A Generalization of Thin-Airfoil Theory for Infinite Wings with Sweep

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Mission Statement

1. Publish fundamental aerodynamic principles.
2. Produce innovative leaders in the aerospace industry.
3. Develop world-class aerodynamic analysis, design, and optimization tools.
Infinite Wings

• Models of infinite wings result in 2D airfoil section properties that can be used in models of 3D finite wings

• Swept wings are creating from “shearing” straight wings
  (Kuchemann and Weber (1953); Kuchemann (1978); Rand and Rosen (1985))

Definition: Infinite Wing
A wing that extends to \( \pm \infty \) along the “span-wise” axis \((z)\)
Thin-Airfoil Theory: Model

• Thin-Airfoil Theory models infinite wings with a sheet of infinite vortices on the camber line of the wing.

• Vortex strength distribution is set to perturb the inviscid flow so the airfoil surface is a streamline of the flow.

• The section lift is found by:

\[ C_L = \frac{2\Gamma}{V_\infty c} \sin(\psi) \rightarrow \frac{C_L'}{C_L} = \cos(\Lambda) \]
Thin-Airfoil Theory: Assumptions

• The assumption $\Gamma' = \Gamma$ requires:
  • Airfoils with negligible thickness
  • Small angles of attack and sweep angles
  • The same airfoil geometry “seen” by the flow for the swept and un-swept wing

**GOAL:** Relax the assumptions made by Thin-Airfoil Theory to create a more accurate model of infinite wings with sweep

$$\Gamma' = \Gamma \rightarrow \frac{C_L'}{C_L} = \cos(\Lambda)$$
Effective Freestream: Magnitude

• Only the flow normal to the vortices is perturbed by the circulation strength

• Span-wise flow does not affect the section lift

\[ V_n = V_\infty \frac{\sqrt{\cos^2(\alpha) \cos^2(\Lambda - \beta) + \sin^2(\alpha) \cos^2(\beta')}}{\sqrt{1 - \sin^2(\alpha) \sin^2(\beta')}} \]
Effective Freestream: Angle of Attack

- By rotating the effective freestream, the effective angle of attack is increased

\[ \alpha' = \tan^{-1} \left( \frac{\tan(\alpha) \cos(\beta)}{\cos(\Lambda - \beta)} \right) \]

**Definition: Angle of Attack**
The angle the freestream makes with the \(x\)-\(z\) plane

**Definition: Side-slip**
The angle the freestream makes with the \(x\)-\(y\) plane
Effective Airfoil: Geometry

• As the infinite wing is swept, the effective airfoil geometry is scaled along its x-axis
Effective Airfoil: Circulation

Thin-Airfoil Theory:

$$C'_L = \frac{2\Gamma}{V_\infty c} \cos(\Lambda)$$

Generalized Prediction:

$$C'_L = \mathcal{R} \frac{2\Gamma}{V_\infty c}$$

where

$$\mathcal{R} = \frac{\sqrt{\cos^2(\alpha) \cos^2(\Lambda - \beta) + \sin^2(\alpha) \cos^2(\beta)}}{\cos(\Lambda) \sqrt{1 - \sin^2(\alpha) \sin^2(\beta)}} \cdot \frac{\Gamma'}{\Gamma}$$

- Calculate $\Gamma'$
  - Vortex Panel Method
  - Computational Fluid Dynamics (CFD)

- Approximate $\frac{\Gamma'}{\Gamma}$
  - Conformal Mapping
  - Curve Fit Airfoil Data

Future Work
Thicknss: Vortex Panel Method

• The Vortex Panel Method approximates a bound vortex sheet on the airfoil surface with a finite number of vortex panels.
Vortex Panel Method: Inputs

Freestream Magnitude
Angle of Attack
Airfoil Geometry

Γ′
Vortex Panel Method: Effectiveness

*CFD cases were inviscid, incompressible simulations, computed using OpenFOAM at three mesh densities to ensure grid convergence.
Conclusions

• **Thin-Airfoil Theory** predicts the section lift of an infinite wing to decrease as the cosine of the sweep angle.

• By *relaxing the assumption* that an infinite wing section produces the same circulation regardless of sweep angle, a more accurate model of section lift was formulated.

• The generalized prediction of the section lift will be used to *increase the accuracy of 3D finite wing models* and optimization routines.

• The next step is to *find accurate approximations* for the change in section circulation with sweep, using *analytical* and *empirical* methods.
Questions
References