A Monte Carlo Study on the Persistence of Variance with Garch

Aristides Romero Moreno
Utah State University

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A MONTE CARLO STUDY ON THE PERSISTENCE OF VARIANCE WITH GARCH

by

Aristides A. Romero Moreno

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Financial Economics

Approved:

Dr. Tyler Brough
Major Professor

Dr. Devon Gorry
Committee Member

Dr. James Feigenbaum
Committee Member

Dr. Mark McLellan
Vice President for Research and Dean of the School of Graduate Studies

UTAH STATE UNIVERSITY
Logan, Utah
2016
ABSTRACT

The GARCH model is widely used to forecast volatility for economic and financial data. There are, however, several shortcomings of using the simple GARCH estimator alone for forecasting volatility. The major issue with the use of the default GARCH model is the persistence of variance that evolves through time and the simple GARCH model fails to address. This paper looks at the GARCH(1,1) model and consistent with Lamoureux and Lastrapes (1990), finds that it overstates the persistence of variance due to model misspecification, specifically the lack of structural shifts.

by

Aristides A. Romero Moreno, Master of Arts

Utah State University, 2016

Major Professor: Dr. Tyler Brough
Department: Economics and Finance

(20 pages)
PUBLIC ABSTRACT

A Monte Carlo Study on the Persistence of Variance with GARCH models

This paper performs a Monte Carlo study to test Lamoureux and Lastrapes (1990)’s hypothesis on the persistence of variance present in the GARCH volatility model.
ACKNOWLEDGMENTS

I would like to thank Dr. Tyler Brough for his guidance and aid throughout this process. I would specially like to thank my committee members: Dr. Devon Gorry, for her helpful comments and support as well as her passion in teaching Econometrics inspired me into pursuing the field further along with other statistical techniques and computer programs; Dr. James Feigenbaum, I always enjoyed a challenge. It was most enjoyable and instructive. I am honored to have them as committee members. Thank you to their support and assistance through-out the entire process.

I give special thanks to my family, friends (near and far), and colleagues for their encouragement, moral support, and patience as I worked my way from the initial proposal writing to this final document. I could not have done it without all of you.

Aristides A. Romero Moreno
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I. INTRODUCTION

Homo-skedasticity, an assumption of the classical linear regression model that the variance of the errors is constant is improbable for financial time series data. If the opposite is true, the variance of the errors is nonconstant, also known as heteroskedasticity, the standard errors will be incorrect. Further, for time series data, the volatility tends to cluster over time. That is, there is a correlation between volatility and its level for subsequent periods. Large changes in asset prices are usually followed by large changes and small changes are usually followed by small changes, regardless of the sign. Therefore, it is recommended to assume heteroskedasticity and instead use a model that describes how the variance of the errors changes over time. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) is one of such models.

The GARCH model is widely used to forecast volatility for economic and financial data. There are, however, several shortcomings with using the simple GARCH estimator for forecasting volatility. The major issue with the use of the default GARCH model is the persistence of variance that evolves through time, which the simple GARCH model fails to address.1 The purpose of this paper is to use Lamoureux and Lastrapes (1990) study as an exercise on econometric methods for estimating variance and to study the shortcomings of using the simple GARCH model over long periods of time.

Lamoureux and Lastrapes (1990) used evidence from daily returns – based on the previous day closing price – on a sample of 30 randomly selected stocks and a stock index

---

(from January 1, 1963 to November 13, 1979) to show that due to deterministic structural shifts, the persistence of variance in the simple GARCH model is overstated. To support this hypothesis, they estimated the generalize variance equation and conduct a Monte Carlo simulation with two scenarios: one that accounts for the structural shifts and one that ignores it. In the generalized equation, Lamoureux and Lastrapes (1990) account for the structural shifts by including 14 dummy variables based following criteria: for observations ranging \((1 + 302i)\) to \([(1 + i)302]\), \(D_{it}(i = 1, \ldots, 13)\) is 1, 0 otherwise. For the Monte Carlo study, the dummy variables are arbitrary. In each case, their study found supporting evidence that when discrete shifts in the unconditional variance are not accounted for, the estimates of the persistence of variance in the GARCH model can be misinterpreted – the estimates are overstated.

