

Maximizing Sustainable Ground-Water Withdrawals—Comparing Accuracy and Computational Requirements for Steady-State and Transient Digital Modeling Approaches

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Abstract

Rigorous models for maximizing sustainable ground-water withdrawals may require more computer memory for their constraint set than is available. In some situations, alternative constraint formulations yield similar or identical answers resulting in great saving in computer memory requirements. In order to evaluate the efficiency of using alternative constraints, maximum ground-water withdrawal pumping strategies were computed by three digital models for a hypothetical area for a five-decade period. Model A maximized steady ground-water withdrawal. Model B maximized unsteady ground-water mining. Model C maximized unsteady ground-water mining subject to a constraint that final pumping be sustainable after the end of the 50-year period. Change in pumping with time was forced to be monotonic (variably increasing or decreasing but not oscillating) in time. The models were tested by assuming constant transmissivity and by using a range of recharge constraints for four scenarios—with stressed and unstressed initial potentiometric surfaces and with constant and changing upper limits on pumping. In situations where upper limits on pumping changed with time, Model A was run repetitively, by using monotonicity constraints. In those situations, optimality of solution is not assured in all cells. Models A and C computed pumping strategies sustainable after the end of the 50-year period. Model C was the most detailed in that it allowed pumping to vary in time and recharge constraints were based both on unsteady-state flow at 50 years and on steady flow after that time. Model A considered only steady pumping and recharge constraints. Pumping strategies from Model B were not necessarily sustainable because it considered only recharge constraints at 50 years. Results indicate that, when recharge through the study area periphery is unconstrained, all models compute identical pumping. For an initially undeveloped aquifer,

or for a developed aquifer if steady pumping is assumed, Model A computes strategies very similar to those computed with Model C and requires only 28 percent of the computer memory and 38 percent of the execution time. For an initially overdeveloped aquifer, Model B computes identical pumping strategies to those computed with Model C and requires 73 percent of the computer memory and 78 percent of the computation time. For that situation, Model A is more conservative and computes less pumping than Model C if pumping in Model C is permitted to vary. Although Model A may compute lower pumping rates during the first 50 years, the sustainable pumping rate thereafter may be greater for Model A than for Model C.

INTRODUCTION

Ground-water availability is an important consideration for agricultural and land-use planners in the United States and abroad. Ensuring the long-term availability of ground water contributes to developing sustainable production. Computer models are used to develop regional land-use plans and agronomic cropping strategies that consider the restraints on ground-water use posed by the physical system. Such models simulate ground-water flow and compute development strategies optimal for particular policy objectives and physical or nonphysical constraints. This paper compares the accuracy and computer-resource requirements of three optimization model formulations to determine their appropriateness for estimating maximum sustainable regional ground-water withdrawals. Each model computes optimal future withdrawals for each decade of a 50-yr planning period. Optimal pumping was computed for a hypothetical region consisting of finite difference cells, each 3 mi by 3 mi in size (fig. 1). Distributed pumping is assumed within the block-centered nodes. Because of the large cell sizes and time steps, the models are more

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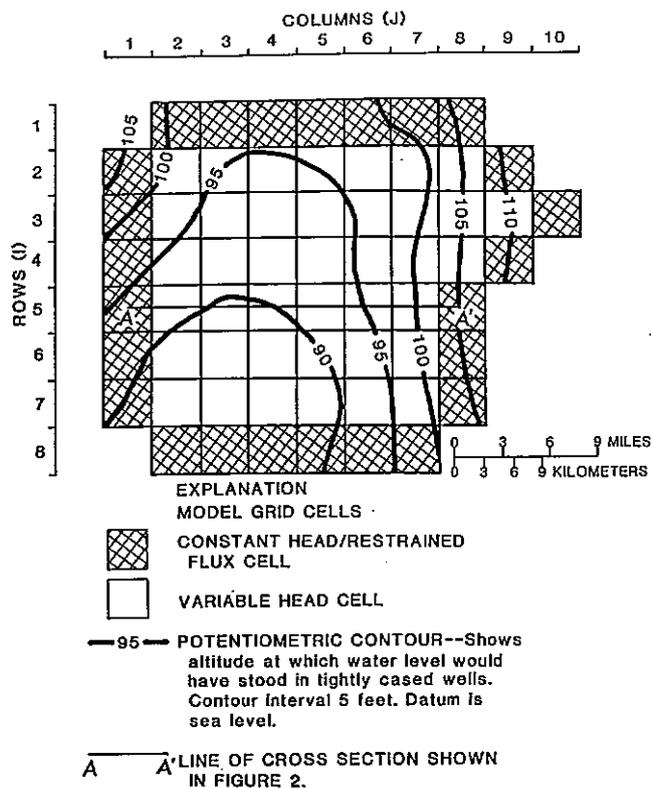


Figure 1. Model grid and initial potentiometric surface in hypothetical study area.

appropriate for planning future crop acreages supportable by ground water than for managing daily pumping operations.

Early uses of distributed parameter computer models to develop optimal volumetric ground-water management strategies are summarized by Domenico (1972), Bear (1979), and Gorelick (1983). Gorelick describes both linear and nonlinear programming models. Because this paper deals only with linear programming to optimize volumetric withdrawals, no studies utilizing quadratic or nonlinear models are cited. Applications of optimization based on economic or water-quality considerations also are omitted, even though several have been reported.

Recent studies using the linear objective function of maximizing ground-water withdrawal are reported by Tung and Koltermann (1985), Tung (1986), Peralta and others (1987), and Yazicigil and others (1987). Other efforts have included maximizing withdrawal within multiobjective optimization (Datta and Peralta, 1986; Peralta and Killian, 1987; Yazicigil and Rasheeduddin, 1987).

For several reasons, planners might prefer to use the linear objective function of maximizing ground-water withdrawal rather than a quadratic function of maximizing economic benefits. The first reason is that a strategy maximizing withdrawal volume may be almost identical to a strategy maximizing net return, depending on the planning

period, cost coefficients, bounds on variables (head, pumping, recharge, streamflow, and so forth), and other restrictions. In fact, a linear objective function is sometimes considered a surrogate for the quadratic function of maximizing net economic return of crop production resulting from ground-water use. For example, Peralta and Kowalski (1988) obtained total ground-water withdrawal and economic value differences of less than 2 percent between strategies that maximize withdrawal and those that maximize economic value. Casola and others (1986) also reported little difference between volumetric and economic solutions. Their optimal economic pumping strategy consisted of pumping at or near its upper bound until the final time steps. A second reason for using the linear objective functions is that the terminology used by a legislature or court, in mandating water management directives, generally is related to volumetric rather than economic constraints. For example, "maximizing use" does not imply economic optimization. A third reason for using linear objective functions is that they require less computer memory and time than similar quadratic problems, since no matrix of quadratic coefficients is needed. Even though quadratic problems are readily solved by using commercially available optimization algorithms, linear objective functions are used to ensure that formulated problems can be solved practically on hardware such as microcomputers.

