Struggling Students' Use of Representation When Developing Number Sense and Problem Solving Abilities

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STRUGGLING STUDENTS’ USE OF REPRESENTATION WHEN DEVELOPING
NUMBER SENSE AND PROBLEM SOLVING ABILITIES

by

Allison L. Roxburgh

A creative project submitted in partial fulfillment of the requirements for the degree of
Masters in
Elementary Education

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2016
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CHAPTER I

INTRODUCTION

Through my experience I have found students often rely on concrete or pictorial strategies to solve mathematical problems. These strategies are great to build an understanding in mathematical concepts. However, using these strategies becomes a tedious task when working with multi-digit numbers to solve problems involving mathematical operations. For example, a student who relies on drawing base ten blocks to solve three-digit addition problems may experience fatigue, as this is not the most efficient means to solve problems everyday. Through my experience I have found that these strategies may hinder students’ abilities to solve a problem correctly because they focus on their drawing and become overwhelmed with how many blocks they have to draw.

Concrete manipulatives allow students opportunities to manipulate concrete objects, which help build a strong foundational understanding of mathematical concepts, such as place value (Wai Lan Chan, Au, & Tang, 2014). When students use their understanding of place value with concrete manipulatives they are able to extend this understanding in their mental math abilities, which will help them abstractly compute problems correctly (Bobis, 2008). If students are able to abstractly solve a problem they would then be able to mentally compute a problem, instead of having to use concrete objects or draw a picture. This would help students be able to focus on what a problem features instead of focusing on drawing a picture.

The purpose of this study was to help me understand how my students’ flexible engagement with concrete experiences can help construct flexibility abstractly.
Furthermore, I wondered if this flexibility would help improve students’ problem-solving abilities in mathematical experiences. Specifically, the purpose of this project was to determine how third grade students (ages 8-9 years old), identified as struggling, flexibly used their concrete experiences to construct flexible abstract strategies when solving mathematical problems involving addition, subtraction, and estimation. Student flexibility was measured through assessments given that involved story problems and numbers lines. It was also measured by student dialogue (Shumway, 2011; Yang & Wu, 2010), whole class counting routines (Shumway, 2011), and number line tasks (Siegler & Booth, 2004.)

**Research Questions**

This study focused on how flexibility in concrete experiences can influence how flexibility in abstract experiences is developed. Problem solving experiences were also a focus of this study to see how students applied their flexibility in concrete and abstract experiences to solve mathematical problems. This focus led to the development of the research questions:

1- How can struggling students concentrating on mathematical flexibility in concrete experiences help foster their operational flexibility in abstract experiences?

2- Can struggling students’ flexibility in concrete and abstract experiences improve their problem solving abilities?

This study used a quantitative and qualitative approach because these forms of data were collected to show multi-faceted change in students’ mathematical thinking over the course of the project. Quantitative and qualitative data was used to better explore the
research questions and determine multiple perspectives when measuring student learning. This exploration helped understand how teacher instruction and task development can help foster flexibility in concrete and abstract experiences for students.

**Impact of This Study**

Ongoing analyses of data collected in this study informed instructional methods in a third grade classroom intervention and whole class structure. Findings also gave insights into how students were able to use concrete experiences and flexibly expand their conceptual understanding development. The literature and analyses from this study provided guidance for the teacher when planning instruction, which positively influenced student flexibility with concrete and abstract mathematical experiences.
CHAPTER II
LITERATURE REVIEW

Number Sense and Problem Solving Abilities

*Number sense* is a foundational understanding children need in order to be successful in future mathematical experiences (Bobis, 2008; Shumway, 2011; Witzel, Ferguson, & Mink, 2012; Yang & Wu, 2010). Bobis (2008) describes a student with number sense as having, “…a thorough understanding of relationships among numbers and operations-being able flexibly to partition and combine numbers in convenient ways to allow appropriate estimations and mental calculations to be made.” (p. 4). When students lack this ability in mathematics they will generally have low achieving scores and struggle because number sense is “linked to future math achievement.” (Witzel et al., 2012, p. 90). To better understand the importance of number sense and mathematical success, I chose to explore how number sense understanding effects how students use contexts and *representations* (external materials used to help students solve mathematical problems such as language, concrete manipulatives, number lines, etc.) to support their flexibility with numbers.

West (2016) suggests that teacher and student interaction is an effective way to see how number sense and problem solving abilities relate because the teacher is able to see if students truly have an understanding of numbers, or if the students have any misconceptions that need to be corrected. Through these interactions findings from the research field propose that students should be given plenty of time to build an understanding of number sense in order to ensure they make connections between mathematical concepts, such as being able to connect their place value concepts to their
adding and subtracting (Blote, Van der Burg, & Klein, 2001; Ulrich, Tillema, Hackenberg, & Norton, 2014; Bobis, 2008; Witzel et al., 2012; Yang & Wu, 2010).

Research (Fuson et al., 1997; West, 2016; Witzel et al., 2012) found that students constructed flexibility with numbers in concrete experiences, by using physical objects, which leads to flexibility with numbers in abstract experiences because students have constructed and are using mental representations (see Figure 1). Instructional practices that promote this development from concrete to abstract multi-digit number understanding will be the focus throughout this paper (as shown in figure 1).

Development in abstract experiences involves students’ reliance on patterns and relationships among numbers (Blote et al., 2001), counting (Witzel et al., 2012), decomposing numbers, breaking numbers apart, (Bobis, 2008) and dialogue between students and a teacher (Yang & Wu, 2010). A focus of how contexts (real world problems and model drawing) and representations (language, concrete manipulatives, and number lines) are used to support this development will also be discussed.