In this paper, I apply the methods and parameters (with a slight modification) of Lamoureux and Lastrapes (1990) to the daily returns – based on the previous day closing price –of the the 30 constituents of the Dow Jones Industrial Average\(^2\) from January 3, 2007 to December 31, 2015.\(^3\) I estimate the generalize variance equation and conduct the Monte Carlo simulation with the same two scenarios: one scenario accounts for structural shifts and the other ignores it. For both, the generalized equation and the Monte Carlo Simulation, I rely on the \textit{garchFit} and \textit{ugarch} functions in the \textit{fGarch}\(^4\) and \textit{rugarch}\(^5\) packages in R to account for the structural shifts, which are commonly used in the Economics and Finance Literature.\(^6\) These functions optimize the discrete shifts to better fit the data. I find that the results remain

\(^2\) Constituents of the Dow Jones Industrial Average as of January 10, 2016.
\(^3\) An index is not included because these stocks already compose the index.
consistent. The GARCH model overestimates the persistence of variance due to time model specification, specifically the lack of structural shifts. These structural shifts confound the volatility persistence by diminishing the degree of ARCH parameters in long time series.7

An understanding volatility and how to estimate it correctly is pivotal for accurate forecasts of economic and financial data. Volatility, as a measure of dispersion and uncertainty, is widely used as gauge for risk. Arguably, the effects of volatility are more readily observed in the financial markets. Volatility causes wild swings in asset prices and returns; and thus, it is no surprise that volatility is one of the most important factors in asset and option pricing models. Similarly, policy makers observe closely factors that could trigger volatility in the financial markets since it can have spillover effects on the national and global economy, such as the market tumult that followed the devaluation of the Chinese yuan or renminbi on August 24th, 2015. Therefore, learning and examining common issues with one of the most common volatility estimators, GARCH, serves as a comprehensive exercise in time series econometric methods and how to address the persistence of variance that when estimating volatility over long periods of time.

To verify the robustness of these results, this paper is structured as follows: section 2 discusses a review of the literature; section 3 explains the GARCH(1,1) model specification and the restricted and unrestricted models proposed by Lamoureux and Lastrapes (1990); section 4, replicates their study using the constituents of the Dow Jones Industrial Average (DJIA). To avoid the dealing with the 2008-2009 recession this paper focuses on data from 2007 to 2015,

7 Ibid.
half the time in Lamoureux and Lastrapes (1990) paper. Their study, however, contains data from the 60’s and 70’s; and thus, this study was due for an update. Section 5, presents the results of the Monte-Carlo study to test the robustness of the empirical results across the two models; the major findings are discussed in section 6; and finally section 7, summarizes the study.

II. LITERATURE REVIEW

The Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Conditional Heteroskedasticity (GARCH) models developed by Engle (1982)\textsuperscript{8} and Bollerslev (1986)\textsuperscript{9} have grown to be of great importance in the analysis of time series data and volatility estimation. However, this study shows that when applying the GARCH model to time series data that span a significant time period, there will be a “high persistence…. Because of the presence of deterministic shifts in the unconditional model.”\textsuperscript{10} Therefore, if true, it is important for the researcher to pay special attention to these structural shifts when working with a long time series.

Many estimators have been developed that address the persistence in variance issues discussed by Lamoureux and Lastrapes (1990). For instance, Baillie and Bollerslev (1996) worked on the “Fractionally integrated generalized autoregressive conditional heteroskedasticity”, in which they claim the persistence of variance decays over time due to the influence of “lagged squared innovations” Bollerslev et al. (1992). This paper assumes that the

\textsuperscript{8} Autoregressive Conditional Heteroskedasticity (ARCH).
\textsuperscript{9} Generalized Autoregressive Conditional Heteroskedasticity (GARCH)
\textsuperscript{10} Lamoureux and Lastrapes: Structural Change and GARCH (1990)
persistence in variance decays over time, but it never fades out and it is also influenced by structural shifts; these results are consistent with this paper’s hypothesis on the persistence of variance.

