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OPTIMAL
PIEZOMETRIC SURFACE MANAGEMENT
FOR
GROUNDWATER
CONTAMINANT CONTROL

by

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SUMMARY: A methodology is described for efficiently
evaluating possible injection/extraction pumping schemes to most economically
contain a groundwater contaminant plume. A multi-objective model is analyzed
with a micro-computer. Simulation and optimization is performed by employing
the response matrix method.

KEYWORDS: groundwater, aquifers, simulation



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BACKGROUND

The Department of Defense (DoD) began its Installation Restoration Program (IRP) in 1975. The IR program is a comprehensive effort to identify and evaluate past hazardous waste disposal sites on DoD installations, and to control the migration of contamination resulting from such operations. This paper is a part of the IRP.

On Aug. 14, 1981, in Executive Order 12316, the President formally made the IRP a part of the "Superfund" project and delegated authority specified in the Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA) to the Secretary of Defense. The Secretary of Defense was given responsibility for:

- Response actions on hazardous wastes (i.e., removal and remedial actions)
- Investigation, monitoring, survey, and testing as needed
- Planning, legal, fiscal, economic, engineering, architectural, and any other studies or investigations as necessary for response actions
- Enforcement of the provisions of CERCLA

The objectives of the DoD restoration program are:

- To identify and evaluate past hazardous material disposal sites on DoD facilities, and to control contamination migration.
- To review and decontaminate land and facilities excess to DoD's mission.

The first phase in the IR program is an installation assessment. In this phase, installation files are examined, current employees and key retirees are interviewed, and the terrain and facilities are examined. Additionally, all available information on past mission, current operations, waste generation, disposal, and geohydrology of the area are collected. Limited soil and water sampling may also be conducted to determine if contaminants are present.

The second phase in the IR program involves confirming that contamination exists. In this phase, a comprehensive survey is conducted to fully define the problem through environmental sampling and analyses. Data are developed to fill information

gaps identified during the installation assessment phase.

In the third phase, technology base development, control technology is developed to solve contamination problems at specific sites to determine the most economical solutions. If control technologies do not exist, they are developed in this phase. This project is a part of phase three.

When required, the DoD IR Program terminates with an operations phase. This phase includes design, construction, and operation of pollution abatement facilities, and the completion of remedial actions.

INTRODUCTION

The Air Force faces many situations in which it will have to remedy or prevent groundwater contamination. Inadequate response to these cases may result in unnecessary damage to groundwater. Excessive response may be unnecessarily expensive. Therefore, Air Force managers wish to systematically develop a group of tools or methodologies useful for optimizing, to the extent possible, response to the groundwater contaminant problems they face. The purpose of this project is to develop one of those methodologies.

The presented methodology is applicable for a groundwater contaminant situation in which the best solution requires modifying the potentiometric surface in the vicinity of the contaminant source. Appropriate modification can:

- prevent groundwater from contacting the source of contamination and becoming contaminated
- prevent contaminated groundwater from spreading beyond the immediate site.

Methods of modification include construction of artificial barriers to groundwater flow and/or extraction/injection of water from/to the aquifer. Cost of installing and maintaining the different types of artificial barriers varies greatly, as does their reliability. Extraction/injection (E/I) methods have comparatively low installation expense and good reliability, but are commonly used as transitional elements of remedial action efforts. They are less often used as long-term solutions.

Our objective is to employ a pumping well configuration around an existing contaminant plume to develop an economically optimal pumping strategy (combination of injection and extraction) to create a zero hydraulic gradient in the vicinity

of the plume. We assume an isotropic aquifer in which the contaminant's dominant mechanism of transport is advection caused by the hydraulic energy gradient.

PREVIOUS WORK

Some earlier efforts to identify strategies for managing groundwater quantity and quality resources focused on simulation of groundwater flow and mass transport in aquifers for which discharge and contaminant input rates were known or assumed (Pinder and Bredehoeft, 1968; Pinder, 1973; Bredehoeft and Pinder, 1973; Konikow and Bredehoeft, 1974; Pickens and Lennox, 1976; Gorelick, 1982).

