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Fatigue Life Analysis of T-38 Aileron Lever Using a Continuum Damage Approach

James D. Gyllenskog
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FATIGUE LIFE ANALYSIS OF T-38 AILERON LEVER USING A CONTINUUM DAMAGE APPROACH

by

James D. Gyllenskog

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Mechanical Engineering

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UTAH STATE UNIVERSITY
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2010
ABSTRACT

Fatigue Life Analysis of T-38 Aileron Lever Using a Continuum Damage Modeling Approach

by

James Gyllenskog, Master of Science
Utah State University, 2010

Major Professor: Dr. Leila Ladani
Department: Mechanical and Aerospace Engineering

In a recent investigation conducted by the United States Air Force, the mechanical failure of the aileron lever, manufactured from 2014-T6 aluminum, caused the fatal mishap of a T-38 trainer aircraft. In general the locations of cracks are unknown and must be determined by simulation. In this study we propose to use a continuum damage modeling approach to determine the degradation and damage in a material as the number of cycles of loading increases. This approach successfully predicts the location of crack initiation, propagation path, and propagation rate. A stress-based model in conjunction with the successive initiation technique is utilized.

Successive initiation is based on the idea that damage will accrue in a material. Each element inside a new material will have a value of 0 damage assigned to it. Over time, the damage that occurs due to stresses on individual elements will add until the damage reaches a value of 1. At that point, failure of the element will occur. A code was developed in ANSYS that can draw, mesh, and apply appropriate forces on the aileron
lever for successive runs. By using the S-N curve for the 2014-T6 aluminum material, the material damage constants are found. This stress-based damage model is then used to determine the state of damage in each element. Each time the elements are stressed, a particular amount of damage will occur. When an element reaches a specific amount of damage, ANSYS will “kill” the element, resulting in the element no longer adding to the stiffness matrix of the material.

Variability is a common occurrence in all aspects of engineering such as manufacturing, testing, and loading. A Monte Carlo simulation is used to determine the sensitivity of the results to variability of input parameters by ±15%. Input parameters include loads, material properties and damage model constants. The Monte Carlo simulation indicates the only significant input in the initiation life of the material is the exponential value in the stress-based fatigue life equation. Material properties and load variations in the ± 15% range will not significantly change the life prediction results.
This thesis is dedicated to my family members and a loving wife who has constantly encouraged and supported me throughout the entire education process.
ACKNOWLEDGMENTS

First and foremost, I am truly grateful and indebted to my advisor, Dr. Leila Ladani, for her unceasing support and guidance throughout this process. Her patience with me has gone far and above the call of duty. The research that I have done has not been beneficial to her or her work in any way, shape, or form, yet she was willing to take me on as a graduate student and put forth the time and effort in helping me achieve something that I never imagined possible. Dr. Ladani is truly an asset to the Utah State University College of Engineering.

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I would like to thank my supervisor, Grant Herring, and lead engineer, Eric Flygare, for providing me with the necessary information to accomplish this undertaking. I will be forever grateful for their support of me while I have been away from work and for giving me the opportunity to better myself as an engineer.

Most of all I am so grateful for my loving, caring, inspiring wife, Wendy, who is, without a doubt, the driving force behind me. If not for her belief in me, when I struggled so often to believe in myself, this project never would have happened.

James D. Gyllenskog
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1.1 Objectives and Problem Statement

On 23 April 2008, at approximately 1226 Central Standard Time, the Mishap Aircraft (MA), with an experienced instructor and student pilot on board, crashed during initial takeoff at Columbus AFB, MS.

After a comprehensive investigation into this mishap, the mishap investigation board president found that, by clear and convincing evidence, the right aileron of the MA failed in the full down position prior to takeoff. This condition resulted in an uncommanded left roll as the aircraft was rotating for takeoff. Initial correction was made momentarily reducing the left bank. However, as airspeed continued to increase, the left roll continued until the MA was completely inverted. Given the mechanical failure and the critical phase of flight, the pilots were unable to maintain control of the aircraft. The MA became inverted within 3 to 4 seconds and struck the ground, sliding off the runway and bursting into flames. Neither pilot could successfully eject unfortunately resulting in the loss of life due to ground impact.

During the investigation, the aileron levers were found and sent to the Hill AFB materials lab for inspection. The aileron lever that caused the mishap was found to have a quarter elliptical crack with a final crack length of 0.09 x 0.06 inches measured down the bore of the hole and along the surface, respectively. Figure 1-1 shows a picture of the failed lever. It was never determined at what load the lever failed prior to takeoff roll.
Material lab findings showed numerous fatigue cracks on the failed surfaces of the lever. Figures 1-2 and 1-3 show the locations and lengths of the multiple cracks. Fatigue phenomenon occurs due to accumulation of damage and can happen at stresses well below the yield stress at which point no plasticity is occurring. Traditionally fatigue has been divided into two categories, low cycle fatigue and high cycle fatigue. High cycle fatigue represents the cases that the stress applied on the structure does not exceed the yield stress and thus no local plasticity occurs. However, in many structures, stresses may exceed the yield stress locally due to stress concentrations caused by stress raisers typically due to sharp points in geometry and shape. Low cycle fatigue represents cases where the applied stresses are beyond the yield point of a material and plasticity has occurred. In low cycle fatigue where plasticity is experienced, during the cycle sequence,
dislocations pile up producing slip bands which can cause either extrusive or intrusive bands that rise above or fall below the surface of the structure. These slip bands leave microscopic steps on the surface that serve as stress risers where cracks can initiate [1].

Figure 1-2: Surface of failed aileron lever showing locations of cracks (Courtesy of Hill AFB Materials Lab).
Failure of a structure that could cause extensive financial loss or loss of human lives must be designed such that it can tolerate a predetermined amount of damage. Continuum damage mechanics is a relatively new concept in the field of engineering science. In 1958, L.M. Kachanov introduced a scalar damage variable $\psi$, called “continuity,” which is considered by many, to be the starting point for continuum damage mechanics. However, it was not until 1972 that the term Continuum Damage Mechanics was first used by J. Hult. J. Lemaitre has been regarded as one of the most distinguished representatives of damage mechanics [2]. According to Lemaitre, damage can be viewed in a couple different ways. Conceptually, “damage is the deterioration which occurs in metals prior to failure” [2]. “Damage, in its mechanical sense in solid materials
is the creation and growth of microvoids or microcracks which are discontinuities in a medium considered as continuous at a larger scale” [3].

The objective of this thesis is to investigate damage initiation and propagation in the T-38 aileron lever and perform a fatigue life analysis in comparison to the study performed by Southwest Research Institute (SwRI) using a continuum damage mechanics approach. This approach models damage initiation and propagation explicitly and provides and identifies crack initiation sites, initiation life and propagation rate and path as opposed to the fracture mechanics approach used by SwRI. One of the shortcomings of the fracture mechanics based approach is the assumptions of crack initiation sites as well as initial crack lengths must be made. In this study, ANSYS software was used to conduct the modeling and simulation of the crack initiation and propagation. The ANSYS code was written in such a way that it would conduct the modeling automatically for many runs and predict the location of the cracks and the number of cycles to grow a crack to a critical length. Based on the findings, recommendations are made as to the number of flight hours between replacements of the aileron levers or inspection intervals and crack locations and expected crack lengths after a particular number of hours. This study will provide a base for maintenance personnel to conduct inspections in places that are more likely to develop cracks.

1.2 Introduction to the T-38

The first flight of the T-38 aircraft occurred on April 10, 1959. Delivery of 1,187 T-38 aircrafts to the United States Air Force occurred from 1961 to 1972. It was the world’s first supersonic trainer. Currently over 500 remain in use to date. The Air Force
utilizes the T-38 in various training scenarios due to the versatility of the aircraft. Air Education and Training Command (AETC) utilize the T-38 for their Joint Specialized Undergraduate Pilot Training (JSUPT), the Air Combat Command (ACC) use the aircraft for its Introduction to Fighter Fundamentals (IFF), Air Force Material Command (AFMC) utilizes it by testing experimental equipment, and the National Aeronautics and Space Administration (NASA) uses it as a trainer for astronauts as well as a chase plane for programs such as the space shuttle.

Currently programs are underway to extend the service life of the T-38 out to 2020 and possibly further. All USAF T-38 models are receiving an avionics and propulsion upgrade along with new structural elements.

1.3 Flight Control System and Function of Aileron Servo Valve Lever

The T-38 is controlled by two separate flight systems, the primary and secondary systems. The primary flight controls consist of the aileron, rudder and horizontal stabilizer systems. Each surface may be controlled from either cockpit. Movement of the control stick or rudder pedals actuates a closed cable system that operates the servo valves for each surface. Since each system is controlled hydraulically, air loads on the control surfaces will not be perceived by the pilots thus artificial feel springs have been included into the system.

The ailerons are located on the outboard trailing edge of the wing and control the roll axis of the aircraft. The ailerons are powered hydraulically by the utility and flight control hydraulic systems and are actuated by dual actuating cylinders in each wing forward of the aileron. These actuators contain two hydraulic cylinders and a dual-
control servo valve. The servo valves direct hydraulic pressure to the cylinders of the actuators to deflect the ailerons. The movement of the servo valves is controlled from a conventional control stick in each cockpit.

When either control stick is moved to the right or left, a mechanical linkage moves the cable quadrant under the control stick. The cable system transmits this movement from the cable quadrants aft along the bottom of the fuselage, down the leading edge of each wing to the aileron actuator quadrant forward of each aileron surface. An overload-relief spring in the interconnecting quadrant assembly under the rear cockpit prevents cable loads from overloading or damaging pulley brackets and operating mechanisms in each wing. Movement of the cable system positions the quadrants of the aileron actuators through mechanical linkage; the quadrants operate the dual servo valves of the aileron actuators to open the pressure and return ports between the valve and cylinders to produce aileron surface movement proportional to control stick position. As the ailerons reach the desired travel, the servo valves return to neutral and hydraulic pressure is again applied to both sides of the actuating cylinder pistons.

The aileron lever of the T-38 is a Critical Single Point Failure Item (CSPFI) that connects directly to a servo valve that controls the movement of the aileron surface. Originally produced from a forging of 2014-T6 Aluminum, the aileron servo valve lever was designed to accommodate a 208 pound static load. The term CSPFI signifies that the component is so critical to the safety of the aircraft, that if it were to fail, total loss of the aircraft would occur.
CHAPTER 2
FATIGUE

2.1 History of Fatigue and Fracture

During his remarkable life, Leonardo da Vinci performed various experiments; some of which consisted of measuring the strength of iron wires. He found that the strength of the wire varied inversely with the length of the wire. These results implied that flaws in the material controlled the strength of the wire (i.e. a longer wire had a higher probability of containing a flaw decreasing the wire strength) [1].

Galileo wrote Two New Sciences [1638] during his time in seclusion, where he describes the results of his earlier studies on the strength of materials. He introduced the concept of tensile strength, which he referred to as “absolute resistance to fracture.” In his observations, he noted that the strength of a bar was proportional to the cross-sectional area and is independent of the length [1].

In the 19th century, a major shift in the theory of strength of materials occurred with the introduction of malleable iron as the primary construction material. With this new type of building material came a new type of failure behavior which now had to be accounted for: fatigue. The failure theory of the day was that the cyclic stresses caused the tough, fibrous, quality of the iron to turn into a brittle, crystalline material. This perception was based on the surface of a material that has failed due to fatigue. When casually observed, a fatigue crack will have a smooth flat region followed by a rough irregular unstable growth region [1].
The earliest recorded inquiry of slow growth fatigue was discussed in 1843 by William John Macquorn Rankine. Rankine was trained as a civil engineer under Sir John Benjamin MacNeill. His scientific work on fatigue in metals of railway axles, led to new methods of construction [4]. However, it was not until 1858 that the first full study of fatigue testing on railroad axles occurred when A. Z. Wöhler built a machine capable of performing cyclic tests on the axles. His study was carried out from 1858-1870 and was eventually published in Zeitschrift für Bauwesen [5].

In 1913 C. E. Inglis, published his work concerning the stress on an elliptical hole in a glass plate. He found that as the hole became longer and thinner, that pulling the plate in the plane perpendicular to the hole caused the stress at the tip of the ellipse to increase significantly. From this and several other observations, he recognized that it was the length and radius of curvature at the tip of the hole that mattered most in the cracking of the plate [6]. Inglis proposed the stress at the tip of an elliptical shape hole could be calculated by equation (2-1):

\[ \sigma_A = 2\sigma \frac{a}{\sqrt{\rho}} \]  

(2-1)

where \( a \) is half the length of the crack and \( \rho \) is the radius of curvature at the tip.

A. A. Griffith began his studies of fracture just prior to 1920. He was aware of Inglis’ prior work calculating the stress concentrations around elliptical holes. He found that Inglis’ solution posed a mathematical difficulty: in the event of a perfectly sharp crack, \( \rho = 0 \), the stress around the crack tip would approach infinity resulting in a near-zero strength regardless of the material. Physically this is not the case. At the tip of the
crack, local yielding takes place to “blunt” the cracktip. Instead of focusing on the stresses at the cracktip, Griffiths employed an energy-balance approach [6]. The Griffith equation can be seen in equation (2-2) where \( \sigma_f \) is the stress at fracture, \( E \) is the modulus of elasticity, \( a \) is the crack length, and \( \gamma_s \) is the surface energy per unit area.

\[
\sigma_f = \left( \frac{2\gamma_s}{\pi a} \right)^{\frac{1}{2}}
\]  

(2-2)

In the early part of the 20\(^{th}\) century, ductile materials were beginning to replace the more brittle iron materials. Nonetheless, failures due to the growth of cracks were still occurring and were attributed to design flaws and not flaws in the materials. The solution in most cases was to add more material. According to A. E. H. Love (1926), his Treatise on the Mathematical Theory of Elasticity, describes safety factors ranging anywhere from 6 for boilers and axles, 6-10 for railway bridges, and 12 for propeller shafts, relative to tensile strength. During World War II, the rise in the use of aircrafts with their increased strength-to-weight requirements, forced out the old method of design and gave way to a new, more efficient design based on a more realistic theory of failure [1].

In an effort to replace the large number of ships lost to the German U-boats, the U.S. and England embarked on a radically new technique in shipbuilding. In a period of 4 years, 1940-1944, 2,708 Liberty ships were constructed relying heavily on welding rather than riveting to assemble the ships. But with the new technology came a new problem: Liberty ships had a tendency to crack in cold weather and rough seas. The
causes of these failures can readily be explained with modern day principles of fracture mechanics.

1. At the time of the Liberty ships, the composition of the steel was such that the transition from ductile to brittle behavior of the metal occurred at the temperatures that the ships were experiencing in the North Atlantic.

2. The design of the ships called for hatch openings with square corners. These corners acted like starter cracks.

The de Haviland “Comet” commercial aircraft was first manufactured in 1952. It was the first twin-jet-engine passenger aircraft to fly at 40,000 ft. with a pressurized cabin. After about a year in service, three aircraft failed, with considerable loss of life. The origin of the failure was identified as a short fatigue crack that started from an overhead observation window causing the fuselage to burst.

Fracture tests performed in the United Kingdom on similar panels of the same aluminum alloy and fatigue cracks comparable to those found on the recovered aircraft failed to correlate with the stress levels experienced in service. Irwin and others at the Naval Research Laboratory argued that the effective crack length should include the diameter of the window. Using the larger value resulted in critical stress levels that accounted for the failures [1].

2.2 Understanding of Classical Fracture Mechanics

Since its inception in the early 1920’s, classical fracture mechanics has been the primary source in performing fatigue analyses and designing structures against failure in the field of engineering. The discipline of fracture mechanics has evolved into an
indispensable tool to design engineers. Through the course of its development, fracture mechanics has shown that three factors control the susceptibility of a structure to fail due to brittle fracture. These factors are: fracture toughness \( (K_c, K_{ic}, K_{Id}) \), crack size \( (a) \), and the stress level \( (\sigma) \) on the material. The theory behind fracture mechanics is; by knowing and understanding these factors, brittle fracture can be predicted and thus designed against.

