S} \rangle_{t,x} = \frac{1}{2L} \int_{-L/2}^{L/2} \vec{E} \times \vec{H}^*$$

(3.14)

To find the propagating electric field, we can once again use the Ampere-Maxwell equation.

$$\nabla \times \vec{H} = \epsilon \frac{\partial}{\partial t} \vec{E}$$

(3.15)

Because we have chosen to deal only with the TM wave, $\vec{H}$ only has a $y$ component. In order to find the reflected or transmitted power, we would take the dot product of the Poynting vector with the normal vector of some screen. It is convenient to choose as a screen a plane with constant $z$. Thus, it is only the $z$ component of $\vec{S}$ that contributes to radiated power, and only the $x$ component of $\vec{E}$, which has no $y$ component. For this reason, the useful relation between $\vec{H}$ and $\vec{E}$ is as follows.

$$E_x = \frac{i}{\omega \epsilon} \frac{\partial}{\partial z} H_y$$

(3.16)

The incident power $P_i$, the transmitted power $P_t$, and the reflected power $P_r$, which includes the power of the reflected plane wave and $H_{ys1}$, can be solved for in terms of their coefficients.

$$P_i = \frac{1}{2\omega \epsilon} q_0 |A_0|^2$$

(3.17)

$$P_t = \frac{1}{2\omega \epsilon} \left( \sum_{n=N_1}^{N_2} q_n |C_n|^2 \right)$$

(3.18)

$$P_r = \frac{1}{2\omega \epsilon} \left( q_0 |A_0| + B_0|^2 + \sum_{n=N_1 \neq 0}^{N_2} q_n |B_n|^2 \right)$$

(3.19)

Here, the bounds $N_1$ and $N_2$ correspond to the modes with real $q_n$ because, for all imaginary modes, real power is not transmitted away from the surface. The form of the reflected power highlights the relationship between the reflected and scattered waves. Conservation of energy requires that the sum of the reflected and transmitted powers be equal to the incident power.
Thus, we expect the coefficient in front of the reflected field $B_n$, specifically $B_0$, to be negative. This represents a phase shift of 180° with respect to the incident wave and results in the magnitude $|A_0 + B_0|^2$ decreasing, so that the total reflected power never exceeds the incident power. The question now regards precisely what form $B_0$ and $C_0$ take on and how they vary with wavelength, angle, and the dimensions of the reflecting surface.

### 3.3. The Form of the Electric Field in the Aperture

As we have seen, the scattered magnetic field is generated by the electric field in the aperture, which in turn is generated by the incident wave. The electric field is constrained by the boundary conditions on the conducting tape, and by the requirement that the electric and magnetic fields be continuous through the aperture. The continuity of the magnetic field can be expressed as follows.

$$H_{yi} + H_{yr} + H_{ys1} = H_{ys2} \quad \text{at } z = 0 \quad (3.20)$$

Including the boundary condition that the electric field must go to zero at $\pm W/2$, this can be written as the following integral equation.

$$A_0 e^{-i\beta_0 x} = -i\omega e^\frac{W}{2} \int_{-W/2}^{W/2} f(x')G(x, x') dx' \quad (3.21)$$

$$G(x, x') = \sum_{n=-\infty}^{\infty} \frac{e^{-i\beta_n(x-x')}}{i q_n L} \quad (3.22)$$

This is a Fredholm integral equation of the first kind, which has the general form.

$$g(x) = \int_{a}^{b} f(x')K(x, x') dx' \quad (3.23)$$

Here, $K(x, x')$ is the integral kernel, and the function to be solved for is $f(x)$. This form of integral equation is particularly difficult to find solutions to because the problem is ill-posed [12], meaning that it may have more than one solution, or the solutions may depend discontinuously on initial data. In this case, this is due to the fact that the equation contains no information about how the electric field goes to zero. In general, this depends on the polarization of the incident wave. It can be shown that for an electromagnetic wave with a magnetic field oscillating along an edge, in
order for the energy density of the field to be finite everywhere, the component of the electric field pointing away from the edge must be proportional to [13]

\[ E_{\rho} \propto \rho^{\frac{\pi}{\varphi_0} - 1} \sin \left( \frac{\pi \varphi}{\varphi_0} \right) \]  