Lamoureux and Lastrapes (1990) attribute the persistence of variance to time-varying factors. To prove their hypothesis, they specify two GARCH(1,1) models: a restricted and an unrestricted model. In the restricted model, they estimate a GARCH(1,1) model without any structural shifts; then, they include 13 dummy variables among various subsamples arbitrarily to account for the structural shifts. They argue the subsamples allow for structural shifts while maintaining sensible statistical inference. Lamoureux and Lastrapes (1990) apply both models to a sample of 30 randomly selected publicly listed companies and test the persistence of variance of each of them. To prove the robustness of their results, they conduct a Monte Carlo Analysis. The results are shown in section 7. Lamoureux and Lastrapes (1990) conclude by maintaining their hypothesis of high persistence of variance for long time series stock return data. Their original paper on this topic in 1990 has been widely discussed in the literature, see Bollerslev et al. (1992); Baillie et al. (1996); Hamilton and Susmel (1994); and Lamoureux and Lastrapes (1990) for a review of the literature.

Lamoureux and Lastrapes (1990) claim that the shocks to the variance using GARCH are persistent due an autoregressive moving average (ARMA) structure that happens to the residuals squared. They acknowledge Engle and Bollerslev (1986)’s paper aiming to address the high persistence of variance, so-called the integrated-GARCH (I-GARCH). Nonetheless, they dismiss it citing that it lacks theoretical motivation.
III. GARCH (1,1) MODEL SPECIFICATION

The ARCH model was first introduced by Engle in 1982 and later generalized by Bollerslev in 1986 as the GARCH model. From this point, the GARCH model has been widely used in its GARCH(1,1) form. The notation (1,1), also known as ARCH terms, indicates the number of autoregressive lags and the moving average lags, respectively.

The GARCH model considered in this paper has the following form:

\[ y_t = x_t \beta + \epsilon_t, \]
\[ (\epsilon_t | \epsilon_{t-1}, \epsilon_{t-2}, \ldots) \sim N(0, h_t), \]

and

\[ h_t = \omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \cdots + \alpha_p \epsilon_{t-p}^2 + \gamma_1 h_{t-1} + \cdots + \gamma_q h_{t-q}, \]

where \( x_t \) is an exogenous variable and \( \epsilon_t \) is a random error, and \( p=q=1 \).

The GARCH\((p, q)\) model in the previous three equations permits the conditional variance of the error term to depend in a linear way with previous squared errors. Lamoureux and Lastrapes (1990) argue that using the above model implies that the squared residuals are generated by an ARMA process. While there are time series methods, developed by Bollerslev
(1996) for instance, to find the optimal values of $p$ and $q$, the GARCH(1,1) model has become the default for most financial times series.

Using their convention the GARCH(1,1) model becomes:

$$h_t = \omega + \lambda h_{t-1} + \alpha v_{t-1}, \quad (4)$$

$$v_{t-1} \equiv \epsilon_{t-1}^2 - h_{t-1}$$ is a serially uncorrelated innovation

$$\lambda \equiv (\alpha + \gamma)$$, measures the persistence of variance

Then by eliminating the $h_{t-1}$ in the right side of equation (4), the expression for the conditional variance is,

$$h_t = \sigma^2 + \alpha [v_{t-1} + \lambda v_{t-2} + \lambda^2 v_{t-3} + \lambda^3 v_{t-4} + \cdots], \quad (5)$$

$$\sigma^2 \equiv \omega / [1 - \lambda]$$, is the unconditional variance

From this (5), the dependence of the persistence of volatility shocks ($v_t$) on the sum of the GARCH parameters, $\lambda$, becomes obvious. For, $\lambda = 1$, the process is said to be an I-GARCH process (Engle and Bollerslev 1986), or integrated in variance. This means that the lag polynomial of conditional variance has a unit root process. In this particular case, shocks to the variance do not decrease over time and the unconditional variance does not exist, Lamoureux and Lastrapes (1990).