2.2.1 Classical Fracture Mechanics

Classical fracture mechanics divides the crack propagation into two groups, static loading and cyclic loading. The most widely accepted model used to predict the crack growth rate under cyclic loading in materials is known as the power or Paris law. The Paris Law predicts the change in the length of a crack per cycle as seen in equation (2-3):

\[
\frac{da}{dN} = C(\Delta K)^n
\]

where \( C \) and \( n \) are material constants and \( \Delta K \) is the stress intensity range. This law requires strict adherence to “ideal” conditions of small-scale yielding, constant amplitude loading, and long cracks. Numerous modifications to this equation have been performed in an effort to suit any departure from these “ideal” conditions [7]. Studies have shown that the growth characteristics for small fatigue cracks are very much different than the characteristics for large cracks in the same material. Tests have shown that small cracks grow much faster than large cracks at the same \( \Delta K \) threshold. It has also been observed that small cracks grow at a \( \Delta K \) threshold lower than that required for large cracks to grow [8-12].
Fracture toughness, $K_c$, is defined as “…the measure of a material’s resistance to brittle fracture when a crack is present” [13]. In most cases, the fracture toughness will be written as, plane strain fracture toughness, $K_{IC}$, where the Roman numeral I represents mode I crack displacement. Figure (2-1) shows modes I, II, and III, crack displacement.

Strain rate is a major factor in the material property $K_{IC}$. $K_{IC}$ testing is conducted at “slow” loading rates, generally within the range of 30-150 ksi $\sqrt{\text{in/min}}$. This is done because some materials are strain-rate sensitive, resulting in a different fracture toughness at faster loading rates. The fact is that the fracture toughness of a material can decrease significantly with an increasing loading rate [14]. The magnitude of $K_{IC}$ diminishes with increasing strain rate; however the magnitude of $K_{IC}$ will increase with reduction in grain size while composition and other microstructural variables are maintained [13].

Fatigue of a material occurs due to a repeated cyclic loading. The stress level does not necessarily have to be higher than the yield stress of the material. If the peak load is higher than the yield stress and causes some plastic deformation, then the fatigue is categorized as low cycle fatigue. When stress levels are lower than the yield stress of the material, this condition leads to high-cycle fatigue. Either loading causes deterioration of the material through micro-cracks/micro-voids that coalesce and propagate through the material. Fatigue is usually represented by S-N curves. S-N curves are generated using experimental data on samples of various materials. Cyclic testing is performed on the samples using constant stress amplitude as calculated by equation (2-4). Depending on the stress amplitude, the number of cycles to failure for individual samples will vary.
\[
\sigma_u = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}
\]  

(2-4)

Figures 2-2 and 2-3 show typical S-N curves for various materials. As indicated on figure 2-2, the middle curve is for 2014-T6 aluminum. Figure 2-3 shows the curve for 7050-T74 aluminum.

Figure 2-1: Modes of crack tip displacement [6].

Figure 2-2: S-N curve for 2014-T6 aluminum alloy [13].
Fatigue is one of various forms of failure and occurs in structures that are subjected to fluctuating stresses. These stresses make it possible for a structure to fail at a stress level significantly lower than the tensile or yield strength of the material. Fatigue is the single largest cause of failure in metals and is catastrophic due to the fact it occurs suddenly and without warning. For materials such as aluminum, the S-N curve continues its downward trend with increasing $N$. Accordingly, fatigue will ultimately occur regardless of the magnitude of the stress [13].

Why do structures fail? Two reasons exist as to why structural failures occur.

1. Negligence during design, construction, or operation of the structure.
2. Application of a new design or material, which produces an unexpected (and undesirable) result.

In the first instance, existing procedures are sufficient to avoid failure, but are not followed by one or more of the parties involved, due to human error, ignorance, or willful misconduct. Poor workmanship, inappropriate or substandard materials, errors in stress
analysis, and operator error are examples of where the appropriate technology and experience are available, but not applied [16].

The second type of failure is much more difficult to prevent. When an “improved” design is introduced, invariably, there are factors that the designer does not anticipate. New materials can offer tremendous advantages, but also potential problems. Consequently, a new design or material should be placed into service only after extensive testing and analysis. Such an approach will reduce the frequency of failures, but not eliminate them entirely; there may be important factors that are overlooked during testing and analysis [13].

When applied correctly, both fracture and continuum damage mechanic approaches not only help prevent Type 1 failures, but also reduce the frequency of Type 2 failures because designers can rely on rational analysis rather than trial and error [13].

One area that seems to be relatively insufficient when dealing with the field of fracture mechanics is fatigue crack initiation.

2.2.2 Literature Reviews

Fracture mechanics has long been the tool for both designing against and analyzing catastrophic failures. Numerous papers have been written on the subject. In an effort to better understand the science of fracture mechanics, several papers were reviewed relating to the topic and are presented here.

The ability to predict the service life of a material is extremely complicated. In general, most fatigue strengths of materials are measured using constant amplitude tests when in reality an alloy will most likely be subjected to random loading. Various
researchers have shown that the use of currently available fatigue life models, tend to predict the lives of samples higher than the experimental fatigue lives for these same specimens [17]. An ASTM Task Group (NASA Langley Research Center) evaluated several different methods for predicting crack growth under random loading and determined that the Root Mean Square (RMS) method showed good results and carried insignificant computational costs [17]. RMS “is a statistical measure of the magnitude of a varying quantity” [18].

Kim et al. [17] presented the concept of using RMS values in Forman’s equation to predict the fatigue life of a high-strength aluminum alloy 7475-T7351. Forman’s equation was developed by Royce Forman while studying the crack growth and the instability of cracks in Vietnam War aircraft while working at Wright-Patterson Air Force Base in Ohio [19]. The RMS model along with the loading history of several specimens was used to determine the maximum and minimum stresses. All tests were conducted with the assumption that testing was performed under constant amplitude loading using the maximum and minimum RMS values. Combination of the RMS values and Forman’s equation resulted in equation (2-5).

\[
\frac{da}{dN} = \frac{C\Delta K_{rms}^n}{(1 - R_{rms})K_c - \Delta K_{rms}}
\]  

(2-5)

\(C\) and \(n\) are empirical fatigue constants of the material, \(\Delta K_{rms}\) is the fracture toughness range, \(K_c\) is the applicable fracture toughness, and \(R_{rms}\) is the stress ratio. Solving for \(dN\) and integrating equation (2-5) results in the number of cycles to failure \(N_f\) as seen in
equation (2-6) where $c_f$ and $c_i$ represent the final crack length and the initial crack length respectively.

$$N_{\text{pred}} = \int_{a_i}^{a_f} \frac{(1 - R_{\text{rms}})K_c - \Delta K_{\text{rms}}}{C\Delta K_{\text{rms}}} = \frac{(1 - R_{\text{rms}})K_c - \Delta K_{\text{rms}}}{C\Delta K_{\text{rms}}} (c_f - c_i)$$  \hspace{1cm} (2-6)

During the testing, all test specimens were fabricated from aluminum plate with surface cracks. Pre-cracking was performed under constant-amplitude loading. Prior to each test, the life of the specimen was calculated. A ratio was calculated at the conclusion of each test as $N_{\text{pred}}/N_{\text{Test}}$. This ratio was a way for Kim et al. to see how their predictions did versus the actual test data. Anything less than one was considered conservative and anything over one was considered non-conservative. The calculated ratios ranged from 3.22 to 1.52 which indicates the predicted values for the fatigue lives of the specimens were larger than the actual test values.

Later use of Forman’s equation in the RMS model showed an increase in accuracy for fatigue life predictions. Kim et al. proposed using the average crack size instead of integrating Forman’s equation from initial crack size to final crack size as seen in equation (2-7).

$$N_{\text{pred}} = \frac{(1 - R_{\text{rms}})K_c - \Delta K_{\text{rms}}}{C\Delta K_{\text{rms}}} \cdot c_{\text{avg}}$$  \hspace{1cm} (2-7)

$C$ and $n$ are empirical fatigue constants of the material, $\Delta K_{\text{rms}}$ is the fracture toughness range, $K_c$ is the applicable fracture toughness, and $R_{\text{rms}}$ is the stress ratio. The only difference from equation (2-6) is the use of the average crack length, $c_{\text{avg}}$. This
suggestion was shown to be more accurate by calculating the fatigue life ratios between 1.35 and 0.62 which ultimately reduced the errors from 222% to 38%.

Fatigue failure in metals has generally been divided into three separate phases consisting of crack initiation, crack growth and fracture. The growth of large cracks has long been characterized through the use of fracture mechanics. It hasn’t been until the past couple of decades that focus has shifted to the study of smaller cracks and their behavior with respect to linear-elastic fracture mechanics [20].

Numerous studies have shown that crack behavior differs in small and large cracks [8-11, 20]. Experimental data is available that shows for cracks that are less than 1mm in length, Linear Elastic Fracture Mechanics (LEFM) concepts will break down [20]. Wu et al. [20] performed several experiments on two high strength aluminum alloys, 7075-T6 bare and LC9cs clad, commonly used on aircraft structures. The two main objectives of the testing were to: (1) obtain crack length against cycle data; and (2) obtain crack shape information.

Experimentation showed that for both aluminum alloys, the initiation life of the cracks in the Single Edge Notched Tensile specimens (SENT) was no more than 10% of the fatigue life of the material. That means that over 80% of the fatigue life was due to crack growth and propagation.

This study was one example of the classical small crack effect, where small cracks grow much faster at the same stress intensity range than large cracks. It was also shown in the study that the small crack effect is much more pronounced at negative stress ratios. Stress ratios are defined by equation (2-8).
\[ R = \frac{S_{\text{min}}}{S_{\text{max}}} \]  

(2-8)

\( S_{\text{min}} \) and \( S_{\text{max}} \) are minimum and maximum stresses respectively. Negative stress ratios will occur when a compression-tension test is applied to a specimen.

Newman et al. used a continuum mechanics concept to predict fatigue life based solely on crack growth from an assumed initial material defect. Newman et al. saw \( \Delta K \)-based analyses being used for large crack growth problems and felt that a similar \( \Delta K \)-based analysis for small crack growth problems would be beneficial.

Because small cracks behave differently from large cracks, Newman et al. [11] investigated crack initiation and the growth of small cracks. The study performed by Newman et al. was a continuation of the study performed by Wu et al. Newman et al. used the analytical crack closure model, FASTRAN, to correlate large-crack growth rate data and develop a baseline effective stress intensity factor range.

The FASTRAN model was developed for use by NASA similar to the AFGROW software which was developed for use by the Air Force for predicting crack growth of various materials. Newman et al. used the FASTRAN model to calculate crack opening stresses under constant-amplitude loading to demonstrate the influence of the initial defect size on crack closure behavior for small cracks emanating from these voids. Studies have indicated that small-crack effects are more pronounced at negative stress ratios and under plane stress conditions where the stress is increased by a factor of 3 [11].

The model was found to be quite successful in predicting growth rate trends for LC9cs clad alloys. Growth rate trends were not as successful for the Al 7075-T6. A
crack closure model was also used to predict the fatigue life of both the Al 7075-T6 and LC9cs clad alloys. In this case, it was found that the measured and predicted fatigue lives agreed quite well.

Huynh et al. [21] investigated the affect of the stress concentration factor ($K_t$) from which fatigue cracking is initiated. The purpose of their research was to develop a crack growth model using both high and low $K_t$ data. Testing of the coupons was done using the variable amplitude loading spectrum representative of the F/A-18 aircraft fleet wing root bending moment loading.

The loading spectrum came from a full-scale fatigue article, but was modified by adding five compressive marker-loads just prior to one of the highest loads and between four preceding tensile loads. The spectrum was equivalent to 324.92 flight hours and 13,475 turning points. All fatigue cracks were initiated naturally, meaning no artificial crack starters were induced. All cracks initiated from the material surface.

The effective block approach (EBA) was used in the study to predict the crack growth using variable amplitude loading. The EBA allows for any fatigue mechanism that influences the cycle-by-cycle crack growth (spectrum effects, retardation, closure mechanism, etc.). The empirical block-by-block variable amplitude model or EBA model can be seen in equation (2-9).

$$\frac{da}{dt} = \Lambda \alpha^j \left(\sigma_{ref}\right)^k$$

(2-9)

There are several forms that can represent equation (2-9) depending on assumptions about the crack growth mechanism. Huynh et al. explored two different
models the first of which is based on the Paris model. The Paris-based crack growth model can be seen in equation (2-10).

\[
\frac{da}{dt} = C(K_{ref})^m
\]  

(2-10)

As expected from the results of the testing, the stress concentration factors play a large role in the fatigue life of a material. The higher the \( K_t \), the lower the stress needed to reach the same fatigue life of the same material with a lower \( K_t \) and higher stress. During their research, Huynh et al. noticed that the Paris-Law is stress dependant for variable amplitude loading. The higher the applied stress, the higher the Paris constant \( C \) will be. They also showed that fatigue crack growth history follows an exponential relationship which is dependent on crack depth and is a function of stress-cubed for lower \( K_t \) values [21].

Using the data collected from the lower \( K_t \) testing and correlating that with the data collected from the higher \( K_t \) testing, a final model was developed using the Paris-Law along with the Frost and Dugdale model to predict the crack growth rate of a \( K_t \) value somewhere between the \( K_{t(low)} \) and \( K_{t(high)} \) values [21]. The final model was found to be quite successful for predicting the crack growth rate of coupons with a stress concentration value in the range between the high and low values used to formulate the model.

Research was performed by D. L. McDowell [9] to correlate the cyclic crack growth rate, \( da/dN \), and the stress intensity factor \( \Delta K \). His research showed that for a given \( \Delta K \), the \( da/dN \) is higher for small cracks than for long cracks. Microstructurally
small cracks generally range on a length scale of 5-10 times the size of the grain diameter where as long cracks generally range between 10-20 times the grain size. As the smaller crack lengths propagate, the $da/dN$ vs. $\Delta K$ curve eventually merges with the long crack response. As the stress amplitude decreases, $da/dN$ becomes more dependent on $\Delta K$. At low stress amplitudes $da/dN$ may decrease with $\Delta K$, and then accelerate prior to merging with the long crack data. And at extremely low stress amplitudes, $da/dN$ may become a non-propagating crack.

Small crack propagation depends significantly on the R-ratio ($\sigma_{\min}/\sigma_{\max}$) and the stress amplitude. During his observations, McDowell [9] discovered that higher stress amplitudes lead to a more constant value for $da/dN$ at a given $\Delta K$ prior to the small crack data merging with the large crack data.

Initiation approaches have been used for a long time (i.e. strain-life, stress-life). These models generally preassign crack lengths of 1mm in length and later correlate this dimension with stress or strain amplitude. The problem with these approaches is that they don’t take into account any kind of crack evolution. What they need to do is allow for some kind of cumulative damage [9].

The energy criterion first proposed by Griffith, is considered one of the pioneering criteria in fracture mechanics [22]. “Basically, the energy criterion is thought of as the balance between the elastic strain energy released and the increase in surface energy as the length of a crack increases in the case of brittle failure, or the work of plastic deformation in the case of quasi-brittle failure” [22:799]. Khoroshun [22] decided to investigate the energy criterion as proposed by Griffith. According to Griffith, the
elastic energy released during the brittle failure of a material is expended in the formation of a new surface and is equal to the increase in surface energy [22].

During his investigation into the energy criterion proposed by Griffith, Khoroshun [22] came to the conclusion that the energy criterion of failure based on the notion of a balance between energy released and the increase in surface energy or plastic deformation as a crack propagates is incorrect for two reasons. “Firstly, it is impossible to obtain a critical-load value as a result of the unboundedness of the maximum stress at which the material begins to fail near the crack tip from the condition whereby the energy released and the work of failure are balanced. Secondly, the notion concerning a balance between the elastic energy released and the work of failure have no real basis for both brittle and quasi-brittle failure” [22:804-805]. These two findings cast a shadow of doubt on a basic position of fracture mechanics.

2.3 Continuum Damage Mechanics

Continuum damage mechanics is a relatively new development in solid mechanics. It deals with the distribution, characterization and growth of microstructural defects in terms of macroscopic state variables. Physically, the continuum damage mechanics concept represents the loss of material integrity thus reducing the ability of the structure to bear applied stresses. Continuum damage mechanics promotes the concept of distributed damage over a continuum solid, such as micro-voids, micro-cracks and defects, generated by material deformation during monotonic or cyclic loading.
2.3.1 Theory of Continuum Damage

Cyclic fatigue in the presence of irreversible plastic deformation poses many complexities such as microstructural evolution, presence of micro defects (dislocations), micro-voids, impurities, and creep fatigue interactions etc that restrict the use of conventional S-N curves. Efforts have been limited to generating models that can predict the number of cycles to failure or rupture for certain materials and many models have been proposed. These models often suffer from limiting characteristics such as loading type, test condition, microstructural state, specimen scale and many others. These aspects of continuum damage and fatigue behavior have been investigated by many scientists and researchers [12, 23-38] but these studies also show that the community still lacks a comprehensive satisfactory approach with general applicability.