![Figure 1. Conceptual framework showing the connection between flexibility with concrete experiences and flexibility with numbers in abstract experiences.](image)

**Flexibility with Multi-Digit Numbers**

Upon review of the literature, three major themes were formed to suggest that students rely on three types of representations; 1-identifying patterns and relationships in
numbers 2-composing and decomposing numbers 3-using appropriate mathematical language to help build understanding of numbers. These topics will be discussed to show the importance of number understanding in order to see how strong number sense ability effects how students use context and representations to support their flexibility.

**Patterns and Number Relationships**

Patterns and relationships in numbers are important to students’ number understanding and problem solving abilities. When students can see relationships between numbers they are more eager to solve problems because they are confident in their mathematical abilities (Dougherty, Bryant, Darrough, & Pfannenstiel, 2015). Being able to use number sense abilities in seeing patterns and relationships among numbers allows students to believe mathematics is about understanding a concept, instead of simply following procedures to get the correct answer (Shumway, 2011). Viewing mathematics as understanding concepts can increase students’ level of confidence in their mathematical abilities because they ground their procedures in the concept, which helps them become more flexible in their number operation abilities (Blote et al., 2001). When students are able to see relationships they can compute mentally, which shows that they are developing number sense abilities (Bobis, 2008; Ellemor-Collins & Wright, 2011). The ability to understand patterns and relationships in numbers allows children to progress to more complex mathematical skills.

Teachers can promote students to see patterns in numbers by asking generalized questions, so students can make statements about particular patterns they see. For instance in asking, “Do you see a pattern in these numbers?” would help students predict answers and check to see if those answers make sense (Dougherty et al., 2015). Another
way to increase students’ pattern recognition is to allow students to provide their own procedure when solving a problem. In doing this, students are also demonstrating their conceptual understanding (how students use what they know in new mathematical situations) (Fuson et al., 1997). Patterns and relationships in numbers should be a focus in teachers’ instructional strategies when helping students develop an understanding of numbers.

**Counting**

The representation of counting helps build number sense. Shumway (2011) found that, “Students who struggle with mathematics often lack counting skills.” (p. 56). By planning meaningful activities such as, count around the circle (Shumway, 2011), students are able to build effective counting skills. Counting strategies help improve reasonable operations when solving story problems and help students engage in a mathematical process that they find meaningful (Clements, 1984). Providing students with concrete manipulatives to help them count allows them opportunities to build a one-to-one correspondence and make the verbal numbers meaningful. As students begin to build a strong foundation of counting they should move toward reliance on more abstract representations, allowing them to conceptually use number sense (Witzel et al., 2012). Olive (2001) suggests that educators should ensure students are given the opportunity to, “develop their mathematical structures and their ways and means of operating mathematically” (p. 4). Olive (2001) explains that these opportunities could involve developing counting routines that focus on number sequences, doubling numbers, and counting by composite units (numbers grouped together, such as counting by groups of
When children are given this opportunity, they are able to build a deeper meaning of mathematical concepts.

**Decomposing Numbers**

Number sense involves place value and the ability to use benchmarks when working with numbers (Bobis, 2008). It is also important for students to see patterns in numbers. When students see patterns they are able to group numbers in groups of five or ten. When students are able to use their place value understanding to group numbers and count them as groups, then they are able to move onto advance mathematical skills (Fuson et al., 1997). Place value understanding helps students to be able to compose or decompose numbers like, part-whole relationships (Bobis, 2008). This is important for students to grasp since this understanding allows them to be flexible when working with numbers.

Students with a more advance form of number sense are able to manipulate numbers such as being able to compose or decompose easily (Witzel et al. 2012). To help promote this deep understanding children should be given ample time and experiences to work with various number quantities (Shumway, 2011). These experiences could involve using concrete manipulatives like place value blocks to make or break apart numbers or using counters to make tens on a ten frame. When students have a deep understanding of place value they can compose or decompose numbers, which helps build a strong foundation of number sense. Student understanding of how to make numbers or break numbers apart in multiple ways allows the teacher to see how students can manipulate numbers. Teachers should use what they find to help guide students to deeper mathematical thinking.
Shumway (2011) found that students are able to compose and decompose numbers when given the opportunity to manipulate different quantities through experiences that provide students with the ability to see relationships within number values. Shumway (2011) also expresses the importance of having meaningful classroom discussions about quantities of numbers because it allows students to identify ideas they have about the different values each place has in the place value system.

**Language**

Mathematical language helps build understanding in numbers, and should be used correctly to ensure students don’t have any misconceptions. When language is used appropriately and connects to concrete manipulative activities children are able to build a stronger number sense (Witzel et al., 2012). When students are able to connect mathematics with appropriate language they are making connections, which helps them build internal mathematical knowledge.

Shumway (2011) explains that, “When students talk about mathematical concepts and strategies, they are using and creating knowledge.” (p.120). Students create and use knowledge by verbalizing their understanding, which can help them clarify their own mathematical understandings (Shumway, 2011). Students who talk through their mathematical thinking are thinking critically about how to solve a mathematical problem. This helps students avoid being thoughtless or quick to solve a problem, which will help avoid mistakes when solving a mathematical problems (Witzel et al., 2012).

**Using Context and Representations of Numbers to Support Flexibility**

Students use context and representations of numbers to support flexibility through using multiple representations (dialogue, concrete manipulatives, and number lines) to
help solve real world problems. Researchers such as West (2016), suggest that using a variety of representations students are able to think about concepts in an abstract way because they can make conjectures and test to see if their conjectures are correct. When given this opportunity students are able to use manipulatives to form conjectures they are able to take that knowledge and apply it to higher mathematical concepts, such as algebra. These topics will be discussed to support the idea that number sense is important to support flexibility in mathematical contexts.

