Anisotropic Compressive Pressure-Dependent Effective Thermal Conductivity of Granular Beds

R. Daniel Garrett
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ANISOTROPIC COMPRESSIVE PRESSURE-DEPENDENT EFFECTIVE THERMAL CONDUCTIVITY OF GRANULAR BEDS

by

Daniel Garrett

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Mechanical Engineering

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2011
ABSTRACT

Anisotropic Compressive Pressure-Dependent Effective Thermal Conductivity of Granular Beds

by

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Utah State University, 2011

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In situ planetary effective thermal conductivity measurements are typically made using a long needle-like probe, which measures effective thermal conductivity in the probe’s radial (horizontal) direction. The desired effective vertical thermal conductivity for heat flow calculations is assumed to be the same as the measured effective horizontal thermal conductivity. However, it is known that effective thermal conductivity increases with increasing compressive pressure on granular beds and horizontal stress in a granular bed under gravity is related to the vertical stress through Jaky’s at-rest earth pressure coefficient. No research has been performed previously on determining the anisotropic effective thermal conductivity of dry granular beds under compressive uniaxial pressure.

The objectives of this study were to examine the validity of the isotropic property assumption and to develop a fundamental understanding of the effective thermal conductivity of a dry, noncohesive granular bed under uniaxial compression. Two experiments were developed to simultaneously measure the effective vertical and horizontal thermal conductivities of particle beds. One measured effective thermal conductivities in an atmosphere of air. The second
measured effective thermal conductivities in a vacuum environment. Measurements were made as compressive vertical pressure was increased to show the relationship between increasing pressure and effective vertical and horizontal thermal conductivity. The results of this experiment show quantitatively the conductivity anisotropy for different materials.

Based on the effective thermal conductivity models in the literature and results of the two experiments, a simple model was derived to predict the increase in effective vertical and horizontal thermal conductivity with increasing compressive vertical applied pressure of a granular bed immersed in a static fluid. In order to gain a greater understanding of the anisotropic phenomenon, finite element simulations were performed for a vacuum environment. Based on the results of the finite element simulations, the simple derived model was modified to better approximate a vacuum environment. The experimental results from the two experiments performed in this study were used to validate both the initial simple model and the modified model. The experimental results also showed the effects of mechanical properties and size on the anisotropic effective thermal conductivity of granular beds.

This study showed for the first time that compressive pressure-dependent effective thermal conductivity of granular beds is an anisotropic property. Conduction through the fluid has been shown to have the largest contribution to the effective thermal conductivity of a granular bed immersed in a static fluid. Thermal contact resistance has been shown to have the largest influence on anisotropic effective thermal conductivity of a granular bed in a vacuum environment.

Finally, a discussion of future work has been included.
ACKNOWLEDGMENTS

I express my gratitude to my major professor and mentor, Dr. Heng Ban, who continues to provide encouragement, guidance, and support. His guidance and wisdom will forever have an impact on my life. I also thank Jingwen Mo and Kurt Harris for their assistance in this work. Beyond the research, I am grateful to Liz, my wife and best friend, who has supported me in all of my endeavors including all of the late nights taking measurements in the lab.

Daniel Garrett
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<td>BCC</td>
<td>body centered cubic</td>
</tr>
<tr>
<td>FCC</td>
<td>face centered cubic</td>
</tr>
<tr>
<td>JKR</td>
<td>Johnson-Kendall-Roberts</td>
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<tr>
<td>SC</td>
<td>simple cubic</td>
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NOMENCLATURE

\( A \)  
Model correction fit parameter

\( A_c \)  
Container cross-sectional area, [m\(^2\)]

\( A_r \)  
Radiation area, [m\(^2\)]

\( A_v \)  
Cylindrical control volume cross-sectional area, [m\(^2\)]

\( B \)  
Model correction fit parameter

\( C \)  
Modified model correction fit parameter

\( D_{\text{average}} \)  
Average particle diameter, [µm]

\( D_{\text{particle}} \)  
Particle diameter, [µm]

\( dA_s \)  
Differential sphere cross-sectional area, [m\(^2\)]

\( dq_f \)  
Differential heat rate through fluid between half-spheres, [W]

\( dq_s \)  
Differential heat rate through sphere, [W]

\( E \)  
Young’s modulus, [GPa]

\( F_c \)  
Contact force, [N]

\( H \)  
Height of cylindrical unit cell, [m]

\( k_{\text{eff}} \)  
Effective thermal conductivity of cylindrical unit cell, [W/m/K]

\( k_f \)  
Fluid thermal conductivity, [W/m/K]

\( k_s \)  
Thermal conductivity of sphere, [W/m/K]

\( K_o \)  
At-rest earth pressure coefficient

\( N \)  
Number of unit cells in container cross-sectional area

\( P \)  
Pressure, [Pa]

\( Q \)  
Total heat rate from finite element simulation, [W]