Some researchers such as Kachanov, not only were interested in fatigue failure, but also investigated the state of damage in materials and tried to find a relationship between the state of damage and change in material properties. Effective stress and strain equivalence concepts introduced by Kachanov [25], are concepts in the continuum damage mechanics field that have been recognized and applied by different researchers to evaluate the state of damage in materials. Chaboche was one of the leading researches who started analyzing damage utilizing mechanistic approaches that employed the effective stress concept. He used the Kachanov’s concept of effective stress to develop an anisotropic damage evolution model using evolution of scalar damage and generalized Kachanov-Robotnovs [39, 23, 40] equation to 3-D [41].
Thermodynamic approaches were another class of techniques that have been used by many researchers [35-38, 40-47]. These approaches usually rely on the concept of internal state variables and the evolution of the variables as damage evolves in materials.

If the damage is considered isotropic and homogeneous a scalar quantity can represent the damage in the material. The damage variable can then be defined as seen in equation (2-11) [39].

\[ D = \frac{A}{A_0} \]  

(2-11)

\( D \) is a positive, monotonically increasing function. \( A \) is the lost area due to damage and \( A_0 \) is the original area. Kachanov introduced a field variable \( \psi \), called continuity which is considered by many as the starting point of continuum damage mechanics (CDM). Over the years the variable \( D = (1-\psi) \) has become accepted as the representative for damage. \( D = 0 \) identifies the undamaged state and \( D = 1 \) is used to identify total failure. None of these two states happen in reality. Most materials and structures have initial micro-cracks and flaws and they always fail before \( D \) ever reaches 1. If we assume that \( \sigma \) is the stress related to the undamaged material or nominal stress, then an ‘actual’ stress is defined by equation (2-12).

\[ \sigma_a = \frac{P}{(A_0 - A)} = \frac{P}{A_0(1 - D)} = \frac{P}{A_0 \psi} = \frac{\sigma}{\psi} \]  

(2-12)

The stress-strain behavior of the damaged material can be represented by the constitutive equation of the undamaged material with the stress in it replaced by this
‘actual’ stress. In general the point of interest is not the state of damage, but the number of cycles that a structure survives.

2.3.2 Literature Reviews

For the past several decades, various damage models derived from CDM center on the micro void/crack development and provide an understanding of the mechanics of fracture in structures by means of damage variables which represent the deterioration of a material element [48]. Lemaitre and Chaboche both tried to explain fatigue damage using continuum damage mechanics.

Jean Lemaitre has been regarded as one of the most distinguished representatives of CDM [2]. Introduced by Kachanov in 1958, continuous damage mechanics was originally used to model creep rupture, but has since been developed for use in modeling low cycle fatigue, high cycle fatigue, coupling between damage and cyclic creep, and creep/fatigue interaction [49]. Lemaitre also indicated that this area of solid mechanics is based on metallurgy and provides a better understanding of rupture in structures by defining a variable that represents the deterioration of the material prior to the initiation of a macrocrack [3].

The damage process is governed by two main events. Elastic damage corresponds to the portion of damage when the applied stress is less than the yield stress. Plastic damage corresponds to the portion of damage where the applied stress is larger than the yield stress. In low-cycle fatigue, the total damage is the summation of the elastic and plastic damages [49].
The researcher Jean-Louis Chaboche has been the author of numerous articles dealing with the continuous damage approach. A select few [50-52] have been considered for this literature review.

Chaboche [50] described CDM as a tool to describe phenomena before crack initiation. According to Chaboche, CDM is based on the framework of irreversible processes and offers complementary possibilities to Linear Elastic Fracture Mechanics (LEFM) [50]. The final state of CDM generally corresponds to the presence of a material discontinuity, crack initiation, which is sufficiently large as compared to the grains and subgrains. The theory of CDM is supported by the physical idea that prior to crack initiation, a progressive internal deterioration of the material occurs [50]. There are several methods that can be used when measuring the damage sustained during loading. Along with density changes, or electric resistivity measurements, there are measures of remaining life which are typically used in creep and fatigue, measures in the reduction of fatigue limits which require numerous tests to define damage evolution curves, and measures of the stress-strain behavior.

In a two part article [51, 52], Chaboche presented the general concepts of CDM, which are; damage growth, crack initiation, and crack growth. According to Chaboche [51], “the damaging process corresponds to localizations and accumulations of the strains and are considerably more irreversible” (p. 59). Defects in materials lead to progressive material deterioration (material damage), crack initiation, and finally fracture. These effects can be measured through the decrease in stiffness, the toughness, the strength, and the residual life of the material after damage has accumulated.
Crack initiation is defined by Chaboche [51] as the “breaking up” of a continuum volume element. Just like fracture mechanics, the objective of damage theory is to predict the life of a structure. Being able to accurately predict the life of a structure is of utmost importance for design engineers.

A major concept that comes from damage theory is that of remaining life. Remaining life is best described as the ratio of $N/N_f$ where $N$ and $N_f$ represent the current number of cycles already applied and the number of cycles to failure respectively. The concept of remaining life can best be illustrated in equation (2-13) where $D$ is the damage after the initial damaging process. The damage constant $D$ will be equal to 0 for the undamaged material and 1 at the time of rupture. After the initial period, the damage accrued is seen in equation (2-13).

$$D = \frac{N_1}{N_{f1}} \quad (2-13)$$

$N_1$ is the number of cycles that has been applied to the material at a stress amplitude $\sigma_a$ and $N_{f1}$ is the number of cycles to failure at the same stress amplitude. The remaining life of the material after the initial damage period is seen in equation (2-14).

$$\frac{N_2}{N_{f2}} = 1 - D \quad (2-14)$$

Fatigue does not have a unique damage evolution curve as a function of the life ratio $N/N_f$. Instead, it is dependant on the applied load.
Both Lemaitre [49] and Chaboche [52] described damage measures using the effective stress concept. The notion behind the effective stress is that a damaged material under an applied stress $\sigma$ shows the same strain response as the undamaged material under the effective stress. Using damage $D$ to represent the loss of effective area, the effective stress can be seen in equation (2-15).

$$\tilde{\sigma} = \frac{\sigma}{1 - D}$$  \hspace{1cm} (2-15)

Lemaitre takes the damage variable $D$ one step further by applying the effective stress concept to the elasticity modulus. Using Young’s modulus $E$, the elasticity modulus of the damaged material could be considered as:

$$\tilde{E} = E(1 - D)$$  \hspace{1cm} (2-16)

With the variable $E$ being known, $\tilde{E}$ can be measured through tension tests. However precautions must be taken to measure the damage. Since ductile plastic damage begins when necking starts, changes in the geometry of the specimen occur rapidly. In order to measure this rapid change, extremely small strain gages on the order of .5 X .5 mm must be used in the area where the most damage is occurring. The most accurate method found by Lemaitre for measuring $\tilde{E}$ is during the unloading phase. According to Lemaitre, if both the use of strain gages in the damaged area along with measuring during the unloading phase, an accuracy of 5% can be expected for the damage variable $D$ [49].

Chaboche [52] outlined life prediction in structures. Life predictions incorporate two aspects that are to be treated independently or successively. These aspects are: (a)
the macrocrack, which is important in design methodologies, and (b) the crack propagation which is used for the “damage tolerance” concept. Chaboche realized that crack growth laws and parameters used by fracture mechanic concepts are practical tools to predict crack growth in “small scale yielding” materials. However fracture mechanic approaches present difficulties when dealing with material nonlinearity such as ductile fracture. When using linear fracture mechanics, the correlation between $K$ and the crack growth rate are no longer a one-to-one correlation. Non-linear fracture mechanics introduce the parameters $J$ and $\Delta J$ for ductile rupture and low-cycle fatigue respectively. Nonetheless these parameters are justified for special inelastic behavior.

Chaboche [52] discussed an alternative method referred to as “local approaches.” These approaches take into consideration the actual behavior at the crack tip in an effort to calculate as accurately as possible, the stress, strain, and subsequent deterioration. CDM treats the damaged zone ahead of the crack tip as a group of material points where the damage has reached its critical value and the points no longer possess any rigidity. However there are a few problems that inhibit the general use of these techniques. The cost of calculations tend to be higher using CDM techniques unless an inelastic fracture mechanics analysis has to be performed, and the dependence on the finite element modeling. “Crack width,” crack growth rate, and the failure load are characteristically dependent on the chosen mesh size.

CDM has been shown to be an exceptionally useful tool, however under multiaxial or more complex loading, problems with these models have been discovered [50].
2.3.3 Damage Models

Several models have been proposed to determine the number of cycles to failure based on stress, strain or energy. A summary of these methods is provided below.

2.3.3.1 Stress-Based Approaches

The stress-based approach to life prediction is the oldest method used in fatigue modeling [53]. In this method the fatigue life of a material is expressed as a function of a strength parameter. Basquin proposed equation (2-17) in which, \( \sigma_a \) is the stress amplitude, \( \sigma_f' \) is called the fatigue strength coefficient and \( b \) is the fatigue strength exponent.

\[
\sigma_a = \sigma_f' (2N_f)^b
\]  

(2-17)

2.3.3.2 Strain-Based Approaches

The strain based approach to fatigue modeling is one of the most widely used approaches for predicting the life of a material and is especially useful in the case of low cycle fatigue. Therefore low cycle fatigue could be considered a strain control phenomena. If only plastic strain is considered, the function that correlates the number of cycles to plastic strain is called the Coffin–Manson [54, 55] relation and is as follows in equation (2-18).
The Coffin-Manson equation shows good correlation with experiment. However, in real life environments, the loading is not always constant amplitude sinusoidal. The total life could be broken into subsets of cycles with different amplitudes, means and frequencies. The Coffin-Manson equation was modified to include the effect of frequency and is available in [57].

Because there is no practical method to separate plastic shear strain from total shear strain during typical accelerated testing, Engelmaier [58] proposed a new formula, based on the Coffin-Manson equation using total shear strain rather than plastic shear strain, with equation (2-20).

\[
N_f = \frac{1}{2} \left( \frac{\Delta \gamma}{2 \epsilon_f'} \right)^{1/c} 
\]  

(2-20)

\( \epsilon_f' \) is the fatigue ductility coefficient, \( N_f \) is mean cycles to failure, and \( c \) is the fatigue ductility exponent. The fatigue ductility exponent includes effects of both temperature
and frequency. A Linear temperature correlation and a logarithmic frequency correlation have seemed to describe the relation best.

Halford et al. [59] were motivated to develop a more sophisticated strain based approach due to various shortcomings of available approaches. Their model is called the strain-range partitioning approach. In this model the total inelastic strain is broken into two parts consisting of plastic strain and creep strain components. In the case of axial tension and compression loading, the two possible inelastic components allow for a maximum of four permutations in basic cycle types: pp (plastic in tension and compression), cp (creep in tension and plastic in compression), pc (plastic in strain tension and creep in compression), cc (creep in tension and compression). To apply the strain range partitioning method, an interactive damage rule is used that relates the four separate strain ranges to life relationships as seen in equation (2-21).

\[
\frac{1}{N_f} = \frac{F_{pp}}{N_{pp}} + \frac{F_{cc}}{N_{cc}} + \frac{F_{cp}}{N_{cp}} + \frac{F_{pc}}{N_{pc}}
\]  

\(N_f\) is the predicted cycles to failure for the given complex hysteresis loop, \(N_{ij}\) is cycles to failure for a given partitioned strain range of type \(ij\) (pp, cc, pc, or cp), and \(F_{ij}\) is fraction of total inelastic strain range that is actually of type \(ij\). This method has been applied widely for many alloys and often resulted in very good correlation with experimental data. This method also was modified by Solomon [60] for the cyclic frequency of the load. His frequency dependent model showed that a family of parallel Coffin-Manson fatigue curves should exist, one for each frequency.
2.3.3.3 Energy-Based Approaches

Energy based models are the largest group of fatigue models [61]. Cyclic hysteresis energy is believed to be a comprehensive metric of cyclic fatigue damage as it includes both stress and strain hysteresis. Energy based models can be used to predict fatigue failure based on hysteresis loops. These models are divided into two groups; unified and partitioned energy. One of the most widely used models is the Darveaux Model [62] which uses the accumulated inelastic strain energy density per thermal cycle and correlates crack initiation time and crack growth to the average energy as follows in equations (2-22) and (2-23).

\[ N_0 = K_1 (\Delta W_{avg})^{K_2} \]  
(2-22)

\[ \frac{da}{dN} = K_3 (\Delta W_{avg})^{K_4} \]  
(2-23)

\( N_0 \) is the number of cycles to initiation and \( K_1, K_2, K_3, \) and \( K_4 \) are crack constants. \( \Delta W_{avg} \) is the volume-weight average the of total inelastic work density accumulated per thermal cycle. For more details about this method please refer to Darveaux et al. [62].

Another example is Akay’s model which was proposed based on total energy [63]. This model can be seen in equation (2-24). The \( \Delta W_{Total} \) is the total strain energy, \( N_f \) is mean cycles to failure, and \( W_0 \) and \( K \) are fatigue coefficients.

\[ N_f = \left( \frac{\Delta W_{Total}}{W_0} \right)^{\frac{1}{K}} \]  
(2-24)
2.3.3.4 Energy Partitioning Damage Model

Plastic and creep deformation result in different types of material damage as seen in various partitioned damage models such as, Strain Range Partitioning [59]. A mechanism based damage model was proposed by Dasgupta et al. [64]. This model assumes that cyclic fatigue damage is due to a combination of creep deformation mechanisms, plastic deformation mechanisms, and elastic deformation mechanisms. The term “plastic” refers to rate-independent inelastic deformation, while “creep” refers to rate-dependent anelastic and inelastic deformations. This model predicts cyclic creep fatigue damage based on deviatoric energy densities: $U_e$ (elastic), $W_p$ (plastic), and $W_c$ (creep) for a typical load cycle. The damage due to each of these deformation mechanisms is determined by using a power law as provided in equations (2-25) to (2-27).

$$U_e = U_{e0}N_f^b$$  \hspace{1cm} (2-25)

$$W_p = W_{p0}N_{fp}^c$$  \hspace{1cm} (2-26)

$$W_c = W_{c0}N_{fc}^d$$  \hspace{1cm} (2-27)

The total energy is obtained by superposition of these contributions as seen in equation (2-28).
Total Energy = \( U_e + W_p + W_c = U_{e0}N_{fe}^b + W_{p0}N_{fp}^c + W_{c0}N_{fc}^d \)  

\( U_{e0}, W_{p0}, \) and \( W_{c0} \) represent the intercept of the elastic plastic and creep energy density plots versus cycles to failure, on a log-log plot; while the exponents \( b, c, \) and \( d \) are their corresponding slopes. These constants are material properties. The variables \( N_{fe}, N_{fp}, \) and \( N_{fc} \) represent the cycles to failure due to elastic, plastic, and creep damage respectively. Subscripts \( e, p \) and \( c \) refer to elastic, plastic and creep damage, respectively. The total number of cycles to failure \( N_f \) is then calculated from equations (2-29) and (2-30), by estimating the total cyclic damage as a superposition of the three individual damage mechanisms (elastic, plastic and creep).

\[
D_{\text{total}} = D_e + D_p + D_c \tag{2-29}
\]

\[
1/N_f = 1/N_{fe} + 1/N_{fp} + 1/N_{fc} \tag{2-30}
\]

### 2.3.3.5 Energy Partitioning Damage Evolution (EPDE) Model

The idea of the Energy Partitioning Damage Evolution Method, which is more appropriate for thermo-mechanical cycling where you have creep fatigue interactions, was inspired by the work of Wen et al. [65] where damage was considered to have a power law relationship with the number of cycles. The power law relationship of damage with number of cycles was also mentioned by Kachanov long before that. The relationship is defined by equation (2-31).

\[
\left( \frac{D}{D_e} \right) = \left( \frac{w}{w_e} \right)^\eta \tag{2-31}
\]
$D$ is the damage ratio in this study, $D_c$ is the critical damage, $w$ is the portion of grains that are damaged, and $w_c$ is the percolation limit according to the percolation theory [65]. The metric $w$ grows with the cycle number $N$ and micro-cracks percolate within the material until $w = w_c$ at which point the structure becomes unstable or completely damaged.

This relationship is plotted in figure 2-4 for different values of $\eta$. The value of $\eta$ depends on the material and is assumed to be constant for specific materials. The number of cycles is related to $w$ through the following, equation (2-32) [65].

$$\left( \frac{N}{N_f} \right) = \left( \frac{w}{w_c} \right)$$

Wen and Keer used a micromechanics approach based on Mura and Nakasone’s [66] dislocation model to predict the micro-crack density and then calculate $w$. Even though some of creep deformation is caused by dislocation motion (climb), there is also the possibility of diffusion. Mura and Nakasone’s dislocation model is generally used for plastic deformation only. In other words, Wen and Keer failed to include damage caused by creep. There is also another issue with the model where they assumed $\eta = 1$, for linear damage evolution.