**Multiple Representations**

Educators should use multiple representations in order for students to develop a strong understanding of a mathematical concepts (West, 2016). The literature found a common themes among the representations to use. These include using dialogue (Dougherty et al., 2015; Shumway, 2011; Yang & Wu, 2010), concrete manipulatives (Fuson et al., 1997; West, 2016; Witzel et al., 2012), and number lines (Kallai & Tzelgov; Siegler & Booth, 2004; Simms et al., 2016; West, 2016). By using a variety of representations students are able to become flexible in abstract experiences (West, 2016).

**Dialogue.** Talking about mathematics is a form of dialogue and an important representation students can use to help build confidence and motivate them to problem solve in mathematics. When students talk about mathematics it is most beneficial if they are asked to identify what a problem is asking and explain their results (Yang & Wu, 2010). It also builds a community within the classroom that helps students use mistakes as a learning opportunity (Shumway, 2011). Mathematical discussions are collaborative ways for students to work through mathematical concepts, which then helps them build a deeper meaning of concepts. For instance, students can discuss their ideas about
composing and decomposing numbers, which can lead to a higher level of confidence with this concept. This is why it is important to incorporate time into classroom instruction for mathematical discussions.

Dialogue in mathematics is a key component of building understanding of numbers. Whole-class discussions are a great way to monitor mathematical understanding because it gives the teacher feedback of what students have grasped about a mathematical skill (Yang & Wu, 2010). Using think-pair-share strategies (students first think about the mathematical concept, then the get with a partner and discuss their thinking) allows students to discuss with a partner their thinking, and helps teachers understand any misunderstandings students might have about the concept being discussed (Dougherty et al., 2015). Discussions really help teachers understand the students’ mathematical abilities.

**Concrete Manipulatives.** Concrete manipulatives help students build conceptual, abstract thinking (Witzel et al., 2012). Abstract thinking is the mental strategies that children use in order to visualize the mathematical concept in their minds. These visualizations depend on the mental structures made through concrete experiences (Fuson et al., 1997; West, 2016). Using physical objects helps students build meaning behind counting. It is important to give students the opportunity to manipulate materials when building a strong foundation in their number understanding.

Teachers should use manipulatives through clear instruction, so students understand how to use the manipulatives correctly. This will help students be able to correct any misconceptions (West, 2016). Having students show their thinking with physical representations helps teachers see what mathematical concepts children already
know. By being aware of students’ mathematical concept knowledge, teachers can design instruction to help further their thinking toward more complex mathematical ideas.

**Number Line.** Researchers such as, Siegler and Booth (2004), have found that number lines help improve a child’s ability to estimate because it allows students to be more accurate as they place numbers on a number line. Siegler and Booth (2004) also found that a mental representation of a number line is what helps students estimate. Number lines also help students engage flexibly with numbers as students expand their mathematical understanding (West, 2016). The relationships that children see when working with number lines help them estimate effectively. As students progress from single-digit number understandings to multi-digit number understandings they use mental number lines. (Kallai & Tzelgov, 2012). Mental number lines can develop when students have repeated experiences with counting on a physical or pictorial number line. This will help students be able to build a foundation with sequencing numbers that can lead to mental representations of a number line (Shumway, 2011).

Research has also found estimations are more accurate when students use a mental number line because they have an internal understanding of numbers (Siegler & Booth, 2004). Students need to be given appropriate tasks to build a mental number lines. These tasks might include using a physical number line from zero to one hundred, and asking students to identify where a specific number could be placed (Siegler & Booth, 2004). It takes time for students to develop this internal representation of numbers. Students need to have a strong number sense to be able to accurately place numbers proportionately accurate on a number line (Simms, Clayton, Cragg, Gilmore, & Johnson, 2016). When students use number lines to estimate they are able to see numbers
proportionately placed on a line. This shows that students have a strong understanding of linear sets of numbers because they place numbers appropriately on a line. For example, they place 156 between 150 and 160 on a number line.

**Real World Problems**

Siegler and Booth, (2004), found that, “…individual children tend to use between three to five strategies” when involved with mathematical thinking (p. 442). Multiple representations help students understand mathematical concepts deeply because they are making connections with various experiences, instead of assuming a concept works only with one representation. Multiple representations also help students be able to think abstractly as they become confident in a mathematical concept (Witzel et al., 2012). It is important for children to gain an abstract understanding in order to have a deep understanding of numbers.

An important way to facilitate number sense is by actively engaging children with real life situations (Yang & Wu, 2010). Students are able to improve their number sense when they can relate to the mathematical concepts and see these concepts in multiple settings. It also allows students the ability to make their own structures of understanding mathematical concepts, helping them be successful in how they operate mathematically (Olive, 2001).

**Model Drawing.** Model drawing is a representation method that helps children progress through real world story problems, especially with problems that have multiple steps to solve (Lei Bao1, 2016). Model drawing involves bar models (rectangles) to help students visualize what the problem is asking. The bar models help student produce a visual model that helps guide them to the operation they will use to solve the problem
Model drawing also helps students progress through stages of a problem by having them focus on understanding the problem, drawing a picture to help them visualize the problem, and help them use equations toward the last phase to solve the problems (Ciobanu, 2015). These phases help students think deeply about a problem, helping them solve the problem appropriately. Model drawing is a representation that helps students become flexible with numbers as they explore relationships in various experiences.