The central issue of this paper is how to best incorporate equations of ground-water flow within models for maximizing volume withdrawal during a hypothetical planning period. The purpose is to compare the accuracy and computational requirements of alternative approaches and to demonstrate situations where one set of simulation constraints is preferable to another.

Three finite-difference digital models were used to test the alternative approaches. Model A incorporated steady-state flow equations embedded directly as constraints (embedding method). The systems engineering concept of not simulating in more detail than is necessary for a particular situation is employed in Model A. Model B utilizes superposition and linear systems theory (response matrix method) to represent unsteady flow. It maximizes ground-water mining, withdrawing in excess of what is recharged for a period of time, and does not assure sustainability of pumping beyond the 50-yr planning period. Model C incorporated both steady-state embedding and transient-response matrix approaches. Model C is a combination of Models A and B. It simulates unsteady flow for the planning period and has additional steady-state flow constraints to ensure that pumping in the final time step can be continued beyond the end of the planning period (50 yr in this test). Model C is the most physically rigorous model and is used as the basis for comparison.

Constraints were tested for a range of acceptable boundary recharge rates by using combinations of four scenarios—with constant and varying upper limits on

pumping and with stressed and unstressed initial potentiometric surfaces. Pumping strategy sensitivity to aquifer parameters and transmissivity was demonstrated.

DESCRIPTION OF MODELS

Two-dimensional saturated ground-water flow is assumed in a hypothetical 585 mi² study area (fig. 1). The aquifer in the study area is merely part of a surrounding, much larger aquifer system. The surrounding aquifer can provide recharge to the study area through each of the boundary cells in the study area. The hypothetical simulations and parameters are representative of alluvial aquifers.

Simulation of the flow system is accomplished by using the finite-difference model code AQUISIM, an acronym for aquifer simulation (Verdin and others, 1981). This model code solves the linearized Boussinesq equation and also is utilized to compute influence coefficients (the drawdown that results at a particular cell at a certain time in response to a unit pumping at some other cell and time) in optimization Models B and C. Finite-difference optimization models have been developed to compute maximum sustainable ground-water withdrawal volumes using the boundary conditions, constraints, and assumptions given below.

The use of constant-head/restrained-flux (CH/RF) cells exhibiting a modified Dirichlet boundary condition has been justified previously (Peralta and Killian, 1985; Yazdanian and Peralta, 1986) and applied to the developed models along their lateral outer limits. For the models tested, boundary heads are assumed to remain at constant elevations as long as the rate of ground-water movement across the boundary does not change significantly. Because of the application of this boundary condition in the models, boundary flow is not permitted to exceed predetermined limiting values, thus justifying applicability of the model computed strategies to field conditions.

In this study, aquifer transmissivity is assumed constant for a particular scenario for comparisons between models. Transmissivity is the same for Scenarios I and II; it is the same for Scenarios III and IV; however, transmissivity of Scenarios I and II differs from that of Scenarios III and IV. In general, transmissivity is assumed to be constant if the aquifer is confined or if the change in saturated thickness with time is small with respect to the initial saturated thickness. The effect of changes in transmissivity in response to changes in assumed saturated thickness is presented later in the section discussing simulation accuracy and sensitivity analysis.

The aquifer is assumed to be overlain by a completely impermeable cap (fig. 2). Except for ground-water pumping from wells, all recharge to or discharge from the aquifer in the study area enters or leaves through the 25 boundary CH/RF cells. No stream-aquifer hydraulic connection or

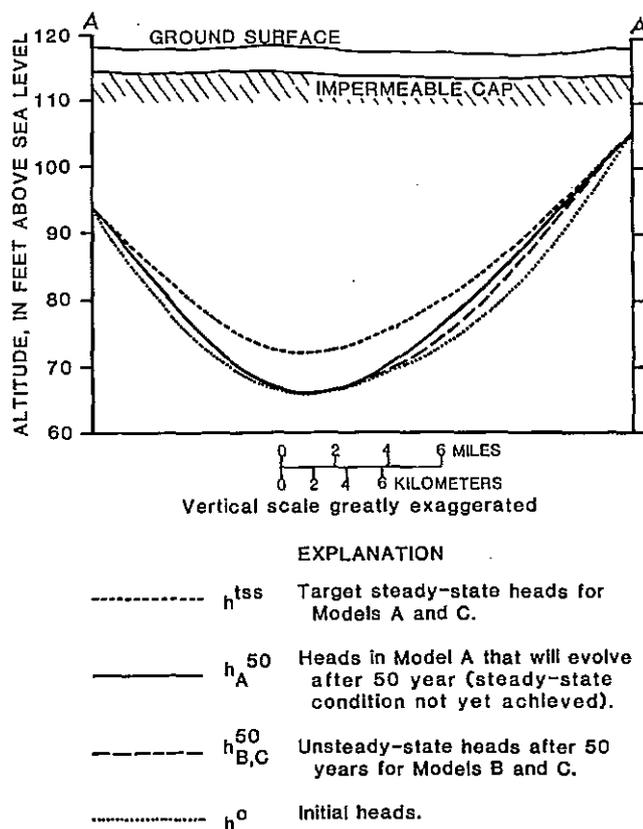


Figure 2. Geologic section A-A' showing initial and ultimate steady-state potentiometric surfaces for strategies developed for Scenario IV by using Models A, B, and C.

surface recharge occurs in any internal cells. Pumping constitutes the only discharge from the aquifer at the 40 internal cells. Because their heads can change with time, all interior cells are termed variable-head (VH) cells.

The technical development of steady-state embedded and unsteady response matrix models are presented below. In the literature, embedded models usually use a row-column notation to identify cells, while transient response matrix models usually include a running-string notation. For clarity, when merging both formulations, a row-column notation was used for all models.

Discussion of each model begins with presentation of the objective function, constraint equations, and bounds. Even though the study area used to compare the models is irregularly shaped, models are described as if they are being applied to a rectangular area of I rows and J columns. The total I×J cells is comprised of some inactive cells and active VH and CH/RF cells. Ground-water pumping occurs only at the internal VH cells, and recharge occurs only at the boundary cells.