(3.24)

where \( \rho \) is the distance from the edge, \( \varphi \) is the angle from the horizontal portion of the edge, and \( \varphi_0 \) is the angle of the edge itself. Both angles are measured around the outside of the edge, such that a corner would have \( \varphi_0 = \frac{\pi}{2} \) and a perpendicular edge would have \( \varphi_0 = \frac{3\pi}{2} \). In the case of our conducting tape, \( \varphi_0 = 2\pi \). We are interested in the \( x \) component of the electric field, which is \( E_{\rho} \) at \( \varphi = \pi \). In the vicinity of the edge of the tape, the field is then proportional to

\[ f(x) \propto \rho^{-\frac{1}{2}}. \]  

(3.25)

This is certainly not an intuitive result. It implies that the electric field goes to infinity as \( \rho \to 0 \), even though the total energy in the vicinity of the edge is finite. This behavior is related to why forks generate plasma when put in a microwave. It is not that the metal itself generates plasma—a fact that is obvious by noting that the interior of a microwave is in fact made of metal. It is rather that conducting objects with sharp corners can generate very large electric fields, until a discharge occurs.

3.4. The Long Wavelength Limit

An approximate solution for the electric field in the aperture can be obtained by assuming a form of the field and solving for a variable parameter. To begin with, the condition that \( f(x) \) is proportional to \( \rho^{-\frac{1}{2}} \) in the vicinity of the edge can be ignored. A simple field that satisfies the requirement that the electric field is zero on the conducting tape is as follows.

\[ f(x) = \begin{cases} 
C e^{-i\beta_0 x}, & |x| < \frac{W}{2} \\
0, & \frac{W}{2} < |x| < \frac{L}{2} 
\end{cases} \]  

(3.26)

The problem can be simplified by assuming normal incidence (\( \beta_0 = 0 \)), which reduces \( f(x) \), to a constant in the aperture, and taking the long wavelength limit (\( k \ll \frac{2\pi}{L} \)). To solve, both sides of
the integral equation are multiplied by $f^*(x)$ and integrated over a period $-L/2 \rightarrow L/2$ to make each side a constant, rather than a function of $x$.

$$A_0 \int_{-L/2}^{L/2} f^*(x) e^{-i\beta_0 x} dx = -i\omega \epsilon \int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} f^*(x) f(x') G(x, x') \hspace{1cm} (3.27)$$

$$C = \frac{iA_0 W}{\omega \epsilon} \left[ \int_{-W/2}^{W/2} dx \int_{-W/2}^{W/2} dx' G(x, x') \right]^{-1} \hspace{1cm} (3.28)$$

Because $G(x, x')$ is inversely proportional to $q_n$, which is given as $q_n = \sqrt{k^2 - \frac{2n\pi}{L}}$, the absolute value of the double integral is expected to decrease quickly for terms with values of $2n\pi/L$ greater than $k$, especially for large wavelengths, such that $k \ll 1$. Further, the integral of a sinusoid over a large length compared with its wavelength is approximately zero, so the increasing values of $\beta_n$ also makes this series rapidly decrease. [Fig. 15] To get an idea of the long-wavelength behavior, it is appropriate to approximate $G(x, x')$ with just the first term. For normal incidence, $\beta_0 = 0$ and $q_0 = k$, so the integral is trivial.

$$C \approx -\frac{A_0 L}{\omega \epsilon W} = -Z \frac{A_0 L}{W} \hspace{1cm} (3.29)$$

where $Z = \sqrt{\mu/\epsilon}$ is the impedance of the dielectric. The minus sign confirms the intuition that the coefficients of the scattered field in region 1, which are proportional to the electric field in the aperture, should be out of phase with respect to the reflected wave. This can be used to solve for the coefficients of the magnetic field associated with the zeroth mode.

![Figure 15. Strength of the terms of the integral of G(x,x'). L = 1, W = 0.9 and (a) λ = 0.3 (b) λ = 10.](image)
\[ B_0 = \frac{W\omega\epsilon}{Lk} C = -A_0 \]  
\[ C_0 = -\frac{W\omega\epsilon}{Lk} C = A_0 \]  

For long wavelengths, the scattered wave in region 1 is equal and opposite to the reflected plane wave, resulting in complete cancellation of all reflected power. The transmitted wave is identical to the incident wave, so that the wave is totally transmitted. Physically, an electromagnetic wave with long wavelength is approximately a static field. As the electric field moves charge onto the edges of the conducting strips, they become dipoles. [Fig. 16] If there is no limit to the number of free charge carriers in the conductor, then a sufficient amount of charge will accumulate for the dipole to completely cancel out the incident field on the conductor. In region 2, the induced dipoles will generate an electric field with strength proportional to the incident field.