Significance of the Literature

After reviewing the literature, it seems that students need a strong foundation in their number sense abilities to help move from concrete mathematical experiences to abstract experiences. There were common themes throughout the research that suggested how students could become more successful in their abstract experiences involving mathematical concepts. These themes are, identifying patterns within a number sequence, using number lines to estimate, discussing mathematical thinking, and using models to problem solve. These themes helped design the research questions 1- How can struggling students concentrating on mathematical flexibility in concrete experiences help foster their operational flexibility in abstract experiences? 2- Can struggling students’ flexibility in concrete and abstract experiences improve their problem solving abilities? These overreaching questions are outlined in figure 2, to show how the research guided the researcher to these questions, and the outcomes the researcher may see at the end of the research period.
Figure 2. Overreaching questions identifying how research helped form the questions and what the results might be.
CHAPTER III
METHODS

The purpose of this action research project was to plan interventions and recognize effective instructional strategies from the literature that would help students become flexible with concrete experiences to support their abstract flexible strategy development. This flexibility involves using an understanding of a mathematical concept and applying it to problem solving situations. For example, being able to add or subtract numbers by using an understanding of place value blocks, but not having to use the physical representations of the place value blocks. As a result of the information found in the literature, I chose to look at how students use concrete materials to build flexible number sense, and how this supports students’ abilities to be flexible in abstract experiences. Tasks were developed from the literature and drew from concrete manipulatives, counting routines, and number lines were used to help engage students in multi-digit concrete experiences.

Methods of Research Used

During a seven-week period I planned and taught eleven lessons to an intervention group comprised of four third grade students. I also implemented eleven whole class-counting activities, for a class of eighteen third grade students. To better understand the effects of instructional decisions in the small group and whole class lessons I gathered qualitative and quantitative data from the four students assigned to the intervention group. To determine how flexibility in concrete and abstract experiences improve students’ flexibility in operational strategies when problem solving. I gave these four students a pretest and posttest (quantitative data). To determine how students
concentrating on mathematical flexibility in concrete experiences help foster flexibility in abstract experiences, I interviewed the students bi-weekly (qualitative data). Essentially, I the quantitative data analysis indicated changes in mathematical achievement and the intended for the qualitative analysis to explain why these changes may have occurred.

Further, student artifacts from the intervention and whole class lessons were collected throughout the seven-week period to inform my instructional decisions in both the small group setting and the whole class lessons. These artifacts were used to explain how students reliance upon concrete and abstract experiences changed overtime. These artifacts included audio recordings, student math journals, and pictures of student work.

Figure 3 shows a summary of the research questions explored through this seven-week period. To answer these questions I designed pretests and posttests to show if students improved their flexibility in concrete and abstract experiences. I also reflected on the artifacts gathered to discern if students’ thinking was relying on concrete manipulatives or abstract experiences.

**Overview of Analysis**

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<th>Data Collected</th>
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<tr>
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<td>Student Artifacts (Qualitative Data)</td>
<td>Number Line talks</td>
</tr>
<tr>
<td></td>
<td>Student Interviews (Qualitative Data)</td>
<td>Reflections on student artifacts</td>
</tr>
<tr>
<td>2. Can struggling students’ flexibility in concrete and abstract experiences improve their problem solving abilities?</td>
<td>Pretest (Quantitative Data)</td>
<td>Student score comparison of pretest and posttest</td>
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<td>Posttest (Quantitative Data)</td>
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<tr>
<td></td>
<td>Student Interviews (Qualitative Data)</td>
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</table>

*Figure 3. Summary of research stating the research questions that were explored, the data used to answer the research questions, and what was found from these questions.*
Participants and Setting

The students who participated in the whole class counting routines were eighteen third grade students, ranging from ages eight to nine years old. I am their regular classroom teacher and teach them at an elementary school with 50% of the students receiving free or reduced lunch fees. Permission granted from the principal of the school. Consent was also granted by all of the parents of the eighteen students. Instructional Review Board was waived (see Appendix A) due to the impact of this project being limited to this particular classroom instruction and pedagogy.

Out of the eighteen students who participated in the whole class counting routines, eight are girls and ten are boys. In this class of eighteen two receive special education services for mathematics, and one receives special education services for reading. Five students receive speech services. None of the eighteen students receive English as a Second Language (ESL) services. The demographic of this particular class can be described as being comprised of 83% is Caucasian, 11% Latino, and 6% African American. The class also has 40% of the students receiving free or reduced lunch indicative of low socio-economic status (SES).

Following a pretest (used here as a screening tool) four students were selected for intervention groups based on the pretest given for number lines and problem solving situations. These students chosen for the intervention group scored between 0% -25% on the word problem pretest. They also scored between 0%-66% on the number line pretest. Compared to the class average these students were considered to be well below in their pretests, which is why they were chosen to participate in the intervention group. Figure 4 describes participants that were involved in the intervention groups.
<table>
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<tr>
<th>Participant</th>
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*Figure 4. Description of the four students involved in the small intervention group.*

This seven-week intervention took place during the months of September and October. The students who participated were enrolled in third grade classroom. Students was chosen purposefully to participate in the research process, as the pretest indicated these four students scored relatively lower than their peers and required a small group intervention outside their whole class mathematics class. Students who participated in the counting routines met as a whole class and used journals to record their thinking. The small group intervention met in the back of the room during a separate time at a small table, allowing students to work closely together without any distractions.