Specific groundwater hydraulic management models were then developed in response to the growing need to systematically relate the hydraulic behavior of the flow system to the cost of utilizing scarce aquifer supplies. This was accomplished through coupling of the physical principles of groundwater flow and optimization theory (Gorelick, 1983).

Aquifer management research has also treated the problem of groundwater pollution control. Groundwater management models can be classified according to attributes, such as objective or formulation. As far as objectives are concerned, the models broadly belong to one of two categories (Gorelick, 1983). In one category are all those models in which management decisions are principally concerned with groundwater hydraulics. The second category includes models designed to evaluate economic and other consequences of water policies.

The groundwater flow equation is an integral part of any numerical groundwater model. Incorporation of this equation into the present model is achieved via either the 'embedding' or 'response matrix' methods (Gorelick, 1983). In the 'embedding' method, numerical approximations of the governing flow equation are directly included as constraints in an optimization model. In such cases drawdowns and pumpings often are the decision variables.

The embedding method was first presented by Aguado and Remson (1974). Using one- and two-dimensional examples, they showed that the physical behavior of the groundwater system could be included as an integral part of an optimization model. They used finite-difference approximations to simulate both steady and unsteady flow.

Moltz and Bell (1977) applied the embedding method to a

hypothetical case involving the steady-state control of hydraulic gradients to insure stationarity of a fluid stored in the aquifer.

Another application of the embedding approach to control hydraulic gradients was reported by Remson and Gorelick (Gorelick, 1982). Its objective was to contain a plume of contaminated groundwater. They did this in the context of other regional management goals, including the dewatering of two excavation areas and obtaining water for export from the system. The objective function was to minimize pumping. The solution selected those nodal locations where either pumping or injection wells should be located. The solution also determined the optimum pumping rates and gave the resulting steady-state hydraulic head distribution over the 99 active nodes.

Datta and Peralta (1986) developed an influence coefficient method for optimally modifying a steady state surface to satisfy a groundwater contaminant concentration criteria. They used the embedding method for a 25 cell subsystem of a larger study area.

In the response matrix method an external groundwater simulation model is used to develop unit responses. Each unit response describes the influence of a unit stimulus (e.g., pumping) upon hydraulic heads at points of interest throughout a system. These coefficients, Dirac delta functions, (Maddock, 1972; Haines and Dreizin, 1977) are also termed discrete kernels (Morel-Seytoux and Daly, 1975; Illangasekare et al, 1984) or response values (Heidari, 1982; Danskin and Gorelick, 1985). An assemblage of the unit responses, a response matrix, is included in the management model. Decision variables often include pumping and drawdowns in the objective function.

Deininger (1970) is perhaps the first who considered the response matrix method for use in groundwater modeling. He considered two objectives, maximization of water production and minimization of the production costs for a well field. Linear and quadratic objective functions were proposed for the first and second objectives respectively. The Theis unsteady-state formula (Todd, 1980, p. 123) was also used to calculate drawdown responses. Constraints were formulated so that drawdowns were controlled according to pump and well facility limitations. For the second objective function, water production costs were assumed to be directly proportional to the products of the lifts and the discharge rates, both of which were initially unknown. Therefore, the use of a quadratic programming routine was proposed. However, no solutions were presented.

Bear (1979, pp. 505-508) presented a hypothetical example of a 25-cell aquifer system. The purpose was to maintain groundwater elevations above specified minimum levels at specific locations in order to prevent poor quality water from a lake to encroach into the aquifer. The objective function sought to determine the pumping locations so as to minimize the cost of water supply to be delivered at a specific point in the basin. A computer simulation model was used to generate response coefficients that were, then, used to find the optimal solution.

Larson et al. (1977) developed a model intended to estimate the safe yield of a groundwater basin in Indiana. The objective function was formulated to select appropriate well sites that would maximize the steady-state pumping. Selections were to be made from 199 potential well sites. Lower limits were imposed on the pumping rates at each active well site. The number of wells at each site was less than or equal to a maximum. Integer variables were used to specify whether a well exists at a certain site (integer variable = 1) or not (integer variable = 0). Other constraints were imposed to keep the pumping rates below specified maximum rates and to limit drawdowns to a maximum of 50 percent of the initial saturated thickness. The solution selected 26 active well sites and identified the spatial distribution of pumping rates.