Inspired by this model and the point that plastic and creep deformation result in different types of material damage as seen in various partitioned damage models such as, Strain Range Partitioning [59] and Energy Partitioning damage model [64], a mechanism based model was proposed to include both plastic and creep damage in separate terms.
Based on this model Damage is broken into two parts; damage caused by plastic deformations and damage caused by creep deformation as follows from equation (2-33).

\[
\left( \frac{D}{D_c} \right) = \left( \frac{N}{N_{f_p}} \right)^{\eta_p} + \left( \frac{N}{N_{f_c}} \right)^{\eta_c}
\]  

(2-33)

\(D\) is called the damage ratio, \(D_c\) is the critical damage ratio, \(N\) is the number of cycles, \(N_{f_p}\) is the number of cycles to failure for plastic damage, \(N_{f_c}\) is the number of cycles to failure for creep damage, \(\eta_p\) and \(\eta_c\) are damage exponents for plastic and creep that can be obtained by fitting experimental data. \(N_{f_p}\) and \(N_{f_c}\) can be calculated using the Energy Partitioning Damage model as it was explained in former sections according to the following, equations (2-34) and (2-35).

\[
N_{f_p} = \left( \frac{W_p}{W_{p0}} \right)^{1/c}
\]  

(2-34)

\[
N_{f_p} = \left( \frac{W_c}{W_{c0}} \right)^{1/d}
\]  

(2-35)

In earlier sections Kachanov's [25] definition of damage was mentioned. The same definition for damage is used in this context which is load drop or decrease in load bearing. The main assumption here is that damage develops isotropically. Therefore load drop can be assessed as a metric for damage.
Figure 2-4: Plot of $D$ vs. $w$ for different values for $\eta$
CHAPTER 3
PREVIOUS STUDIES

3.1 Southwest Research Institute

Immediately following the catastrophic mishap of the T-38, SwRI was contracted to perform a “quick and dirty” analysis to determine the loads being applied to the lever as well as a durability and damage tolerance analysis. The original designer of the T-38, Northrup Grumman, did not have any data on the assumed loads, let alone the actual loads which lead SwRI to perform some on ground testing of the lever arm. Reports from the manufacturer were found that contained calculations for the maximum design load of 208 lbs on the aileron lever arm. The following sections discuss the study and work performed by SwRI.

In order for SwRI to perform the analysis, they had to first determine what kind of loading the aileron lever was experiencing. This effort began with the instrumentation of an aileron lever. Six strain gages were placed in various locations to measure the strain on the lever with a known applied load. Figure 3-1 shows the location of the strain gages. Gages SG3 and SG6 were not used in the load calibration. Their position on the curved surface caused several problems during calibration. The curved surface caused non-linear responses that resulted in hysteresis and the strain gages were not always aligned with the principal stress. Figure 3-2 shows the lever as installed for on aircraft testing. Figure 3-3 shows the calibration of the strain gages with a known applied load.

With hydraulic power applied to the aircraft, testing consisted of any stick or aileron movement that would apply a load to the lever. This could range from
maintenance bumping the stick in the cockpit or people leaning against the aileron, but the testing mostly involved stick movements from the pilot community simulating flight profiles as effectively as they could in the given situation.

Once the data had been collected from the strain gages, the loads on the lever had to be calculated. Using the data from the testing and the information from the calibration, SwRI was able to produce a sequence of loads including the maximum and minimum loads of 92.450 and -105.187 lbs.

The Finite Element Analysis (FEA) analysis began with a stress-to-load ratio to calculate the stress at the center of the stop screw bolt hole depending on the load applied throughout the flight. This was achieved using a simple static model and calculating the bending moment (Mc/I) about the center stop screw hole for a baseline value. Using two idealized rectangular cross sections, the moment of inertia was calculated for the lever along with the distance from the neutral axis. The moment arm was determined to be 1.54 inches. Using equation (3-1), the stress-to-load reference stress was determined where \( \sigma_r \) is the reference stress (Ksi) and \( L \) is the load (lbs).

\[
\sigma_r = 0.266078 \cdot L
\]  
(3-1)

Typically geometric correction factors are readily available for most common geometries. Since the geometry of the lever arm is not common, SwRI had to perform an analysis that would give them the proper stress intensity factor. Using a Finite Element Model (FEM) of the lever arm, a solid model with an identical crack to the mishap aircraft was created in order to determine the stress intensity factor. A 100 lb. load was applied and the resulting stress intensity factors were extracted from that data. Using the
calculated stress intensity factor, the geometric correction factor was calculated from equation (3-2) and later used in a durability and damage tolerance analysis. $SIF$ is the stress intensity factor found from the FEM, $\sigma_r$ is the reference stress found in equation (3-1), and $C$ is the surface crack length.

$$\frac{SIF}{\sigma_r \cdot \sqrt{\pi \cdot C}} \quad (3-2)$$

Simulations were made for both 2014-T6 and 7050-T74 materials. A fracture toughness of 16 ksi $\sqrt{\text{in}}$ and 30 ksi $\sqrt{\text{in}}$ were used for the materials, respectively. A durability analysis was performed on the original material with an assumed initial flaw size of 0.005 in. SwRI determined that the minimum residual stress for the 2014-T6 material, with the crack in the same location as the mishap aircraft, was 27.99 ksi. The residual strength was calculated by equation (3-3) where $\beta$ is the geometric correction factor, $K_k$ is the average fracture toughness and $a$ is the crack length.

$$\frac{K_k}{\beta \sqrt{\pi a}} \quad (3-3)$$

Using the above information, along with AFGROW, the durability and damage life tolerances for both the 2014-T6 material as well as the 7050-T74 material were calculated. Durability and damage tolerance relates to a structure's ability to resist the onset of damage and perform to required parameters in the presence of damage. For the 2014-T6 material these values were calculated to be 2,341 IFF flight hours and 42 IFF flight hours respectively and 1,319 IFF flight hours and 62 IFF flight hours for the
7050-T74 material, respectively. It should be noted that the 7050-T74 material can operate in the presence of a longer crack due to its higher fracture toughness value; however, cracks grow much quicker in this material than the 2014 material. From these findings, SwRI recommended the replacement of the levers due to the fact that the critical crack size of 0.09 x 0.06 inches, as determined from the failed lever, cannot be detected in the center hole using NDI techniques.

Figure 3-1: Location of strain gages (Courtesy of SwRI).

Figure 3-2: Instrumented lever installed for on aircraft testing (Courtesy of SwRI).
3.2 Gaps or Deficiencies of Previous Work

The SwRI analysis focused solely on the pre-existing crack at the stop screw hole. A study of 275 parts performed by the materials lab at Hill AFB showed that 127 were cracked in one location, 74 were cracked in multiple locations, and 74 parts had no evidence of any cracks. Of the parts studied, only one lever showed evidence of cracking in the location SwRI focused their study on. A crack length of 0.03 inches was detected in the same area as the mishap aircraft crack. In the analysis performed by SwRI, it was assumed that the levers would have to be replaced instead of inspected because NDI techniques could not detect cracks smaller than 0.10 inches. The material lab findings contradict the assumption made by SwRI that cracks smaller than 0.10 inches can not be
detected by NDI techniques. How can a recommendation be made by analyzing one crack in a location that is not common or truly represents the majority of the crack locations?

During the FEA analysis of this model, the way the model was constrained seemed flawed a couple of reasons. First, constraining the lever arm right at the location of the crack will cause even higher stresses at the crack tip than what may be occurring in the real world application. Second, the location of the constraints in the FEM was not consistent with the real world constraints.
CHAPTER 4
AILERON LEVER MODEL AND ANSYS PROGRAM

4.1 Damage Model

Fatigue is a phenomenon that weakens a material through cyclic loading. The end result can very well be the catastrophic failure of the material at stress levels below the yield stress. Due to the large amounts of time required for cracks to initiate and propagate through a material in environmental conditions, several durability models have been proposed in literature that are designed to predict the life of the material. These models typically use the stress-strain behavior of the material or the amount of work done on the material during one cycle of loading. In order for these models to predict the life of the material, the cyclic stress or strain history is needed. This information can be found through direct experimentation or the use of finite element analysis [67].

Fatigue of a material can be classified into two separate modes, low cycle fatigue and high cycle fatigue. Low cycle fatigue occurs when cyclic deformations are large enough that the material has passed the elastic region and crossed into the inelastic region of behavior. When the deformations are small, the material remains in the elastic region of behavior, and high cycle fatigue will occur. The T-38 aileron lever is an excellent example of high cycle fatigue. The loads are such that the maximum stress on the lever is below the yield strength thus resulting in high cycle fatigue.

Numerous models exist that have been used in performing analyses or efforts to predict the fatigue life of a material and have been discussed in previous sections. The model chosen for this study was a stress-based model. Since the stress-based model is
one of the oldest and most used models, there is substantial experimental data that can be obtained to develop material constants required to perform a fatigue life analysis. The remainder of this chapter briefly discusses the model used to predict the fatigue life of the materials, as well as a comparison and justification to the model chosen for this study.

4.2 Selected Materials

Since the design of the aircraft, the aileron lever has been manufactured from forged 2014-T6 aluminum and then machined to its final dimensions. Following the mishap, the entire fleet of T-38 aircraft were grounded until all aileron levers could be replaced. The lead time to procure levers manufactured from the same 2014-T6 material was far too long. Instead, an alternative material was proposed for use by the Aircraft Structural Integrity Program (ASIP) group. The new levers were machined from a solid block of 7050-T74 aluminum. It was assumed that this alternate material would be an acceptable replacement for the 2014 material. Comparison of the material properties and compositions were made. Table 4-1 lists the major material properties along with the chemical composition percentages of other materials found in both the 2014 and 7050 aluminum. From the table, it’s clear the compositions are substantially different; however the major material properties are relatively similar with the alternate 7050 material being slightly stronger.
Table 4-1: Material and Chemical Composition of 2014-T6 and 7050-T74 Aluminum Alloys

<table>
<thead>
<tr>
<th>Material Properties Comparison</th>
<th>2014-T6</th>
<th>7050-T74</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate Tensile Strength</td>
<td>70.0 ksi</td>
<td>76.0 ksi</td>
</tr>
<tr>
<td>Yield Strength</td>
<td>60.0 ksi</td>
<td>68.0 ksi</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>10500 ksi</td>
<td>10400 ksi</td>
</tr>
<tr>
<td>Poisons Ration</td>
<td>0.330</td>
<td>0.330</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chemical Composition Comparison</th>
<th>2014-T6</th>
<th>7050-T74</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>90.4 – 95.0 %</td>
<td>87.3 – 90.3 %</td>
</tr>
<tr>
<td>Chromium</td>
<td>&lt;= 0.100 %</td>
<td>&lt;= 0.040 %</td>
</tr>
<tr>
<td>Copper</td>
<td>3.90 – 5.00 %</td>
<td>2.0 – 2.6 %</td>
</tr>
<tr>
<td>Iron</td>
<td>&lt;= 0.700 %</td>
<td>&lt;= 0.150 %</td>
</tr>
<tr>
<td>Magnesium</td>
<td>0.200 – 0.800 %</td>
<td>1.90 – 2.60 %</td>
</tr>
<tr>
<td>Manganese</td>
<td>0.400 – 1.20 %</td>
<td>&lt;= 0.100 %</td>
</tr>
<tr>
<td>Other</td>
<td>&lt;= 0.150 %</td>
<td>&lt;= 0.150 %</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.500 – 1.20 %</td>
<td>&lt;= 0.120 %</td>
</tr>
<tr>
<td>Titanium</td>
<td>&lt;= 0.150 %</td>
<td>&lt;= 0.060 %</td>
</tr>
<tr>
<td>Zinc</td>
<td>&lt;= 0.250 %</td>
<td>5.70 – 6.70 %</td>
</tr>
<tr>
<td>Zirconium</td>
<td>-</td>
<td>0.080 – 0.150 %</td>
</tr>
</tbody>
</table>
4.3 Stress-Based Damage Model

Beginning with the S-N curve found in figure 2-2, digital software was used to retrieve data points from this curve and create a plot in Excel. A power law curve was fit to the data in order to obtain the equation of the line. Solving for $N_f$ gave equation (4-1).

$$N_f = C(\sigma_a)^m$$  \hspace{1cm} (4-1)

$\sigma_a$ is the stress amplitude and $C$ and $m$ are material damage constants calculated as 6.2874 E+47 and -9.33707 respectively. Figure 4-1 depicts the original S-N curve used to calculate the damage constants along with the plot of equation (4-1) using the same stress amplitudes as the original curve for the 2014-T6 aluminum.

Using a similar technique and figure 2-3, three logarithmic curves were produced to calculate the damage constants of the alternate 7050-T74 material. The digital data for the S-N curve was input into Excel, numerous tries were made to fit a curve to the data; however the data was such that no curves could accurately represent the digitized data to produce the necessary equations. The only approach that could depict the original curve was employing three separate logarithmic functions for different stress ranges. Figure 4-2 shows the original digitized data with the logarithmic curves and the stress ranges each equation is responsible for. This also provided an additional challenge in the ANSYS code to select the proper damage constants depending on the stress.

The equations used can be seen in equations (4-2, 4-3, and 4-4). Table 4-2 shows the value and stress range for the damage constants used in the previous equations. Equation (4-2) was used to calculate $N_f$ for any element with a stress greater than or
equal to 50,000 psi. Similarly equation (4-3) was used to calculate $N_f$ for any element with a stress less than 26,000 psi. And finally, equation (4-4) was used to calculate $N_f$ for any element with a stress between 26,000 and 50,000 psi. Both figures 4-1 and 4-2, clearly show that the calculated material constants give a reasonably accurate values for the cycles to failure for a given stress amplitude. Table 4-2 shows the value, type of curve fit, and the stress range they were used in to calculate the fatigue life of the elements.

$$N_f = \exp\left(\frac{-\left(\sigma_a - A_1\right)}{B_1}\right)$$

(4-2)

$$N_f = \exp\left(-5 \cdot \frac{-\left(\sigma_a - A_2\right)}{B_2}\right)$$

(4-3)

$$N_f = \exp\left(\frac{-\left(\sigma_a - A_3\right)}{B_3}\right)$$

(4-4)

It should be noted that figures 2-2 and 2-3, used to create the original curves as seen in figures 4-1 and 4-2, were not created for this specific fatigue analysis and may not necessarily reflect the damage characteristics of the material used by manufacturers for production of the aileron lever for use by the Air Force.
Table 4-2: Material Damage Model Constants

<table>
<thead>
<tr>
<th>Variable Used</th>
<th>Damage Constant Value</th>
<th>Type of Curve</th>
<th>Stress Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2014-T6</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>6.2874E+47</td>
<td>Power Law</td>
<td>All</td>
</tr>
<tr>
<td>m</td>
<td>-9.33707</td>
<td>Power Law</td>
<td>All</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7050-T74</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>237090</td>
<td>Logarithmic</td>
<td>50,000 psi and up</td>
</tr>
<tr>
<td>B1</td>
<td>23540</td>
<td>Logarithmic</td>
<td>50,000 psi and up</td>
</tr>
<tr>
<td>A2</td>
<td>39965</td>
<td>Logarithmic</td>
<td>Below 26,000 psi</td>
</tr>
<tr>
<td>B2</td>
<td>7423</td>
<td>Logarithmic</td>
<td>Below 26,000 psi</td>
</tr>
<tr>
<td>A3</td>
<td>139931</td>
<td>Logarithmic</td>
<td>26,000 – 49,999 psi</td>
</tr>
<tr>
<td>B3</td>
<td>11351</td>
<td>Logarithmic</td>
<td>26,000 – 49,999 psi</td>
</tr>
</tbody>
</table>

Figure 4-1: S-N curve for 2014-T6 comparing original digital data to the equation found from a power law fit.
Figure 4-2: S-N curve for 7050-T74 comparing original digital data to three separate logarithmic curves.