\( q_{\text{contact}} \)  
Heat rate through contact area, [W]
<table>
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<tr>
<td>( q_{\text{eff}} )</td>
<td>Effective heat rate through cylindrical unit cell, [W]</td>
</tr>
<tr>
<td>( r )</td>
<td>Coordinate in radial direction, [m]</td>
</tr>
<tr>
<td>( R )</td>
<td>Sphere/particle radius, [m]</td>
</tr>
<tr>
<td>( r_c )</td>
<td>Contact radius [m]</td>
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<tr>
<td>( R'_{\text{contact}} )</td>
<td>Thermal contact resistance, [m²K/W]</td>
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<tr>
<td>( R_{\text{contact}} )</td>
<td>Total thermal contact resistance, [K/W]</td>
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<tr>
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<td>( R_f )</td>
<td>Total thermal resistance through fluid between half-spheres, [K/W]</td>
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<tr>
<td>( R_{\text{rad}} )</td>
<td>Total thermal resistance from radiation, [K/W]</td>
</tr>
<tr>
<td>( R_s )</td>
<td>Total thermal resistance through sphere, [K/W]</td>
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<tr>
<td>( R_{sp} )</td>
<td>Modified total thermal resistance through spheres, [K/W]</td>
</tr>
<tr>
<td>( T )</td>
<td>Temperature, [K]</td>
</tr>
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<td>( T_b )</td>
<td>Temperature of bottom half-sphere, [K]</td>
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<td>( T_c )</td>
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<td>( z )</td>
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<tr>
<td>( \gamma )</td>
<td>Free surface energy, [mJ/m²]</td>
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<tr>
<td>( \theta_c )</td>
<td>Contact angle, [rad]</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\rho_{\text{bulk}}$</td>
<td>Bulk density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann constant</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>Horizontal stress</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Vertical stress</td>
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CHAPTER 1

INTRODUCTION

1.1. Background and Significance

Planetary heat flow values are of particular interest to planetary scientists. These values can tell a great deal about a planetary body’s current state and history. Thermal conductivity and a temperature profile of soils near the planetary surface must be known for heat flow calculations. In situ measurements of effective thermal conductivity on planetary bodies typically involve the use of a long needle-like probe or cable inserted into the planetary surface such as those used for Apollo 15 and 17 [1,2], the MUPUS probe for the Rosetta’s PHILAE lander [3], and other proposed probes [4]. These needle probes utilize the transient hot wire method to measure thermal conductivity. They consist of a heating element which sends a heat pulse radially from the probe into the material to be measured. The temperature change is monitored at the center of the probe. The temperature change data can then be used to determine thermal conductivity by fitting a line to the straight line portion of the log-linear plot of temperature change versus time [5]. Because the heat pulse is sent radially from the probe, the measured directional thermal conductivity is in the radial direction. The desired thermal conductivity for heat flow measurements is in the vertical direction and may be much different from the measured thermal conductivity.

Effective thermal conductivity of granular materials is of interest in industrial applications such as insulation, packed beds for chemical reactions, fusion reactor blankets, and powder metallurgy. Many analytical models for effective thermal conductivity of granular beds in the presence of a static gas have been developed for many of these applications. None of these models, however, consider the effective thermal conductivity of a granular bed to be anisotropic.
1.2. **Heat Transfer in Granular Beds**

Thermal conductivity, which is the measure of a material’s ability to transport thermal energy, is an intrinsic property of any material. It is defined as the rate of thermal energy transmitted per unit area per unit distance per unit temperature change in the direction of heat transfer. It is highly dependent on the chemical composition, physical structure, and state of the material. Because of the presence of multiple granular particles and an interstitial fluid, the thermal conductivity which can be measured is not the thermal conductivity of an individual particle but rather an effective thermal conductivity through the composite material.

Heat transfer through granular beds is highly dependent on the contact area and contact resistance between granular particles. The effective thermal conductivity through a granular bed increases as the contact area between granular particles increases with increasing applied compressive pressure to the granular bed.

The analytical models derived in the literature determine effective thermal conductivity in the same direction as the compressive force. These models can be adapted to estimate anisotropic effective thermal conductivity if they include the effect of compressive stress distributions in granular beds. Soil mechanics show that the lateral (horizontal) stress within a dry, noncohesive soil under gravity is less than but related to the applied vertical pressure through Jaky’s at-rest earth pressure coefficient [6]. The difference in the horizontal and vertical compressive stress should be reflected by the effective horizontal and vertical thermal conductivities of an anisotropic granular bed. However, there has been no research on the anisotropic thermal conductivity in dry granular beds under uniaxial compression.

The results of this study can lead to a greater understanding of heat transfer in a granular bed and the anisotropic nature of the effective thermal conductivity within the bed.
CHAPTER 2

OBJECTIVES

The principal objective of this study is to show that effective thermal conductivity is an anisotropic property of granular beds and to develop a greater understanding of the phenomenon. This principal objective may be broken down into the following specific objectives:

- Obtain experimental data by simultaneously measuring effective vertical and horizontal thermal conductivities to show that effective thermal conductivity is an anisotropic property of granular beds under uniaxial compression

- Derive simple models to predict the anisotropic effective thermal conductivity of granular beds based on the experimental results obtained and effective thermal conductivity models presented in the literature

- Perform finite element simulations of effective thermal conductivity of granular beds in a vacuum environment to gain a greater understanding of the physics and improve the derived model

- Validate the derived models with the experimental data obtained in air and vacuum
CHAPTER 3
LITERATURE REVIEW

Effective thermal conductivity of granular beds has been studied in many fields. This literature review gives a brief overview of some of the modeling techniques and experimental results that have been used to model this phenomenon for various applications.