Leikoff and Gorelick (1985) minimize costs of containing and treating a contaminant plume. Using the response matrix method, extended to velocity responses and specifying a time period by which hydraulic goals were to be completed, the model determines location, timing, and rates of pumping.

Although hypothetical and site-specific optimizations of E/I pumping have been reported, no systematic procedure for optimizing the design of E/I solutions to groundwater contaminant problems has been found in the literature. Presenting such a methodology is the broad purpose of this paper. A systematic and time efficient approach is being proposed to economically deal with a contamination plume. When the limits of a contaminant plume have been found different extraction/injection schemes can quickly be analyzed for efficiency and economics.

The presented model uses discrete kernels (influence coefficients) that explain the response of a potentiometric surface to pumping stimulus. The Theis equation was used to generate these point to point influence coefficients via the procedure described by Morel Seytoux and Daly (1975). Once these coefficients are generated, water level response can be expressed as an explicit linear function of the pumping rates and the

coefficients.

MODEL FORMULATION

Use of the model depends upon being able to define the size of the contaminant plume when the E/I strategy is to be implemented. The initial task in any containment problem is to assess the nature and magnitude of the contaminant plume. Site characterization must determine the extent of the plume and its velocity of travel. An estimate must be made of when the proposed E/I system will be functioning. With this knowledge we can predict the size of the plume at the time of E/I start-up.

The predicted shape for the boundary of the contaminant plume is an ellipse. From the equation for an ellipse (1) and using Darcy's law we can predict the limits of our plume.

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad (1)$$

a = point of intersection of ellipse and x - axis (L)

b = point of intersection of ellipse and y - axis (L)

An X-Y coordinate system is established with the contaminant plume source at the origin and the positive X-axis down gradient from the source. We predict, using seepage velocity, where the farthest downgradient limits of our contaminant plume will cross the X axis and the Y axis ('a' and 'b' in our ellipse equation). Begin with the Darcy velocity, q.

$$q = -Ki \quad (2)$$

K = hydraulic conductivity (L/T)

i = hydraulic gradient (L/L)

$$\text{The seepage velocity is computed by: } v = q/\bar{\Phi} = Ki/\bar{\Phi} \quad (3)$$

$\bar{\Phi}$ = porosity

Therefore the downgradient limits of the plume are predicted as:

$$a = (K_x i_x t) * s.f. / \bar{\Phi} \quad b = (K_y i_y t) * s.f. / \bar{\Phi} \quad (4)$$

K_x, K_y = hydraulic conductivity in X and Y direction (L/T)

i_x, i_y = hydraulic gradient in X and Y direction (L/L)

t = time from initial contaminant discharge to activation of pumping containment system (T)

s. f. = Appropriate safety factor based on the uncertainty of the geologic and aquifer data.

The containment well system is arranged in an octagonal shape completely encircling the assumed elliptically shaped contaminant plume.(fig.1) An octagonal (regular or elongated) shape was selected because it can be configured to closely encircle an elliptical plume. Its straight sides and 45 degree angles promote easy calculation of the coordinates of the proposed wells. This also simplifies well installation in the field. The length (L) of each side of a regular octagon is a function of 'a'.

$$L = a / (0.5 + \cos 45) \quad (5)$$

The first model assumes a well-point system. We neglect losses in the system and assume pumping values (q) at all well points are equal in a particular time step. Future models may assume a different q at different wells and may use side (L) values that vary depending on the elongation of the plume.

Our objective is to contain the plume by producing a horizontal hydraulic gradient (i.e. as near as possible to horizontal) at a specific time for a minimal cost. Ideally, a target potentiometric surface would be attained precisely when it is most convenient for planning and management purposes. Physically, depending on the situation, there may be no conceivable sequence of pumping that can cause complete convergence to a horizontal surface within the desired time. It may be that the best that can be achieved is to minimize the difference between a horizontal target and that which is actually attained by the end of the specified period.

The model attempts to develop a strategy that minimizes operating and maintenance (O&M) costs of pumping and also minimizes the difference between water table levels achieved at observation wells and the water table elevation at the mid point of our octagon (i.e. the plume source). Simultaneous consideration of both goals makes this a multi-objective optimization. Hydraulic equilibrium will be maintained in the plume vicinity by constraining total extraction to equal total injection for each time period.