According to Lamoureux and Lastrapes (1990), the sum of GARCH parameters approaches one, generally, when using daily stock data spanning long periods of time. They
argue that these estimates are overstated because they do not account for structural shifts over
time. This paper seeks to replicate and verify findings.

They claim there is a model misspecification when using the simple GARCH model and
propose to include dummy variables for estimating the general model. Their specification
follows,

$$\begin{align*}
h_t &= \omega' + \delta_1 D_1 + \cdots + \delta_k D_{kt} + \lambda h_{t-1} + \alpha v_{t-1} + \alpha v_{t-1}, \\
D_{it}(i = 1, \ldots, k) &\text{ are dummy variables for periods where the GARCH is stationary.}
\end{align*}$$

Equation (6) specifies \(k+1\) periods. This paper estimates equation (4), the restricted
model, and then compares it to equation (6), unrestricted model.

One of the most important points to address is how to choose the timing of the structural
shifts. Given the available econometric tools available, this paper used a method that optimizes
the number of dummy variables. The packages \textit{fGarch}\textsuperscript{11} and \textit{rugarch}\textsuperscript{12} in R provide an algorithm
for these purposes. Therefore, this paper departs slightly from Lamoureux and Lastrapes (1990)
and instead of building an algorithm from scratch, this paper relies on the \textit{garchFit} and \textit{ugarch}
functions in the \textit{fGarch} and \textit{rugarch} packages in R. While dummy variables are not reported, the
function accounts for these time-varying factors.

Test statistics are calculated taking into consideration the non-normality of the standard
errors. Additionally, robust standard errors are used for every estimate of the parameters.

\textsuperscript{11} Diethelm Wuertz and Yohan Chalabi. 2013. \textit{Rmetrics - Autoregressive Conditional Heteroskedastic Modelling}. R
package version 3010.82.

\textsuperscript{12} Alexio Ghalanos. 2014. \textit{Univariate GARCH Models}, R package version 1.3-5.
The test conducted determines whether or not the persistence of variance, $\lambda$, in both the restricted and unrestricted models is equal against the alternative that there is less persistence of variance in the unrestricted model. The immediate test that comes to mind to test the hypothesis is a likelihood ratio test. Nonetheless, the R functions used limit the linear hypothesis testing.

IV. GARCH (1,1): APPLICATION TO DOW JONES (DJIA) CONSTITUENTS

Table 1 shows the components of the stocks in the Dow Jones Industrial Average. The information presented from these 30 stocks was obtained from the Center for Research in Security Prices (CRISP) at the Wharton Research Data Services database. The information in table 1 contains the permanent number assigned in CRISP, the stock symbol (ticker), and CUSIP.

For each stock this paper estimates equation (4), the restricted model, and compares it with equation (6), the unrestricted model. The returns used in the models are the arithmetic returns obtained from CRISP from January 03, 2007 to December 15, 2015.

Table 1. Companies in the DJIA\textsuperscript{13}, used in estimation of the GARCH model

<table>
<thead>
<tr>
<th>PERMNO</th>
<th>Ticker</th>
<th>Company Name</th>
<th>CUSIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>10107</td>
<td>MSFT</td>
<td>MICROSOFT CORP</td>
<td>59491810</td>
</tr>
<tr>
<td>11308</td>
<td>KO</td>
<td>COCA COLA CO</td>
<td>19121610</td>
</tr>
<tr>
<td>11703</td>
<td>DD</td>
<td>DU PONT E I DE NEMOURS &amp; CO</td>
<td>26353410</td>
</tr>
<tr>
<td>11850</td>
<td>XOM</td>
<td>EXXON MOBIL CORP</td>
<td>30231G10</td>
</tr>
<tr>
<td>12060</td>
<td>GE</td>
<td>GENERAL ELECTRIC CO</td>
<td>36960410</td>
</tr>
<tr>
<td>12490</td>
<td>IBM</td>
<td>INTERNATIONAL BUSINESS MACHS CC</td>
<td>45920010</td>
</tr>
</tbody>
</table>