Model A (Steady-State Embedded Constraints)

Using a steady-state modeling approach is appropriate if one wishes to compute maximal pumping rates that will cause acceptable heads, sustainable for an infinite length of time. The objective function of this model maximizes sustainable ground-water withdrawals (eq 1), while simultaneously satisfying constraints in the ground-water flow equation (eq 2) and bounds on variables (head, recharge, and withdrawal) (eqs 3-5).

$$\max Q = \sum_{i=1}^I \sum_{j=1}^J q_{i,j}^{tss} \quad \text{cell } i,j \text{ is a VH cell} \quad (1)$$

Subject to:

$$T_{i,j}^i h_{i+1,j}^{tss} + T_{i-1,j}^i h_{i-1,j}^{tss} + T_{i,j}^j h_{i,j-1}^{tss} + T_{i,j}^j h_{i,j+1}^{tss} - h_{i,j}^{tss} (T_{i,j}^i + T_{i-1,j}^i + T_{i,j}^j + T_{i,j-1}^j) - q_{i,j}^{tss} - r_{i,j}^{tss} = 0 \quad i=1,2,\dots,I, j=1,2,\dots,J \quad (2)$$

$$h_{i,j}^{L,tss} \leq h_{i,j}^{tss} \leq h_{i,j}^{U,tss} \quad i=1,2,\dots,I, j=1,2,\dots,J, \text{ and cell } i,j \text{ is a VH cell} \quad (3)$$

$$r_{i,j}^{L,tss} \leq r_{i,j}^{tss} \leq r_{i,j}^{U,tss} \quad i=1,2,\dots,I, j=1,2,\dots,J, \text{ and cell } i,j \text{ is a CH/RF cell} \quad (4)$$

$$q_{i,j}^{L,tss} \leq q_{i,j}^{tss} \leq q_{i,j}^{U,tss} \quad i=1,2,\dots,I, j=1,2,\dots,J, \text{ and cell } i,j \text{ is a VH cell} \quad (5)$$

where

Q = total ground-water pumping (L^3/T);

I and J = number of rows and columns of the area grid system;

$q_{i,j}^{tss}$ and $r_{i,j}^{tss}$ = ground-water pumping (+) and recharge (-) in cell i,j that will maintain $h_{i,j}^{tss}$ (L^3/T); there is only one ground-water flux variable per cell;

$h_{i,j}^{tss}$ = target steady-state potentiometric head that will ultimately evolve at each internal cell i,j if each is stressed by rate $q_{i,j}^{tss} + r_{i,j}^{tss}$ (L);

$T_{i,j}^i$ = geometric mean transmissivity between cells i,j and $i+1, j$ (L^2/T); $= K[(b_{i,j}) (b_{i+1,j})]^{1/2}$

$T_{i,j}^j$ = geometric mean transmissivity between cells i,j and $i, j+1$ (L^2/T); $= K[(b_{i,j}) (b_{i,j+1})]^{1/2}$

L and U = lower and upper bounds on superscripted variables.

The model simulates steady-state flow (eq 2) to compute constant ground-water withdrawal rates. Stressing internal cells at a particular constant rate (q^{tss}) ultimately produces a unique "target" head (h^{tss}) for each cell. Acceptable final target heads are assured by equation 3, and equations 2 and 4 assure that final steady-state recharge rates are acceptable.

Heads for a specified time and location can be predicted by using a transient simulation model after optimal withdrawals have been determined. Heads might not attain their steady-state values for many years. It can be

assumed that if initial and final heads are acceptable, transitional heads also will be satisfactory if withdrawals are constant in time. For purposes of comparing model performance, transient flow was simulated with the optimum withdrawals for a 50-yr period. The simulated heads and boundary flows were then used in evaluating the performance of the various models.

Model B (Transient Response Matrix Constraints)

The response matrix method (Morel-Seytoux and Daly, 1975) in Model B uses the linear systems theory, analogous to well image theory or superposition, where a simulation model or set of equations is used to compute the head change that occurs at a specific cell at a specific time in response to a unit pumping at some cell at some time. The computed head change may reflect the result of pumping at a different location and time.

The objective function of Model B (eq 6) maximizes ground-water mining (withdrawal of more water than is recharged for a period of time), subject to constraints and bounds (eqs 7-12).

$$\max Q = \sum_{k=1}^K \sum_{i=1}^I \sum_{j=1}^J q_{i,j,k} \quad \text{cell } i,j \text{ is a VH cell} \quad (6)$$

Subject to:

$$h_{i,j,K} + \sum_{k=1}^K \sum_{m=1}^I \sum_{n=1}^J \delta_{i,j,m,n,K-k+1} (q_{m,n,k} - q_{m,n}^{ass}) = h_{i,j}^0 \quad i=1,2,\dots,I, j=1,2,\dots,J, \text{ and cell } i,j \text{ is a VH cell and cell } m,n \text{ is a pumping cell} \quad (7)$$

Boundary flows are computed by:

$$T_{i,j}^i h_{i+1,j,k} + T_{i-1,j}^i h_{i-1,j,k} + T_{i,j}^j h_{i,j-1,k} + T_{i,j}^j h_{i,j+1,k} - h_{i,j,k} (T_{i,j}^i + T_{i-1,j}^i + T_{i,j}^j + T_{i,j-1}^j) - r_{i,j,k} = 0 \quad i=1,2,\dots,I, j=1,2,\dots,J, \text{ and cell } i,j \text{ is a CH/RF cell} \quad (8)$$

$$q_{i,j,k} \leq q_{i,j,k+1} \quad i=1,2,\dots,I, j=1,2,\dots,J, k=1,2,\dots,(K-1), q_{i,j,1} \geq q_{i,j}^0 \text{ and cell } i,j \text{ is a VH cell} \quad (9a)$$

$$q_{i,j,k} \geq q_{i,j,k+1} \quad i=1,2,\dots,I, j=1,2,\dots,j, k=1,2,\dots,(K-1), q_{i,j,1} \leq q_{i,j}^0 \text{ and cell } i,j \text{ is a VH cell} \quad (9b)$$

$$h_{i,j,K}^L \leq h_{i,j,k} \leq h_{i,j,K}^U \quad i=1,2,\dots,I, j=1,2,\dots,J, \text{ and cell } i,j \text{ is a VH cell} \quad (10)$$

$$r_{i,j,K}^L \leq r_{i,j,K} \leq r_{i,j,K}^U \quad i=1,2,\dots,I, j=1,2,\dots,J, \\ \text{and cell } i,j \text{ is a CH/RF cell} \quad (11)$$

$$q_{i,j,k}^L \leq q_{i,j,k} \leq q_{i,j,k}^U \quad i=1,2,\dots,I, j=1,2,\dots,J, \\ k=1,2,\dots,K, \\ \text{and cell } i,j \text{ is a VH cell} \quad (12)$$

where

K =total number of time steps;