Whether considering well point systems or individual wells, the radius of influence, for predicted pumping rates, determines maximum spacing. Spacing can be varied with consecutive model runs to determine the best spacing. Observation wells (where achieved water table elevations will be monitored) are located mid-way between pumping wells. From the theory of superposition these mid-point water table elevations are least effected by an extraction and injection scheme. Therefore, minimizing the final difference between these water table elevations and the elevation at a selected point within the system yields as nearly level a potentiometric surface as possible within our specified time frame.

The objective function used in this model minimizes the present value of groundwater extraction/injection and the squares of deviations from a final horizontal piezometric surface for a predetermined time period:

$$\min: \sum_{i=1}^I \sum_{t=1}^T [c'(h_{i,t}) q_t + c'' q_t] + W \sum_{j=1}^J [(h_{o,T} - h_{j,T})^2] \quad (6)$$

Based on the following constraints:

$$q_L < q_i < q_U \quad (7)$$

$$h_L < h_{i,t} < h_U \quad (8)$$

where:

$$h_{i,T} = h_{i,0} - \sum_{j=1}^J \sum_{t=1}^T [B_{i,j,T-t+1} q_t] \quad (9)$$

$h_{i,t}$ = head at pumping well i at time T

$h_{o,T}$ = head at contaminant source at end of modeling period T

$h_{j,T}$ = head at observation well j at the end of the modeling period T

$B_{i,j,T-t+1}$ = the drawdown at a well i caused by a unit

volume of pumping at well j. The subscript T-t+1 provides the correct coefficient to be multiplied by the correct pumping value

c' = cost of pumping a unit volume of water a unit vertical distance ($\$/L^3/L$)

c'' = maintenance cost per unit volume pumped ($\$/L^3$)

W_f = weight factor to convert the square of hydraulic head differences to dollars. This value will vary based on economic factors and physical parameters ($\$/L^2$)

In addition to the upper and lower limits on pumping (7) total injection can never exceed total pumping during any one time period. This eliminates need for disposal or acquisition of water.

$$\sum_{i=1}^{I1} q_{i,t} \text{ (extraction)} = \sum_{i=1}^{I2} q_{i,t} \text{ (injection)} \quad (10)$$

where: $I1 + I2 = I$ (total pumping wells)

The hydraulic head term is not summed over time because we are concerned solely with the final piezometric surface.

The first step in developing an optimal strategy is to calculate the 'influence' coefficients using the Theis equation. They are a function of transmissivity, effective porosity, time and the distance between wells. The coefficients are used to calculate heads which in turn effect operating costs and final hydraulic gradient. The influence coefficients are calculated using equation (11) (Morel-Seytoux & Daly 1975).

$$B_{i,j,t} = (W(U_{i,j,t}) - W(U_{i,j,t-1})) / (4\pi T) \quad (11)$$

$$U_{i,j,t} = (r_{i,j}^2) / (4Tt) \quad (12)$$

B = Influence coefficient. These values are positive for extraction wells and negative for injection wells (T/L)²

$W(U)_t$ = Theis well function at time t (dimensionless)

T = transmissivity (L²/T)

U = Boltzman variable at time t (dimensionless)

$\bar{\phi}$ = effective porosity (dimensionless)

r = distance from stimulus i to point of interest j (L)

Head (h) is eliminated as an unknown by substituting the right hand side of equation (7) for all head terms in the objective function. The final objective function is obtained by squaring the hydraulic term of the objective function to avoid using absolute values.

GAMS/MINOS (Manne 1986) is the code used to solve the optimization problem. It determines the optimal pumping (extraction and injection) value to contain the contaminant plume at a minimum cost. GAMS (General Algebraic Modeling System) is a preprocessor which converts input data into standard MPS format for the optimization program MINOS (Modular In/Core Nonlinear Optimization System).

It should be emphasized that extraction/injection is rarely a permanent solution but is a cost effective method for immediate action to contain a contaminant plume. It permits time to determine a permanent solution to the contamination problem.