4.4 Finite Element Analysis

Using ANSYS, a finite element model was built in order to calculate the stress on individual elements with a known applied load. The ANSYS code was written in such a way that it would conduct the modeling, meshing, applications of boundary conditions and solutions automatically for many runs and predict the location of the cracks and the number of cycles to grow a crack to a critical length. The original drawings were provided by Hill AFB in order to create a solid model as accurately as possible. For ease in modeling, features that were determined to have no effect on the stress, initiation, and crack propagation were removed from the model. Figure 4-3 shows a 3-D image of the solid model drawn in ANSYS.
4.4.1 Mesh

A SOLID186 element was chosen from the ANSYS library to use in the meshing sequence. A SOLID186 is a higher order 3-D 20-node element that exhibits quadratic displacement behavior. Each node has 3 degrees of freedom: translation in the x, y, and z directions. The SOLID186 element supports plasticity, creep, stress stiffening, large displacement and large strains. This element is well suited to meshing the irregular shapes created in modern CAD systems. Figure 4-4 shows the lever with the mesh applied. In total, 53,422 elements were created. The ability to mesh irregular shapes was necessary for this model because of the curved surfaces around the center of the lever. A finer mesh was applied to the areas of most interest, and where stresses were assumed to be the highest.
4.4.2 Boundary Condition and Loading

Constraints were applied to two different locations on the model. Figure 4-5 shows those locations. Figure 4-5a shows the location where the lever attaches to the servo valve. Figures 4-5b and 4-5c show the constraints applied on the lower and upper surfaces respectively while the load is applied.

In figure 4-5a, constraints were applied to individual nodes restricting the model from translating in the x, y, and z directions. The constraints in figure 4-5b and 4-5c were applied to lines around the center bore of the lever simulating the bolt head and nut that prevent the lever from over travel and constraining the motion in the positive and negative z directions only.

Cyclic loading using the maximum and minimum loads provided by SwRI were used to calculate the fatigue life of the lever. A load of 83.25 lbs was uniformly applied.
distributed to the nodes on the rod end of the lever in the positive z direction as seen in figure 4-6a. This configuration created compressive and tensile stresses on the lower and upper surfaces of the lever respectively. Extreme loading of the lever while on the aircraft only happens for a brief second when full stick deflection is applied by the pilot. In an effort to simulate the loading of the lever, as occurs on the aircraft, the load was applied for a time of 1 sec. and then released. All loads and constraints were removed from the model and reapplied in a similar configuration with the differences being the upper surface was constrained from moving in the z direction, and a load of 93.7 lbs was uniformly distributed to the nodes on the rod end in the negative z direction as seen in figure 4-6b. This configuration created compressive and tensile stresses on the upper and lower surfaces of the lever respectively. Again, the load was applied for a time of 1 sec. and then released. Figure 4-7 is a graph of the cyclic loading depicting the maximum and minimum forces. During each time step, the equivalent stresses for individual elements were calculated and stored. Using ETABLE commands in ANSYS allowed for the calculated stresses to be operated on as necessary.
Figure 4-5: View of constraints applied. (a) Splined end of lever is constrained from translating in x, y, and z directions. (b) Lower surface constrained from z displacement during positive z direction loading. (c) Upper surface constrained from z displacement during negative z direction loading.

Figure 4-6: View of applied loads on lever. (a) 83.25 lb load being applied in the positive z direction. (b) 93.7 lb load being applied in the negative z direction.

Figure 4-7: Graph of cyclic loading depicting the maximum and minimum forces.
4.5 Successive Initiation and Propagation

The finite software ANSYS was used to create a simplified 3-D model of the T-38 aileron lever. A code was created that would draw, mesh, solve and perform post processing for successive runs. Okura [68] introduced a method called successive initiation which can be used with any damage model. The successive initiation analysis requires several steps and is implemented by utilizing finite elements in this study. The flowchart of the successive initiation process can be seen in figure 4-8 with of the process following a description.

Figure 4-8: Damage initiation and propagation using successive initiation
Beginning with the stress-based model discussed previously in section 4.3, the damage initiation site is first identified using constant amplitude cyclic loading. A set of finite elements surrounding the initiation site with damage above a selected threshold are identified as the damage initiation zone. These values are “killed” in ANSYS which results in the elements no longer adding to the stiffness matrix of the solution.

Starting with equation (2-4), the stress amplitude for each element was calculated. Substitution of the stress amplitude into equations (4-1 thru 4-4) resulted in the number of cycles to failure, \( N_f \), for each element. After calculating \( N_f \), the damage per cycle, \( D_c \), was found using equation (4-5).

\[
D_c = \frac{1}{N_f}
\]  

(4-5)

In order to calculate the time required for cracks to initiate, a damage criterion had to be set. After the damage per cycle was calculated, the table was sorted from maximum to minimum damage received per cycle. Using the max damage value, the limiting damage was determined using equation (4-6).

\[
\lim_{dmg} = 0.05 \cdot \max_{dmg}
\]  

(4-6)

Several limiting damage values were tested. The limiting value used in equation (4-6) was determined to be acceptable. This limiting damage value allowed for a large enough group of elements to be selected and calculate a reasonable initiation life. Using the limiting damage as calculated above, any element with damage greater than the limiting damage was selected and “killed.” In ANSYS, when an element is “killed,” the element
remains in the model, but does not add any stiffness to the overall stiffness matrix. This will result in the stresses of the subsequent runs to be distributed to the remaining elements surrounding the “killed” element. The elements that were selected and “killed” were used to calculate the number of cycles for crack initiation. This was done using equation (4-7) where *summation* was the summation of $D_c$ and *numelem* was the total number of elements selected using the limiting damage criteria. For successive runs, the average life of each run was calculated in an identical fashion as the initiation. At the conclusion of the program, the total life of the lever was calculated by the sum of the average and initiation lives.

\[
\text{initiation} = \frac{1}{\left(\frac{\text{summation}}{\text{numelem}}\right)} \quad (4-7)
\]

Successive initiation causes damage to accumulate on the elements and therefore must be monitored. The accumulated damage was calculated using equation (4-8) where $N$ is the number of cycles completed which in the case of the first run equals *initiation* as calculated in equation (4-7). Subsequent runs used the average life to compute the damage sustained during that period of time.

\[
D_{\text{accum}} = N \cdot D_c \quad (4-8)
\]

Using the accumulated damage found in equation (4-8), the residual damage, $D_r$, was calculated using equation (4-9).

\[
D_r = 1 - D_{\text{accum}} \quad (4-9)
\]
The calculated values of the accumulated and residual damages were stored in vector arrays to be used in subsequent runs. Once the damage values were saved, the process was repeated for a total number of 10 successive runs. The cycles to failure for the subsequent runs was calculated by equation (4-10) where \( i = 1, 2, \ldots, 10 \).

\[
N_i = \frac{D_r}{D_{c_i}} \quad (4-10)
\]

The selection criterion for the remainder of the program was slightly different than the initiation run. After the completion of each successive run, the values for the accumulated and residual damages were updated and saved in place of the old data.

Table 4-2 is a portion of the ETABLE from the initiation run sorted from the smallest to the largest number of cycles to failure. \( S1 \) is the stress on the elements during the loading of the material in the positive \( z \) direction. \( S2 \) is the stress on the elements during the loading of the material in the negative \( z \) direction. \( Amp \) is the stress amplitude calculated by equation (2-4). \( NF \) is the calculated number of cycles to failure using equations (4-1 thru 4-4). \( DC \) is the damage that occurs to the elements per cycle as calculated using equation (4-5). \( DAC \) is the damage accumulated from equation (4-8) and \( DR \) is the residual damage found using equation (4-9).

<table>
<thead>
<tr>
<th>ELEM</th>
<th>S1</th>
<th>S2</th>
<th>AMP</th>
<th>NF</th>
<th>DC</th>
<th>DAC</th>
<th>DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>26980</td>
<td>45349</td>
<td>52660</td>
<td>49004</td>
<td>10125</td>
<td>9.88E-05</td>
<td>4.2309</td>
<td>-3.2309</td>
</tr>
<tr>
<td>39180</td>
<td>44527</td>
<td>52246</td>
<td>48387</td>
<td>11399</td>
<td>8.77E-05</td>
<td>3.7581</td>
<td>-2.7581</td>
</tr>
<tr>
<td>20605</td>
<td>52880</td>
<td>40635</td>
<td>46758</td>
<td>15694</td>
<td>6.37E-05</td>
<td>2.7296</td>
<td>-1.7296</td>
</tr>
<tr>
<td>34429</td>
<td>42393</td>
<td>50537</td>
<td>46465</td>
<td>16642</td>
<td>6.01E-05</td>
<td>2.5742</td>
<td>-1.5742</td>
</tr>
<tr>
<td>27567</td>
<td>42644</td>
<td>49608</td>
<td>46126</td>
<td>17818</td>
<td>5.61E-05</td>
<td>2.4043</td>
<td>-1.4043</td>
</tr>
</tbody>
</table>
After each successive run, the ETABLE was sorted by the number of cycles to failure, \( N_f \). The program was forced to select the elements that had less than 200,000 cycles to failure remaining. This was done for two reasons:

1. To ensure the program was able to pick several elements after each run
2. Speed up the overall crack propagation cycle

Previous test runs showed that a selection criterion of less than 200,000 cycles would eventually cause the program to stop running due to divide by zero errors from equation (4-7). Following the selection step, ANSYS would again “kill” the selected elements and calculate the average life. The life was added to the life calculated in the previous run until the program had completed the specified number of cycles. The initiation life and the cumulative average life of the successive runs were finally combined to give the total cycles to failure.

4.6 Monte Carlo Simulation

This section gives an overview of the Monte Carlo simulation as a mathematical tool and as a complement or an alternative to real experiments. The Monte Carlo simulation was used in this study to observe the effects variability of select parameters on the life of the lever. These parameters consist of the damage constants as found using the S-N curve for the materials, the material properties such as the modulus of elasticity, and the loads applied to the lever.

4.6.1 Introduction and History

The Monte Carlo (MC) method is a general name of any method that uses a sequence of random numbers to perform different types of calculations. The method can
be used to approximate solutions of problems in different areas. It was developed during the twentieth century, but there are some older experiments from the second half of the nineteenth century which showed that different kinds of deterministic problems can be solved by using random processes. A systematic development of the method started about 1944 and was named during World War II, by a team working with the development of the atomic bomb. The method was named after the city of Monte Carlo, a center of gambling, because of the similarity between the games of chance played in the casinos, and the random numbers in a statistical simulation. In the beginning, the method was used in order to solve nuclear physics problems, but soon became a well known method that could be applied in different fields such as mathematics, physics, economy, demography, etc. [69, 70].

4.6.2 Why a Simulation?

The objective of a simulation is to understand how something works in reality. Simulations are experiments that are performed on designed models instead of real objects. In order to understand how something works and to prove the correctness, a model and its assumptions must be tested by repetitions of the behavior. Repetitions can be achieved by performing several physical experiments. But, in many cases the restrictions of both time and money makes it impossible to perform the experiments in reality. A simulation is then a very competitive alternative to an experiment, since it is more cost and time effective in most cases. In some cases it might even be impossible to perform real experiments. It is possible to simulate how fast a mortal disease could be spread across the world and how large casualties that could be expected. Yet, no one
would perform a real experimental study on a human population. A simulation is also a
good tool to produce new artificial data from data collected by measurements, this
technique is known as resampling [70, 71].

4.6.3 The Method

MC methods can handle two types of problems. They can either be probabilistic or deterministic. The difference between the two types is whether or not they are dealing with objects, operations or processes that involve randomness. A deterministic simulation deals with a process where all of the object and operations are non-random. In a probabilistic simulation, on the other hand, there are random objects or processes that cannot be predicted. In the real world there’s a lot of randomness, therefore it is hard to find a real situation that can be described as purely deterministic. Probabilistic simulations can deal with these variations. There is only one requirement for probabilistic problems solved by MC simulations. They must have solutions that can be described by probability density functions or probability mass functions [69, 70].

A mathematical model that describes the problem is created, and the input and output variables are defined. The input values are randomly picked out of their probability distribution functions, which are used to describe the input variables. One set of input values will result in one set of output values, which will be stored in one way or another. The law of large numbers is one of the fundamental things which the MC method is based on. A short version of the law of large numbers given by Marek et al. [70] ultimately says, that when dealing with random variables, the empirical distribution will converge with the theoretical one if the number of samples increases to infinity. The
consequence of this law is that mathematical models are looped as many times as necessary in order to get the desired accuracy of the results.

4.6.3.1 The Central Limit Theorem

The central limit theorem (CLT) states why results from simulations can be very good approximations of answers to some problems. The CLT states that the sum of \( n \) independent random variables can be approximated by a normal distribution when \( n \) is large [69]. This theorem is not general and there are some limitations. The random variables are not allowed to vary in size too much. For instance, the theorem is not valid if one of the variables is bigger than the sum of all other variables. Another disadvantage of CLT is the “tail problem.” If \( n \) goes towards infinity, the distribution function of the sum adopts the same shape as a normal distribution in the region around the mean value. However, the values that are much lower or higher than the mean value (values in the tail) do not adopt the normal distribution shape as quickly as the central regions.

4.6.3.2 Random Variables

A classification of random variables can be made in three basic types: attributes, counts and measurements. Attributes are criteria which a sample either fulfills or doesn’t. The number of samples that fulfill an attribute can be summarized in order to create counts. As an example, the attribute could be “heavier weight than 70 kg.” If this attribute was tested on 1000 students, the sample, one could create counts showing how the distribution is between students that weigh more or less than 70 kg. Attributes and counts are both discrete variables. Measurements on the other hand are continuous variables, which can be described by numerical values. For instance length, weight,
temperature, loads, etc. [70] are all considered measurements. A random variable can be purely discrete, purely continuous or a combination of both discrete and continuous. A purely discrete variable, which are defined at discrete values only, can be represented by a Probability Mass Function (PMF) which shows how the variable is distributed. Rolling a dice would generate a PMF. Figure 4-9a shows a PMF for a discrete variable. The sum of the heights of the bars shall be equal to 1. Random variables that are purely continuous are described by continuous functions. These functions are called Probability Density Functions (PDF). Figure 4-9b shows a PDF for a continuous variable, the area under the curve shall equal 1.

Figure 4-9: Illustration of distributions for discrete and continuous variables.
4.6.3.3 Limitations

MC simulations based on experimental data are dependent on how good the experimental data are. The sample can be more or less suitable for the problem, and the result of the simulation can never be better than the input. If the input contains invalid data, then the output will contain invalid data as well. This is known as the axiom “garbage in garbage out.” Simulations will never give answers that are absolutely correct. The method can nevertheless be very useful in order to solve complicated problems. The questions that must be answered are how likely is it that the answer is wrong and how wrong might the answer be. If the uncertainties can be controlled then it will be possible to achieve an answer which can be treated as valid with a certain probability.

4.6.4 Structure of Monte Carlo Simulations

The structure of a MC simulation is almost the same, no matter what type of problem is studied. Figure 4-10 is an illustration of the structure of a simulation, and describes how the different components and steps are connected. The following sections summarize how Marek et al. [70] described the major components and the basic structure of MC simulations.

4.6.4.1 Random Input Values

The probability distributions of the input values must be known in one way or another. One way of obtaining the probability distribution is to perform real experiments and record the data from the measurements that are done. If no tests can be performed and no historical data are available, it might be possible to create a probability
distribution using theoretical knowledge. If there is little knowledge available about the problem that is studied, then it might be necessary to use some general theoretical distribution, like uniform, normal, triangular etc. The character of the problem governs which type of distribution that will be selected.

4.6.4.2 Random Number Generation

To be able to perform a MC simulation there must be a source from which random numbers can be generated. The source is the probability distribution that is created in one way or another (see 4.6.4.1 Random Input Values). The generation process is normally accomplished in two steps. The first step is to generate random numbers out of a uniform distribution. The second step is to convert the result from the first step into the specified probability distributions. The outcome of the second step is single values from the probability distributions of the input variables. These values will be used as input data in the system model, see figure 4-10.

![Monte Carlo simulation scheme](image)

Figure 4-10: Monte Carlo simulation scheme, after Marek et al. [70].
4.6.4.3 System Model

Problems taken from real life are often very complex and involve a large amount of variables. The real-life situation must be translated into a mathematical model that describes the real situation as accurately as possible. The creation of a system model involves a compromise between the complexity of the model and the accuracy. The model should also be designed in a way which makes it possible to perform time effective calculations. The studied problem must be described by at least one variable that has a probability density function or a probability mass function. If no probability function is involved in the model, then there will be no need for a MC simulation.

4.6.4.4 Recording the Results

Results from simulations can be recorded and presented in different ways, depending on the type of problem and the aim of the simulation. In many cases there is no need of information from a specific calculation, a few statistical parameters are often the only values that are interesting.

Sometimes it can be useful to illustrate the results from the simulations by some sort of diagram. Histograms are one way of presenting the results. Some important choices must be made when a histogram is created. The number of distribution categories and their intervals must be decided. Each simulation step will result in one value which will be sorted into one of the distribution categories, each category will be illustrated by a histogram bin. The choice of the number of histogram bins will affect the result and cause some errors if the graphical result is compared to the numerical result. If a continuous variable is involved in the calculation then it will always be a loss of
information using histograms. The loss of information can be reduced by using more histogram bins.