$h_{i,j}^0$ =initial potentiometric surface elevation, (L);

$\delta_{i,j,m,n,K-k+1}$ =nonnegative-valued influence coefficient describing the effect on hydraulic head in cell i,j by period K of a unit pumping in cell m,n in period k (T/L^2). The computed influence coefficient includes the effect of storage in the hydraulic behavior of the aquifer;

$q_{m,n}^{ass}$ =ground-water pumping that must occur in each time step in cell m,n for that cell to maintain its initial head (L^3/T);

$q_{i,j}^0$ =pumping prior to beginning of planning period (L^3/T).

Arrays and equations containing the time dimension are analogous to some found in Model A. As in Model A, no pumping occurs in CH/RF cells, and no recharge occurs in VH cells.

Superposition is used (eq 7) to compute the total head change or response at a particular cell and time to the pumping in all active cells. The head change is termed an influence coefficient and is used in equation 7. The response matrix is the matrix of influence coefficients as they are found within the constraint equations of an optimization model.

The head change caused by two units of pumping will be twice the head change caused by one unit of pumping. To determine the head response to a particular pumping rate of q (some multiple of unit pumping), the influence coefficient is multiplied by q . In equation 7, however, the influence coefficients are multiplied by $q - q^{ass}$ to account for the initially stressed potentiometric surface used in Scenarios III and IV. In Scenarios I and II, when the initial potentiometric surface is unstressed, $q^{ass}=0$, and the influence coefficient is multiplied by q in equation 7. Equations 8 and 11 assure that the heads resulting by the end of the planning period do not induce unacceptable values of recharge through CH/RF cells.

Although ground-water withdrawal can vary with time, equation 9a or 9b assures that acceptable pumping rates do not oscillate. From an agricultural planning and management perspective, if ground-water withdrawals are expected to change with time, the change generally is monotonic. If the aquifer is initially undeveloped (no prior pumping) pumping might be expected to increase with time. Thus equation 9a applies to Scenarios I and II discussed later. If the aquifer is initially overdeveloped, pumping

might be expected to decrease with time. Therefore equation 9b applies to Scenarios III and IV.

Model C (Transient Response Matrix Constraints With Embedded Terminal Steady-State Constraints)

This model is designed to maximize unsteady pumping during the planning period, while assuring that the pumping values of the final time step are sustainable beyond that period. It is a combination of approaches used in Models A and B. It includes the same objective function (eq 6) as Model B, equations 2–5 from Model A, and equations 7–12 from Model B. It also contains an embedding method (Tung and Koltermann, 1985) that includes finite difference or finite element equations describing ground-water flow included directly as a constraint equation (eq 13) within the optimization model, assuring that pumping in period K does not exceed a hypothetical steady pumping value, q^{ss} .

$$q_{i,j,K} \leq q_{i,j}^{ss} \quad i=1,2,\dots,I, j=1,2,\dots,J, \\ \text{and cell } i,j \text{ is VH cell} \quad (13)$$

It is assumed that q^{ss} can be sustained by feasible recharge rates (eqs 2 and 4) and will cause acceptable heads to develop (eqs 2 and 3). By using this model, a management agency can avoid having to reduce ground-water withdrawal after the end of the planning period.

APPLICATION AND RESULTS

Tested Scenarios and Utilized Data

Optimal regional ground-water withdrawal strategies were developed for a five-decade planning period using all three models. Each model was tested for a range of recharge constraints for four scenarios. The scenarios differ depending on whether the aquifer was already being utilized and whether upper limits on ground-water withdrawal might change with time. These scenarios are as follows:

- I. Initially undeveloped aquifer, constant limits on pumping,
- II. Initially undeveloped aquifer, changing limits on pumping,
- III. Initially developed aquifer, constant limits on pumping, and
- IV. Initially developed aquifer, changing limits on pumping.

In all optimizations, except some performed for sensitivity analysis, transmissivities were assumed to be constant in time (fig. 3). These values were used to compute the finite difference terms in equations 2 and 8 and also were used, in conjunction with a specific yield of 0.3, to compute the influence coefficients for equation 7.

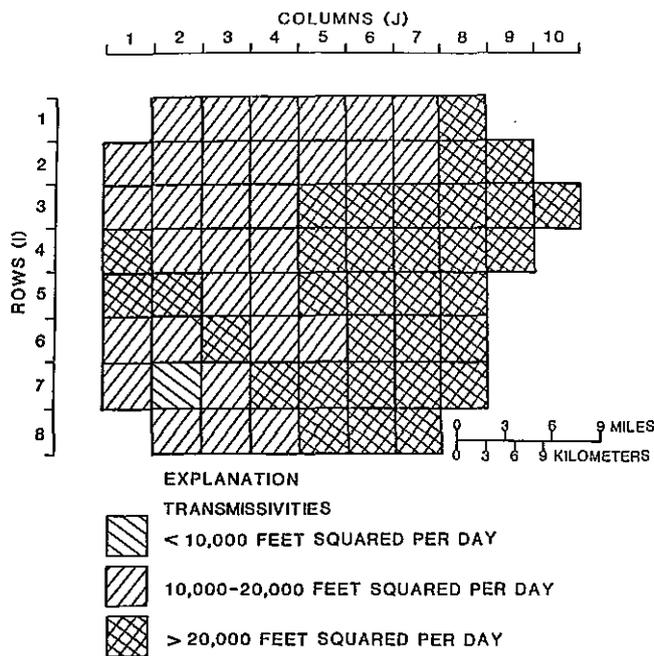


Figure 3. Transmissivities used in model comparisons.

Target steady-state and transient heads are constrained by equations 3 and 10. The initial unstressed potentiometric surface, h^0 in equation 7, for Scenarios I and II is shown in figure 1. Initial heads for Scenarios III and IV (initially developed aquifer) are the h^{tss} computed by using Model A for Scenario II and unconstrained recharge.

