

Groundwater Recharge Planning Using Resolvent Discrete Kernels

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INTRODUCTION

IN the Arkansas Grand Prairie (Fig. 1), wells are becoming inoperable, and the threat of litigation is increasing because of decreasing groundwater availability. Groundwater provides over half of the irrigation water currently used in this important rice and irrigated soybean producing area. Most groundwater is obtained from a relatively shallow Quaternary aquifer, part of the Mississippi Plain alluvial aquifer. That portion of the aquifer underlying the Grand Prairie is recharged primarily from surrounding extensions of the same aquifer system. No doubt, rivers peripheral to the study area contribute recharge, but their effect is considered to be lumped with that of the surrounding aquifer. Very little vertical recharge occurs within the Grand Prairie because of a relatively impermeable clay cap. As a result, groundwater levels have been declining in the unconfined central portion of the Grand Prairie and saturated thickness has decreased alarmingly in some locations.

The Grand Prairie is a likely candidate to be the first region designated as a critical groundwater area in Arkansas. Recent legislation has given the Arkansas Soil and Water Conservation Commission responsibility for identifying such regions. Selected areas may experience more intensive state control and management than is the norm in Arkansas.

Large-scale diversion of water from the Arkansas and White Rivers is the most likely means of reducing reliance on groundwater. Enhancing aquifer recharge is a complementary, though partial, solution. State and federal agencies have cooperated in evaluating the feasibility of diverting river water. However, at least 10 years would be required to bring proposed diversion systems into operation and reduce reliance on Quaternary groundwater. In the meantime, enhanced recharge of the aquifer may help alleviate the adverse impact of continued groundwater use.

Previous studies determined that recharge via injection was impractical for the central Grand Prairie (Sniegocki, 1963; Sniegocki et al., 1965; Griffis, 1976). An

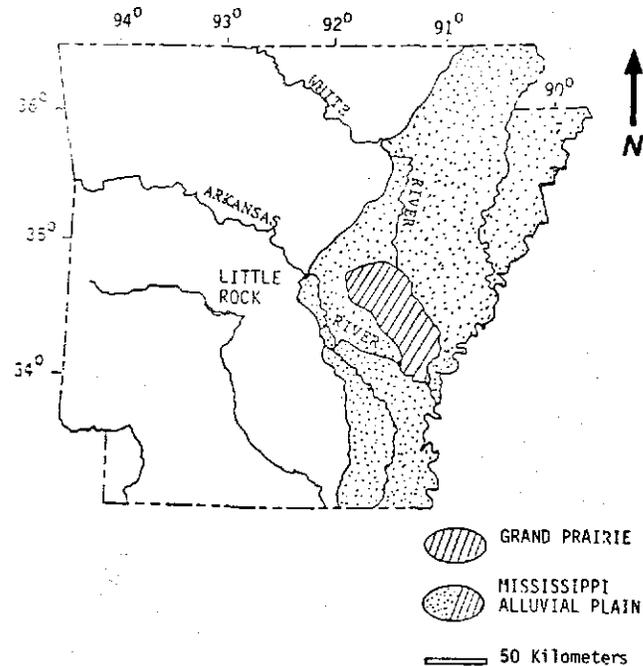


Fig. 1—Arkansas, the Mississippi Alluvial Plain and the Grand Prairie.

alternative is the use of recharge basins near peripheral streams where aquifer material outcrops. The primary purpose of this report is to quantify the increase in optimal groundwater extraction that would occur if two such basins were installed. The increase is determined using a computer model that calculates maximum extraction volume for a specific planning period, subject to constraints. For efficiency, the linear model utilizes the discrete kernel (algebraic influence coefficient) concept.