\textsuperscript{13} Dow Jones Industrial Average (DJIA), stocks that are component of the Index as of January 24, 2016.
The parameters \( \alpha \) and \( \gamma \) are optimized using the package \textit{rugarch} in R\textsuperscript{14}. The \textit{ugarch} function in the \textit{rugarch} package is widely used in the Economics and Finance literature to maximize the likelihood function. To allow for a direct comparison, Figure 1 presents Lamoureux and Lastrapes (1990) results. The results in table 2 are consistent with Lamoureux and Lastrapes (1990), see figure 1. The estimated models show an average persistence of variance of 0.991, as measured by \( \lambda = \alpha + \gamma \). The half life (HL) of the volatility of shocks, which depends only on \( \lambda \), is computed to provide some perspective on the persistence of

\textsuperscript{14}Alexio Ghalanos. 2014. Univariate GARCH Models, R package version 1.3-5
variance and measures the number of days a shock to volatility decays to half its original size,

\[ HL = 1 - \left(\frac{\log(2)}{\log(\lambda)}\right). \]

Unlike Lamoureux and Lastrapes (1990) this paper does not perform a bootstrap estimation due to computing limitation. The purpose of this study is to understand volatility and how to estimate and test it using Monte Carlo simulations. It is now widely accepted in the literature that structural breaks are needed in the GARCH model. Therefore, since the results of this study show the same statistical results than Lamoureux Lamoureux and Lastrapes (1990), it relies on their bootstrap to claim statistical difference.

For every stock in the Dow Jones Industrial Average (DJIA) the persistence of variance is overstated in the restricted model. All but 6 parameters for alpha are not significant at the 5% level, which would theoretically lower the estimated persistence of value calculation. Thus, the persistence of variance is not the same regardless of structural shifts. These results are consistent with Lamoureux and Lastrapes (1990). Due to the lack of time-varying structural shifts, the restricted GARCH(1,1) model overstates the persistence of variance.

V. MONTE CARLO STUDY

To check the robustness of our results, a Monte Carlo study is conducted. The objective is to replicate the results obtained by Lamoureux and Lastrapes on the nature of the misspecification bias by controlling for extraneous factors.
I simulate the GARCH(1,1) process with the following parameters:

\[ \beta_0 = -0.15, \beta_1 = 0.01, \alpha = 0.15, \gamma = 0.45, \epsilon_0 = 0. \]

The dummy variables are applied automatically by the \textit{fGarch} function in R, thus, no specification was used (to Lamoureux and Lastrapes specified the dummies as \( \delta_1 = 20, \delta_2 = -10 \) in the first environment and \( \delta_1 = -10, \delta_2 = -15 \) in the second environment). Since the dummy variables are applied automatically, only one environment is considered, as opposed to Lamoureux and Lastrapes (1990) who considered two environments.

To simulate equations (4), (the restricted model) and (6), (the unrestricted model), one thousand simulations are performed in one environment.

Figure 1. Lamoureux and Lastrapes (1990): Persistence in Variance for 30 stocks: Restricted and Unrestricted GARCH(1,1) Specifications
Table 2. Persistence of Variance for DJIA Stocks, GARCH (1,1) Specifications