3.1. Structure-Dependent Effective Thermal Conductivity Models

Many analytical models have been developed for the effective thermal conductivity of granular particle beds in the presence of a static gas. Of these models, many focus on particular aspects of the structure of the bed itself.

3.1.1. Volume Fraction Models

Equations for three main groups of materials are presented in the literature with respect to volume fraction: low volume fraction materials (volume fraction of spheres up to 10%), medium volume fraction materials (volume fraction of spheres from 15-85%), and high volume fraction materials (volume fraction of spheres larger than 90%). Maxwell’s solution [7] for effective thermal conductivity of randomly distributed and non-interacting spherical particles in a homogeneous continuous medium has been shown to predict effective thermal conductivity very well. Chiew and Glandt [8] proposed an improved form of Maxwell’s equation for medium volume fraction materials. Gonzo [9] presented an equation for high volume fraction materials. The high volume fraction equation could be used for granular beds, however determining the volume fraction of the bed from the applied pressure distribution would be a challenging task.
3.1.2. Packing Structure Models

Cheng et al. [10] presented a method to evaluate effective thermal conductivity of a packed bed of mono-sized spheres by using Voronoi polyhedra to include the packing structure of spherical particle beds. The structure of the packed bed was determined by the results measured by Finney [11]. They showed that when the solid to fluid conductivity ratio is low, the dominant heat transfer mechanism is the solid-fluid-solid conduction between point- and area-contacted particles.

3.1.3. Mixing Law Models

An extensive literature review was done by Abdulagatova et al. [12] on mixing law models and the dependence of effective thermal conductivity on temperature, porosity, and gas pressure. Mixing law models combine values of the solid and fluid thermal conductivity, typically as a function of volume fraction, to determine an effective thermal conductivity. Mixing law models tend to be general in nature and have limited applicability. These models can, however, provide convenient, simple predictions for the physical limits of effective thermal conductivity.

3.2. Pressure-Dependent Effective Thermal Conductivity Models

Heat transfer through granular beds is highly dependent on the contact area and thermal contact resistance between granular particles. The effective thermal conductivity through a granular bed increases as the contact area between granular particles increases with increasing applied compressive pressure to the granular bed. It is important to include the effects of applied pressure to a model of effective thermal conductivity.
3.2.1. **Pressure-Dependent Results**

Increasing effective thermal conductivity with increasing pressure has been shown in many materials. Demirci et al. [13] built an experimental system with a hydraulic press to apply uniaxial and triaxial pressure. A steady state method was used to measure the pressure-dependent effective thermal conductivity of rocks. Reimann and Hermsmeyer [14] measured the pressure-dependent effective thermal conductivity of metatitanate and orthosilicate ceramic breeder pebble beds using the hot wire method. Tehranian and Abdou [15] measured the pressure-dependent effective thermal conductivity of aluminum, lithium zirconate and beryllium particle beds using a steady-state method.

Attempts have been made to model the effects of compressive pressure on the effective thermal conductivity. Tehranian and Abdou used the Hertz elastic equation and the Bauer, Schlünder, and Zehner model to predict the effective thermal conductivity as a function of external pressure [15].

Siu and Lee [16] found that the ratio of effective thermal conductivity to bulk granular material thermal conductivity for simple cubic (SC), body centered cubic (BCC), and face centered cubic (FCC) packing arrangements is a linear function of the ratio of contact radius to particle radius as shown in Figure 3-1. This result allows the following models to model effective thermal conductivity by modeling only the SC packing arrangement and multiplying this by a constant.

Slavin’s model [17] assumed a primitive tetragonal packing of spheroids with a gap near the points of contact between the spheres due to long-range surface undulations. The individual spheres were considered isothermal with heat transfer occurring by conduction through the points in direct contact, gap inside the contact points, fluid between particles and radiation between the particles. Weidenfeld’s model [18] used a cylindrical control volume containing two spheres.
Figure 3-1. Trends of the ratio of effective thermal conductivity to bulk particle thermal conductivity against the contact radius to particle radius ratio from Siu and Lee [16].

The model assumes that the thermal conductivity of the spheres is much greater than that of the fluid between the spheres and accounts for surface roughness in extending Slavin’s model. Bahrami et al. [19] modeled simple cubic (SC) and face-centered cubic (FCC) packings of rough mono-sized spheres to yield upper and lower bounds for the effective thermal conductivity of rough mono-sized spheres immersed in a stagnant gas.

3.2.2. Soil Mechanics

All of the models discussed in section 3.2.1. were developed for effective thermal conductivity in the same direction as the compressive force. These models can be adapted to estimate anisotropic thermal conductivity if they include the effect of compressive stress distributions in granular beds. Soil mechanics show that the lateral (horizontal) stress within a dry, noncohesive soil under gravity is less than but related to the applied vertical pressure through Jaky’s at-rest earth pressure coefficient. This coefficient is defined as the horizontal-to-vertical
stress ratio in loose deposits and normally consolidated clays and is a function of the friction angle or angle of repose of the soil [6]. The difference in the horizontal and vertical compressive stress should be reflected by the effective horizontal and vertical thermal conductivities of an anisotropic granular bed. However, there has been no research on the anisotropic thermal conductivity in dry granular beds under uniaxial compression.
CHAPTER 4
THEORETICAL MODELING