**Instructional Procedures**

Throughout this seven-week period I followed the scheduled materials from my district-adopted curriculum, Go Math! Grade 3 Common Core Edition (Houghton Mifflin Harcourt). These lessons were taught along with implementing interventions I designed and whole group counting routines. The chapters from the Go Math! Curriculum that were taught focused on place value (adding, subtracting, and estimating) and collecting data to form graphs. Multiplication strategies for single-digit whole numbers (i.e. skip counting) was also introduced during this seven-week period.
Whole class counting routines were implemented during a ten-minute session before enacting instruction from the Go Math chapters. These ten minutes were used to engage students in counting in a sequence, writing down the sequence, and reflecting on the patterns seen within the counting sequence. Students were encouraged to discuss with a partner the patterns they saw. Students would then share with the class what they discussed with their partners. These routines were designed from the literature reviewed by Shumway (2011). The counting routines were used to aide students in the small intervention group to identify patterns in a number sequence, and discuss with classmates outside of the intervention group about patterns.

Small group interventions were done during fifteen-minute math rotations. Students who weren’t in the intervention group were working on practice problems from the lesson taught that day. The small intervention group discussed patterns they saw during the counting routines. The tasks were then performed for locating numbers on a number line or performing a counting task.

Figure 5 shows an outline of the counting routines and intervention tasks that were used over the seven-week session. Counting routines and intervention groups were done twice a week during the regular weeks schedule, and once a week during the shorter week schedules. Counting routines involved discussing patterns that were found during the sequence counting or comparing patterns from previous routines. The counting routines also involved students identifying the next three numbers in the sequence (i.e., 223, 233, 243, __, __, __). These routines were to aide the students in the intervention group to help them identify patterns within a number sequence. Small intervention groups would focus on either a counting or number line task.
Whole class problem solving experiences was also used in the Whole Class Counting Routines, which was used as a supplement for the students in the small intervention group. Students were given a problem to solve and explain their reasoning behind how they solved the problem. Students used models to solve the problems during these experiences. Problems included addition and subtraction word problems.

<table>
<thead>
<tr>
<th>Week</th>
<th>Whole Class Counting Routine Day 1 of the Week</th>
<th>Whole Class Counting Routine Day 2 of the Week</th>
<th>Intervention Task (number line and counting tasks) Done During Day 1 of the Week</th>
<th>Intervention Task (number line and counting tasks) Done During Day 2 of the Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Pretest given</td>
<td>Counting routine modeled with counting by ones and patterns discussed</td>
<td>Counted by one hundred and discussed similarities to counting by ones</td>
<td>Group discussion on what a number line is and how can it help us in math</td>
<td>Locate 5 and 10 on a number line</td>
</tr>
<tr>
<td>Two</td>
<td>Counted by ones starting at 83</td>
<td>Counted by tens starting with 83</td>
<td>Counting from 0-100 And making groups of 10</td>
<td>Locate 50 on a number line And making groups of 100</td>
</tr>
<tr>
<td>Three</td>
<td>Counted by 100s starting with 83</td>
<td></td>
<td>Beaded Number line compared to a open number line locating the number</td>
<td></td>
</tr>
<tr>
<td>Four</td>
<td>Counted by threes then by 30s</td>
<td>Counted by fours and 40s</td>
<td>Locate 75 on a number line</td>
<td>Locate missing numbers from 0-100</td>
</tr>
<tr>
<td>Five</td>
<td>Counted by 40s starting with 340</td>
<td>Counted by sevens starting with 71</td>
<td>Locate 90 on a number line</td>
<td>Locate 50 and 100 on a number line</td>
</tr>
<tr>
<td>Six</td>
<td>Counted back by sevens starting with 197</td>
<td></td>
<td>Locate 125 on a number line</td>
<td></td>
</tr>
<tr>
<td>Seven Posttest given</td>
<td>Counted back by 6 starting with 124</td>
<td></td>
<td>Locate 35 and 65 on a number line</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 5. Overview of tasks completed over the seven-week session.*
**Data and Analysis**

Quantitative data was collected through pretests and posttests. Qualitative data was collected through student dialogue from whole class discussions and student interviews and through student artifacts that were collected. These situations allowed me to infer how students were using flexibility in concrete and abstract experiences. I was able to infer this because students could show me their understanding through concrete manipulatives or explain their thinking in abstract experiences.

**Student Samples**

**Quantitative data**

**Pretest and Posttests.** Students were given a pretest during week one and a posttest at the end of week seven. These samples helped analyze if students improved flexibility in concrete and abstract experiences, and if this flexibility helped problem solving abilities. Figure 6 shows the pretest and posttest gathered from each student in that participated in the research study. The problem solving tests had similar characteristics, and were developed from the Go Math! Curriculum. The number lines were added to the assessments to best determine how these students utilized their estimating abilities when solving multi-digit problem (Kallai & Tzeglov, 2012). Further, numbers for each assessment were chosen to align with prerequisite knowledge, as described by the CCSSM in second grade. Second grade was chosen so the students would not experience levels of frustrations when answering questions throughout the assessment.
Figure 6. Pretest and Posttest

Qualitative Data

Student Dialogue. Students discussed their mathematical thinking in partners formats, whole class formats, and small groups formats. They discussed patterns they noticed during counting routines by answering what they noticed about the numbers, how the counting routine related to previous routines, and how they could continue the sequence. They also explained how they could identify the next numbers in a sequence.
Students were also able to discuss if a number in the sequence was wrong and identify the correct number.

Students in the intervention group used dialogue to discuss their counting and number line tasks. They talked with partners and as a small group by answering questions about why they placed the number where they did on the number line, why is it important to group numbers, and why are benchmarks helpful in mathematics.

**Open Number Line.** Students drew number lines on white boards to locate specific numbers within a range. They drew a mark on the number line where they believed the number would be. They then had to explain why they would place that number where they did. A beaded number line was also used to compare a concrete number line to a pictorial number line on the whiteboard. Photos were taken of number lines that students drew to help analyze how students were building on their flexibility with concrete experiences.

