Optimizing the Rapid Evolution of Target Groundwater Potentiometric Surfaces

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ABSTRACT

The long-term availability of adequate groundwater for sustainable irrigated agricultural production can be assured by causing the evolution and maintenance of an appropriate potentiometric (piezometric) surface in utilized aquifers. By providing adequate saturated thickness and hydraulic gradients and heads, such a surface affords some protection from the effects of drought, economic hardship, and environmental or legal complications. Procedures for determining desirable or optimal regional steady-state groundwater potentiometric surfaces have been reported by several researchers. An existing potentiometric surface can be forced to evolve into a desirable ‘target’ surface if it is subjected through time to the proper combination of recharges and discharges. Depending on the difference between initial and target surfaces, steady pumping strategies may require extended periods to complete the evolutionary process. The evolutionary period may be significantly shortened by using a time-varying pumping strategy. Such a strategy may be calculated by combining goal programming and simulation. The presented technique maximizes attainment of target potentiometric surface elevations within a predetermined planning period. It is a valuable tool for a management agency that is seeking to determine how to most rapidly achieve desirable groundwater conditions in either an undeveloped region or a region that has an existing pattern of groundwater pumping.

INTRODUCTION

In many regions, sustainable agricultural production depends on the long-term availability of groundwater. Groundwater accessibility depends on potentiometric head, since head affects pumping lift, saturated thickness, and recharge to discharge from an aquifer system. (The term ‘potentiometric,’ is preferable to ‘piezometric,’ which was once used commonly (Lohman, 1979).) Thus, the sustained availability of groundwater can be assured by causing the evolution and maintenance of an appropriate potentiometric surface.

The idea of maintaining a steady-state potentiometric surface in some aquifers or portions or aquifer systems is gradually gaining popularity (Knapp and Feinerman, 1985). For example, the Arkansas State Water Plan includes a presentation of the physical and legal feasibility of maintaining a specific regional potentiometric surface in an intensively irrigated rice and soybean producing area (Peralta and Peralta, 1984b). Knapp and Feinerman (1985) discuss the desirability of attaining economically optimal steady-state groundwater levels for aquifer systems. In addition, as seen in the literature review, several other methods have been reported for determining either optimal or nonoptimal ‘target’ steady-state potentiometric surfaces for specific regional objectives.

The question naturally arises as to how a water management agency can cause an existing potentiometric surface to evolve into a more desirable target surface. Morel-Seytoux et al. (1981), Peralta and Peralta (1984a, 1984b) and Knapp and Feinerman (1985) state that implementation of a strategy of steady-state hydraulic discharges and recharges can eventually cause the development of a particular unique steady-state potentiometric surface.

Thus, assuming constant parameters, a target steady-state surface can be attained if the same annual pumping volume that would maintain the surface are pumped throughout the surface evolution era.

Depending on how different an initial potentiometric surface is from a desired target surface, it may take many years for steady pumping rates to achieve the transformation. Peralta and Kilian (1985) report two simulated cases of the very gradual evolution of groundwater levels toward target elevations. Each case utilized a different set of initial elevations for a 204 5-by 5-km cell system describing the Arkansas Grand Prairie region. During the evolutionary era both cases utilized the steady spatially-distributed pumping strategies that would maintain the target surface once it was achieved. In the two cases the elevations of the initial surface were, respectively, an average of 1.9 and 5.6 (absolute values) different than the target surface elevations. After 50 simulated years of pumping, the average difference between simulated and target elevations were 0.8 and 2.8 m respectively. Clearly, transformation of an initial potentiometric surface into a target surface may take a number of years if the evolution is achieved through the use of a ‘steady’ pumping strategy in which the same volume of groundwater is withdrawn from a cell year after year.

It was surmised that the use of an appropriate unsteady pumping strategy could result in more rapid target surface attainment than that possible using steady pumping. Therefore, the purpose of this paper is to present an optimization methodology for determining the time-varying groundwater pumping strategy that will
maximize the transformation of regional groundwater levels into a regional 'target' potentiometric surface during a specific planning period. Application of the method is demonstrated for a hypothetical study area using assumed initial and target levels. In the process, the target level attainment abilities of a steady pumping strategy and four optimal unsteady pumping strategies are compared. The optimal strategies are developed using different sets of bounds and constraints.