<table>
<thead>
<tr>
<th>Company</th>
<th>Restricted Model</th>
<th></th>
<th>Unrestricted Model</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ticker</td>
<td>( \alpha )</td>
<td>( \gamma )</td>
<td>( \lambda )</td>
<td>HL</td>
<td>% CV</td>
</tr>
<tr>
<td>---------</td>
<td>------------------</td>
<td>---</td>
<td>-------------------</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>MSFT</td>
<td>0.0065**</td>
<td>0.9925**</td>
<td>0.999</td>
<td>692.79</td>
<td>0.0491**</td>
</tr>
<tr>
<td>KO</td>
<td>0.0000**</td>
<td>0.9999**</td>
<td>0.999</td>
<td>692.79</td>
<td>0.1003**</td>
</tr>
<tr>
<td>DD</td>
<td>0.0040</td>
<td>0.9949**</td>
<td>0.9989</td>
<td>678.42</td>
<td>0.1458**</td>
</tr>
<tr>
<td>XOM</td>
<td>0.0214**</td>
<td>0.9775**</td>
<td>0.999</td>
<td>692.77</td>
<td>0.0886**</td>
</tr>
<tr>
<td>GE</td>
<td>0.0062**</td>
<td>0.9927**</td>
<td>0.9989</td>
<td>668.38</td>
<td>0.1133**</td>
</tr>
<tr>
<td>IBM</td>
<td>0.0067**</td>
<td>0.9923**</td>
<td>0.9989</td>
<td>673.52</td>
<td>0.1189**</td>
</tr>
</tbody>
</table>

**HL is the half-life of variance; HL = 1 - (log 2) / log \( \lambda \). The bootstrap 5% critical value is denoted by 5% CV.

* Because \( \gamma \) exceeded 1 in this case, HL cannot be interpreted. Note that HL approaches infinity as \( \gamma \) approaches 1, so persistence should be considered large for this company.

† The mean HL is obtained by applying the half-life formula to the mean of \( \lambda \).
| Symbol | CVX | AAPL | UTX | PG | CAT | BA | PFE | JNJ | MMM | MRK | DIS | MCD | JPM | WMT | NKE | AXP | INTC | TRV | VZ | HD | CSCO | GS | V | UNH | Index |
|--------|-----|------|-----|----|-----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|-----|-----|-----|-----|-----|-----|-----|
|        | 0.0094** | 0.0002** | 0.0006 | 0.0109** | 0.0018 | 0.0036** | 0.000 | 0.0029 | 0.0054** | 0.0084** | 0.0001** | 0.0208** | 0.0042** | 0.0050** | 0.0151** | 0.0053** | 0.0131** | 0.0103** | 0.0092** | 0.0176** | 0.0173** | 0.0099** | 0.0072** | 0.0098** | 0.0111 |
|        | 0.9895** | 0.9990** | 0.9989 | 0.9880** | 0.9971** | 0.9953** | 0.999 | 0.9961** | 0.9936** | 0.9906** | 0.9989 | 0.9781 | 0.9947** | 0.9939** | 0.9838 | 0.9937** | 0.9859** | 0.9887** | 0.9897** | 0.9814 | 0.9816** | 0.9890** | 0.9890 | 0.9888 | 0.9877* |
|        | 0.9989 | 0.9999 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 |
|        | 687.99 | 682.04 | 687.33 | 692.60 | 690.59 | 688.77 | 692.79 | 692.65 | 692.73 | 692.67 | 691.08 | 692.71 | 664.07 | 692.70 | 691.63 | 684.96 | 692.60 | 691.64 | 689.44 | 691.26 | 672.69 | 689.98 | 688.36 | 590.29 |
|        | 0.0940** | 0.0869** | 0.0622** | 0.1344** | 0.0641** | 0.073** | 0.0642 | 0.1472** | 0.0455* | 0.1115** | 0.0891** | 0.0427 | 0.1161** | 0.0396 | 0.0836** | 0.0804* | 0.0509** | 0.0925* | 0.0820** | 0.0820** | 0.0778** | 0.1193** | 0.0509** |
|        | 0.8945** | 0.8848** | 0.9231** | 0.7885** | 0.9258** | 0.9114** | 0.9239** | 0.8770** | 0.9400** | 0.8584** | 0.8770** | 0.9469** | 0.9927 | 0.9532** | 0.9928 | 0.9118** | 0.9309** | 0.9867** | 0.8933** | 0.8656** | 0.8558** | 0.8656** | 0.8656** |
|        | 0.9967 | 0.9717 | 0.9853 | 0.9231 | 0.9886 | 0.9850 | 0.9981 | 0.9502 | 0.9855 | 0.9717 | 0.9299 | 0.9896 | 0.9969 | 0.9961 | 0.9961 | 0.9548 | 0.9922 | 0.9892 | 0.9892 | 0.9927 | 0.9967 | 0.9336 | 0.9929 | 0.9929 |