The second objective of this report is to discuss development of influence coefficients that permit calculation of aquifer response to simultaneous groundwater pumping and interflow between recharge basin and aquifer. This is not trivial since interflow is a function of water levels which are in turn functions of pumping. Use of these coefficients in simulation or optimization models replaces the somewhat inaccurate procedure of computing interflow in a time step using groundwater levels existing at the end of the previous time step. (Illangasekare and Morel-Seytoux, (1982), also describe computation of stream/aquifer influence coefficients.)

RELATION TO PREVIOUS WORK

The linear influence coefficient approach has long been used in groundwater simulation or management optimization (Maddock, 1972; Morel-Seytoux et al.,

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1973; Maddock and Haines, 1975; Morel-Seytoux, 1975a,b,c,d; Morel-Seytoux and Daly, 1975; Haines and Dreizin, 1977; Morel-Seytoux et al., 1981; Heidari, 1982; Illangasekare and Morel-Seytoux, 1984; Illangasekare et al., 1984; Colarullo et al., 1984; Danskin and Gorelick, 1985; Morel-Seytoux, 1985; Peralta et al., 1985; Peralta and Kowalski, 1986a, b). Gorelick (1982) provides an excellent review of early papers on that subject.

Peralta and Kowalski (1986a) used discrete kernels to determine optimal groundwater extraction strategies for the Grand Prairie. By appropriate recharge constraints, they assured that the developed strategies would not disrupt the surrounding regional groundwater flow patterns. They developed strategies maximizing groundwater extraction and maximizing the present value of net economic return resulting from extraction. Four different sets of constraints affecting acceptable drawdown and change in pumping with time were used.

They found that both objective functions yielded essentially the same total pumping and net return. This probably results from three facts: (a) all net return is assumed to be generated from groundwater, (b) there are no foregone costs of unsatisfied demand for water and (c) the planning period is relatively short. The major difference in results was that maximum return strategies extracted more groundwater in the early part of the planning period, while the maximum pumping strategies pumped more in the latter part.

Based on the findings of Peralta and Kowalski (1986a), only strategies maximizing extraction are analyzed in this paper. We use the same objective function, but selectively add recharge basins to demonstrate the effect of those basins on maximum extractable groundwater. The same sets of constraints are used in this paper as were used in the previous work.

To demonstrate the effect of recharge basins on groundwater extraction, we must consider how best to linearly model interflow between basin and aquifer. In general, influence coefficients utilized by other researchers have been designed to describe groundwater level response to specific extraction or injection stimuli. This is not entirely satisfactory when stimuli are themselves functions of existing groundwater levels, as for example in a connected surface water/aquifer system. Interflow between a reservoir and an aquifer is affected by the difference in head between the surface water and the groundwater (Morel-Seytoux et al., 1974; Morel-Seytoux and Daly, 1975). In such a situation, common practice is to estimate interflow based on levels existing in a preceding time step, or to estimate and then recalculate until heads and interflow are in harmony. Clearly, a need exists for discrete kernels that can express groundwater level response to both pumping and interflow based on simultaneously existing groundwater and surface water heads. The presented discrete kernels accomplish this.

THEORY AND MODEL FORMULATION

The simple model used in this study maximizes total groundwater extraction, Z , subject to constraints and bounds (Heidari, 1982).

$$\max Z = \sum_{k=1}^K \sum_{i=1}^J g_{i,k} \dots \dots \dots [1]$$

where

K = the number of time steps in the planning period

J = the number of variable-head cells in the study area

$g_{i,k}$ = groundwater extraction in cell i , time step k , L^3

Subject to

$$0 \leq g_{i,k} \leq w_{i,k} \quad \text{for } i = 1 \dots J, k = 1 \dots K \dots \dots [2]$$

$$s_{i,k} \leq s_{i,k}^U \quad \text{for } i = 1 \dots J, k = 1 \dots K \dots \dots [3]$$

$$e_{\ell,k}^L \leq e_{\ell,k} \leq e_{\ell,k}^U \quad \text{for } \ell = 1 \dots L, k = 1 \dots K \dots \dots [4]$$

and, if it is desirable that the annual pumping volume in a cell not increase after it has decreased from current pumping (unidirectional change):

$$g_{i,k+1} \leq g_{i,k} \quad \text{for } i = 1 \dots J, k = 1 \dots K - 1 \dots \dots [5]$$

where

$w_{i,k}$ = the volume of groundwater required for irrigation to support current (1982) acreages in cell i under average climatic conditions in a single time step L^3