LITERATURE REVIEW OF REGIONAL TARGET POTENTIOMETRIC SURFACE DESIGN

The Target Level Approach (TLA) to groundwater management is one in which a management agency first selects a desirable, steady-state regional potentiometric surface and then uses a computer model to calculate a spatially distributed set of annual groundwater withdrawal rates that will approximately maintain that surface (Peralta and Peralta, 1984a). The desirable ('target') surface may be selected based on legal, economic or drought-protection criteria (Peralta et al., 1986). The calculated groundwater pumping rates comprise a sustained yield groundwater withdrawal strategy as long as required recharge and discharge between the study area and the surrounding system are physically feasible at the assumed boundary conditions. Using the definitions of Todd (1959) and Lohnian (1979), the sustained yield constitutes a "safe yield" as long as no significant undesirable consequence results from implementation of the strategy.

Peralta and Peralta (1984b) demonstrate the physical and legal feasibility of applying the TLA in Arkansas. In a simulated example they develop a pumping strategy that would approximately maintain the current potentiometric surface for the Arkansas Grand Prairie, while satisfying sustained yield and safe yield criteria. To accomplish this, their model assures that the computed annual groundwater recharge/withdrawal volumes, and the resulting aquifer saturated thicknesses, are physically practical. They show that the temporal variation in groundwater pumping that would occur during a year, if such a strategy were implemented, would not cause significant long-term deviation from the springtime "target" groundwater levels. In their example, groundwater levels returned to the springtime "target" elevations (within 0.2 m) year after year despite deviation from those levels during the year.

Not all arbitrarily conceivable regional potentiometric surfaces are physically attainable or sustainable. However, since TLA models incorporate the laws of groundwater flow, the pumping strategies that they develop are physically feasible (subject to error inherent in any steady-state approach). For this reason, TLA models develop pumping strategies that will 'approximately' maintain the initially conceived surface. In other words, a target steady-state surface assumed by a TLA model may be somewhat different than the initially desired surface.

In the process of designing a groundwater management strategy an agency may feel that it is especially important to assure a specific target groundwater level elevation in one part of an aquifer system. The ability to do this is found in an enhanced TLA methodology that uses quadratic optimization and weighting coefficients (Yazdanian and Peralta, 1986).

A similar concept in regional potentiometric surface management is the Target Objective Approach (TOA). It is used when, rather than preselecting a desirable surface, an agency prefers to first select a regional policy objective and then to design a potentiometric surface and sustained yield strategy that best achieves that objective (Peralta and Killian, 1985). A TOA surface is conceptually similar to the 'optimal steady state' developed to maximize sustainable net economic return (Knapp and Fienerman, 1985). Optimal regional surfaces have been determined for alternative water policies (Peralta, A., et al, 1985) and for conflicting multiple policy objectives (Datta and Peralta, 1986).

In conclusion, TOA surfaces have been designed for agricultural areas of either 4600 or 8200 km² for the following regional policies: minimization of cost of coordinating the use of groundwater and diverted river water, maximization of groundwater use, minimization of unsatisfied demand for water, minimization of the uniform proportionate reduction in current groundwater use necessary to achieve a sustained yield, and multiobjective optimization.

THEORY AND MODEL FORMULATION

Ideally, a target potentiometric surface would be attained precisely when it is most convenient for planning and management purposes. Physically, depending on the situation, there may be no conceivable sequence of pumping that can cause complete convergence to target levels within the desired time. It may be that the best that can be achieved is to maximize attainment of target levels by the end of the period. A groundwater withdrawal strategy that can accomplish this can be computed using a goal programming approach. In a goal programming model the objective is to minimize the sum of the differences of J achieved values from their corresponding target values. To formulate such a model the first step is to adapt standard goal programming terminology (Goicoechea et al, 1982) to describe target hydraulic head attainment for an individual cell:

$$h_{i,k} = h_i + d_i^{-} - d_i^{+}$$

where

- $h_i$ = the hydraulic head that is attained in cell i by the end of K times steps in a specific planning period, L
- $d_i^{+}$ = a nonnegative overachievement variable (i.e. the difference between the attained head and the target head if the attained head is the larger of the two), L
- $d_i^{-}$ = a nonnegative underachievement variable (i.e. the difference between the target head and the attained head if the attained head is the lesser of the two), L

It should be noted that for a single cell, either the overachievement or the underachievement variable can have a nonzero value, but not both.