Notes: **, * indicate 0.01 (t-stat >1.960), 0.05 (t-stat> 2.576) significance levels, respectively. t-statistics correspond to robust standard errors computed yet omitted due to space limitations. Please see code in the Appendix for replication. HL is the half life of the variance: $HL = 1 - [(\log 2)/\log \lambda]$. The half life was computed using the `halflife` function in R. Rows may not add up due to rounding.

I found that, generally, the results are consistent with Lamoureux and Lastrapes (1990), see figure 2. The persistence of variance measured by $\alpha + \gamma = 0.581$, unrestricted model, against the true value $\lambda = 0.6$. Accounting for structural shifts in the unrestricted model prevents overstating the variance of stock returns in long time series. These results contrast sharply with the restricted model.

In the restricted model $\lambda$ is largely overstated with a value of roughly 0.683. In each parameter in the simulation, in the restricted model overstates the true value, particularly $\alpha$ and $\gamma$. 


as a measure of persistence of variance. These results are consistent with the hypothesis that including structural shifts in the model helps in not overstating the persistence of variance.

Figure 2. Lamoureux and Lastrapes (1990): Monte Carlo Simulation Results

<table>
<thead>
<tr>
<th>Environment 1</th>
<th>Unrestricted model</th>
<th>Restricted model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>-0.162</td>
<td>0.224</td>
</tr>
<tr>
<td></td>
<td>(0.218)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.010</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\omega'$</td>
<td>21.346</td>
<td>8.541</td>
</tr>
<tr>
<td></td>
<td>(6.388)</td>
<td>(4.520)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.146</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.422</td>
<td>0.187</td>
</tr>
<tr>
<td></td>
<td>(0.182)</td>
<td>(0.090)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>21.601</td>
<td>10.005</td>
</tr>
<tr>
<td></td>
<td>(9.128)</td>
<td>(5.278)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-10.629</td>
<td>4.905</td>
</tr>
<tr>
<td></td>
<td>(4.877)</td>
<td>(2.472)</td>
</tr>
</tbody>
</table>

Environment 2

<table>
<thead>
<tr>
<th>Environment 2</th>
<th>Unrestricted model</th>
<th>Restricted model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>-0.148</td>
<td>0.158</td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.099</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\omega'$</td>
<td>21.909</td>
<td>8.873</td>
</tr>
<tr>
<td></td>
<td>(8.993)</td>
<td>(3.949)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.149</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.409</td>
<td>0.195</td>
</tr>
<tr>
<td></td>
<td>(0.194)</td>
<td>(0.082)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-10.831</td>
<td>5.084</td>
</tr>
<tr>
<td></td>
<td>(5.121)</td>
<td>(2.125)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-16.404</td>
<td>6.885</td>
</tr>
<tr>
<td></td>
<td>(6.950)</td>
<td>(3.067)</td>
</tr>
</tbody>
</table>

NOTE: Means and standard deviations are calculated across 1,000 simulations. The numbers in parentheses are the summary statistics calculated for the estimated coefficient standard deviations.