The most time and space consuming way of recording the results is to record all output values and sometimes also all input values. If this is done, every single calculation can be reconstructed and studied. This type of recording can give enormous amounts of data, which has to be stored. Therefore, it is mostly used in cases where the solution of the problem is not as important as the understanding of the whole system.

4.6.4.5 Monte Carlo Simulation in ANSYS

The Monte Carlo method in ANSYS allows for the probability analysis to simulate how virtual components behave. One simulation loop represents one manufactured component subjected to a particular set of loads and boundary conditions. There are two major methods used in ANSYS, the Direct Sampling method and the Latin Hypercube Sampling method.

The Direct Sampling method is the most commonly used form of the Monte Carlo analysis. It mimics natural processes that can be easily understood or imagined. However, it is not the most efficient technique. The fact that the sampling process has no “memory” means that there is a good chance that two samples can be selected from sampling points that occur extremely close together. In this event, no new information or insight into the behavior of a component will be gained by simulating two samples that are the same or very close to the same.

The second sampling method is the Latin Hypercube Sampling (LHS) method. The LHS is a more advanced and efficient technique differing only from the Direct
method in that the sampling process has “memory.” This means that clustering of multiple samples is avoided and the tails of the distribution are forced to participate in the sampling. The LHS method typically requires 20% – 40% fewer simulation loops to deliver the same results with the same accuracy.
CHAPTER 5
RESULTS AND CONCLUSIONS

5.1 Stress Distribution in the Lever

The following section shows the results obtained by utilizing the successive initiation technique described in Chapter 4. To begin, figure 5-1 shows a view looking down on the right and left aileron lever and gives time designations pertaining to each lever. These time designations will be used throughout the remainder of the chapter as a reference to the locations of the cracks found from ANSYS as well as the experimental data acquired from the inspections performed by the Hill AFB materials lab.

Figures 5-2a and 5-2b are side views of the lever showing the direction the load is applied. Each figure shows an exaggerated deflection as well as the edge of the undeformed material. Figure 5-2a has a load of 83.25 lbs. applied in the positive z direction. Earlier in Chapter 2, a force of 92.45 lbs. was reported by SwRI as the maximum load. Although use of the maximum load during the successive initiation program would present the worst case scenarios, current observation of the lever has found that the maximum load is seldom attained during actual flight. Another reason a slightly lower value for the maximum load was used, was to allow the Monte Carlo simulation to show the significance of the load on the initiation life of the cracks. The Monte Carlo results will be reviewed in a later section.

Figure 5-2b shows the side view of the lever with a load of 94.704 lbs. applied in the negative z direction. Again as mentioned in Chapter 2, a force of 105.187 lbs. was
reported by SwRI. For the same reasons as discussed in the previous paragraph, a slightly lower load was used.

Figures 5-3a and 5-3b show the equivalent stress distribution around the center bore of the lever for the lower and upper surfaces respectively for the positive z load (time step 1). Figures 5-4a and 5-4b show the equivalent stress distributions around the center bore of the lever for the lower and upper surfaces respectively for the negative z load (time step 2). It’s clear from these images that the highest amount of stress occurs at the 11 and 1 o’clock time positions on both the upper and lower surfaces. Higher amounts of stress will result in more damage to the elements in these areas resulting in a higher probability that cracks will initiate and begin to grow in these locations.

Figure 5-1: View looking down on upper surface of aileorn levers (Courtesy SWRI).
Figure 5-2: View from side showing the deformed and undeformed edge of the lever. (a) Equivalent stresses and distribution for positive z load (time step 1). (b) Equivalent stresses and distribution for negative z load (time step 2).

Figure 5-3: View of stress distribution on upper and lower surface of lever for positive z load (time step 1). (a) Compressive stresses distributed on lower surface. (b) Tensile stresses distributed on upper surface.

Figure 5-4: View of stress distribution on upper and lower surface of lever for negative z load (time step 2). (a) Tensile stresses distributed on lower surface. (b) Compressive stresses distributed on upper surface.
5.2 Successive Initiation Results

This current study was conducted to determine the crack sites more accurately without presumptions and guesses and to clarify the contradiction between the SWRI report and the real data available using the computer software ANSYS to predict the location of the cracks, propagation path, and propagation rates.

After the simulations were performed in ANSYS, plots were created showing the locations and propagation paths of the cracks. The following figures look at only the original 2014 aluminum material. Similar figures of the alternate 7050 aluminum material can be seen in Appendix I. Figures 5-5a through 5-5i show the crack propagation path beginning with the undamaged state in figure 5-5a and ending with a complete crack in figure 5-5i connecting the 11:00 and 1:00 o’clock positions. Naturally, a completely cracked lever would never occur. Catastrophic failure would happen long before the crack had a chance to grow to the length seen below.

It can be seen in figure 5-5 that the crack locations occur at the 11:00 and 1:00 o’clock positions. During the cyclic loading, a majority of the damage happens at various positions between the crack initiation sites due in part to the large compressive stresses depicted in figure 5-3a. These stresses are caused by the physical constraints on the lever. This data shows very good agreement with the data provided from the Hill AFB materials lab discussed in the following section. Figure 5-6 is the side view of the lever depicting the crack initiation and propagation during the same sequence as figure 5-5.
Figure 5-5: Crack initiation and propagation on the upper surface of the aileron lever. The red elements represent the “killed” elements during each successive sequence of the ANSYS code.

In order to calculate the number of cycles to grow a crack to a similar length as the mishap crack, a built in function in ANSYS was used. ANSYS has the ability to measure the distance between nodal points. Utilizing this ability was a straightforward approach to figuring out how long the cracks were and how much they had propagated between cycles. Figure 5-7a and 5-7b show the location and length of the cracks as measured between nodal points on the upper surface and side of the lever, respectively.
The lengths depicted in figure 5-7 were established to be critical based on the data provided and the previous research done by SwRI. The successive initiation technique outlined in the previous chapter was used to calculate the total number of cycles to grow a crack to the lengths seen in the figure 5-7.

A total fatigue life for the original 2014-T6 material was calculated to be 208,135 cycles. Figure 5-5b depicts the initiation life and locations of cracks. The number of cycles required to initiate a crack was 36,630. In comparison, the alternate 7050-T74 material had a much shorter initiation and fatigue life. The number of cycles required for the alternate material to initiate a crack was calculated as 4,122 cycles with a total fatigue life of 23,314 cycles.

In addition to the location and propagation paths, the propagation rates were calculated for both materials. This was again done using the built in tools in ANSYS. Beginning with figure 5-5b, a length of crack was measured after the initiation cycle. The two most extreme nodal points at the 11:00 o’clock position on both the upper surface and the side of the lever were selected. These values were found to be 0.03922 and 0.03784 inches, respectively.

Figures 5-7a and 5-7b show the measurements at the final critical crack lengths of the original 2014 material. These measurements along the upper surface and side were taken after 171,505 additional cycles. The final critical crack lengths were measured at 0.06408 inches on the upper surface and 0.06304 inches along the side. The crack propagation rates were calculated using equation (5-1) where $a_i$ is the crack length after initiation, $a_f$ is the final crack length, and $\Delta N$ is the number of cycles required for the crack to propagate from the initial to the final crack length.
\[
\frac{da}{dN} = \frac{a_f - a_i}{\Delta N}
\]  

(5-1)

Using this equation (5-1), the maximum crack propagation rates were calculated for both materials. The original 2014 material had a maximum propagation rate of 1.4693 x 10^{-7} in./cycle. The maximum crack propagation rate for the alternate 7050 material was calculated as 4.7325 x 10^{-6} in./cycle.

Figure 5-6: Side view of lever depicting crack location and propagation path.
Figure 5-7: (a) Critical crack length of 0.06408 inches measured between nodal points on the upper surface after a total of 208,135 cycles. (b) Critical crack length of 0.06304 inches measured between nodal points on the side after a total of 208,135 cycles.

The alternate material, as previously shown, has a much shorter initiation life, and the propagation rate is at least one order of magnitude higher than the original material. The findings from the successive initiation method and the crack propagation calculations clearly show that the original 2014 material is a superior choice to the alternate 7050 material. Similar figures to figures 5-5, 5-6, and 5-7 for the 7050 material can be seen in Appendix I.

5.3 Comparison with Field Data

Data of the location and lengths of the cracks found was provided by the Hill AFB Materials Lab. The data was compiled from analyzing 275 levers using Non-destructive Investigation (NDI) techniques. From those levers, a total of 294 cracks were located and documented. Figures 5-8 through 5-11 show the approximate locations, number, and lengths for the majority of cracks detected. Additional cracks were located during the investigation and can be seen in figures 5-12 through 5-14. From the ANSYS simulation discussed previously, and the experimental data that follows, it is evident that
the majority of the damage and therefore the majority of the cracks were occurring at the 11:00 and 1:00 o’clock positions for both the upper and lower surfaces of the lever.

Although the mishap aircraft had a crack occur on the right aileron lever at approximately the 9:00 o’clock position, the probability of a crack beginning at either the 3:00 (left aileron lever) or 9:00 (right aileron lever) o’clock positions is extremely small. As seen in the following figures, the locations where the most cracks occurred were the 11:00 and 1:00 o’clock positions on the both the upper and lower surfaces. The ANSYS model predicts the most damage occurring on the upper surface which agrees quite well with the experimental data.

Figure 5-8: 11:00 o’clock position on the upper surface of the lever (Courtesy Hill AFB Materials Lab).
Figure 5-9: 11:00 o’clock position on the lower surface of the lever (Courtesy Hill AFB Materials Lab).

Figure 5-10: 1:00 o’clock position on the upper surface of the lever (Courtesy Hill AFB Materials Lab).
Figure 5-11: 1:00 o’clock position of the lower surface of the lever (Courtesy Hill AFB Materials Lab).

Figure 5-12: 2:30 position of the upper surface of the lever (Courtesy Hill AFB Materials Lab).
Figure 5-13: 3:00 o’clock position on the upper surface of the lever (Courtesy Hill AFB Materials Lab).

Figure 5-14: 7:00 o’clock position on the upper surface of the lever (Courtesy Hill AFB Materials Lab).
Comparison of figures 5-8 and 5-10 shows that the upper surface of the lever has a much higher frequency of cracks than the lower surface at the 11:00 o’clock position. Similarly, comparison of figures 5-9 and 5-11 shows that the upper surface has a much higher frequency of cracks than the lower surface at the 1:00 o’clock position.

5.4 Monte Carlo Simulation Results

Using the Monte Carlo simulation in ANSYS required some key information. As has been discussed in Chapter 4, in order to run a Monte Carlo simulation random input and random output variables are required. The random input variables used in this model were: Modulus of Elasticity, Force applied (each direction), and Damage constants (coefficient and exponent). Table 5-1 shows the random input variables along with the mean and standard deviation as input into ANSYS.

The random output variable was the initiation life of the model. The purpose of running the simulation was to show the effect the input variables had on the overall initiation life of the lever. A 15% deviation was examined to see what variables had the largest impact on the initiation life.

The modulus of elasticity was looked at in an effort to rule out any major effects the manufacturing process may have on the material properties of the component. Since the values that are reported for material properties are generally averages, it can be expected that there will be some amount of variation in each individual lever material. By defining the modulus of elasticity as a variable input parameter, depending on the results, a conclusion could be drawn as to whether or not a variation in the modulus of elasticity would affect the initiation and fatigue life of the lever.
Table 5-1: Monte Carlo Variable Inputs from ANSYS Simulation

<table>
<thead>
<tr>
<th>Input Variable</th>
<th>Input Value</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity (psi)</td>
<td>1.05 X 10^7</td>
<td>1.575 X 10^6</td>
</tr>
<tr>
<td>Force 1 (lbs)</td>
<td>4.1625</td>
<td>0.62438</td>
</tr>
<tr>
<td>Force 2 (lbs)</td>
<td>-7.8920</td>
<td>-1.1838</td>
</tr>
<tr>
<td>Coefficient</td>
<td>1.33147 X 10^-29</td>
<td>1.33147 X 10^-28</td>
</tr>
<tr>
<td>Exponent</td>
<td>-9.33707</td>
<td>.933707</td>
</tr>
</tbody>
</table>

Along with the modulus of the material, the forces applied in each direction were examined. This was originally done in an effort to show some sort of variation in the loading sequence since the code that was written for the current study consisted of constant amplitude cycling only.

The last variables considered in the Monte Carlo simulation were the coefficient and exponent values from equation (4-1). In order to calculate the fatigue life of the lever, equation (4-1) uses the stress amplitude found from the FEA, raises that to an exponent, and multiplies that value by a constant coefficient. Since the fatigue life of the lever depends on the coefficient and exponential values, it can be assumed that these values will have a reasonable impact on the life of the part.

Table 5-2 shows the values used by ANSYS to perform the Monte Carlo simulation. It is quite obvious that the values in Table 5-2 are slightly different than the original values from Table 5-1. This occurs for the simple fact that ANSYS uses the input information provided by the user and randomly selects values that are within the STD deviation provided by the user. In this case, the STD deviation for each input variable was ±15%. The values in Table 5-2 are the mean and STD deviations of those 30 values used for this simulation along with the minimum and maximum values picked by ANSYS.
Table 5-2: Monte Carlo Variable Inputs Results Used from ANSYS Simulation

<table>
<thead>
<tr>
<th>Input Variable</th>
<th>ANSYS Mean</th>
<th>STD Deviation</th>
<th>Min Value</th>
<th>Max Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity</td>
<td>1.05 X 10^7</td>
<td>1.5539 X 10^6</td>
<td>7.283 X 10^6</td>
<td>1.37 X 10^7</td>
</tr>
<tr>
<td>Force 1</td>
<td>4.1512</td>
<td>0.67356</td>
<td>2.3085</td>
<td>5.6610</td>
</tr>
<tr>
<td>Force 2</td>
<td>-7.8993</td>
<td>1.1812</td>
<td>-10.266</td>
<td>-5.2131</td>
</tr>
<tr>
<td>Coefficient</td>
<td>1.3280 X 10^-9</td>
<td>1.9734 X 10^-8</td>
<td>8.7564 X 10^-8</td>
<td>1.7055 X 10^-9</td>
</tr>
<tr>
<td>Exponent</td>
<td>-9.3323</td>
<td>1.5245</td>
<td>-13.036</td>
<td>-5.5418</td>
</tr>
</tbody>
</table>

5.5 Sensitivity Analysis

The purpose behind performing a sensitivity analysis was to be able to identify which input variables in this study could affect the initiation life of the lever. By using the variable inputs described in the previous section of this chapter, the sensitivity results can be seen in figure 5-15. This figure lists each of the input variables and gives the user a quick glance at which parameters significantly affect the initiation life of the lever.

It is quite obvious from the data that the only significant factor in the initiation life of the aileron lever is the exponential value from equation (4-1). Figures 5-16 and 5-17 were created to show the impact these values have on the fatigue curve of the material. Figure 5-16 shows plots of the original data, the curve fit (equation (4-1)), and a 15% increase and decrease in the coefficient value from equation (4-1). Figure 5-17 shows how only a 5% variation in the exponential value changes the fatigue curve of the 2014-T6 material.
Figure 5-15: Sensitivity analysis using the Monte Carlo simulation.

Figure 5-16: Plot showing the variability of the coefficient from equation (4-1). A 15% variability appears to have little impact on the initiation life of the lever.
Figure 5-17: Plot showing the variability of the exponential value from equation (4-1). A 5% variability in the value was used to show the impact on the initiation life of the lever.

From Table 5-2, the forces applied on the lever seem very small. This occurs for the simple fact that the force shown in the table is applied to a set number of nodes. For example, a force of 4.1512 applied to 20 nodes equates to a total force of 83.024 lbs. Similarly, a force of -7.8993 applied to 12 nodes would equate to a total force of -94.792 lbs. The maximum forces applied during the simulation were 113.22 lbs. and 123.19 lbs. in the positive z and negative z directions, respectively. It was originally assumed that the force applied on the lever would contribute significantly to the initiation life.