Use of equation [1] in developing a groundwater withdrawal (pumping) strategy requires the ability to express the hydraulic head that results at the end of the
planning period as a function of hydraulic stimuli. Response matrix methods used by several researchers provide this capability (Maddock and Haimes, 1975; Morel-Seytoux and Daly, 1975; Dreizin and Haimes, 1977; Haimes and Dreizin, 1977; Verdin et al., 1981; Heidari, 1982; Gorelick, 1983). From among these, the discrete kernel approach (Morel-Seytoux and Daly, 1975) is used in this paper. Once discrete kernels are calculated, the hydraulic head at cell i that results from stimuli at J cells for K time periods may be determined from (Verdin et al., 1981; Illangasekare et al., 1984):

\[ h_{i,k} = h_{i}^{0} - \sum_{k=1}^{K} \sum_{j=1}^{J} \delta_{i,j,K-k+1} (q_{j,k} - q_{j}^{ss}) \]  

where

- \( K \) = the number of time steps in the planning period
- \( J \) = the number of internal (variable-head) cells in the study area includes cell i
- \( h_{i}^{0} \) = the initial hydraulic head at cell i, L
- \( \delta_{i,j,K-k+1} \) = a nonnegative-valued discrete kernel (linear influence coefficient) that describes the effect on the hydraulic head at cell i in the time step K caused by \((q_{j,k} - q_{j}^{ss})\). The temporal subscript \( K-k+1 \) is used to insure that the proper \( \delta \) is utilized in each time step, \( T/L^{2} \)
- \( q_{j,k} \) = the net vertical hydraulic stimulus in cell j in time step k. It is the sum of all discharges (+) from the aquifer and recharges (-) to the aquifer from the ground surface, \( L^{2}/T \)
- \( q_{j}^{ss} \) = the net vertical hydraulic stimulus that must occur in each time step in cell j in order for that cell to maintain its initial head. It is calculable using the linearized Boussinesq equation for steady-state two-dimensional flow through porous media (Illangasekare et al., 1984; Peralta and Peralta, 1984a, L^{2}/T).

A discussion of the theoretical development of the discrete kernel approach and the means by which kernels (influence coefficients) are calculated is found in the citations and is outside the scope of this paper. However, a brief review of how equation [2] functions is appropriate. By inspection, it is apparent that there will be no change in head at cell i by time K if either all discrete kernels \( \delta_{i,j,K-k+1} \) equal zero or if all \( q_{j,k} \) equal \( q_{j}^{ss} \). A discrete kernel as subscripted above is zero only if a stimulus at cell j in time period k has no effect on the water level in cell i by time period K. Simply speaking, a discrete kernel may be zero if cell j is distant from cell i, but is probably nonzero if they are in proximity with each other. Thus, the water level at cell i in a system of stimulated cells will change unless the actual stimuli at all cells j, \( q_{j,k} \), equal the steady stimuli that will maintain initial levels, \( q_{j}^{ss} \). Assume a simple 2-cell system in which only one cell, j, is stimulated. Let \( \delta_{i,j,1} > 0 \) and both \( q_{j} \) and \( q_{j}^{ss} = 0.0 \). In this case, the water level in cell i will decline during a first time step if \( q_{j} \) is greater than \( q_{j}^{ss} \). On the other hand, if \( q_{j} \) is less than \( q_{j}^{ss} \), the water level in cell i will rise during the time step.

Before proceeding with model development, it is appropriate to mention pertinent hydrogeologic assumptions.