### 3.5. The Free-Space Limit

It is important to understand the behavior of this approach for very long periodicity and very wide apertures i.e., very small and very widely spaced conducting lines which approach free space. Recall that the continuity of the fields through the aperture and the boundary conditions of the fields on the conductor are contained in a single equation.

\[ A_0 e^{-i\beta_0 x} = -i\omega\epsilon \int_{-W/2}^{W/2} f(x') G(x, x') dx' \]  

There is one case in which this equation is generally solvable: when the kernel is a function only of the difference of its arguments, \( x - x' \), and the limits of the integral are from \(-\infty\) to \(\infty\). In this case, the function \( A_0 e^{-i\beta_0 x} \) is the simply the convolution of \( f(x) \) and \( G(x) \), and the solution is given by [14]
\[ f(x) = \mathcal{F}_\beta^{-1} \left[ \frac{\mathcal{F}_x[A_0 e^{-i\beta_0 x}]}{\mathcal{F}_x[G(x)](\beta)} \right] \]  

(3.33)

where \( \mathcal{F}_x \) is the Fourier transform, and \( \mathcal{F}_\beta^{-1} \) is the inverse Fourier transform.

\[ \mathcal{F}_x[f(x)] = \int_{-\infty}^{\infty} f(x) e^{i\beta x} \, dx \]  

(3.34)

\[ \mathcal{F}_\beta^{-1}[g(\beta)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\beta) e^{-i\beta x} \, d\beta \]  

(3.35)

This solution is valid in the limit that \( W \) goes to infinity. Because \( W \leq L \), \( L \) necessarily goes to infinity also. In this case, the discrete values of \( \beta_n = \beta + \frac{2\pi n}{L} \) become continuous, and the Green’s function becomes an integral, rather than a sum.

\[ G(x - x') = \lim_{L \to \infty} \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \frac{e^{-i\beta_n (x-x')}}{i q_n} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-i\beta (x-x')}}{i\sqrt{k^2 - \beta^2}} \, d\beta \]  

(3.36)

The function \( G(x - x') \) is already an inverse Fourier transform of the function \(-i(k^2 - \beta^2)^{-1/2}\).

\[ f(x) = \mathcal{F}_\beta^{-1} \left\{ \mathcal{F}_x \left[ \frac{i}{\omega\epsilon} A_0 e^{-i\beta_0 x} \right] \right\} = -\frac{A_0}{\omega\epsilon} e^{-i\beta_0 x} \sqrt{k^2 - \beta^2} \]  

(3.37)

The magnetic field coefficients \( B_n \) also become a continuous function \( B(\beta) \).

\[ B(\beta) = \lim_{L \to \infty} \frac{1}{L} \int_{-L/2}^{L/2} \frac{\omega\epsilon}{\sqrt{k^2 - \beta^2}} e^{i\beta x} f(x) \, dx = -A_0 \delta(\beta - \beta_0) \]  

(3.38)

Here, \( \delta(\beta - \beta_0) \) is the Dirac delta function, which is zero everywhere except for \( \beta = \beta_0 \). This means that the only nonzero mode of the reflected wave is \(-A_0 e^{-i\beta_0 x}\), which completely cancels the reflected wave. Similarly, from \( C_n = -B_n \), the wave in region 2 is \( A_0 e^{-i\beta_0 x} \), which is simply the incident wave. As expected, in the free-space limit, the entire wave is transmitted.