The lower limit on pumping, $q^{L,tss}$ and q^L , is 0.0 in equations 5 and 12 for all optimizations. The constant upper limits on pumping in equations 5 and 12 for Scenarios I and III are shown in figure 4. The arbitrary upper limits on pumping in Scenario II for the five decades are 0.8, 0.95, 1.0, 1.05, and 1.2 times the constant values of Scenarios I and III; the upper limits for Scenario IV are 1.2, 1.05, 1.0, 0.95, and 0.8 times the constant values, respectively. In other words, the constant upper bounds on pumping in Scenarios I and III are the same as the upper bounds of the third decade for Scenarios II and IV and also are the average values of all five decades for Scenarios II and IV. The total upper limit on pumping for all five decades is the same for all Scenarios.

The goal of having a monotonic change in pumping applies additional lower limits on pumping in Scenarios I and II (initially underdeveloped aquifer) and additional upper limits on pumping for Scenarios III and IV (initially developed aquifer). For Scenarios I and III, which have constant upper bounds on pumping, monotonicity is assured by Model A since it can compute only a single steady withdrawal for each cell.

In order to use Model A for Scenarios with changing bounds on pumping (II and IV), it is run sequentially, one distinct optimization after the other in five separate opti-

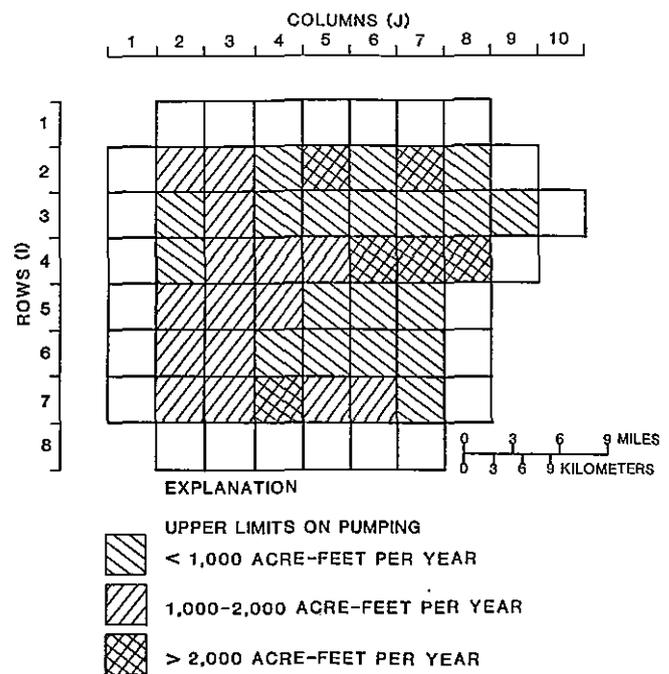


Figure 4. Assumed upper limits on ground-water pumping for Scenarios I and III. No pumping exists in boundary cells.

zations. In this process, the optimal pumping from one optimization affects the bounds on pumping in the next optimization. The lower bound on pumping in a cell for decade k in Scenario II is the pumping in decade $k-1$ (an equivalent of eq 9a). The upper bound on pumping in Scenario IV in decade k is the pumping in decade $k-1$ (equivalent to eq 9b).

This repetitive optimization is similar but not the same as a "staircase" procedure mentioned by Dantzig (1963), Gorelick, (1983), and Tung and Koltermann (1985), because the heads at the end of one decade are not used as the initial condition of the second decade (the optimal steady-state heads from one decade could not be used as the initial heads of the next decade because heads in a real system probably may not evolve to equilibrium by the end of a decade). This results from the fact that for constant transmissivity a volumetrically optimal steady-state strategy is independent of the initial heads of the internal VH cells. When using this approach, the "target" steady-state potentiometric surface changes every decade. None of the target surfaces was attained during the five-decade management period. However, the actual surface that would result from strategy implementation during the management era would always be evolving toward the target.

Because recharge is negative in sign, the greatest volume of flow permitted to enter the study area through a CH/RF cell is $r_{i,j}^{L,tss}$ in equation 4 or $r_{i,j,k}^L$ in equation 11. Upper limits on recharge are large positive numbers to

permit discharge if it enhances the value of the objective function. The lower limits on recharge through each cell are specified for each optimization and range from being constrained to being fairly restrictive at $-2,000$ acre-yr per CH/RF cell.

Volumetric Comparison of Optimal Strategies

Models A, B, and C are compared below on the basis of optimized pumpage and not pumping rate. The optimized constant pumping rate for Model A has been converted to a 50-yr volume for comparison with volumes calculated in Models B and C. Optimal total pumping volumes computed by each model (table 1) are shown in figure 5. Recharge constraints of unconstrained, 5,000, 3,750, and $2,500 \times 10^3$ acre-ft shown in the first column are the product of unconstrained, 4,000, 3,000, or 2,000 acre-ft/yr per CH/RF cell recharges times 50 yr times 25 CH/RF cells, respectively. These total maximum conceivable recharges are used to aid comparison with total pumping. As previously described, the actual constraints in the models are on a cell-by-cell basis. No constraint on total recharge is used.

Before comparing models, some general observations are in order. Care must be used when making comparisons because the assumed transmissivities and initial heads of Scenarios I and II are different than those of III and IV. Also it should be noted that, as available recharge is reduced (as the recharge constraint becomes increasingly restrictive), pumping decreases for all models.

Comparing strategies in table 1, it is apparent that the pumped volume from Model A is greater for scenarios in which there are constant upper bounds on pumping (I and III) than for those with varying upper bounds (II and IV), even though the total of all upper bounds is the same. This occurs because in Scenarios I and III pumping is effectively restricted primarily by the recharge constraint since the upper bound is an average value as previously stated. On the other hand, in Scenarios II and IV, pumping in the decades with a large upper bound on pumping is restricted by the recharge constraint, and pumping in the decades with a small upper bound on pumping is restricted by that upper bound.

If the initial potentiometric surface is unstressed, Models B and C also compute greater pumping if bounds are constant than if they are varying. The reasoning is the same as that presented for Model A. However, if the initial surface is stressed, the opposite trend is observed for those two models. This probably results from the fact that Models B and C emphasize pumping as early in the planning period as possible in order to have as much time as possible for the water levels to adjust to recharge constraints. Scenario IV permits the models to accomplish this because the upper bounds on pumping are greatest in the early decades.

Induced recharge/pumping ratios at the end of a 50-yr simulation period describe the proportion of total pumping that is replaced by recharge through the boundaries and are given in table 1, and trends in these and other ratios are given in table 2. It should be emphasized that Model A simulated transient flow for a 50-yr period using optimum withdrawals from a steady-state optimization model. In table 1, these ratios are less than 1.0 for a decrease in aquifer volume (Scenarios I and II) and greater than 1.0 for an increase (Scenarios III and IV). Within a Scenario, as total pumping increases, the recharge/pumping ratio decreases.