- The entire hypothetical study area (Fig. 1) is underlain by an aquifer of such large saturated thickness that transmissivities are essentially the same for the initial and target potentiometric surfaces. Hence discrete kernels do not change significantly during the period of surface evolution. (If variation of discrete kernels with time were large, corrections suggested by Jacob (1944), Maddock (1974) or Heidari (1982) may be utilized.) Thus, the aquifer can be modelled using linear systems theory and equation [2], representing linear superposition of responses to stimuli, can be appropriately used.
- The study area's aquifer is completely surrounded by a much larger aquifer system of which it is merely a portion. Therefore, the surrounding aquifer can act as a source of recharge to the study area through each of the study area's peripheral cells. The study area's peripheral potentiometric surface can be maintained at constant elevations as long as physical feasibility constraints are satisfied. In other words, as long as the rate of groundwater movement through the periphery does not exceed certain predetermined values, peripheral elevations can be considered as being relatively unchanging with time. (Peralta and Killian (1985) and Yazdanian and Peralta (1986) describe methods for designing potentiometric surfaces that satisfy such boundary conditions.) In applying the developed methodology, both the initial and target potentiometric surfaces satisfy the constraints on groundwater flow through the periphery. Therefore, it is assumed that any transitional surface will also satisfactorily maintain the boundary conditions.
- Except for groundwater pumping via wells, all recharge to or discharge from the study area's aquifer
and, when it is desirable that pumping in a cell is consistent in either decreasing or increasing during the evolutionary era, either

\[ p_{i,k+1}^o \leq p_{i,k} \quad \text{for } i=1,J \text{ and } k=1,K-1 \]

if \( p_{i}^o \geq p_{i}^{\text{tss}} \) .............................. [7a]

or

\[ p_{i,k+1}^o \geq p_{i,k} \quad \text{for } i=1,J \text{ and } k=1,K-1 \]

if \( p_{i}^o \leq p_{i}^{\text{tss}} \) .............................. [7b]

where

- \( W \) = the sum of the absolute values of the differences between target levels and attained levels for \( J \) cells at the end of a planning period consisting of \( K \) time steps, \( L \).

- \( p_{i,k}^l \) and \( p_{i,k}^u \) = the lower and upper bounds on pumping, respectively, in cell \( i \) in time step \( k \), \( L/T \).

- \( p_i^o \) = the volume that is being pumped during a time step prior to the initiation of a target attainment strategy, \( L/T \).

- \( p_i^{\text{tss}} \) = the volume that must be pumped from cell \( i \) during each time step in order for the target potentiometric surface to be maintained once it has fully evolved. It is the sustained yield (steady-state) pumping rate that may well be utilized once the target surface is attained. \( L^2/T \).

Sequential use of the objective function, Equation 4, with a different number of time periods, \( K \), being considered in each successive optimization, allows one to determine the number of years required to satisfactorily attain the target potentiometric surface.

It should be realized that increasing the number of time steps used in successive optimizations does not necessarily result in improved target surface attainment. For example, consider an optimization model in which the number of constraints increases with number of time steps. Whenever a constraint is added, some previously feasible solutions are made unacceptable. A solution that was optimal for an optimization run of \( K \) time steps may no longer be feasible for a run of \( K + 1 \) steps. Thus, if an optimization run using \( K \) time steps achieves very good target surface attainment, a run using \( K + 1 \) steps may achieve less attainment. An example of this is illustrated later.

**PROCEDURE**

In this paper we compare target surface attainment using one steady pumping approach (Approach A) and four optimal unsteady pumping approaches (Approaches B-E). Approach A consists of simulating achieved water levels by using equation [2] for predetermined pumping values and simulation time periods. Approaches B-E use equation [4] and appropriate constraint equations to develop optimal pumping values for simulation time periods of predetermined duration. The unsteady approaches all use constraint equation [3]. They differ
from each other only in the bounds placed on pumping in equation [6] and in whether or not constraint equation [7a] is used. In the presented examples $p^e$ is never less than $p^0$, so equation [7b] never applies.

All five approaches are applicable for that period of time when water levels are to evolve from current values, $h^e$, toward target values, $h^t$. This period of time is considered to be a transition or evolutionary era. In the presented examples the model for Approach A was used for eras of one, two and three 3-year time steps (3, 6 and 9 years). Models for Approaches B and C were utilized for eras comprised of two and three time steps. Models for Approaches D and E use two time steps because the optimization subroutine in the models develops numeric difficulties and ‘blows up’ when applied for three steps.

The cause of the numeric instability is not known, although similar difficulties have been reported by Elango and Rouve (1980), Evans and Remson (1982) and Gorelick (1983) in other models that optimize groundwater management. The models presented in this paper use an optimization subroutine based on the General Differential Algorithm (Liefson et al., 1981) and were run on the University of Arkansas mainframe computer, an Amdahl 470. It is surmised that the use of a different linear optimization subroutine or possibly a different computer may remedy the instability problem.