4.1. Effective Thermal Conductivity Model

Based on the experimental and theoretical findings in the literature, a simple quasi one-dimensional model was developed for a unit cell of the granular system. This model was derived to be simpler than those found in the literature by including fewer model parameters. Heat transfer was modeled using a cylindrical unit cell containing two smooth half-spheres in contact with a static fluid filling the rest of the unit cell (Figure 4-1). The following assumptions were made:

- Half-spheres are perfectly round with a flat, Hertzian contact interface
- No thermal contact resistance
- Heat flows in one direction only
- Heat transfer by convection and radiation can be neglected

4.1.1. Geometry

Values for the geometry shown in Figure 4-1 for θc, H, Δys, and Δyt are calculated by Equations (4.1-4.4).

\[
\cos(\theta_c) = \frac{r_c}{R} \tag{4.1}
\]

\[
H = 2R \sin(\theta_c) \tag{4.2}
\]

\[
\Delta y_s = R \sin(\theta) \tag{4.3}
\]

\[
\Delta y_t = 2R [\sin(\theta_c) - \sin(\theta)] \tag{4.4}
\]

where R is the sphere radius, θ is an angle defined in Figure 4-1, rc is the contact radius, and θc is the angle defined in Figure 4-1. rc is calculated by the Hertz contact equation [20]
Figure 4-1. Two-dimensional rendering of cylindrical unit cell containing contacting spheres.

\[ r_c = \sqrt[3]{\frac{3F_c R (1 - \nu^2)}{4E}} \]  \hspace{1cm} (4.5)

where \( F_c \) is the contact force, \( E \) is Young’s modulus, and \( \nu \) is Poisson’s ratio.

4.1.2. **Sphere-Contact-Sphere Path**

Heat transfer by conduction through the spheres is included in this model in contrast to the models of Slavin [17], Weidenfeld [18], and Bahrami [19] who assumed the spheres to be isothermal. Such a treatment will allow this model to be applicable to particles with a wide range of thermal conductivity, especially those with low thermal conductivity. The heat flow is modeled through a resistance network of conduction through the lower sphere, contact area, and upper sphere in parallel with conduction through the lower sphere, fluid, and upper sphere.

The heat transfer through the sphere-contact-sphere conduction path \((q_{\text{contact}})\) is calculated by Fourier’s law of heat conduction.
where $k_s$ is the sphere’s thermal conductivity, $H$ is shown in Figure 4-1 and calculated by Equation (4.2), and $\Delta T$ is the temperature difference over $H$. After substituting Equation (4.2) into Equation (4.6) and rearranging Equation (4.6) to yield an Ohm’s law expression for the heat transfer, the thermal resistance for the heat transfer through the sphere-contact-sphere conduction path ($R_{contact}$) becomes

$$R_{contact} = \frac{2R \sin(\theta_c)}{k_s \pi r_c^2} \quad (4.7)$$

### 4.1.3. Sphere-Fluid-Sphere Path

The differential heat transfer through the half-sphere ($dq_s$) is found by

$$dq_s = k_s dA_s \frac{\Delta T}{\Delta y_s} \quad (4.8)$$

where the distance $\Delta y_s$ is shown in Figure 4-1 and calculated by Equation (4.3) and the differential ring area $dA_s$ through which heat is conducted is

$$dA_s = 2\pi R^2 \cos(\theta) \sin(\theta) \, d\theta. \quad (4.9)$$

After substituting Equation (4.3) and Equation (4.9) into Equation (4.8), integrating from $\theta=0$ to $\theta_c$, and rearranging Equation (4.8) into an Ohm’s law expression of heat transfer, the thermal resistance through the half-spheres ($R_s$) becomes

$$R_s = \frac{1}{k_s 2\pi R \sin(\theta_c)} \quad (4.10)$$

The differential heat transfer through the fluid ($dq_f$) [18] is found by

$$dq_f = k_f dA_s \frac{\Delta T}{\Delta y_f} \quad (4.11)$$
where $k_f$ is the fluid’s thermal conductivity, $dA_s$ is calculated by Equation (4.9), $\Delta T$ is the temperature difference across the fluid, and $\Delta y_f$ is shown in Figure 4-1 and calculated by Equation (4.4).

Equation (4.11) cannot be integrated from $\theta=0$ to $\theta_c$ because the heat flow becomes infinite at $\theta_c$. Equation (4.11) will be integrated from $\theta=0$ to $\theta_c+\Delta\theta$ where $\Delta\theta$ is a small angle and found by equating the heat flux through the half-sphere ($dq$, given by Equation (4.8)) and the heat flux through the fluid ($dq_f$ given by Equation (4.11)) at $\theta=\theta_c+\Delta\theta$ [18]. This asymptotic heat flux limit for the fluid gap approaching zero results in

$$\Delta\theta = \theta_c - \sin^{-1}\left[\sin(\theta_c) - \frac{k_f \sin(\theta_c)}{k_s}ight].$$

(4.12)

Equation (4.11) is then integrated from $\theta=0$ to $\theta_c+\Delta\theta$. After integrating, simplifying, and rearranging to form an Ohm’s law type expression, the thermal resistance through the fluid ($R_f$) becomes

$$R_f = \frac{1}{k_f \pi R \sin(\theta_c) \cdot \left[\frac{k_f}{k_s} - 1 + \ln\left(\frac{k_s}{k_f}\right)\right]}.$$