$s_{i,k}$ = the mean drawdown that has occurred in cell i by the end of time step k , L

$s_{i,k}^U$ = the upper bound on acceptable drawdown in cell i by the end of period k , L

$e_{\ell,k}$ = the volume of groundwater that will enter the study area aquifer in peripheral cell ℓ and time step k from extensions of the aquifer outside the study area, L^3

$e_{\ell,k}^L$ and $e_{\ell,k}^U$ are lower and upper bounds on the volume of groundwater flowing between the aquifer underlying cell ℓ and extensions of the aquifer outside the study area in time step k , L^3

L = the number of peripheral cells surrounding the variable-head cells of the study area. In this study the peripheral cells are all constant-head/restrained flux cells.

In actuality, neither $s_{i,k}$ nor $e_{\ell,k}$ are explicitly used as variables within the models. Since groundwater movement is a function of water levels, e is represented as a function of s (Peralta and Kowalski, 1986a). Drawdown s , a function of pumping, is developed in the following way. First, adopting the convolution equation described by Morel-Seytoux et al (1981) and Illangasekare et al (1984), the drawdown in water level since initial time in cell i by the end of time period N is:

$$s_{i,N} = \sum_{k=1}^N \sum_{j=1}^J \delta_{i,j,N-k+1} (q_{j,k} - q_j^{ass}) \dots \dots [6]$$

where

$\delta_{i,j,N-k+1}$ = a nonnegative-valued discrete kernel (linear influence coefficient) that describes the contribution to the hydraulic head at cell i in time step N caused by a unit ($q_{j,k} - q_j^{ass}$). The temporal subscript $N-k+1$ merely insures that the proper δ is utilized ($1/L^2$)

$q_{j,k}$ = the net vertical hydraulic stimulus in cell j in time step k . It is the sum of all discharges (+) from the aquifer and recharges (-) to the aquifer from the ground surface, L^3

q_j^{ass} = the net vertical hydraulic stimulus that must occur in each time step in cell j for that cell to maintain its original head, L .

The steady-state stimulus needed to maintain the original, possibly artificial, potentiometric surface, q^{ass} , is calculable using the linearized Boussinesq equation for steady-state two-dimensional flow through porous media. (The steady state stimulus at a cell is a function of the heads and transmissivities at itself and four adjacent cells.) For an m cell study area (including both internal and boundary cells) all q^{ass} values are computed simultaneously from:

$$\{Q^{ass}\} = [T] \{H^0\} \dots \dots \dots [7]$$

where

$\{Q^{ass}\}$ = an $m \times 1$ vector of net steady-state stimuli, L^3

$[T]$ = an $m \times m$ matrix of finite difference transmissivities, L^2

$\{H^0\}$ = a vector of initial potentiometric heads, L .

Assume that $q_{j,k}$ equals groundwater pumping ($g_{j,k}$) minus recharge basin/aquifer interflow ($\sigma_{j,k}$), where we assume movement of water from surface to aquifer. Saturated basin/aquifer interflow at cell j in time step k equals the reach transmissivity, Γ , times the difference in heads between the reservoir and the underlying water table. Thus,

$$\sigma_{j,k} = \Gamma_j (\sigma_{j,k} - h_j^0 + s_{j,k}) \dots \dots \dots [8]$$

where

Γ_j = reach transmissivity, (L^2). It is zero for all cells without surface water resources in hydraulic connection with the aquifer

$\sigma_{j,k}$ = the elevation of the free water surface in the reservoir, L

h_j^0 = the initial water groundwater table elevation, L .