It is assumed that prior to the evolutionary era, each cell is experiencing a particular 3-year groundwater withdrawal rate, $p^i$. After the evolutionary era, 3-year groundwater pumping in each cell will be at that rate that will maintain its target level, $p^m$.

The purpose of evaluating different approaches is to determine which transition period sequence will cause the desired potentiometric surface to be approached most closely. Fig. 2 is presented to aid in visualizing the manner in which pumping at a sample cell is constrained for the five different approaches. It displays the pumping that is occurring prior to the evolutionary era, $p^e$, the pumping that will occur after the evolutionary era, $p^m$, and the manner in which pumping can vary during the era. The way in which $p^e$ and $p^m$ are created is mentioned in subsequent paragraphs which describe determination of the constant values required for constraint equations [3], [6] and [7a].

Fig. 1 displays assumed initial water table elevations in the aquifer underlying the study area. These initial water levels are the $h^e$ values of equation [3]. The cell-by-cell volumes of groundwater that must be pumped from the aquifer during each three year period ($q^m$) in order to maintain those water levels are calculated using assumed transmissivities and the linearized Boussinesq equation as described by Yazdanian and Peralta (1986). The transmissivities are derived from the initial water levels, saturated thicknesses and a hydraulic conductivity of 82 m/day.

The necessary $\delta$, discrete kernels, are computed based on an effective porosity of 0.3 and cell-by-cell transmissivities. Computation of the discrete kernels is performed using an algorithm of Verdin et al. (1981). As previously stated, we assume that the change in transmissivity with time during the simulated evolution era is not significant enough to justify recalculation of the discrete kernels during that period.

Fig. 3 displays the difference between assumed target potentiometric surface elevations, $h^t$, and initial water table elevations. Note that the target level is above the initial level for some cells and below the initial level in other cells. This was done purposely in order to demonstrate the facility of the modelling approach to design both groundwater mining strategies (for situations in which target levels are below initial levels) and recovery strategies (for cases in which target levels are above initial levels).

The three-year pumping needed from each cell in order to maintain the target elevations are calculated

![Fig. 2A-E—Feasible variation in annual pumping rates with time for a typical cell in Approaches A-E, (L^3/T).](image-url)

![Fig. 3—Target water table elevations minus initial water table elevations in the study area, (m).](image-url)
using the approach of Yazdanian and Peralta (1986). These $p_i^n$ values are the steady 3-year pumping values used in each 3-year time step of the evolutionary era in Approach A (Fig. 2). Fig. 2 shows that they are the steady 3-year pumping values that will be utilized after the evolutionary era for all approaches. They are also the lower bounds on pumping, the $p_i^n$ values in equation [6], for two of the four unsteady approaches, Approaches B and C. Approaches D and E use 0.0 as the lower bound on pumping in all cells. All four unsteady approaches use recent historic pumping from those cells as upper bounds, $p_i^n$, in equation [6].

As previously stated, approaches B and D use constraint equation [7a] for each internal cell. In these approaches, pumping in a cell cannot increase with time, i.e. the change in pumping is negatively unidirectional. Fig. 2 indicates that Approaches C and E are not constrained in this manner.

RESULTS

For the assumptions utilized in this study, it is not expected that perfect surface attainment can be achieved. Recall that initial and target surfaces are selected such that the potentiometric surface elevations must rise in one portion of a study area, and decline in another portion. This would not create difficulty if the effect of pumping at a cell were not felt outside that cell. For the presented cell size and assumed aquifer parameters however, pumping in a cell affects potentiometric head within a five-by-five grouping of nine cells, including the pumped cell itself. The pumping needed to help attain a target elevation in one cell may hurt attainment of the target in a different cell. Thus, the problem of attempting to achieve target surface elevations in multiple cells is a multi-objective problem, with conflicting goals. Approaches B-E simply resolve the conflicts by minimizing the sum of the absolute values of all the differences between achieved and target elevations.