(4.13)

The spheres can be considered isothermal when the conduction resistance through the fluid and the contact resistance of the interface are large. The isothermal sphere approximation can be used for all but highly insulating materials as gas thermal conductivities are generally very low. For the case of isothermal spheres, or if the sphere thermal conductivity is much greater than the fluid thermal conductivity, the contribution due to the $k_f/k_s$ term in $R_f$ approaches zero and can be neglected to form

$$R_f = \frac{1}{k_f \pi R \sin(\theta_c) \cdot \left[\ln\left(\frac{k_s}{k_f}\right) - 1\right]}.$$

(4.14)
4.1.4. Effective Thermal Conductivity

The effective heat transfer through the control volume \( q_{eff} \) [17,18] is given by

\[
q_{eff} = k_{eff} \pi R^2 \frac{\Delta T}{H} \tag{4.15}
\]

Rearranging Equation (4.15) to form an Ohm’s law expression for the effective heat transfer, the effective thermal resistance \( R_{eff} \) becomes

\[
R_{eff} = \frac{2 \sin(\theta_c)}{k_{eff} \pi R} \tag{4.16}
\]

The effective thermal resistance is solved for by summing the sphere-contact-sphere resistance in parallel with the sphere-fluid-sphere resistance

\[
\frac{1}{R_{eff}} = \frac{1}{R_{contact}} + \frac{1}{2 \cdot R_s + R_f} \tag{4.17}
\]

This yields an expression for the effective thermal conductivity

\[
k_{eff} = \frac{2 \sin(\theta_c)}{\pi R} \left[ \frac{1}{R_{contact}} + \frac{1}{2 \cdot R_s + R_f} \right] \tag{4.18}
\]

To account for different packing configurations other than the assumed simple cubic packing configuration, particle shape and size distributions, three dimensional heat flow paths, and surface roughness, Equation (4.18) is modified to add fit correction parameters A and B, yielding Equation (4.19) [17,18]. A and B can be found by linear regression to empirical data.

\[
k_{eff} = \frac{2 \sin(\theta_c)}{\pi R} \left[ A \frac{1}{R_{contact}} + B \frac{1}{2 \cdot R_s + R_f} \right] \tag{4.19}
\]
In the case of isothermal spheres, or sphere thermal conductivity much greater than the fluid thermal conductivity, the thermal resistance through the spheres approaches zero ($R_s$ in Equation (4.19) becomes zero) and $R_f$ is calculated by Equation (4.14).

4.1.5. **Model Discussion**

This simple heat transfer model underestimates the thermal resistance for the sphere-contact-sphere conduction path, $R_{\text{contact}}$, when compared with the actual thermal contact resistance. The assumption of two perfectly round spheres in Hertzian contact results in calculating a larger contact area than that of real spheres with surface roughness in contact. A larger contact area results in a smaller $R_{\text{contact}}$ than the real case. The model corrects for the smaller $R_{\text{contact}}$ calculated using this assumption by including the fit correction parameter $A$. Including the $A$ parameter in the model allows the model to incorporate the surface roughness factor and simply use the average radius of the particles. This results in a simpler model requiring fewer input parameters than those found in the literature.

Heat transfer by radiation has been neglected by this model. The contribution to the heat transfer by radiation is much smaller than that of the particles or the surrounding fluid. The smallest possible radiation thermal resistance is the blackbody radiation case calculated by Equation (4.20), where $\sigma$ is the Stefan-Boltzmann constant, $T_{\text{sur}}$ is the surrounding temperature and $A_r$ is the surface area [21].

$$R_{\text{rad}} = \frac{1}{4\sigma T_{\text{sur}}^4 A_r} \tag{4.20}$$

A comparison of this thermal resistance to the thermal resistance of the fluid shows that the radiation thermal resistance is much larger than the fluid thermal resistance in the experiment. This larger thermal resistance has a much smaller contribution to the overall heat transfer and can be neglected.
Figure 4-2 shows a comparison of the model fit developed in this study to Tehranian and Abdou’s empirical effective vertical thermal conductivity data [15] and model predictions made to this data using the Bauer, Schlünder and Zehner [15] and Weidenfeld [18] models. The data come from an experiment using 2 mm diameter beryllium particles in air at atmospheric pressure. The model fit developed for this study compares favorably with the Bauer, Schlünder and Zehner model used by Tehranian and Abdou [15] and the model developed by Weidenfeld [18], but diverges slightly at lower applied pressures. This divergence is due to the calculation of $R_{\text{contact}}$ in the model. The effect of the lower calculated contact resistance is greater at lower applied pressures than at higher applied pressures.

![Figure 4-2. Model comparisons of effective vertical thermal conductivity for Tehranian and Abdou’s data [15] on 2 mm beryllium spheres in air at atmospheric pressure.](image-url)
The model developed for this study can predict effective horizontal or vertical thermal conductivity from experimental data of the effective horizontal thermal conductivity with the inclusion of Jaky’s at-rest earth pressure coefficient, $K_o$. $K_o$ is defined as the ratio between the horizontal and vertical stress within a granular bed [6]

$$K_o = \frac{\sigma_h}{\sigma_v} \quad (4.21)$$

$K_o$ should be included in the calculation of the horizontal contact force to yield appropriate results of $r_c$ and $k_{eff}$ for effective horizontal thermal conductivity.