### 3.6. An Approximate Solution using the Edge Condition

An appropriate form of the electric field with the inclusion of the edge condition could be the following.
\[ f(x) = \begin{cases} 
C e^{-i\beta_0 x} & |x| < \frac{W}{2} \\
\left(\frac{W^2}{2} - x^2\right)^{1/2} & \frac{W}{2} < |x| < \frac{L}{2} \\
0 & \text{otherwise}
\end{cases} \] (3.39)

Once again, \( C \) is a constant which is determined by constraining the magnetic field to be continuous at \( z = 0 \). In this case, the strength can be solved for analytically. [10]

\[ C = -A_0 \left[ i\omega \varepsilon \sum_{n=-\infty}^{\infty} \frac{\pi}{i q_n L} J_0^2 \left( \frac{n \pi w}{L} \right) \right]^{-1} \] (3.40)

Here, \( J_0 \) is the zeroth-order Bessel function of the first kind. This function has an interesting behavior at values of \( \lambda \) such that \( k = \beta_n \), at which point \( q_n = 0 \). At these values, the value of the sum goes to infinity, and \( C \) goes to zero. For \( \beta_0 = 0 \), this resonance happens at values of \( k = \frac{2n \pi}{L} \).

[Fig. 17] This drop-off of the field in the aperture corresponds to points where the transmittance goes to zero and the reflectance goes to unity. For these wavelengths, the periodicity of the grating is an integer multiple of the wavelength. That is, \( \lambda = L/n \) for integer \( n \). No resonances occur for values of \( \lambda \) greater than \( L \). While the reflectance and transmittance plots have discontinuities at relatively small wavelengths, these smooth out at long wavelengths. In general, the strength of the field increases with \( \lambda \), corresponding to a gradual increase in transmittance and decrease of reflectance, approaching the long wavelength limit where all of the light is

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**Figure 17.** Resonances of an incident field with a grating. (a) The incident, reflected, and transmitted power as a function of wavelength. (b) The strength factor of the field in the aperture as a function of wavelength. \( L = 1, W = 0.5, \theta = 0 \). The lines correspond to \( \lambda = L/n \).
transmitted and none is reflected. The sum of the scattered and transmitted powers is always equal to the incident power, which is necessitated by the conservation of energy.

The diffraction patterns generated by this approximation can be compared with the scalar approximations from section 2. In this case, both diffraction patterns are only calculated using modes below the cutoff wavelength. For longer wavelengths, this makes the diffraction patterns more regular, and the two approximations are similar for certain screen distances. [Fig. 18] For shorter wavelengths, the patterns generated using electromagnetic scattering become more complicated. [Fig. 19]

The reflected power depends on the wavelength of the incident light, the dimensions of the grating, and the angle of incidence. For a linear, isotropic, homogeneous media, the dependence of reflectance and transmittance on incident angle is given by the Fresnel equations. These equations are derived using only the boundary conditions of a wave on a medium, and do not consider any material properties other than the refractive index $n$, which is the ratio of the
speed of light in free space to the phase velocity inside a material. For an electromagnetic wave with the electric field in the plane containing the normal of the surface and the propagation wavevector, the $x$-$z$ plane in our case, the coefficient of reflection of the wave is given as follows.

\[ R_{||} = \left( \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \right)^2 \]  

(3.41)

where $\theta_i$ is the angle of incidence with respect to the normal of the surface, and $\theta_t$ is the angle of transmission, which is related to the indices of refraction of the incident and transmission media.

\[ \theta_t = \arcsin \left( \frac{n_i}{n_t} \sin \theta_i \right) \]  

(3.42)

Consider the case of incidence nearly parallel to the surface such that $\theta_i \approx \frac{\pi}{2}$, and $\cos \theta_i \approx 0$.

\[ R_{||} \approx \left( - \frac{\cos \theta_t}{\cos \theta_i} \right)^2 = 1 \]  

(3.43)

For a uniform dielectric, as the incident angle increases, more light will reflect from the surface. For a periodic conductor, the scattering calculation predicts the peculiar result that, as the angle of incidence increases, the reflectance at short wavelengths decreases and the transmittance increases. [Fig. 20] This behavior could be related to the fact that at near-parallel incidence, $\beta_0 \approx k$, and the induced dipole reaction from the conducting strips becomes prominent, generating a strong electric field in region 2.

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**Figure 20.** Reflectance and transmittance of grating with varying angle of incidence. L = 1, $\theta = 0$, W = 0.3 (a) $\theta = 45^\circ$, (b) $\theta = 60^\circ$. 
As the aperture width is increased, while holding the periodicity constant, the surface area of the conductor in a period $L$ decreases. For this reason, it is expected that an increase in aperture width would cause transmittance to increase. Figure 21 shows that, as the width of the aperture is increased from $0.2L$ to $0.4L$, the transmittance increases faster at shorter wavelengths. The analytical solution for free space indicates that, as the aperture width is further increased, this intersection between the reflected and transmitted powers will continue to move to shorter wavelengths until the transmittance is 1 for all wavelengths.