Fig. 4 displays the degree to which target levels are attained by the end of the transition eras tested for the different approaches. The ordinate axis shows the sum of the absolute values of the differences between the target water levels and the levels attained by optimization and/or simulation. The abscissa displays the duration of the simulated transition periods. Numeric subscripts to Approaches A-E designate the number of time steps for which each modelling run was performed. Note that Approach A is simulated for eras of one, two and three time steps (36 and 9 years), and Approaches B and C are optimized for eras of both two and three time steps (6 and 9 years respectively). As mentioned previously, the longest era that Approaches D and E are optimized for is two time steps in duration.

Note that the more constrained the pumping is, the more slowly the target surface is approached. Approach A, in which the steady-state target sustained yield pumping is pumped during all time steps, approaches the target more slowly than any of the unsteady approaches. The next slowest evolution is achieved by Approach B, in which pumping is unidirectional in change with time step and has the smallest range of permitted values. The most rapid evolution is attained by Approach E, which is not subject to directional constraint and in which pumping can range between 0 and the initial pumping.

Comparing the attainment of runs B1 and B3 in Fig. 4 shows that the pumping strategy developed for a three-step evolutionary era results in slightly less target level attainment than the strategy developed for a two-step era. As explained previously, this is not unexpected. For the 40 internal cells. Approach B attained the target surface within an average of less than 0.1 m per cell for the two-step optimization. Running the Approach B model for three time steps requires 1 more constraints via equation [7a] than running it for two time steps. Since the three-step problem has more constraints than the two-step problem, and target attainment is excellent for the two-step run, it is not surprising that target attainment is slightly less good for the three-step run. The utilized discrete kernels and constraints simply do not permit as close a fit for the 9 year simulation as for the 6 year simulation, although the difference in target attainment is insignificant.

When considering which of the four optimization approaches is most useful for an actual management scenario, one may feel that it is not particularly desirable for permitted pumping to decrease one year and increase the next. A unidirectional change is preferable for a water user who may be planning the gradual conversion of irrigated to nonirrigated acreages to adapt to a regional sustained yield pumping strategy. Therefore, Approach B, in which pumping unidirectionally changes from current pumping toward target pumping, is the preferred model for designing a target surface attainment strategy. The speed of evolution of Approach B is not much less than that of the other unsteady approaches and it is socially and economically more tractable for planning purposes.

It is desirable to affirm that the presented modelling approach develops pumping strategies that can cause water levels to rise in some cells and decline in others. To
accomplish this one compares Fig. 3 and 5. Fig. 5 displays the difference in elevation between target water levels and those attained after 6 years using Approach B. The only nonzero values in Fig. 5 are positive, indicating that the attained water levels are below the target elevations in those cells. Clearly the simulated pumping strategy caused some water level declines. In addition, levels rose noticeably in cells (1,1) = (5,4) and (5,5). One notes in Fig. 3 that the target level was 1.2 m above the initial level in both those cells. From Fig. 5 one sees that after two time steps the water level is only 0.3 below the target level in each cell. Therefore, the proposed approach can simultaneously cause both the decline and increase in water table elevations within different parts of a single study area.

**SUMMARY**

Water management agencies can assure the sustained availability of groundwater for agricultural or other uses by causing the maintenance of appropriate regional potentiometric surfaces. Under some conditions, an initially existing surface can be caused to evolve into such a target surface by implementing the particular sustained yield pumping strategy that will maintain the target surface once it is achieved. Unfortunately, target surface attainment may be quite slow when that steady pumping strategy is used. Much more rapid target surface attainment can be achieved by using a goal programming methodology that combines optimization and simulation. The technique utilizes discrete kernels to develop the time varying pumping strategies that can maximize target surface attainment within a predetermined planning period. The methodology is applicable for the situation in which initial water levels are higher than target water levels (a groundwater mining scenario) as well as the case in which initial levels are lower than target levels (a recovery scenario).

Use of the appropriate constraints in the goal programming model assures that the resulting optimal pumping strategy is useful for management purposes. Pumping in each cell can be constrained such that it unilaterally either increases or decreases during the attainment era. This ability is important for a management agency that is seeking to develop a particular steady-state potentiometric surface in a region that has an existing pattern of groundwater pumping. For example, it facilitates planning the gradual increase or decrease in acreages that should be irrigated with groundwater while a target potentiometric surface is evolving.

**References**