An example of the predicted results for effective horizontal and vertical thermal conductivity is shown in Figure 4-3 using properties for 0.5 mm diameter aluminum spheres in air. The properties used for the aluminum spheres are shown in Table 4-1. A 25 mm layer of particles over the site of effective thermal conductivity measurement was assumed. The weight of the 25 mm layer of particles over the site of effective thermal conductivity measurement is not included in the applied pressure which causes a slight compressive load at zero applied pressure. This slight load creates a difference in the vertical and horizontal effective thermal conductivities.

Table 4-1. Properties Used for 0.5 mm Diameter Aluminum Spheres in Air

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk Density</td>
<td>$\rho_{\text{bulk}}$</td>
<td>1665</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>$E$</td>
<td>70</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>$\nu$</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>Sphere Thermal Conductivity</td>
<td>$k_s$</td>
<td>237</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Fluid Thermal Conductivity</td>
<td>$k_f$</td>
<td>0.025</td>
<td>W/m/K</td>
</tr>
<tr>
<td>At-Rest Earth Pressure Coefficient</td>
<td>$K_o$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Fit Parameter</td>
<td>$A$</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Fit Parameter</td>
<td>$B$</td>
<td>0.8</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4-3. Model prediction of effective vertical and horizontal thermal conductivity due to increasing applied vertical pressure for 0.5 mm aluminum spheres in air.

Figure 4-3 shows that the effective vertical and horizontal thermal conductivities should increase at different rates because of the pressure difference between the horizontal and vertical directions. Effective vertical thermal conductivity will always be larger than the effective horizontal thermal conductivity for the range of 0 to 20 kPa applied vertical compressive pressure.

The contributions of the parallel conduction paths to the effective thermal conductivities as predicted by the model (including A and B parameters) are shown in Figure 4-4 using the same properties as those for Figure 4-3.
Although the sphere thermal conductivity is much greater than the fluid thermal conductivity, the greatest contribution to the effective vertical and horizontal thermal conductivity at the applied pressure interval of 0 to 10 kPa comes through the sphere-fluid-sphere conduction path and not the sphere-contact-sphere conduction path. This is because the contact area through which heat is transferred directly from sphere to sphere is very small, only 0.005% of the unit cell cross-sectional area at 10 kPa pressure. The contact area between particles increases with pressure and the contribution to the effective thermal conductivity through the sphere-contact-sphere path increases. The weight of the particles above the measurement location is not included in the applied pressure. This slight load creates a contact force and resulting contact area through which heat is conducted resulting in the sphere-contact-sphere path beginning at a value higher
than 0% of the total contribution to effective thermal conductivity and the sphere-fluid-sphere path beginning at a lower value than 100% of the total contribution to effective thermal conductivity.

4.2. Finite Element Simulation

4.2.1. Finite Element Setup

The commercial software package, COMSOL Multiphysics [22], was used for steady-state heat transfer analysis. Figure 4-5 presents a schematic of the geometry and boundary conditions of the problem to be solved. Because of the axially symmetric geometry, a cylindrical coordinate system can be used to solve the problem posed. This allows the three-dimensional problem to be reduced to a two-dimensional problem. Heat transfer is modeled similar to the previously derived model in that two half-spheres in contact are modeled. The top and bottom boundaries are prescribed as constant temperatures $T_h$ and $T_c$, where $T_h$ is defined as larger than $T_c$.

Thermal contact resistance at the interface between the two half-spheres is modeled by creating a “pair” on the interface to link the two half-sphere domains. If thermal contact resistance is not defined, the default continuity condition is applied on the interface where the temperatures and fluxes across the interfaces are equal. However, when thermal contact resistance is considered, a thin, thermally resistive layer is turned on causing a temperature ‘jump’ across the interface while the flux across the interface is still equal. Mathematically, the boundary condition can be expressed as

$$-k_s \frac{\partial T}{\partial z} \bigg|_{\text{bottom interface}} = \frac{T_h - T_t}{R_{\text{contact}}} = -k_s \frac{\partial T}{\partial z} \bigg|_{\text{top interface}}$$

(4.22)

where $T_h$ is the temperature of the bottom sphere and $T_t$ is the temperature of the top sphere at the interface.
Figure 4-5. Schematic of geometry and boundary conditions for finite element simulation.

Inside the domain, the overall temperature distribution is calculated from the steady-state heat conduction equation,

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( k_r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) = 0. \tag{4.23}
\]

Once the temperature distribution has been calculated, the heat rate can be determined by performing a surface integration of the heat flux through either the top or bottom surface. Once this heat rate is calculated, the effective thermal conductivity can be calculated by the method of Fiedler et al. [23]

\[
k_{\text{eff}} = \frac{Q \cdot H}{\pi R^2 (T_h - T_c)} \tag{4.24}
\]

where \(Q\) is the calculated heat rate, \(H\) is defined in Figure 4-1, \(R\) is the sphere radius, \(T_h\) is the temperature of the hot surface, and \(T_c\) is the temperature of the cold surface.
Two-dimensional axisymmetric grids were used in the simulations. The triangle free mesh option in COMSOL was chosen for the element shape. The element maximum size can be specified as a free mesh parameter. For this study, the maximum element size on the subdomain of the two half-spheres and the contact interface between the half-spheres were refined until grid independence was achieved. The heat rates out of one of the surfaces were computed as the maximum element sizes were refined for the case of 0.5 kPa applied pressure to the model with properties of alumina. The results of the grid independence study are plotted in Figure 4-6. The maximum element sizes for the half-sphere subdomains and contact interface were determined to be 2\times10^6 and 1\times10^9. The selected mesh parameters led to above 35,000 mesh elements. These mesh parameters were used in all the finite element simulations used in this study.