**Problem Solving Experiences.** Students solved problems with models to show their mathematical thinking. These models included bars, circles, tally marks, and place value blocks. After students solved their problems with models they would explain their thinking to a partner, the teacher, or to the class. These problems were addition and subtraction problems that involved one or two steps to solve.

**Analysis**

**Quantitative Analysis**

Descriptive statistics were used to compare results from pretest and posttests. These test comparisons helped inform me how the interventions that were planned effected students’ mathematics achievement. If the percentage scores increased, then I
could assume that the interventions were successful in using flexibility in concrete experiences to foster flexibility in abstract experiences. If the scores stayed the same or declined, I would assume that my interventions didn’t foster flexibility in abstract experiences through concrete experiences. I would then need to revisit the tasks and instructional strategies used to determine a more effective way to promote flexibility in concrete and abstract experiences.

**Qualitative Analysis**

Weekly conceptual analysis was examined by revisiting the literature to best explain how students mathematical thinking changed in response to the tasks that were designed. Throughout the intervention time frame, these forms of analyses informed each week’s intervention focus and material that individual students required to better support their own development of abstract mathematical concepts. As student academic for this development needs were met they were able to discuss their thinking to show how their flexibility in concrete experiences was fostering flexibility in abstract experiences, and how these experiences were influencing their problem solving abilities.

**Student Dialogue.** Student discussions were recorded to help analyze how mathematical thinking was changing over the course of the seven-week interventions. Students would explain how they arrived at their mathematical understanding by explaining how they used a mathematical strategy (grouping numbers, using benchmarks, etc.) to help them solve the problem. Listening to student explanations helped infer the change that was occurring in student mathematical thinking because I could analyze their thinking through their explanations.
**Open Number Line.** Students number lines were photographed to help compare their thinking from the beginning of the interventions to the end. It expressed how students thinking was changing from concrete representations to abstract representations. If students used more complex strategies, such as using a benchmark to help them place a number, then I could see that their thinking was moving toward more abstract flexibility in their mathematical thinking.

**Problem Solving Experiences.** Problem Solving was analyzed in a qualitative manner by listening to student explanations about how they solved the problems. This gave insight into what strategies (concrete, pictorial, or abstract) were being used to solve the problems. By analyzing the strategies being used I could plan tasks that would foster flexibility in either concrete or abstract experiences.
CHAPTER IV

RESULTS

Students were given a pretest and posttest to help measure changes in their mathematical thinking (see Figure 7). These scores were compared to the class average and to the students who participated in the intervention research process. From this analysis three out of four of the students (Rodger, Sally, and Daisy) showed an increase from pre to post-test scores. The fourth student, Frank increased in his world problem score from pre to post-test, but his line score test remained the same from his pre to post-test score. Furthermore, Rodger and Daisy performed better than the class average in the post-test line test, both scoring 100% These scores show that students improved in at least one area of the assessment, suggesting that their mathematical thinking changed overtime to allow for more flexible, abstract strategy development. Their explanations in the following sections provided insight into their thinking processes to support this change.

<table>
<thead>
<tr>
<th>Student</th>
<th>Pretest Number Line Score</th>
<th>Posttest Number Line Score</th>
<th>Pretest Word Problem Score</th>
<th>Posttest Word Problem Score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Class Average: 85%</td>
<td>Class Average: 95%</td>
<td>Class Average: 60%</td>
<td>Class Average: 82%</td>
</tr>
<tr>
<td>Frank</td>
<td>2/3 66%</td>
<td>2/3 66%</td>
<td>0/4 0%</td>
<td>2/4 50%</td>
</tr>
<tr>
<td>Rodger</td>
<td>0/3 0%</td>
<td>3/3 100%</td>
<td>1/4 25%</td>
<td>2/4 50%</td>
</tr>
<tr>
<td>Sally</td>
<td>1/3 33%</td>
<td>2/3 66%</td>
<td>¼ 25%</td>
<td>2/4 50%</td>
</tr>
<tr>
<td>Daisy</td>
<td>2/3 66%</td>
<td>3/3 100%</td>
<td>¼ 25%</td>
<td>¾ 75%</td>
</tr>
</tbody>
</table>

*Figure 7. Pre and Posttest scores showing the percentages of correct answers.*
Student Case Studies

Frank. On the pretest Frank scored 66% on the number line tasks and 0% on the problem solving questions (see Figure 8). This was well below the class average of 85% on the number line tasks and 60% on the problem solving, which is why he was chosen to participate in the intervention group. Frank was present for all eleven whole class-counting routines and the intervention sessions throughout the seven-week period.

<table>
<thead>
<tr>
<th>Pretest for Number Line</th>
<th>Posttest for Number Line</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Pretest Image" /></td>
<td><img src="image2.png" alt="Posttest Image" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pretest for Problem Solving</th>
<th>Posttest for Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Pretest Image" /></td>
<td><img src="image4.png" alt="Posttest Image" /></td>
</tr>
</tbody>
</table>

Figure 8. Frank’s test scores showing the work that Frank did to complete the pre and post-tests.

Frank showed change in his thinking when solving real world problems. At the beginning of the seven-week session Frank relied a lot on concrete objects and counting
objects individually. During week six Frank and I had a conversation about how to solve a problem that asked to add 10+10+10+10+2. The conversation below explains Frank’s thinking after the problem was discussed as a group and students identified that they needed to add 10+10+10+10+2.

Teacher: “Frank, how did you solve this problem so fast?”

Frank: “I counted by tens.”

Teacher: “How did you count by tens to get 42?”