If the width of the conductor is held constant, and the periodicity of the array is altered, then the surface area of the conductor in one period does not change. The increase in length decreases the value of $\beta_n$ for a particular $n$, which in turn allows $q_n$ to be a real quantity for more modes. The effect on reflectance and transmittance is that longer wavelengths of light can resonate with the array, which also has a longer periodicity. [Fig. 22] The effect of resonance on reflectance and transmittance raises an interesting question about the behavior of more complicated interfaces. For example, an interface with multiple periodicities, such that the conducting strips are staggered and grouped, may have many more resonances, each corresponding to a different periodicity, and an overall higher reflectance. A random distribution
of strips, with infinite resonances, may even increase reflectance for a wide range of wavelengths. Similarly, a two dimensional periodic structure would have periodicities from $L_x$ to $\sqrt{L_x^2 + L_y^2}$, which may produce complex or oscillatory behavior in the reflectance, similar to the oscillatory behavior predicted by the FDTD simulation.

4. CONCLUSION

Two models have been constructed to understand the interaction of an incident plane wave with an inhomogeneous two dimensional conducting interface. Using the Huygens-Fresnel principle, the light reflected from conducting patches has been modeled as light reflected from point sources. In general, the interference of the light from these point sources decreases the amount of power reflected from the interface. An increase in wavelength decreases the amount of destructive interference and increases the total reflected power, so that the reflectance oscillates around a minimum value and starts to increase monotonically at an onset wavelength. This onset wavelength increases with an increase in spacing between patches, which agrees with the observation that a decrease in CNT forest density increases the onset wavelength of reflectance. [9] This scalar wave model indicates how interference could allow for an interface to reflect or transmit light with a dependence on wavelength. However, a decrease in wavelength which increases destructive interference in the reflected wave is expected to be accompanied by destructive interference in the transmitted wave as well. Conservation of energy requires that an increase in the reflectance be matched by a decrease in transmittance, so the interference in the reflected wave must be compared with the interference in the transmitted wave to explain why a certain interface preferentially reflects or transmits.

Using electromagnetic scattering theory, the reflection of TM polarized light from a one dimensionally periodic conducting interface has been analyzed. It has been found that an increase in aperture width results in a faster rise in transmittance, approaching the free-space limit, at which point the wave is totally transmitted. In the limit that the angle of incidence approaches incidence parallel to the interface, the wave is preferentially transmitted. In general, transmittance is predicted to increase with wavelength, approaching the long wavelength limit, at which point the wave is totally transmitted. This agrees with the intuitive interaction of a static electric field with a series of dipoles but disagrees with the prediction of the Huygens-Fresnel
principle. This result may indicate that long wavelength light is not reflected preferentially from the interface of a broadband absorber but instead passes through the interface and is reflected by the substrate. However, this does not explain and cannot account for the observation that many broadband absorbers can have an extremely low reflectance of short wavelength light. It has also been shown that an increase in lattice periodicity, with conductor width being held constant, does not decrease the envelope of transmittance, but allowed longer wavelengths of light to resonate with the interface, resulting in dips of the transmittance at longer wavelengths. This indicates that resonance may have a role to play in the absorption of short wavelength light. Future work will extend this calculation to the case of two dimensionally periodic apertures to investigate the properties of materials that can have many periodicities and more complex resonances with incident light.

Word Count: 8273
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AUTHOR’S BIOGRAPHY

Christian Lange grew up in Evanston, Wyoming and graduated from Seton Home Study School in 2017. He enrolled in Utah State University with majors in physics and piano performance and was named an honorable mention for the USU 2017-2018 concerto competition. In spring 2019, he made the decision to replace the music major with minors in computer science and mathematics. In summer 2019, he was awarded an URCO grant and a COS minigrant to support his research and, in summer 2020, he was named a Peak Summer Research Fellow.