![Grid independence test for heat rate at 0.5 kPa applied pressure with alumina properties.](image)
4.2.2. **Importance of Thermal Contact Resistance**

The objective of this section is to present the results of finite element simulations of the two half-sphere system in a vacuum environment to better understand the effect of Young’s modulus and size on the effective thermal conductivity. Effective vertical thermal conductivity is considered in this analysis, however the results can be extended to effective horizontal thermal conductivity by using the at-rest earth pressure coefficient, \( K_o \) multiplied by the applied pressure.

Two-dimensional axially symmetric geometries were generated and steady-state conduction analysis was performed using COMSOL Multiphysics. Early analysis showed that including heat transfer by radiation increased the value of effective thermal conductivity less than 5%. Because the contribution to the effective thermal conductivity due to radiation heat transfer is so small, heat transfer by conduction is the only mode of heat transfer considered in the following analysis.

In the absence of a surrounding fluid due to a high vacuum environment, the spheres become almost isothermal. A jump in temperature at the interface between two spheres is due to the thermal contact resistance between the spheres. A comparison of the temperature distributions resulting from a 0.1 mm radius sphere model in vacuum and air environments with the properties listed in Table 4-2 is plotted along the z-axis at the axisymmetric \( r = 0 \) position in Figure 4-7. Because of the larger jump in temperature and virtually no temperature gradient after the jump in temperature for the vacuum environment, the thermal contact resistance becomes the most important factor in effective thermal conductivity of granular beds in a vacuum environment.
Figure 4-7. Temperature distribution along the axis of two half-spheres in a vacuum environment ($k_f = 0$) with properties defined in Table 4-2.

Table 4-2. Properties of Stainless Steel Used in the Finite Element Simulation for Figure 4-7

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere Radius</td>
<td>R</td>
<td>0.1</td>
<td>mm</td>
</tr>
<tr>
<td>Applied Pressure</td>
<td>P</td>
<td>10</td>
<td>kPa</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>E</td>
<td>200</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>ν</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Free Surface Energy</td>
<td>γ</td>
<td>36</td>
<td>mJ/m²</td>
</tr>
<tr>
<td>Sphere Thermal Conductivity</td>
<td>$k_s$</td>
<td>16</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Fluid Thermal Conductivity</td>
<td>$k_f$</td>
<td>0.025</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Temperature of hotter side</td>
<td>$T_h$</td>
<td>301</td>
<td>K</td>
</tr>
<tr>
<td>Temperature of colder side</td>
<td>$T_c$</td>
<td>300</td>
<td>K</td>
</tr>
<tr>
<td>Thermal Contact Resistance</td>
<td>$R_{contact}^0$</td>
<td>$10^{-6}$</td>
<td>m²K/W</td>
</tr>
</tbody>
</table>
The properties used in the finite element simulations investigating the effect of Young’s modulus and sphere size were chosen to be similar to the properties of the particles used in the experiments performed in vacuum. Although the packing arrangements of the finite element simulations and experimental data are different, the trends in effective conductivity are similar [16].

4.2.3. **Effect of Young’s Modulus**

The effect of Young’s modulus on effective thermal conductivity was studied using finite element simulation in COMSOL by generating models using the properties in Table 4-3. The value of Young’s modulus was varied from 100 GPa to 500 GPa in 100 GPa increments. The values of Young’s modulus were chosen to be comparable to the values of Young’s modulus of the spheres used in the experiments performed in vacuum (200 GPa and 375 GPa). The steady-state temperature distribution was solved, the heat rate through the top surface was determined by surface integration, and the effective thermal conductivity was found by the procedure of Fiedler et al. [23]. Figure 4-8 shows the resulting curves of effective thermal conductivity against applied pressure.

Decreasing Young’s modulus has the effect of increasing effective thermal conductivity under compressive pressure. Increasing Young’s modulus also decreases the slope of the resulting curve of effective thermal conductivity against applied pressure. Effective thermal conductivity is a function of thermal contact resistance and contact area (as seen in Equation 4.30). The contact radius is inversely proportional to Young’s modulus to the one-third power in both the Hertz, Equation (4.5), and JKR, Equation (4.25), contact models. Thus when Young’s modulus is increased, a decrease in contact radius and effective thermal conductivity is seen.
Table 4-3. Properties of Stainless Steel Used in Finite Element Simulations Investigating the Effect of Young’s Modulus on Effective Thermal Conductivity