Table 3. Monte Carlo Simulation for Garch(1,1) process

<table>
<thead>
<tr>
<th>Restricted Model</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Unrestricted Model</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>True Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>-0.189</td>
<td>0.147</td>
<td>-0.1867</td>
<td>0.14</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.062</td>
<td>0.257</td>
<td>0.021</td>
<td>0.158</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.527</td>
<td>0.021</td>
<td>0.429</td>
<td>0.065</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>$\omega'$</td>
<td>25.42</td>
<td>5.848</td>
<td>17.657</td>
<td>0.257</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.156</td>
<td>0.047</td>
<td>0.152</td>
<td>0.026</td>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>

Means and Standard Deviations are computed from 1,000 simulations. Dummies are not reported, but are included in the computation using the fGarch package and garchFit function in R.
VI. DISCUSSION OF THE RESULTS

Given the results obtained in section 3 and 4, the initial hypothesis is maintained. The persistence of variance is overstated if time-varying structural shifts are not included in the model. In table 2, demonstrates that the results obtained by Lamoureux and Lastrapes in 1990 still hold today. Date form the 30 components of the Dow Jones show a very large persistence of variance, which this paper argues is overstated. The Monte Carlo simulation supports this hypothesis.

Although a slightly different approach was used to account for dummy variables, the results are essentially the same. This approach, due to the size of our data set, is more appropriate today. Furthermore, the results from the Monte Carlo simulation show that even after controlling for extraneous factors the results remain robust. The simulation parameters in the unrestricted model closely follow the true values, while the parameters in the restricted model are overstated.

VII. CONCLUSION

This study reaffirms the hypothesis tested by Lamoureux and Lastrapes (1992). When dealing with time series data for stock prices spanning for a long time, the persistence of variance estimated by the regular GARCH(1,1) model is overstated due to the lack of structural
shifts misspecification. This paper used market data and performed a Monte Carlo analysis to show consistency with the hypothesis.

This study served as an exercise that aimed at using advanced econometric models and the use of Monte Carlo simulation methods to estimate the persistence of variance in stock prices. The results were successful, not only in replicating Lamoureux and Lastrapes (1990) study, but also applying today’s method for GARCH estimation and obtaining similar results. Further research should focus on reviewing and applying recent GARCH estimators that address the structural misspecification and its practical use for forecasting economic and financial data.

REFERENCES


APPENDIX

Title: R CODE FOR MONTECARLO SIMULATIONS
MODEL SPECIFICATION FOR DOW JONES INDUSTRIAL AVERAGE COMPONENTS

DATA

# Set Directory

#setwd("~/Documents/FALL 2015/Independent Research")

# Unrestricted Model

require(fGarch)

## Read CSV file (header assumed), then put that into "csv.data" data object (any name is ok).

xData = read.csv("masterData.csv")

# Select Stock from DataSet

#returns= subset(xData, ticker=="V")

rets.vec = as.numeric(data.matrix(returns[,4], rownames.force = NA))

# Include dummy interaction

rets.timeSeries = dummyDailySeries(matrix(rets.vec), units = "GARCH11")

require(rugarch)

#### Specify a standard GARCH(1,1) model with mean-equation being a constant.

specU = ugarchspec(variance.model=list(model="sGARCH"),

 mean.model=list(armaOrder=c(0,0)))

### Estimation command

fitU=ugarchfit(spec=specU, data=rets.timeSeries)
show(fitU) #Print the model

persistence(fitU) #Compute the persistence of variance

halflife(fitU) #Compute the half-life

coeff(fitU) #Print coefficients

MODEL SPECIFICATION FOR MONTE CARLO STUDY IN R

Package: fGarch

# A numeric Vector from default GARCH(1,1)

N = 1000

x.vec = as.vector(garchSim(garchSpec(model=list(mu=-0.15, omega=.45, alpha=0.15,
                                gamma=.45, beta=0.01, cond.dist=c("norm"))), n = N)[,1])
garchFit(~ garch(1,1), data = x.vec, trace = TRUE)

# An univariate timeSeries object with dummy dates:

x.timeSeries = dummyDailySeries(matrix(x.vec), units = "GARCH11")
garchFit(~ garch(1,1), data = x.timeSeries, trace = FALSE)