In spring 2021, he will graduate and move to West Lafayette, Indiana to pursue a PhD in atomic, optical, and molecular physics at Purdue University. He hopes to eventually contribute to the research of quantum information technologies.
REFLECTIVE WRITING

This project is the accumulation of almost three years of research and study. The study of electromagnetic scattering theory is the fifth project that I have worked on under the mentorship of Dr. Shen. Our first project was the fabrication of a carbon-nanotube-forest-based bolometer, which introduced me to the optical properties of carbon-nanotube forests and other broadband absorbers. These remarkable properties inspired our second project, which was concerned with the fabrication of forests that were height-modulated, so that they would be dark to longer wavelengths of light. This was based on research that was done in Dr. Shen’s lab before I ever came to USU, which showed that a random height-modulation could increase the onset wavelength of reflectance. This second project, and the results of earlier research, raised the important question of why exactly carbon-nanotube forest is dark.

In pursuit of an answer to this question, this project in some sense began as early as the fall of 2019, when we started using commercial software to numerically calculate the reflectance of light from nanorod arrays. The results of this research agreed with our understanding of the relationship between nanorod density and reflectance at long wavelengths, but we still did not have a full physical understanding of the phenomenon, so we spent the summer of 2020 studying first the reflectance of light from conductors using the Lorentz-Drude model, and then investigating the role of interference in the reflectance of light from nanostructures using the scalar wave Huygens-Fresnel principle. The results from the Huygens-Fresnel principle were promising and gave us a much better understanding of the phenomenon. By the beginning of the 2021 school year, I had plenty of material to write about, but some questions were still nagging us, so we decided to take one more step and investigate the reflectance of light from periodic structures using scattering theory.

This is by far the most ambitious project I have undertaken for a number of reasons. First of all, the scattering theory was based on a treatment given in chapter 7 of Akira Ishimaru’s *Electromagnetic Wave Propagation, Radiation, and Scattering*, which is a graduate-level textbook. I have very few of the prerequisites for studying such a high-level book, and the fact that I was skipping to the middle of the book made the derivation almost incomprehensible, as I had to constantly go to previous chapters to understand notation and assumptions that he
established earlier. Second, our decision to study a fresh topic for my capstone project meant that I was essentially beginning the main body of research for my project in the same semester that it was due, putting me under an enormous amount of time pressure to understand the theory and get a result. Third, Dr. Shen was far too busy to read the text and help me to understand it, which meant that I was virtually on my own.

When I finally calculated my first result using scattering theory, its prediction was precisely the opposite of what we were expecting. Dr. Shen and I were convinced that I had made a mistake somewhere. And to make matters worse, I uncovered what I believed to be a few typos in the text, which completely shook my faith in the book’s derivation and results. This began some of the most challenging weeks of my undergraduate career. For two whole weeks I was obsessed with this problem. I was unable to think about anything else, and I could not focus on any of the other things that I used to be interested in. I was thinking about it while working out, while practicing music, and even while trying to do homework. I would think about it while lying awake in bed and as soon as I got up in the morning. I tried test cases and limits. I tried messing with the equations and thinking up alternate ways of doing the problem. Nothing worked. Eventually, I had to force myself to write up the results, even if I did not trust them. It was during the writing process that I began to sort out the physical meaning of the theory, and finally began to gain some confidence in the results.

This project has taught me a number of invaluable lessons. First, it exposed me to my first truly difficult physics problem. In real research, there is no solutions manual, and there is no recitation. I learned how to find and study resources independently to understand a topic more in depth. Real research is composed mostly of frustration and failure, and I had my first taste of the life of a scientist during this project. Second, through my obsession over this problem and its physical interpretation, I learned the hard lesson of how important it is to separate oneself from one’s research. I had to learn how to discipline myself to study the problem at the appropriate time, and how to stop studying and do something else when I worked myself into a hole. Learning this skill will be absolutely necessary for me to function as a researcher. Third, the contradictory result taught me that research does not always give the answers that are expected. All results need to be evaluated with a critical eye. In order to be an effective scientist, I must always
understand every assumption I have made and only accept the results in the context of such assumptions. It is still not clear the implications that this research has on the physical cause for the reflectance of long wavelength light from high aspect-ratio nanostructures, but the surprising result has made me think more carefully about the way that we have viewed the problem so far. Finally, this project taught me persistence. I took on a more ambitious project than anything I have faced so far, and in persevering until the end, I gained a confidence in my own abilities that will serve me for the rest of my life.

Word Count: 991