Other ratios in tables 1 and 2 describe differences in results with respect to those from Model C. The pumping ratio describes the ratio between total pumping for Model A or B and that of Model C. For unconstrained recharge, the pumping ratio of all models are virtually identical. Subsequent discussion deals only with optimizations in which recharge is constrained.

Note that the pumping ratios of Models A and B in table 1 are never both less than 1.0 for any single situation. This indicates, as does table 2, that Model C never computes higher total pumping than Models A or B for the same situation—an expected result since C includes the constraints of Models A and B.

In all but one Scenario, Model B computes at least as much total pumping as Models A and C, because it optimizes mining, rather than sustainable withdrawal (table 1). The difference in pumping between B and A or C increases as the recharge is progressively constrained. For an unstressed initial surface, Model B may compute pumping that is up to 12 percent greater than that from Model C and 11 percent greater than that from Model A. For these Scenarios (I and II), Model B is not useful for sustained yield analysis. However, for Scenarios III and IV, pumping from Model B is distributed identically with that from Model C (fig. 6). Although it is difficult to predict exactly when this degree of similarity will occur, Model B may in some instances be used in place of Model C. However, if sustainable pumping rates after the management era are to be at least as great as the rates in the final decade, then Model C must be used.

Total pumping in Model A is always within 2 percent (greater) of that of Model C for Scenarios I and II. Distribution of pumping in time also is similar. Dissimilarity is partially due to Model C having more constraints. It may result also from difference in the form of the flow constraints in Models A and C, despite the fact that in both models these constraints link all cells. In Model A, the steady-state equation 2 for a particular cell includes heads and transmissivities of only five cells. In Model C, equation 7 for each cell potentially includes influence coefficients for all cells in the study area.

For Scenarios III and IV, Model A withdrawals are much less than Models B or C. The major reason for the

Table 1. Summary of ground-water pumping and ratios of recharge/pumping and sustainable pumping
[NA, not available]

Maximum total recharge constraints for a 50-yr period (10 ³ acre-ft)	Model	Total pumped volume in 50 yr (10 ³ acre-ft)	Induced recharge/pumping ratio at the end of a 50-yr simulation	Pumping ratio Model (A or B) Model C	Sustainable pumping ratio
Scenario I					
Unconstrained	A	2,265	0.62	1.00	1.00
	B	2,265	.62	1.00	NA
	C	2,265	.62	NA	NA
5,000	A	2,131	.62	1.01	1.01
	B	2,112	.62	1.00	NA
	C	2,112	.62	NA	NA
3,750	A	1,936	.64	1.00	1.00
	B	1,954	.64	1.01	NA
	C	1,936	.64	NA	NA
2,500	A	1,499	.71	1.02	1.02
	B	1,601	.71	1.09	NA
	C	1,468	.75	NA	NA
Scenario II					
Unconstrained	A	2,265	0.60	1.00	1.00
	B	2,265	.60	1.00	NA
	C	2,265	.60	NA	NA
5,000	A	2,067	.61	1.00	1.00
	B	2,105	.60	1.02	NA
	C	2,067	.61	NA	NA
3,750	A	1,867	.64	1.00	1.00
	B	1,952	.62	1.05	NA
	C	1,867	.64	NA	NA
2,500	A	1,436	.72	1.01	1.03
	B	1,593	.69	1.12	NA
	C	1,416	.74	NA	NA
Scenario III					
Unconstrained	A	2,265	1.02	1.00	1.00
	B	2,265	1.02	1.00	NA
	C	2,265	1.02	NA	NA
5,000	A	2,145	1.04	.96	1.00
	B	2,239	1.03	1.00	NA
	C	2,239	1.03	NA	NA
3,750	A	1,970	1.09	.92	1.06
	B	2,139	1.07	1.00	NA
	C	2,139	1.07	NA	NA
2,500	A	1,574	1.28	.85	1.02
	B	1,855	1.19	1.00	NA
	C	1,855	1.19	NA	NA
Scenario IV					
Unconstrained	A	2,263	1.04	1.00	1.00
	B	2,265	1.04	1.00	NA
	C	2,265	1.04	NA	NA
5,000	A	2,084	1.07	.93	.99
	B	2,248	1.05	1.00	NA
	C	2,248	1.05	NA	NA
3,750	A	1,905	1.13	.88	1.07
	B	2,152	1.08	1.00	NA
	C	2,152	1.08	NA	NA
2,500	A	1,513	1.33	.81	.96
	B	1,874	1.20	1.00	NA
	C	1,874	1.20	NA	NA

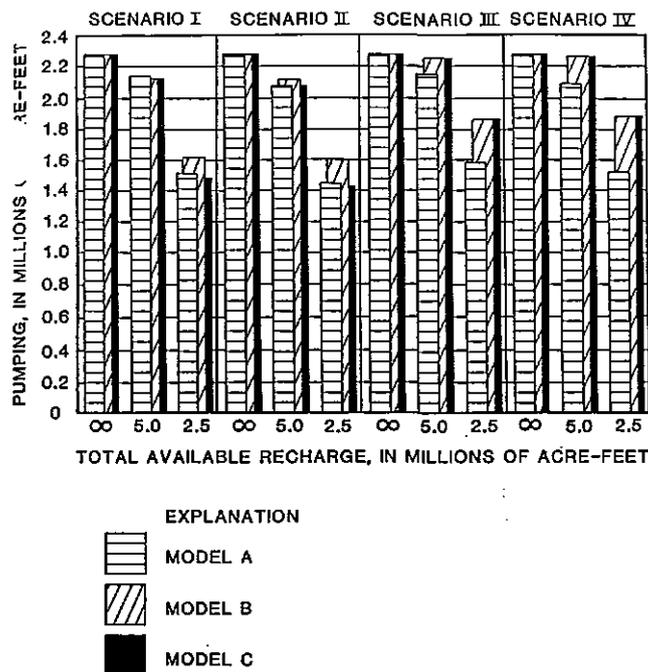


Figure 5. Total 50-yr ground-water pumping for all models and scenarios.