APPLICATION AND RESULTS

A hypothetical situation was tested. Parameters used were a transmissivity of 1255 m²/d (13,500 ft²/d), an effective porosity of 0.3 and a time period of 5 days. The original hydraulic gradient is 0.25%. The 'a' dimension of the contaminant plume (its farthest extent from the source) is approximately 330 meters. Therefore, the sides of the octagon are 274 meters in

length. Our optimal well spacing is one half the side length (137 meters). The economic coefficients used are: $c' = \$0.44/\text{HA-M}/\text{M}$ ($\$0.18/\text{AC-FT}/\text{FT}$) and $c'' = \$1.24/\text{HA-M}$ ($\$1.65/\text{AC-FT}$). Using these constants we varied the weight factor (W_f) with these results:

W_f	1.0	10.0	100.0 *NOP	1000.0
Pumping(L/s)				
day 1	36.1	36.1	36.1	36.1
day 2	33.7	33.7	33.7	33.7
day 3	14.82	15.1	15.2	15.2
day 4	0.0	0.0	0.0	0.0
day 5	0.0	0.0	0.0	0.0
Avg gradient(%)	.017	.017	.017	.017
Sum of head differences(m)	0.658	0.658	0.658	0.658
Obj. function	4.58	43.5	433.	4326.
O & M costs(\$)	272.00	273.00	273.00	273.00

* NOP= not optimal

The tight constraint for all the runs turned out to be the upper limit on head at the pumping wells (ground elevation) which prevents pressurized injection (Gorelick and Lefkoff 1985). These upper limits were reached for all weight factors at the same two wells: one well at day 1 and the second well at day 2.

The run using a weight factor of 100.0 is a non-optimal solution. The nonoptimality is produced at day 3 for the upper water table constraint. This is the day immediately following the tight constraint on the same well. The marginal value (measure of sensitivity) of the non-optimal constraint is a factor of 103 smaller than the marginal values for the optimal tight constraints. This indicates that even though the solution is not optimal relaxing the constraint would have very little effect on

the objective function value. Unfortunately the reason for the non-optimality at a weight factor of 100.00 can not be explained.

Weight factors of 0.1 and 0.01 were also used. These runs resulted in pumping for all five days and for day five only respectfully. The overall costs for both runs were less than those run previously. However, the final gradients were almost 3 times the final gradients for those runs with weight factors of 1 and greater. These gradients are unacceptable.

Other optimizations were performed using a weight factor of 10. spacings of 274 meters and 58 meters (twice and one half of the original spacing). The optimal solution for the larger well spacing was tightly constrained by the upper water table limit for injection. This resulted in an unacceptable final gradient of 10 times that produced with a spacing of 137 meters. The spacing of 58 meters produced a gradient equal to that of the 137 meter spacing at an O & M cost of one fourth of the costs that were previously run. However, it must be kept in mind that the capital cost would be twice that of the 137 meter spaced wells.

CONCLUSIONS

A time-efficient method has been devised to evaluate extraction/injection pumping strategies for containment of a contaminant plume. This multiobjective procedure uses a weighting factor to provide a common basis for simultaneous evaluation of both economic and hydraulic criteria. Optimal extraction/injection strategies were developed for a hypothetical contamination problem using a range of weight factors. Weight factors smaller than one resulted in unacceptable final gradients. In other words, those strategies emphasized economics at the expense of plume containment. Weight factors equal to or greater than one produced a gradient of less than .02%. This gradient could not be reduced further without causing water table levels to rise above those compatible with unpressurized injection.

The ideal weight factor is dependent on many factors and may be problem specific. A major factor is the maximum acceptable increase in water table elevation at an injection site. This constraint is based on the desire to avoid pressurized injection. In a contamination problem with a water table of greater depth than that used in this hypothetical situation weight factors of 10, 100 and 1000 would produce increasingly smaller gradients.

The optimal pumping strategy developed for this hypothetical

problem has greater pumping at the beginning of the modeling period than at any other time. This causes large head changes which subsequently recede over the remainder of the testing period so that by the end of 5 days the piezometric surface is nearly horizontal. Of course, this pumping scheme would have to be continually repeated until an alternative, perhaps more permanent, remediation scheme were implemented.

