A Generalization of Thin-Airfoil Theory for Infinite Wings with Sweep

Jackson T Reid
Utah State University
1. Publish fundamental aerodynamic principles.

2. Produce innovative leaders in the aerospace industry.

3. Develop world-class aerodynamic analysis, design, and optimization tools.
Infinite Wings

• Models of infinite wings result in **2D airfoil section properties** that can be used in models of 3D finite wings

• Swept wings are **creating from “shearing” straight wings**
  
  (Kuchemann and Weber (1953); Kuchemann (1978); Rand and Rosen (1985))

**Definition: Infinite Wing**
A wing that extends to ±∞ along the “span-wise” axis (z)
Thin-Airfoil Theory: Model

• Thin-Airfoil Theory models infinite wings with a sheet of infinite vortices on the camber line of the wing.

• Vortex strength distribution is set to perturb the inviscid flow so the airfoil surface is a streamline of the flow.

• The section lift is found by:

\[ C_L = \frac{2\Gamma}{V_\infty c} \sin(\psi) \rightarrow \frac{C_L'}{C_L} = \cos(\Lambda) \]
Thin-Airfoil Theory: Assumptions

• The assumption $\Gamma' = \Gamma$ requires:
  • Airfoils with negligible thickness
  • Small angles of attack and sweep angles
  • The same airfoil geometry “seen” by the flow for the swept and un-swept wing

**GOAL:** Relax the assumptions made by Thin-Airfoil Theory to create a more accurate model of infinite wings with sweep

\[ \Gamma' = \Gamma \rightarrow \frac{C_L'}{C_L} = \cos(\Lambda) \]
Effective Freestream: Magnitude

- Only the flow normal to the vortices is perturbed by the circulation strength.
- Span-wise flow does not affect the section lift.

\[ V_n = V_\infty \frac{\sqrt{\cos^2(\alpha) \cos^2(\Lambda - \beta) + \sin^2(\alpha) \cos^2(\beta')}}{\sqrt{1 - \sin^2(\alpha) \sin^2(\beta')}} \]
Effective Freestream: Angle of Attack

- By rotating the effective freestream, the effective angle of attack is increased

\[ \alpha' = \tan^{-1} \left( \frac{\tan(\alpha) \cos(\beta)}{\cos(\Lambda - \beta)} \right) \]

**Definition:** Angle of Attack
The angle the freestream makes with the x-z plane

**Definition:** Side-slip
The angle the freestream makes with the x-y plane
Effective Airfoil: Geometry

• As the infinite wing is swept, the effective airfoil geometry is scaled along its x-axis.
Effective Airfoil: Circulation

Thin-Airfoil Theory:

\[ C_L' = \frac{2\Gamma}{V_\infty c} \cos(\Lambda) \]

Generalized Prediction:

\[ C_L' = R \frac{2\Gamma}{V_\infty c} \]

\[ R = \frac{\sqrt{\cos^2(\alpha) \cos^2(\Lambda - \beta) + \sin^2(\alpha) \cos^2(\beta)}}{\cos(\Lambda) \sqrt{1 - \sin^2(\alpha) \sin^2(\beta)}} \cdot \frac{\Gamma'}{\Gamma} \]

• Calculate \( \Gamma' \)
  • Vortex Panel Method
  • Computational Fluid Dynamics (CFD)

• Approximate \( \frac{\Gamma'}{\Gamma} \)
  • Conformal Mapping
  • Curve Fit Airfoil Data

Future Work
Thickness: Vortex Panel Method

• The Vortex Panel Method approximates a bound vortex sheet on the airfoil surface with a finite number of vortex panels
Vortex Panel Method: Inputs

- Freestream Magnitude
- Angle of Attack
- Airfoil Geometry

\[ \Gamma' \]
Vortex Panel Method: Effectiveness

*CFD cases were inviscid, incompressible simulations, computed using OpenFOAM at three mesh densities to ensure grid convergence.
Conclusions

• Thin-Airfoil Theory predicts the section lift of an infinite wing to decrease as the cosine of the sweep angle

• By relaxing the assumption that an infinite wing section produces the same circulation regardless of sweep angle, a more accurate model of section lift was formulated

• The generalized prediction of the section lift will be used to increase the accuracy of 3D finite wing models and optimization routines

• The next step is to find accurate approximations for the change in section circulation with sweep, using analytical and empirical methods
Questions
References