Table 2. Trends of pumping, recharge/pumping, and sustainable pumping ratios for optimizations using constrained recharge

	Scenario			
	I	II	III	IV
Pumping ratio	$C \leq B$ $C \leq A$	$C \leq A \leq B$	$A \leq B = C$	$A \leq B = C$
Recharge/pumping ratio.	$A = B \leq C$	$B \leq A \leq C$	$B = C \leq A$	$B = C \leq A$
Sustainable pumping ratio.	$C \leq A$	$C \leq A$	$C \leq A$	no trend

difference in pumping between Models A and C is that pumping can vary with time in Model C. An additional optimization was performed by using a modified Model C for Scenario III and each of the three recharge-constraint situations. In these three optimizations, pumping in Model C was forced to be constant in time. The resulting pumping totals are 2,144,614, 1,969,710, and 1,544,491 acre-ft for recharge constraints of 4,000, 3,000, and 2,000 acre-ft/yr per CH/RF cell, respectively. Model A computes pumping identical to that computed in Model C for recharge constraints of 5,000 and $3,750 \times 10^3$ acre-ft. For the $2,500 \times 10^3$ acre-ft recharge constraint, Model A pumping is 2 percent higher than that from Model C. This illustrates that, if temporally constant pumping is assumed, Model A is a viable substitute for Model C whether the initial surface is stressed or unstressed.

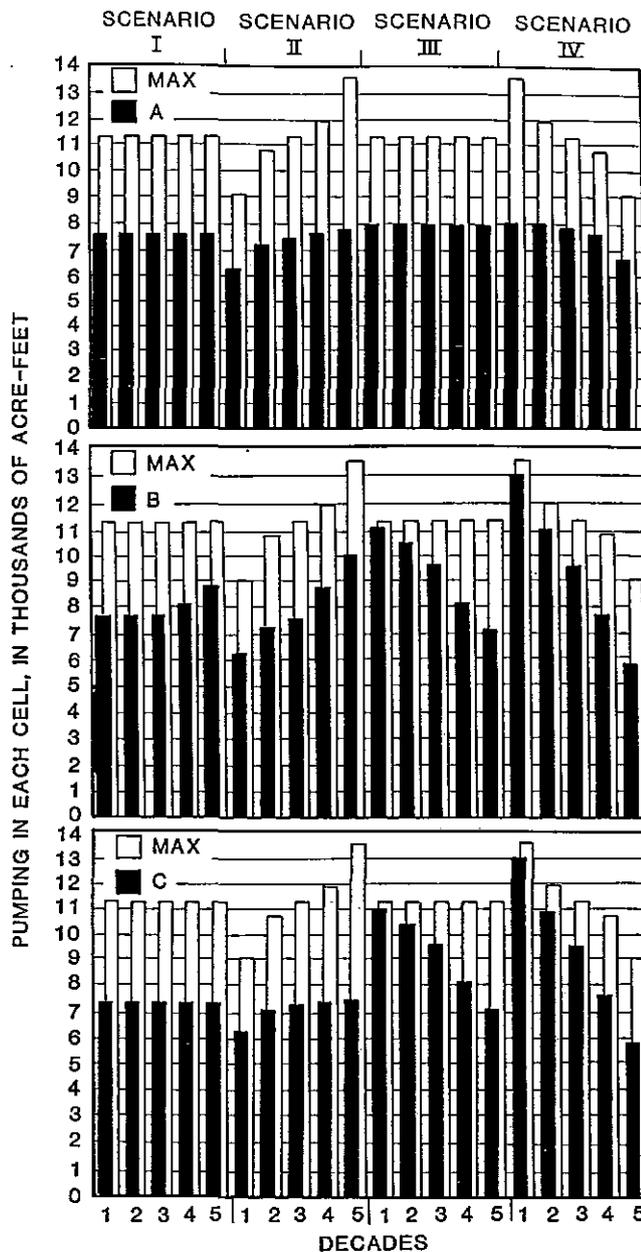


Figure 6. Average pumping by decade for Models A, B, and C and all scenarios for recharge not exceeding 2,000 acre-ft/yr in each boundary cell. "Max" pumping is the upper limit on pumping in a decade.

Trends in sustainable pumping ratios are evident from the data in tables 1 and 2. These ratios compare the steady pumping ratio that can continue after the 50-yr planning period as computed by Model A, with that computed by Model C. No value is shown for Model B because it includes no steady-state constraints. For Scenarios I and II, sustainable pumping for Model A is no more than 3 percent greater than that for Model C. For Scenarios III and IV,

sustainable pumping for Model A ranges between 96 and 107 percent of that for Model C.

The trends in table 2 do not show results of runs using unconstrained recharge because coefficients are identical for all three models. Also, no sustainable pumping is computed for Model B. If sustainable pumpage was computed, it would not exceed that for Model A.

Simulation Accuracy and Sensitivity Analysis

The accuracy of the unsteady head computation achieved by Models B and C through equation 7 was verified by comparing predicted heads from Scenario I with simulated values from AQUISIM. All heads predicted by equation 7 to exist after 50 yr are within 0.002 to -0.046 ft of the heads (0.01 percent to -0.4 percent) predicted by AQUISIM.

The sensitivity of optimization models to changes in parameters is sometimes unexpected. For example, note the 2,131,000 acre-ft pumping volume obtained in Scenario I by Model A constrained such that recharge never exceeds 4,000 acre-ft/yr at any boundary cell. Analysis of the sensitivity of that strategy shows that if, transmissivity is globally decreased by 25 percent, optimal pumping increases slightly by 0.9 percent. Optimal heads are lower but do not reach the lower limits. The new transmissivity coefficients in equation 2 are apparently better for the objective of optimizing pumping than the previous values. If optimal heads had reached their lower limits in the run using a reduced transmissivity, total pumping would be expected to be less. A more intuitively consistent result occurs when transmissivity is globally reduced by 50 percent. Heads reach their lower limits, and pumping is decreased by 4 percent.

Computational Considerations

All models are written by using the Generalized Algebraic Modeling System (GAMS) (Kendrick and Meeraus, 1985). Linked to GAMS is (MINOS) (Murtagh and Saunders, 1987), which accomplishes the optimization. Processing is performed within a VM/SP environment by using a conversational operating system (CMS) (International Business Machines Corp., 1980).

Table 3 indicates that Model A requires significantly less processing time than the others (62 percent less than Model C). This is the result of having fewer constraints and variables (such as rows and columns in a linear programming constraint set). Using the equations in table 4, Model A requires only 28 and 27 percent of the variables and constraints required by Model C, respectively, for the test problem. In contrast, Model B requires 72 and 73 percent of the variables and constraints used by Model C.