Combining equations [6] and [8] yields

$$s_{i,N} = \sum_{k=1}^N \sum_{j=1}^J \delta_{i,j,N-k+1} (g_{j,k} - \Gamma_j (\sigma_{j,k} - h_j^0 + s_{j,k}) - q_j^{ass}) \dots \dots \dots [9]$$

Moving all s values to the left side and rearranging the right-hand side slightly yields:

$$s_{i,N} = \sum_{k=1}^N \sum_{j=1}^J \delta_{i,j,N-k+1} \Gamma_j s_{j,k} = \sum_{k=1}^N \sum_{j=1}^J \delta_{i,j,N-k+1} (g_{j,k} - q_j^{ass} - \Gamma_j (\sigma_{j,k} - h_j^0)) \dots \dots \dots [10]$$

Defining the right-hand side of equation [10] as $f(\delta,y)$, where y equals all terms within parentheses, and rearranging yields:

$$s_{i,N} (1 + \Gamma_j \delta_{i,i,N}) + \sum_{k=1}^N \sum_{j=1}^J \delta_{i,j,N-k+1} \Gamma_j s_{j,k} = f(\delta,y) \dots \dots \dots [11]$$

$j \neq i$ if $k=N$

There is one equation [11] for each time step for each cell. The resulting system of K times J linear equations can be expressed as:

$$[A] \{E\} = [D] \{Y\} \dots \dots \dots [12]$$

$[A]$ = a $(K \times J) \times (K \times J)$ matrix containing the known values in the left-hand side

$[E]$ = a $(K \times J) \times 1$ vector of s terms from the left-hand side

$[D]$ = a $(K \times J) \times (K^2 \times J)$ matrix of known discrete kernels, δ , from $f(\delta,y)$

$[Y]$ = a $(K^2 \times J) \times 1$ vector of the known y terms in $f(\delta,y)$.

Assuming that we know all values in $[A]$, $[D]$ and $[Y]$, drawdowns in all cells at the end of all time periods can be computed by Gauss-Jordan reduction. In this case, one replaces $[D][Y]$ with an equivalent $(K \times J) \times 1$ vector, $[Z]$, before using Gauss-Jordan elimination. The augmented matrix that is solved is $[A:Z]$.

One proceeds differently, if, instead of directly calculating drawdowns, one wishes to compute discrete kernels that can be used for simulation within an optimization model. One begins by computing an intermediate $(K \times J) \times (K^2 \times J)$ matrix $[Z']$ that is the product of $[D][Y]$. Each row of $[Z']$ consists of an uncombined summation of δy products. (Adding all terms in each row, after substituting numeric values for δ and y terms, would cause $[Z']$ to collapse into $[Z]$.) Record is kept of which $y_{j,k}$ is associated with each product.

Next, one performs the same row reductions on $[Z']$ that are performed on $[A]$. This is accomplished term by term while keeping the modified δy products separate. After row reduction is complete, all terms, in a row, that are associated with a common $y_{j,k}$ are combined. The result is a $(K \times J) \times (K \times J)$ matrix, $[Z'']$. Each i,N row of $[Z'']$ can be expressed as the right-hand side of equation [13].

$$s_{i,N} = \sum_{k=1}^N \sum_{j=1}^J \beta_{i,j,N-k+1} y_{j,k} \dots \dots \dots [13]$$

where

$\beta_{i,j,k}$ = a resolvent influence coefficient, L^2 .

Division of each product in equation [13] by the appropriate $y_{j,k}$ yields the $\beta_{i,j,N-k+1}$ influence coefficients.