However, as shown above, the forces applied exceed the loads recorded by SwRI and are still insignificant in the initiation life of the aileron lever. This analysis has shown that a deviation larger than the 15% that was looked at would be required to have any significant impact on the initiation life of the lever.
5.6 Thesis Contribution

This thesis establishes and documents an alternative solution to the standard fracture mechanics approach for analyzing fatigue of a material. Aluminum 2014-T6 and 7050-T74 were examined using continuum damage theory coupled with a successive initiation technique. Accumulated and residual damage to individual elements were calculated and monitored based on the stress attained from the FEA model. The main thesis contributions are as follows:

1. A methodology to predict crack initiation and propagation as a result of cyclic mechanical loading that could be applied to different applications
2. Applying this methodology to predict the number of cycles to failure for the aileron lever and recommendations for inspection and replacement of the lever
3. Obtaining insights on how damage initiates and propagates on the lever in the most likely locations
4. Understanding the impact of random variable inputs on simulation results
5. Developing a methodology to include the random variable inputs into life cycle calculations using a probability analysis in ANSYS

5.7 Recommendations

An analysis on two separate materials used to manufacture the T-38 aileron lever was performed using the successive initiation technique. Constant stress amplitude was calculated using an FEA model in ANSYS. The original 2014-T6 aluminum material was found to be far superior in regards to crack initiation and propagation rate than the alternative 7050-T74 aluminum material. A crack initiation life of 1500 hours was
calculated for the original 2014-T6 material compared to an initiation life of 170 hours. The total time required for the alternate 7050 material to reach a critical length of .112 x .07 inches along the upper surface and the side respectively was 964 hours. The time required for the original 2014 material to reach a critical crack length of 0.06408 x 0.06304 along the upper surface and side respectively was 8600 hours. The crack propagation rates of both materials were calculated as $1.4693 \times 10^{-7}$ in./cycle and $4.7325 \times 10^{-6}$ in./cycle for the 2014 and 7050 materials, respectively. From the information presented, though the alternative 7050 material has a higher fracture toughness value, and can withstand higher stresses in the presence of a crack, cracks will grow and propagate by an order of magnitude faster than the original 2014 material. As has been shown throughout the current study there is a significant amount of difference in the final severity of damage amassed by the different materials. The recommendations for the T-38 are as follows:

1. Maintain the current replacement interval of 900 hours while the 7050-T74 alternate material is in use.
2. Remove the 7050-T74 alternate material and replace with original 2014-T6 material as soon as the supply system becomes parts supportable.
3. Once all parts are replaced with the original 2014-T6 material, perform NDI inspection at 1800 hours.
4. Once all parts replaced with original 2014-T6 material, move replacement interval to 3600 hours.
5.8 Conclusion

Following the catastrophic failure of the T-38, Southwest Research Institute performed a fatigue life analysis on the lever. SwRI’s analysis focused only on the crack location of the lever. Experimental data provided by Hill AFB materials lab showed that the lever failed in a location uncommon to the remaining levers that were inspected using NDI techniques. The conflicting data had raised questions as to the locations and directions of crack growth. The current study stemmed from the uncertainty caused by the conflicting data and has shown that deficiencies exist in the previous analysis that need to be improved on.

An FEA model was produced using ANSYS. A Solid 186 element was used during the meshing sequence for its exceptional ability to model irregular shaped objects. A code was written that would draw, mesh, and apply forces for successive runs to the FEA model. Using a continuum damage model coupled with a successive initiation technique, damage to the individual elements was monitored. Using an appointed damage criteria, elements were killed resulting in initiation sites and propagation paths of multiple cracks. The current research paper has shown the locations and propagation paths of cracks without presumption. Comparison of the experimental crack data has been shown to agree extremely well with the location of the cracks predicted by simulation.

An initiation and fatigue life of the 2014-T6 material was calculated as 1500 and 8600 hours or 36,630 and 208,135 cycles respectively. An initiation and fatigue life of the 7050-T74 material was calculated as 170 and 964 hours or 4,122 and 23,314 cycles respectively.
Crack propagation rates for both materials were calculated as $1.4693 \times 10^{-7}$ in./cycle and $4.7325 \times 10^{-6}$ in./cycle for the 2014 and 7050 materials respectively. The evidence has shown that the original 2014-T6 aluminum material is far superior to the alternative 7050-T74 aluminum material.

Recommendations have been made and will be presented to the Hill AFB mechanical systems engineering group concerning the replacement of the T-38 aileron levers.

5.9 Future Work

Based on the insights gained throughout this study, future works are suggested for the improvement of the experiment, FEA modeling, and analytical procedures conducted in this thesis.

1. Future work could consist of designing a test set up to perform experiments on actual fatigue lives of the levers and compare that to the simulated data found during this study.

2. A finer mesh could be applied to the FEA model and see what impact that may have on the crack initiation and propagation rate of the lever.

3. A different material could be examined for future use in place of the two aluminum materials that were investigated.

4. A continuum damage model could be coupled with fracture mechanic techniques that could utilize the location and initiation capability of the damage mechanics and the crack growth models from fracture mechanics.
5. Regular cyclic constant amplitude loading was used for this study. Adding randomness to the load profile by utilizing a rain flow counting technique to more accurately predict number of cycles to failure.

6. This study only used elastic material properties for the model. Update material properties as a function of damage. As damage accumulates, the material will soften. Also include inelastic behavior as lever experiences loads beyond the yield stress.

7. Perform sensitivity analysis using a wider range for random input parameters to thoroughly evaluate the sensitivity of the results.
REFERENCES


APPENDICES
Appendix I: 7050-T74 images

Crack initiation and propagation on the upper surface of the aileron lever. The red elements represent the “killed” elements during each successive sequence of the ANSYS code.

Side view of lever depicting crack location and propagation path.
(a) Critical crack length of 0.112 inches measured between nodal points on the upper surface after a total of 23,314 cycles. (b) Critical crack length of 0.07 inches measured between nodal points on the side after a total of 23,314 cycles.
Appendix II: ANSYS Codes

2014-T6 Code:
/PREP7
BLC4,0,-.375/2,.87,.375
K,5,0,0,0,
K,6,-1.073,243,0,
K,7,-1.073,0.243+.125,0,
K,8,-1.073,0.243-.125,0,
K,9,0,125,0,
K,10,0,0,125,0,
LSTR,7,9
LSTR,9,10
LSTR,10,8
LSTR,8,7
AL,5,6,7,8
APLOT
CYL4,-1.073,.243,.32
K,15,-2.575,.583,0,
K,16,-2.575,0.583+.19,0,
K,17,-2.575,0.583-.19,0,
K,18,-1.073,0.243-.19,0,
K,19,-1.073,0.243+.19,0,
LSTR,16,19
LSTR,19,18
LSTR,18,17
LSTR,17,16
AL,13,14,15,16
CYL4,-2.575,.583,.38
AADD,1,2,3,4,5
LFILLT,30,32,.25,,
LFILLT,29,31,0.25,,
LFILLT,31,26,0.25,,
LFILLT,32,27,0.25,,
LFILLT,28,24,0.25,,
LFILLT,25,23,0.25,,
LFILLT,24,21,0.03,,
LFILLT,23,22,0.03,,
AL,7,8,9
AL,4,5,6
AL,10,11,12
AL,13,14,15
AL,33,34,35
AL,16,17,20
AL,39,40,41
AL,36,37,38
AADD,1,2,3,4,5,6,7,8,9
VOFFST,10,1.735,,
CYL4,-1.073,.243,.375/2,,,.1.735
CYL4,-2.575,.583,.4375/2,,,.1.672
VSBV,1,2
VSBV,4,3
CYL4,-2.575,.583,.316/2,,,.1.735
VSBV,1,2
KWPLAN,-1,6,5,12
KWPAVE,15
CSYS,4
K,100,0,0,-1.360
KWPAVE,100
K,101,0,32,0,0
KWPAVE,101
K,102,-.75,0,0
K,103,-.75,.01.360
K,104,.6342,0.1.360
LSTR,101,102
LSTR,102,103
LSTR,103,104
LSTR,101,104
LFILLT,103,100,0.12,,
AL,100,101,102,103,104
AGEN,,10,,1,,1
VEXT,10,0,2,0,,
VSBV,3,1
KWPLAN,-1,91,90,101
LGEN,2,134,45,0
KWPAVE,7
LGEN,2,4,75,0
LSTR,7,13
LDELE,7,1,
LDELE,4,1
KWPLAN,-1,6,5,12
KWPAVE,91
K,105,0,0,.52
K,106,3,0,.52
K,107,3,0,-2.25
LSTR,105,106
LSTR,106,107
LSTR,107,13
LSBL,18,4
LSTR,8,106
LDELE,29,1
LFILLT,30,4,0.12,,
AL,4,7,18,19,30
AGEN,,1,5,1
VEXT,1,,0,2,0,,,
LESIZE,24,,10,,,1
LESIZE,114,,10,,,1
LESIZE,23,,10,,,1
LESIZE,105,,1,,,1
LESIZE,21,,1,,,1
LESIZE,158,,6,,,1
LESIZE,15,,7,,,1
LESIZE,115,,10,,,1
LESIZE,104,,10,,,1
LESIZE,90,,,10,,,1
LESIZE,113,,10,,,1
LESIZE,110,,10,,,1
LESIZE,95,,10,,,1
LESIZE,71,,8,,,1
LESIZE,197,,8,,,1
LESIZE,36,,2,,,1
LESIZE,155,,2,,,1
LESIZE,39,,2,,,1
LESIZE,156,,2,,,1
LESIZE,1,,10,,,1
LESIZE,3,,10,,,1
CMDELE_Y2
!************************************************************************
!****************Start Analysis**************************************
!/UIS, MSGPOP, 3!Suppress all warnings
/solu
ALLSEL,ALL
!Restore full element set
outres,erase
antype,static,new
!Specifies new static analysis
!lswrite,init
!Reset load step file number
FDELE,all,FZ
DDELE,all,
!************************************************************************
ptime1=1
!Define time parameter 1
time,ptime1
!Time at the end of step 1
NROPT,FULL,,ON
!Setting the Newton-Raphson option
INLGEOM,ON

NSEL,S,NODE,,75693,75707,2
NSEL,A,NODE,,75711,75717,2
NSEL,A,NODE,,75692,75692,0
NSEL,A,NODE,,75709,75709,0
NSEL,A,NODE,,75720,75722,2
D,all,,,,UX,UY,UZ,,

DL,34,2,UZ,,
DL,43,6,UZ,,
DL,46,4,UZ,,
DL,18,22,UZ,,
!Selecting lines for constraints
NSEL,S,NODE,,14413,14419,2
NSEL,A,NODE,,14421,14427,2
NSEL,A,NODE,,14429,14437,2
NSEL,A,NODE,,14440,14446,2
NSEL,A,NODE,,14408,14409,1
NSEL,A,NODE,,14411,14411,0
F1=4.1625
F,all,FZ,F1
!Selecting nodes for loads
oures,all
!lswrite
allsel,all
solve
FDELE,all,FZ
DDELE,all,
!***************************************************************************
ptime1=2
!define time parameter1
time,ptime1
!time at the end of step 1
NSEL,S,NODE,,75693,75707,2
NSEL,A,NODE,,75711,75717,2
NSEL,A,NODE,,75692,75692,0
NSEL,A,NODE,,75709,75709,0
NSEL,A,NODE,,75720,75722,2
D,all,,,,,,UX,UY,UZ,,,,
DL,41,70,UZ,,,,
DL,62,11,UZ,,,,
DL,82,18,UZ,,,,
DL,101,3,UZ,,,,
!Selecting lines for constraints
NSEL,S,NODE,,14920,14920,0
NSEL,A,NODE,,14926,14936,2
NSEL,A,NODE,,14995,15003,2
F2=7.892
F,all,FZ,F2
!Apply the load,distribution
oures,all
!lswrite
allsel,all
solve
!solve load steps
finish
!***************************************************************************
!**********
!*dim,Fname,string,1,20,
*sread,Fname,Filename,txt
/post1
SET,,,,,1
ETABLE,S1,S,EQV
!Filling Element Table with stresses
SET,,, ,,, ,2
!Reading step 2 information
ETABLE,S2,S,EQV !Filling Element Table with stresses

C=6.2874E47
x=-9.33707

SADD, AMP, S1, S2, .5, .5, ,
!Calculating Stress Amplitude, adding together and divide by 2
SEXP, AMP, x,
!Raising AMP to exponent -9.33707
SMULT, Nf, N, C,
!Multiplying N table by constant 6.2874E47 to get Nf
SEXP, Dc, Nf, -1,
!Calculating Damage per cycle
ESORT, ETAB, Nf, 1,
!Sorting Nf table in ascending order
SMULT, Dac, Dc, 1, 1,
!NF remaining on element using residual damage and damage per cycle
ESORT, ETAB, Nf, 1,
!Sorting Nf table in ascending order

ESORT, ETAB, Dc, 0,
!*GET, maxdmg, SORT, MAX
limdmg=0.05*maxdmg
!*CREATE a new parameter by multiplying maxdmg by some factor
ESEL, R, ETAB, Dc, LIMDMG, ,
ssum
!*SELECTS a subset of elements with damage greater than limdmg
ssum
!*SUMS up all values of selected elements
*GET, sum, ssum, 0, item, Dc
!*CREATING a vector from the summation of the elements
*GET, numelem, elem, 0, count
initiation=1/(sumation/numelem) !Calculating initiation life
!****************************************************************************
/SOLU

ekill, ALL
!Kills selected elements
allsel, all
!Selects all elements
!****************************************************************************
/POST1
eusort

*GET, minnum, elem, 0, num, min
*GET, maxnum, elem, 0, num, max
life = initiation
numelements = maxnum - minnum
!Value used to dimension future vectors
*DIM, Dr, numelements + 1
*vfill, Dr, Ramp, 1, 0
!Allocating size of residual vector
!Creating residual damage vector
Etable, Dac
SADD, Dac, Dc, initiation, ,
Etable, Dr
*vput, Dr(1, 1), elem, 1, etab, Dr
SADD, Dr, Dr, Dac, , -1,
!Calculating the residual damage from 1st run
*dim,Dac,,numelements+1  !Allocating size of dcycle1 vector
*vget,Dac,elem,1,etab,Dac  !Creating accumulated damage vector
*vget,Dr,elem,1,etab,Dr  !Creating residual damage vector

!*dim,life,,numelements+1
!*dim,oldorder,,numelements+1
!*dim,dsort,,numelements+1
!*dim,posttab,array,numelements+1,2,,element number,dcycle
!*voper,life,residual,div,dcycle1

Finish

/filename,Fname(1,1,1),1
save,,,,MODEL
q=1

!*******************************************************************************
!***************************************************2nd Run************************************************************
!*******************************************************************************
totlife=0
count=0
*do,i,1,20,1
/solu
ALLSEL,ALL  !Restore full element set
outres,erase
antype,static,new  !specifies new static analysis
!lswrite,init  !reset load step file number

FDELE,all,FZ  !Deletes all forces
DDELE,all,  !Deletes all constraints

!*******************************************************************************
!************************************************************************************
ptime1=1  !define time parameter1
time,ptime1  !time at the end of step 1
NROPT,FULL,,ON  !Setting the Newton-Raphson option
INLGEOM,ON

NSEL,S,NODE,,75693,75707,2
NSEL,A,NODE,,75711,75717,2
NSEL,A,NODE,,75692,75692,0
NSEL,A,NODE,,75709,75709,0
NSEL,A,NODE,,75720,75722,2
D,all,,,,,,UX,UY,UZ,,,
DL,34,2,UZ,,
DL,43,6,UZ,,
DL,46,4,UZ,,
DL,18,22,UZ,,  !Selecting lines for constraints
NSEL,S,NODE,,14413,14419,2
NSEL,A,NODE,,14421,14427,2
NSEL,A,NODE,,14429,14437,2
NSEL,A,NODE,,14440,14446,2
NSEL,A,NODE,,14408,14409,1
NSEL,A,NODE,,14411,14411,0

NSEL,A,NODE,,75763,75765,2
NSEL,A,NODE,,75767,75767,2
NSEL,A,NODE,,75768,75768,2
NSEL,A,NODE,,75769,75769,2
NSEL,A,NODE,,75770,75770,2
NSEL,A,NODE,,75771,75771,2
D,all,,,,,,UX,UY,UZ,,,
DL,34,2,UZ,,
DL,43,6,UZ,,
DL,46,4,UZ,,
DL,18,22,UZ,,  !Selecting lines for constraints
NSEL,S,NODE,,14413,14419,2
NSEL,A,NODE,,14421,14427,2
NSEL,A,NODE,,14429,14437,2
NSEL,A,NODE,,14440,14446,2
NSEL,A,NODE,,14408,14409,1
NSEL,A,NODE,,14411,14411,0
F,all,FZ,F1  !Apply the load,distribution

outres,all  !specifies the results file
!lswrite  !write the load step file fco50-1.S1
allsel,all  
solve