Frank: “10+10 is 20 and 2+2=4, but it’s 42.”

Teacher: “But why would 2+2=4 let us know the answer is 42?”

Frank: “Because um, It is like 10+10+10+10, but I did 10+10=20 and then did 2+2=4, but it’s 40.”

This shows that Frank was doubling an amount to help him solve the mathematical problem. He used relationships between addition problems he already knew to solve a problem (10+10= 20 and 2+2=4). He was able to use this strategy without counting on his fingers or using individual objects to count. This supports that Frank’s mathematical flexibility with concrete experiences (using manipulatives and fingers to solve mathematical problems) helped him construct flexibility in abstract experiences (doubling a quantity in his mind.)

Frank developed an understanding of benchmarks as he worked with number lines during the intervention sessions. He used benchmarks to help him locate numbers on a number line (see Figure 9). Frank was able to explain his benchmark to support that he understood why he would use it as the middle number.
Teacher: “Frank, why did you place 50 where you did?”

Frank: “Um, because 50 goes there.”

Teacher: “Frank, why do you say it goes there?”

Frank (as he points to the number line): “Because it is in the middle of the line and 50+50=100.”

*Figure 9.* Frank’s benchmark used and explain why 50 was used as the benchmark.

Through the benchmark that Frank used and his explanation of why he chose 50 as the middle of the range 0-100, shows that his thinking became more flexible in his abstract abilities because he isn’t relying on concrete manipulatives.

**Rodger.** On the pretest Rodger scored 0% on the number line tasks and 25% on the problem solving questions. This was well below the class average of 85% and 60%, which is why he was chosen to participate in the intervention group. Rodger was present for all eleven whole class-counting routines and the intervention groups throughout the seven-week period. Figure 10 shows the work that Rodger did in his pre and posttests.
Figure 10. Rodger’s test scores showing the work that Rodger did to complete the pre and post-tests.

Rodger began to use benchmarks toward week four of the interventions, and became more comfortable with benchmarks by week seven. A conversation occurred within the group during week six that showed Rodger understood how to locate a benchmark when using a greater range by connecting to the range that was previously used. This showed that Rodger was making connections with previous mathematical skills and constructing meaning with new mathematical skills. A student had identified that 100 was placed in the middle of 200 because $100+100=200$.

Teacher: “Why else would 100 be in the middle of 200?”
Rodger: “Because it’s like when we add 50+50 if that was 100, so we would take half of 200 which is 100.”

Teacher: Why did you put fifty between 0 and 100?

Rodger: “Because it is like when we only had 100. 50+50 is 100, so 50 is in the middle.”

Rodger used the benchmarks to help him locate the number 125 on a number line that ranged from 0-200 (see Figure 11).

*Figure 11.* Rodger’s benchmarks showing the work that Rodger was discussing in the conversation about benchmarks.

Rodger was also able to use this benchmark understanding to improve his rounding skills. This change in mathematical thinking was supported by the explanation Rodger gave while rounding a number to the nearest hundred.

Teacher: “What is 166 rounded to the nearest hundred?”

Rodger: “100. No, 200.”

Teacher: “Why would we round to 200?”

Rodger: “Because it is closer to 200?”

Teacher: “How did you know it was closer to 200?”

Rodger: “Because it is bigger than the middle, so it is 200.”

Teacher: “What do you mean it is bigger than the middle?”

Rodger: “If we used a number line it would be bigger than the middle.”
Teacher: “How do you know what the middle number is?”

Rodger: “50 is in the middle, and 66 is bigger than 50.”

Teacher: “50 is the in the middle of 100 and 200?”

Rodger: “Yes. No, wait um it would be 150.”

This shows that Rodger used his understanding of benchmarks on a number line to help him estimate to the nearest hundred. This was a positive change in Rodger’s mathematical thinking because he was able to use his flexibility with number lines to help him estimate.

**Sally.** On the pretest Sally scored 33% on the number line tasks and 25% on the problem solving questions. This was well below the class average of 85% and 60%, which is why she was chosen to participate in the intervention group. Sally was present for all eleven whole class-counting routines and the intervention groups throughout the seven-week period. Figure 12 shows the pre and posttests that Sally completed during intervention experience.
Figure 12. Sally’s test scores: showing the work that Sally did to complete the pre and post-tests.

Sally was able to see patterns when working with numbers during the whole class counting routines. A pattern she identified when counting back by sevens was that the numbers followed an even-odd pattern. This helped her identify if the next two numbers in the sequence (63 and 56) was correct. She made sure the numbers followed the even-odd pattern (see Figure 13). Her explanation supported the change in her mathematical thinking because she describes how she used a pattern to help her decide if her mathematical thinking was correct.

Figure 13. Sally’s even-odd pattern used to identify the next two numbers in the sequence.

Sally explained the importance of benchmarks when working with number lines. She described a benchmark as, “helping me see where to put a number, like if I was dealing with a number smaller than 50 then I would know it had to be below the 50
mark.” (see Figure 14). This suggests that her flexibility in concrete experiences were improved with the number line tasks performed in the intervention group.

*Figure 14.* Sally’s benchmarks used to identify where 50 and 125 were located on the number line.

**Daisy.** On the pretest Daisy scored 66% on the number line tasks and 25% on the problem solving questions (see Figure 15). This was well below the class average of 85% and 60%, which is why she was chosen to participate in the intervention group. Daisy was absent for two days out of the eleven whole class-counting routines and the intervention groups throughout the seven-week period.
Daisy’s test scores showing the work that Daisy did to complete the pre and post-tests.