Table 3. Average computer processing time of optimization algorithm

[In seconds; processing time for Model A is the total for five separate optimizing runs (one for each decade)]

Model	Scenario			
	I	II	III	IV
A	29.97	32.1	30.4	32.4
B	63.3	63.2	63.1	62.2
C	81.7	81.4	80.4	80.1

Table 4. Comparison of number of decision variables and constraint equations

[M_c = number of constant-head cells; M_v = number of variable-head cells; K = total number of time periods; number of variables and equations reported for A is the number required to perform a single optimization, not the cumulative number required to perform separate optimizations for K time periods]

Model	Number of decision variables	Number of constraint equations
A	$M_c + 2M_v$	$2M_c + 5M_v$
B	$M_c + (K+1)M_v$	$2M_c + (3K+1)M_v$
C	$2M_c + (K+3)M_v$	$4M_c + (3K+6)M_v$

Note that transient heads in Models B and C are computed and constrained only for the end of the final time step. To constrain heads at other points in time, more computer memory and processing time are required. As long as heads do not need to be constrained at particular finite times, Model A offers a computational advantage. A user wishing to know the interim heads that result from implementing an optimal strategy can always compute them by using a transient simulation model after optimization is performed.

Presented in table 4 is a comparison of the total number of decision variables and constraint equations used in the simulations of Models A, B, and C. The variable M_c is the number of constant-head cells; variable M_v is the number of variable-head cells; variable M is the sum of the constant-head and variable-head cells ($M = M_c + M_v$). For example:

For Model A,

For M_c cells, recharge is a variable,

For M_v cells, pumpage and heads are variables,

Sum = $M_c + 2M_v$ = total number of decision variables.

Model A consists of one objective function (eq 1), M of equation 2, M_v of equations 3 and 5, and M_c of equation 4. Model B consists of one objective function (eq 6), M_v of equation 7, M_c of equation 8, $M_v \times (K-1)$ of equation 9, M_v of equation 10, M_c of equation 11, and M_v of equation 12 if ground-water pumping is to be constant or $M_v \times K$ of

equation 12 if ground-water pumping is to vary with time. Model C has the objective function of equation 6 plus all variable bounds and constraints mentioned for A and B, as well as Mv of equation 13.

The fairly small system described in this paper represents a Model A problem of 250 rows and 105 columns of decision variables. Model B uses 690 rows and 265 columns, while Model C requires 940 rows and 370 columns of decision variables. For a larger imaginary system of 300 CH/RF cells and 1,300 VH cells, Model C requires 27,300 rows and 11,000 columns of decision variables. Model A needs only about 26 percent of that, 7,000 rows and 2,900 columns. Yazdanian and Peralta (1986) report that processing time increases exponentially with problem size for tested optimization models. Thus, Model A becomes increasingly attractive as problem size increases. In preliminary testing, an IBM/AT with 640 K of Random Access Memory had inadequate memory to run Model C, although it could easily run Model A for one decade at a time.

Depending on the optimization solution algorithm that is used, Models A and C may be subject to a weakness sometimes ascribed to the embedding technique—susceptibility to computational instability for large systems. The tendency for the model to fail to converge increases as the size of the pentadiagonal constraint matrix (containing steady flow constraint equations) increases. This possible limitation may affect C more than Model A because Model C has $(2Mc + Mv)$ such equations while A has only $Mc +$

Models B and C compute globally optimal solutions for all situations. Model A solutions are globally optimal for Scenarios I and III. For Scenarios where upper bounds on pumping change and Model A is run for each decade, solutions may not be globally optimal.

SUMMARY

Three management model formulations (Models A, B, and C) were compared to evaluate which are appropriate for computing maximum sustained yield ground-water withdrawal strategies for alternative scenarios and a range of recharge constraints. Model A contains embedded steady-state flow equations. Model B contains only response matrix-type transient equations. Model C combines constraints from both models plus a restriction that pumping in the final time period is sustainable. The models were tested for four Scenarios that included the combinations of having an initially unstressed or stressed potentiometric surface and having constant or changing upper limits on withdrawal.

Model C is the most rigorous model and is preferred for computing sustained yield strategies when sufficient data and computer capability are available. However, in

some cases, Models A and B compute comparable strategies while offering significant reduction in computational effort. To evaluate whether Models A or B can be substituted for Model C, results from both were compared with the results from Model C.

A five-decade planning period was used in all models. Models B and C can optimize for the entire period even if upper bounds on pumping change with time. For situations in which these bounds change, Model A can be run for each consecutive decade. In this situation, an optimal balance between withdrawal and recharge of Model A cannot be assured in every cell. All other scenarios in this study achieved optimal solutions in every cell.

If the initial potentiometric surface is relatively unstressed, Model A is an appropriate substitute for Model C. Pumping strategies computed by Model A are almost identical to those computed by C but require much less computer processing execution time and memory. Pumping strategies from Model B exceed those from Model C. Because they are not sustainable, Model B is not appropriate for this situation.

Model A is a conservative substitute for Model C for the situation where the initial potentiometric surface is stressed to the extent that pumping rates cannot be sustained for specified recharge constraints. If pumping in Model C is forced to be constant in time, pumping strategies from Models A and C are very similar. Pumping in Model A is less than that in Model C because pumping in Model A is not allowed to vary with time. Pumping strategies computed by Model A are sustainable and provide the same reduction in computational effort mentioned previously.

For situations where the potentiometric surface is initially stressed, optimal pumping strategies from Model B are almost identical to those from Model C. However, because Model B contains no sustained-yield constraints, there is no guarantee that pumping will always be sustainable. Therefore, Model B may not be appropriate, even though it requires less computational resources than Model C.

Pumping rates computed by Models A and C to be sustainable beyond the 50-yr planning period also were compared. For the initially unstressed aquifer, rates from Model A may be slightly greater than those computed by Model C. For the initially stressed aquifer, sustainable pumping computed by Model A is within several percent of that computed by Model C.

For computing maximum sustained-yield ground-water withdrawal strategies, a model that employs only embedded steady-state equations compares favorably with a more detailed model that combines steady-state and unsteady response-matrix formulations. In this comparison, the attempt was made to constrain heads during the planning period. Interim heads during the planning period can of course be computed by using transient modeling subsequent to the optimization modeling.

The steady-state formulation is especially competitive when available computer time or memory is limited, a condition that frequently occurs in projects in which computations are performed on personal computers. In the presented example, time requirements were 62 percent lower with the embedded steady-state equation approach relative to the more complex response matrix approach. The number of variables and constraints (affecting computer memory) were 72 and 73 percent, respectively, lower with the steady-state equation approach.

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