In order to differentiate somewhat between the effect of pumping and stream/aquifer interflow, each row in $[Z']$ can also be defined as:

$$s_{i,N} = \sum_{k=1}^N \sum_{j=1}^J \beta_{i,j,N-k+1} (g_{j,k} - q_j^{ass}) - \mu_{i,j,N-k+1} (\sigma_{j,k} - h_j^0) \dots \dots \dots [14]$$

where

$\mu_{i,j,k}$ = a dimensionless coefficient that equals $\Gamma_j \beta_{i,j,k}$. Equation [14] is appropriate for use in constraints on drawdown or groundwater levels.

In summary, the model consists of one objective function, equation [1]; $J \times K$ variable pumping values bounded via equation [2]; $J \times K$ drawdown variables bounded by combining equations [3] and [14]; L constraints on recharge, equation [4]; and either none or $J \times (K-1)$ of equation [5], depending on whether the change in pumping is to be unidirectional.

METHODOLOGY

Data Development

As previously mentioned, the Grand Prairie is only a portion of an extensive aquifer system. Since it is economically impractical to develop optimal groundwater extraction strategies for the entire system, some boundaries assumed in this study are not hydrologic in nature. Justification of the use of constant-head/restrained flux boundary conditions is provided by Peralta and Kowalski (1986a). They also discuss bounds on flux across peripheral cells, e^l and e^u in equation [4], necessary to prevent disruption of regional flow.

Aquifer parameters assumed for computation of q^{ass} , δ , β , and μ are an effective porosity of 0.3 and finite difference transmissivities. Transmissivities are calculated from kriged saturated thicknesses (based on measurements at over 100 wells) and a hydraulic conductivity of 82.3 m/day (270 ft/day) (Engler et al., 1945; Griffis, 1972; Peralta et al., 1985). Influence coefficients, δ , are computed using an algorithm of Verdin et al (1981). β and μ are computed from the δ . Changes in saturated thickness resulting from the optimal extraction strategies do not exceed the standard error of the estimate of the initially estimated saturated thickness. Therefore, initially computed influence coefficients are valid throughout the optimization period.

Values of w used as upper bounds on pumping in equation [2] are the volumes of groundwater currently being withdrawn from the aquifer under average climatic conditions. It is assumed that water currently provided from other sources will continue to come from those sources, and that no expansion of irrigated acreages is likely.

Upper bounds on drawdown in equation [3] are those values that will leave a minimum acceptable saturated thickness remaining at the end of each time step. Optimizations are performed using either 3 m or 6 m (10 or 20 ft) as the minimum acceptable terminal saturated thickness.

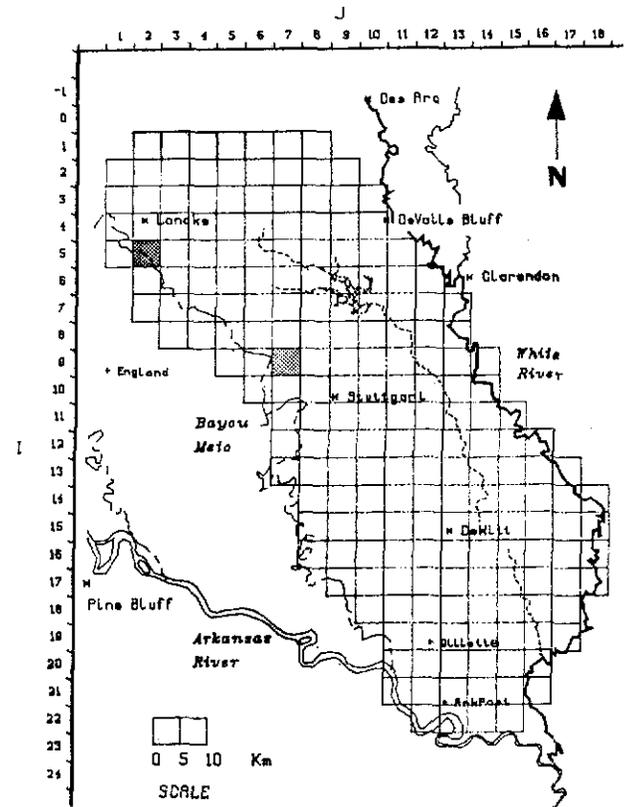


Fig. 2—Cells containing assumed recharge basins, (shaded).