FDELE,all,FZ
DDELE,all,
|***************************************************************************
ptime1=2  !define time parameter 1
time,ptime1  !time at the end of step 1

NSEL,S,NODE,,75693,75707,2
NSEL,A,NODE,,75711,75717,2
NSEL,A,NODE,,75692,75692,0
NSEL,A,NODE,,75709,75709,0
NSEL,A,NODE,,75720,75722,2
D,all,,,,,UX,UY,UZ,,

DL,41,70,UZ,,,,
DL,62,11,UX,,,,
DL,82,18,UX,,,,
DL,101,3,UX,,,,
!Selecting lines for constraints

NSEL,S,NODE,,14920,14920,0
NSEL,A,NODE,,14926,14936,2
NSEL,A,NODE,,14995,15003,2
F,all,FZ,F2  !Apply the load,distribution

outres,all  !specifies the results file
!lswrite  !write the load step file fco50-1.S1
allsel,all  
solve  !solve load steps

finish
|***************************************************************************
/Post1
Allsel,ALL

SET,,,,1  !Reading step 1 information
ETABLE,S1,S,EQV  !Filling Element Table with stresses

SET,,,,2  !Reading step 2 information
ETABLE,S2,S,EQV  !Filling Element Table with stresses
eusort

Etable,Dac
*vpud,Dac(1,1),elem,1,etab,Dac
Etable,Dr
*vpud,Dr(1,1),elem,1,etab,Dr  !Filling etables with Dac and Dr from previous run
SADD,AMP,S1,S2,.5,5,   !Calculating Stress Amplitude, adding together and divide by 2
SEXP,N,AMP,,x,        !Raising AMP to exponent -9.33707
SMULT,Nf,Nf,,C,       !Multiplying N table by constant 6.2874E47 to get Nf
SEXP,Dc,Nf,,-1        !Damage per cycle for successive runs
SMULT,Nf,Nf,Dr,,      !Calculating Number of cycles to failure for cycle 2
ESORT,ETAB,Nf,1,,     !Select all elements with cycles to failure
SEXP,Dc,Nf,,          !between 1 and 100000
SMULT,Nf,                !Multiplying N table by constant 6.2874E47 to get Nf
SADD,Dr,Dr,Dr,,      !Calculating Number of cycles to failure for cycle 2
ESORT,ETAB,Nf,1,200000,,  !Select all elements with cycles to failure

ssum               !Sums up all values of selected elements
*get,sumation,ssum,0,item,Dc  !Creating a vector from the summation of the elements
*get,numelem,elem,0,count

avglife=1/(sumation/numelem)  !Calculating avg life with newly created vectors

******************************************************************************
/SOLU

ekill,ALL            !Kills selected elements
allsel,all           !Selects all elements
******************************************************************************
/post1

eusort

SMULT,D,Dc,avglife,,   !Damage for successive runs
SADD,Dac,D,Dac,,       !Accumulated Damage
SADD,Dr,Dr,Dac,,,-1,,  !Residual Damage for successive runs
!*dim,Dac,,numelements+1  !Allocating size of dcycle1 vector
!*dim,Dr,,numelements+1  !Allocating size of residual vector
*vget,Dr,elem,1,etab,Dr
*vget,Dac,elem,1,etab,Dac

totlife=totlife+avglife
count=count+1

finish

/filename,Fname(q+1,1,1),1
save,,,,MODEL

q=q+1

*enddo

totlife=life+totlife
Num_runs=count

7050-T74 Code: Uses the same modeling commands as the 2014-T6 code

!**********************************************************************
!******************************************************************************
!**********************************************************************
/solu
ALLSEL,ALL  !Restore full element set
outres,erase
antype,static,new  !specifies new static analysis
!lswrite,init  !reset load step file number

FDELE,all,FZ
DDELE,all,

***************************************************************************
ptime1=1  !define time parameter1
time,ptime1  !time at the end of step 1
!NROPT,FULL,,ON  !Setting the Newton-Raphson option
!NLGEOM,ON

NSEL,S,NODE,,75693,75707,2
NSEL,A,NODE,,75711,75717,2
NSEL,A,NODE,,75692,75692,0
NSEL,A,NODE,,75709,75709,0
NSEL,A,NODE,,75720,75722,2
D,all,,,,,,UX,UY,UZ,,

DL,34,2,UZ,,
DL,43,6,UZ,,
DL,46,4,UZ,,
DL,18,22,UZ,,  !Selecting lines for constraints

NSEL,S,NODE,,14413,14419,2
NSEL,A,NODE,,14421,14427,2
NSEL,A,NODE,,14429,14437,2  !Selecting nodes for loads
NSEL,A,NODE,,14440,14446,2
NSEL,A,NODE,,14408,14409,1
NSEL,A,NODE,,14411,14411,0
F1=4.1625
F,all,FZ,F1  !Apply the load,distribution

outres,all  !specifies the results file
!lswrite  !write the load step file fcoc50-1.S1
allsel,all
solve

FDELE,all,FZ
DDELE,all,

***************************************************************************
ptime1=2  !define time parameter1
time,ptime1  !time at the end of step 1

NSEL,S,NODE,,75693,75707,2
NSEL,A,NODE,,75711,75717,2
NSEL,A,NODE,,75692,75692,0
NSEL,A,NODE,,75709,75709,0
NSEL,A,NODE,,75720,75722,2
D, all,,,,,UX,UY,UZ,,,,

DL, 41, 70, UZ,,,,
DL, 62, 11, UZ,,,,
DL, 82, 18, UZ,,,,
DL, 101, 3, UZ,,,,  !Selecting lines for constraints

NSEL, S, NODE,, 14920, 14920, 0
NSEL, A, NODE,, 14926, 14936, 2  !Selecting nodes
NSEL, A, NODE,, 14995, 15003, 2
F2 = -7.892

f, all, FZ, F2  !Apply the load, distribution

outres, all  !specifies the results file
!lswrite  !write the load step file fcoc50-1.S1
allsel, all
solve  !solve load steps

finish

!**************************************************************************
!***********Find initiation life
!**************************************************************************
*/
post1
SET,,, 1  !Reading step 1 information
ETABLE, S1, S, EQV  !Filling Element Table with stresses

SET,,, ,2  !Reading step 2 information
ETABLE, S2, S, EQV  !Filling Element Table with stresses

SADD, AMP, S1, S2, .5, .5, ,  !Calculating Stress Amplitude, adding together and divide by 2

*IF, AMP, GE, 5.00E04, THEN

A = 237090
B = 4.24809E-05

*ELSEIF, AMP, LT, 2.60E04, THEN

A = 30096
B = 6.73582E-04

*ELSE

A = 139931
B = 8.8098E-05

*ENDIF

*get, minnum, elem, 0, num, min
*get, maxnum, elem, 0, num, max
numelements = maxnum - minnum  !Value used to dimension future vectors
*dim, e, numelements + 1  !Allocating size of exponent vector
*vfill, e, Ramp, 2.718281828, 0  !Creating residual damage vector
Etabl
*vput,e(1,1),elem,1,etab,e

SADD,N,AMP,,-1,,A  !Subtracting amplitude from constant A
SMULT,Q,N,,B,,  !Multiplying difference between AMP and A by inverse of B
SEXp,Nf,,Q,,  !Raising value exp to value calculated in previous steps to get Nf
SEXp,Dc,Nf,,,-1,,  !Calculating Damage per cycle

!SMULT,Dac,D,,100,,  !Damage accumulated per cycle times 100 cycles
!SADD,Dr,D,,,-1,,1,,  !Residual damage
!ISEXP,Dc,D,,,-1,,  !Inverting damage per cycle
!SMULT,Nfn,Dr,Dc,1,1,,  !Nf remaining on element using residual damage and damage per cycle

IESORT,ETAB,Nf,,1,  !Sorting Nf table in ascending order
IESEL,S,etab,Nf,0,150,1

ESORT,ETAB,Dc,0,,  !Sorting table in descending order with regards to damage/cycle 1
*GET,maxdmg,SORT,,MAX  !Creates a vector named maxdmg
limdmg=0.05*maxdmg  !Creates a new parameter by multiplying maxdmg by some factor
ESEL,R,ETAB,Dc,LIMDMG,,,  !Selects a subset of elements with damage greater than limdmg
ssum  !Sums up all values of selected elements
*get,sumation,ssum,0,item,Dc
*get,numelem,elem,0,count

initiation=1/(ssum/numelem)  !Calculating initiation life with newly created vectors

I/post1
!SET,,,1  !Reading step 1 information
IETABLE,S1,S,EQV  !Filling Element Table with stresses

ISET,,,2  !Reading step 2 information
IETABLE,S2,S,EQV  !Filling Element Table with stresses

!C=6.2874E47
!x=-9.33707

!SADD,AMP,S1,S2,5,,5,,  !Calculating Stress Amplitude, adding together and divide by 2
!ISEXP,N,AMP,,x,  !Raising AMP to exponent -9.33707
!SMULT,Nf,N,,C,  !Multiplying N table by constant 6.2874E47 to get Nf
!ISEXP,Dc,Nf,,,-1,,  !Calculating Damage per cycle

!!ESORT,ETAB,Nf,1,,  !Sorting Nf table in ascending order
!!SMULT,Dac,D,,100,,  !Damage accumulated per cycle times 100 cycles
!!SADD,Dr,Dac,,,-1,,1,,  !Residual damage
!!ISEXP,Dc,D,,,-1,,  !Inverting damage per cycle
!!SMULT,Nfn,Dr,Dc,1,1,,  !Nf remaining on element using residual damage and damage per cycle

!!ESORT,ETAB,Nf,1,,  !Sorting Nf table in ascending order
!!IESEL,S,etab,Nf,0,150,1
!!ESORT,ETAB,Dc,,  !Calculating Damage per cycle

!*GET,maxdmg,SORT,,MAX  !Creates a vector named maxdmg
!limdmg=0.05*maxdmg  !Creates a new parameter by multiplying maxdmg by some factor
!ESEL,R,ETAB,Dc,LIMDMG,,,  !Selects a subset of elements with damage greater than limdmg
!ssum  !Sums up all values of selected elements
!*get,sumation,ssum,0,item,Dc
!*get,numelem,elem,0,count
initiation=1/(sumation/numelem)  !Calculating initiation life
******************************************************************************
/SOLU

ekill,ALL  !Kills selected elements
allsel,all  !Selects all elements
******************************************************************************
/POST1
eusort

*get,minnum,elem,0,num,min
*get,maxnum,elem,0,num,max
life = initiation
numelements = maxnum-minnum  !Value used to dimension future vectors

*dim,Dr,,numelements+1  !Allocating size of residual vector
*vfill,Dr,Ramp,1,0  !Creating residual damage vector

Etable,Dac
SADD,Dac,Dc,,initiation,,  !Damage accumulated for initiation
Etable,Dr
*vput,Dr(1,1),elem,1,etab,Dr
SADD,Dr,Dr,Dac,,-1,  !Calculating the residual damage from 1st run
*dim,Dac,,numelements+1  !Allocating size of dcycle1 vector
*vget,Dac,elem,1,etab,Dac  !Creating accumulated damage vector
*vget,Dr,elem,1,etab,Dr  !Creating residual damage vector

!*dim,life,,numelements+1
!*dim,oldorder,,numelements+1
!*dim,dsort,,numelements+1
!*dim,posttab,array,numelements+1,2,,element number,dcycle
!*voper,life,residual,div,dcycle1

Finish

/filename,Fname_2(1,1,1),1
save,,,,MODEL
q=1
******************************************************************************
SADD,Dc,,totlife,0,0,0  !Calculating initial damage for initiation
******************************************************************************
/totlife=0
count=0

*do,i,1,1,1

/solu
ALLSEL,ALL  !Restore full element set
outres,erase
antype,static,new  !Specifies new static analysis
FDELE, all, FZ
DDELE, all,

******************************************************************************
ptime1=1

time, ptime1

NROPT, FULL, ON

INLGEOM, ON

NSEL, S, NODE,, 75693, 75707, 2
NSEL, A, NODE,, 75711, 75717, 2
NSEL, A, NODE,, 75692, 75692, 0
NSEL, A, NODE,, 75709, 75709, 0
NSEL, A, NODE,, 75720, 75722, 2
D, all, UX, UY, UZ,

DL, 34, 2, UZ,
DL, 43, 6, UZ,
DL, 46, 4, UZ,
DL, 18, 22, UZ,

******************************************************************************
ptime1=2

time, ptime1

NSEL, S, NODE,, 75693, 75707, 2
NSEL, A, NODE,, 75711, 75717, 2
NSEL, A, NODE,, 75692, 75692, 0
NSEL, A, NODE,, 75709, 75709, 0
NSEL, A, NODE,, 75720, 75722, 2
D, all, UX, UY, UZ,
NL,101,3,UZ,,

!Selecting lines for constraints

NSEL,S,NODE,,14920,14920,0
NSEL,A,NODE,,14926,14936,2
NSEL,A,NODE,,14995,15003,2
F,all,FZ,F2

!Apply the load, distribution

outres,all
!lswrite
allsel,all
solve

finish

!*******************************************************************************
/Post1

Allsel,ALL

SET,,,,1
ETABLE,S1,S,EQV

SET,,,,2
ETABLE,S2,S,EQV

eusort

Etable,Dac
*vput,Dac(1,1),elem,1,etab,Dac
Etable,Dr
*vput,Dr(1,1),elem,1,etab,Dr

SADD,AMP,S1,S2,,.5,.5,,

*IF,AMP,GE,5.00E04,THEN

SADD,N,AMP,A1,-1,1,,
SEXP,Q,B1,,1,
SMULT,Z,N,Q,\

*ELSEIF,AMP,LT,2.60E04

SADD,N,AMP,A2,-1,1,,
SMULT,P,N,,25,\
SEXP,Q,B2,,1,
SMULT,Z,P,Q,\

*ELSE

SADD,N,AMP,A3,-1,1,,
SEXP,Q,B3,,1,
SMULT,Z,N,Q,\

*ELSEIF,AMP,LT,2.60E04

SADD,N,AMP,A4,-1,1,,
SMULT,P,N,,25,\
SEXP,Q,B4,,1,
SMULT,Z,P,Q,\

*ELSE

SADD,N,AMP,A5,-1,1,,
SEXP,Q,B5,,1,
SMULT,Z,N,Q,\

*ELSEIF,AMP,LT,2.60E04

SADD,N,AMP,A6,-1,1,,
SMULT,P,N,,25,\
SEXP,Q,B6,,1,
SMULT,Z,P,Q,\

*ELSE

SADD,N,AMP,A7,-1,1,,
SEXP,Q,B7,,1,
SMULT,Z,N,Q,\

*ELSE

SADD,N,AMP,A8,-1,1,,
SMULT,P,N,,25,\
SEXP,Q,B8,,1,
SMULT,Z,P,Q,\

*ELSE

SADD,N,AMP,A9,-1,1,,
SEXP,Q,B9,,1,
*ENDIF

SEXP,Nf,exp,,Z,,, !Raising value exp to value calculated in previous steps to get Nf
SEXP,Dc,Nf,-1,,, !Calculating Damage per cycle
!SEXP,N,AMP,,x, !Raising AMP to exponent -9.3370
!SMULT,Nf,N,,C, !Multiplying N table by constant 6.2874E47 to get Nf
!SEXP,Dc,Nf,-1 !Damage per cycle for successive runs
SMULT,Nf,Nf,Dr,,, !Calculating Number of cycles to failure for cycle 2
ESORT,ETAB,Nf,1,,, ESEL,S,ETAB,Nf,1,200000,,, !Select all elements with cycles to failure !between 1 and 100000
ssum !Sums up all values of selected elements
*get,sumation,ssum,0,item,Dc !Creating a vector from the summation of the elements
*get,numelem,elem,0,count
avglife=1/(sumation/numelem) !Calculating avg life with newly created vectors

********************************************************************************
/SOLU

ekill,ALL !Kills selected elements
allsel,all !Selects all elements
*********************************************************************************/post1
eusort

SMULT,D,Dc,,avglife,, !Damage for successive runs
SADD,Dac,D,Dac,,, !Accumulated Damage
SADD,Dr,Dr,Dac,-1,,, !Residual Damage for successive runs
!*dim,Dac,,numelements+1 !Allocating size of dcycle1 vector
!*dim,Dr,,,numelements+1 !Allocating size of residual vector
*vgget,Dr,elem,1,etab,Dr !Creating the residual vector
*vgget,Dac,elem,1,etab,Dac
totlife=totlife+avglife
count=count+1

finish

/filename,Fname_2(q+1,1,1),1 save,,,MODEL
q=q+1

*endo
totlife=totlife+totlife
Num_runs=count