The posttest that Daisy took was compared to the pretest scores and suggests that her mathematical thinking changed and became more flexible with concrete and abstract experiences. This is true for the problem solving assessment. Daisy relied on a pictorial representation of place value blocks in her pretest, but did not rely on them for her posttest (see Figure 15). There was improvement in her test scores (25% to 50%), which suggests that the interventions had a positive influence on Daisy’s mathematical thinking.

Daisy supported her change in thinking through explaining number patterns while skip counting with the class. She informed the class that skip counting by fours could help us understand how to skip count by 40’s because they are similar. The conversation went as follows:

Teacher: “If we wanted to skip count by 40’s, is there another number that could help us do that? Yes, Daisy.”

Daisy: “We could use fours.”

Teacher: “Why do you say we could use fours?”
Daisy: “Because fours are like 40’s.”

Teacher: “How are fours like 40’s?”

Daisy: “Because skip counting by fours is in the one place, but skip counting by 40’s is like the four is in the tens place.”

This showed that Daisy was seeing relationships between numbers by using her understanding of place value. She hadn’t used this strategy at the beginning of the seven-week session, but used it during week six. Through her explanation I could see that her flexibility with concrete experiences helped aid her in her abstract thinking when dealing with counting by fours and 40’s.

Daisy also used benchmarks to help her locate numbers on her number lines. She explained benchmarks as helping her be able to locate numbers because she knows what is in the middle of the range that was used. By using 100 as a benchmark she was able to locate 50 and 125 on a number line (see Figure 16). Her explanation shows that she increased flexibility in her concrete experiences using numbers on a number line.

![Figure 16. Daisy’s benchmark on a number line used to locate the numbers 50 and 125 on a number line.](image)

**Common Strategies**

The qualitative data collected over the seven-week period showed common mathematical strategies that students used to solve mathematical problems and explain their mathematical thinking. These common strategies were benchmarks on a number line.
and skip counting by a quantity. This helped students use their flexibility with concrete experiences to increase their flexibility in abstract experiences. These common strategies also showed that students used the flexibility they constructed to help them solve problem solving experiences.

**Benchmarks.** The four students in the intervention groups used benchmarks to help them locate numbers on a number line. These benchmarks helped students estimate and round numbers during problem solving experiences. This shows a positive influence in the change that students experienced in their mathematical thinking. The benchmarks showed students were becoming more flexible in their abstract thinking because they were able to use number lines to show magnitudes of numbers. These magnitudes are more abstract since students weren’t using physical representations, such as place value blocks. Students were able to locate numbers on a pictorial representation of a number line, which is a more abstract mathematical concept.

**Skip Counting.** The four students used skip counting to help them solve mathematical problems. By using skip counting students were able to explain how they saw the relationship among quantities of numbers. For example, students skip counted by quantities of ten to solve addition problems. Students were able to use their skip counting strategies to recognize numbers that came next in a sequence during whole class counting routines. This strategy also aided them in the multiplication abilities because they could easily group numbers and skip by the appropriate quantity to solve problems using the multiplication operation.
Conclusion

Through this study I found that students were able to construct a strong flexibility in their conceptual mathematical understanding when given multiple experiences with concrete representations. The relationships students found in concrete representations could be attributed to flexibility in their mathematical thinking, but it is unclear if it is a change in their mathematical thinking, or if their performance was influenced by being in a small group setting, allowing them more one-on-one time with the teacher. These concrete representations involved physical number lines, place value blocks, and counters. The number line activities that students used allowed them to develop a mathematical understanding of benchmarks. This understanding led students to be flexible in their concrete experience, aiding in the construction of mental representations of number lines. For example, they were able to use these benchmarks to visualize where to place a number on a number line, which is how they developed flexibility in abstract experiences. Students were also able to group numbers by certain quantities with concrete objects, later aiding them in their ability to skip count abstractly in their problem solving experiences.

Students concentrated on mathematical flexibility in concrete and abstract experiences, which helped foster their operational flexibility in abstract experiences. This was shown through the explanations students gave when solving problems. They were able to explain their mathematical thinking when they discussed where they would put numbers on a number line, and when they explained why skip counting helped them solve a problem. Through their explanations I was able to infer that their operational
flexibility in abstract experiences improved. I was also able to use their pre and post-tests to support my inference.

I was able to see that problem solving abilities improved through the explanations students had about locating numbers on a number line and skip counting quantities. I also saw improvement in their pre and post-tests, which is why I can infer that problem solving abilities improved. Although these problem-solving abilities improved, there are other factors that could have influenced improved scores. These factors include more one-on-one time with the teacher, which could have increased student motivation in mathematics leading them to the improved scores. I infer that students improved their flexibility, but it is unclear if that is what helped improve their problem solving abilities.

The research questions that were explored showed that students’ mathematical abilities to solve problems were possibly improved through flexibility in concrete and abstract experiences. These results will be used to develop instructional experiences that will allow students the ability to have multiple experiences to construct flexibility in their abstract abilities. I will also use this information to collaborate with my team when planning lessons for mathematics. It will also guide my research in the future when performing active research in my classroom.
References


APPENDICES
APPENDIX A

Institutional Review Board (IRB) Certificate of Waiver Approval Form
Institutional Review Board

Request for Determination of Non-human Subjects Research

Approved

FROM: Melanie Domenech Rodriguez, IRB Chair
Nicole Vouvalis, IRB Administrator

To: Beth Loveday MacDonald

Date: August 30, 2016

Protocol #: 7779

Title: Math Interventions For Third Grade Students’ Development Of Place Value

Based on the information provided to USU’s IRB, it has been determined that this project does not qualify as human subject research as defined in 45 CFR 46.102(d) and (f) and is not subject to oversight by USU’s IRB.