The initial heads used in equation [14] are those existing in spring 1982. Free water surfaces in the basins are at ground level and are assumed constant with time.

An identical recharge basin is assumed for each of two cells near the Bayou Meto on the western edge of the Grand Prairie (Fig. 2). These are cells: (a) at which aquifer material outcrops, based on records of water well construction, (b) proximal to a surface water resource, and (c) adjacent to cells at which groundwater recharge to the area limits achievable groundwater extraction. Cells satisfying the third criterion are not identified until after optimizations are performed without considering recharge basins.

Rectangular basins 70 m \times 35 m (200 ft \times 100 ft) respectively, are assumed. The conservatively estimated aquifer saturated thickness beneath the basins is 4.6 m (15 ft). The result of these values and an aquifer hydraulic conductivity of 82.3 m/day is a reach transmissivity of 880 m²/day (9500 ft²/day), computed using the procedure of Peters and Morel-Seytoux (1980). No reduction in reach transmissivity due to siltation is considered.

Results and Discussion

Eight optimizations maximizing groundwater extraction are presented (Table 1). Four utilize recharge basins and four do not. Optimization is performed using the QPTHOR code (Liefsson et al., 1981). Each group of four optimizations consists of possible combinations of: (a) constraining saturated thickness to be at least 6 m or at least 3 m, and (b) either forcing pumping to be unidirectional in change with time or letting it change freely within initial bounds.

The optimization problems become less constrained and maximum pumping increases from top to bottom of

TABLE 1. INCREASE IN MAXIMUM FEASIBLE 10-YEAR GROUNDWATER EXTRACTION RESULTING FROM RECHARGE BASIN USAGE

Strategy	Total groundwater extraction (million cubic meters)		Percent increase
	Without recharge basins	With recharge basins	
Unidirectional* Pumping			
MST† = 6 m	2026	2150	6.1
MST = 3 m	2144	2266	5.7
Free Pumping			
MST = 6 m	2155	2306	7.0
MST = 3 m	2300	2449	6.5

*In 'unidirectional' strategies, pumping in a cell cannot increase from one time step to the next.

†MST is the minimum saturated thickness acceptable in any cell in a given strategy.

Note: to convert from cubic meters to acre-feet divide by 1233.5

Table 1. As is expected, maximum pumping that is directionally constrained is less than the maximum pumping obtainable for freely-varying pumping. Similarly, less pumping is possible if final saturated thickness must be at least 6 m than if saturated thickness can be reduced to 3 m.

In all cases, recharge basin utilization increases maximum regional pumping by at least 5%. This is probably greater than it would be if the recharge basins were located in other cells, although the difference is not quantified in this paper. In design practice an interactive procedure for refining recharge basin siting and design is desirable—constraints that are tight, based on one set of assumptions, may no longer be tight if assumptions are changed slightly. An example computer graphics-based program for rapidly modifying optimal strategies is presented by Killian and Peralta (1985).

CONCLUSIONS

Appropriately located recharge basins can contribute significantly to groundwater availability for pumping. Hydraulically desirable sites can be identified by hydrogeologic screening and by performing preliminary optimizations. These optimizations are performed without using recharge basins. Examination of resulting constrained derivatives identifies locations at which the availability of additional recharge would most greatly increase the total pumping volume.

To develop optimal groundwater extraction strategies for systems that include recharge basins, it is desirable to utilize discrete kernels that describe the effect on water levels of pumping and interflow based on simultaneously existing groundwater and surface water heads. This assures that saturated interflow between reservoir and aquifer is modeled efficiently. Discrete kernels that accomplish this are presented.

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