Over an extended period, operating and maintenance costs would not remain constant as has been assumed. As a result the proposed injection/extraction strategy may not be economically practical for long operation. It is, however, an economic and efficient method for short term containment.

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PICTORIAL MODEL

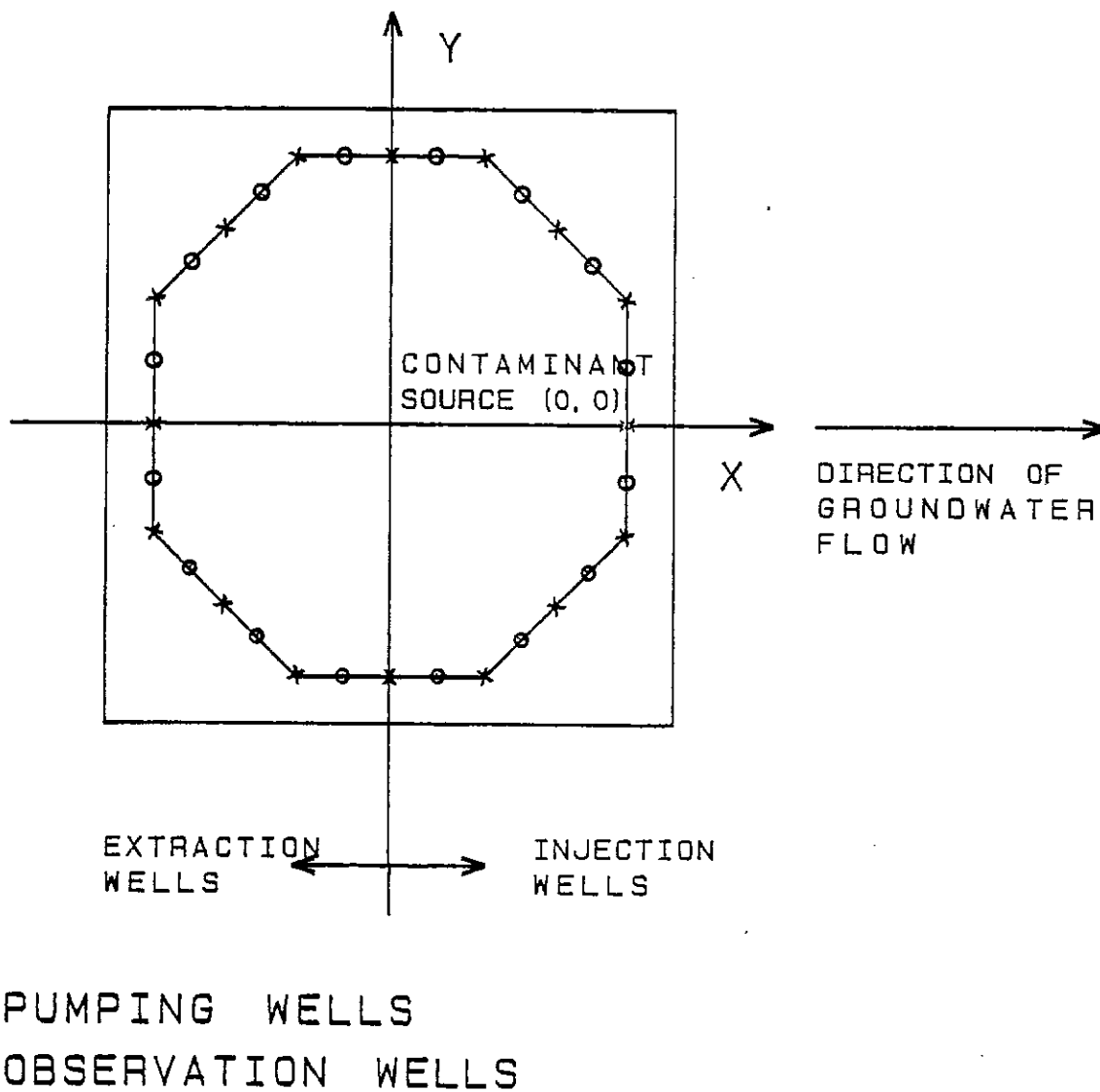


FIG. 1

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