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere Radius</td>
<td>R</td>
<td>0.1</td>
<td>mm</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>ν</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Free Surface Energy</td>
<td>γ</td>
<td>36</td>
<td>mJ/m²</td>
</tr>
<tr>
<td>Sphere Thermal Conductivity</td>
<td>kₚ</td>
<td>16</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Temperature of hotter side</td>
<td>Tₕ</td>
<td>301</td>
<td>K</td>
</tr>
<tr>
<td>Temperature of colder side</td>
<td>Tₖ</td>
<td>300</td>
<td>K</td>
</tr>
<tr>
<td>Thermal Contact Resistance</td>
<td>Rₜₙ</td>
<td>10⁻⁶</td>
<td>m²K/W</td>
</tr>
</tbody>
</table>

Figure 4-8. Effect of Young’s modulus on effective thermal conductivity.
4.2.4.  Effect of Sphere Size

The effect of sphere size on effective thermal conductivity was studied using finite element simulation in COMSOL by generating models using the properties in Table 4-4. Sphere radius values were studied from 0.05 mm to 0.25 mm in increments of 0.05 mm. The radius range was chosen to be comparable with the size of particles used in the experiments performed in vacuum (0.2 mm and 0.5 mm diameter spheres). The steady-state temperature distribution was solved, the heat rate through the top surface was determined by surface integration, and the effective thermal conductivity was found by the procedure of Fiedler et al. [23]. Figure 4-9 shows the resulting curves of effective thermal conductivity against applied pressure.

Table 4-4. Properties of Stainless Steel Used in Finite Element Simulations Investigating the Effect of Sphere Size on Effective Thermal Conductivity

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>E</td>
<td>200</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>ν</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Free Surface Energy</td>
<td>γ</td>
<td>36</td>
<td>mJ/m²</td>
</tr>
<tr>
<td>Sphere Thermal Conductivity</td>
<td>kₛ</td>
<td>16</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Temperature of hotter side</td>
<td>Tᵢ</td>
<td>301</td>
<td>K</td>
</tr>
<tr>
<td>Temperature of colder side</td>
<td>Tᵢ</td>
<td>300</td>
<td>K</td>
</tr>
<tr>
<td>Thermal Contact Resistance</td>
<td>Rcontact</td>
<td>10⁻⁶</td>
<td>m²K/W</td>
</tr>
</tbody>
</table>
Increasing particle radius has the effect of increasing effective thermal conductivity under compressive pressure. Increasing particle radius also increases the slope of the resulting curve of effective thermal conductivity against applied pressure. The models derived in Chapter 4 give insight as to why this occurs. Effective thermal conductivity is a function of thermal contact resistance and contact area (as seen in Equation (4.30)). The contact radius increases as sphere radius increases for both the Hertz, Equation (4.5), and JKR, Equation (4.25), contact models. Increasing contact radius with increasing sphere radius results in an increase in effective thermal conductivity.
4.3. Modified Vacuum Model

The model derived in section 4.1 works well for the case where granular particles are immersed in a static fluid. For the case of granular particles in a vacuum environment, the $B$ parameter from Equation (4.19) could be set to zero and a reasonable fit to data would be achieved. A more detailed model is desirable and achieved by making only minor modifications to the model derived previously.

The same assumptions and geometry as in the previously derived model apply except that a constant thermal contact resistance is assumed. The finite element simulations performed in section 4.2 showed that thermal contact resistance is the most important parameter in determining the effective thermal conductivity in a vacuum environment. Conduction through the spheres themselves makes a very slight contribution to the effective thermal conductivity. It would then seem important to quantify the effect of thermal contact resistance between the particles.

It has been shown that the Hertz contact equation is less appropriate for small particles under small contact forces [24]. The Hertz equation is appropriate for the static fluid case because the sphere-fluid-sphere conduction path has the largest contribution to the effective thermal conductivity. Adhesion and the free surface energy of the spheres play a larger role in contact for spheres under small contact forces as in the experiments performed in this study. The Johnson-Kendall-Roberts (JKR) contact model is used to calculate the contact radius for the vacuum case in Equation (4.25) [25]

$$r_c = \sqrt[3]{\frac{3R(1-v^2)}{2E} \left[ F_c + 6\gamma \pi R + \sqrt{12 \gamma \pi R F_c + (6\gamma \pi R)^2} \right]}$$

(4.25)

where $\gamma$ is the free surface energy (usually in units of mJ/m$^2$). The JKR contact model calculates a slightly larger contact radius than the Hertz equation due to the adhesive forces between particles. Use of this contact model in the modified model yields a more accurate calculation of the important contact radius.
4.3.1. Conduction Through Spheres

The thermal resistance through the spheres is found similar to the static fluid case. $R_s$ from Equation (4.10) is summed with the resistance through the region between the sphere center and the contact area. The new conduction resistance through the spheres, $R_{sp}$, results in

$$R_{sp} = \frac{1}{k_s 2\pi R \sin(\theta_c)} + \frac{R \sin(\theta_c)}{k_s \pi r_c^2}$$

(4.26)

4.3.2. Contact Resistance

The thermal contact resistance, $R''_{contact}$, is assumed to be constant. Thermal contact resistances have the units of $m^2K/W$, to make the units consistent with $R_{sp}$ and to reflect the changes in effective thermal conductivity due to increasing contact area, $R''_{contact}$ is divided by the contact area. $R_{contact}$ now becomes

$$R_{contact} = \frac{R''_{contact}}{\pi r_c^2}$$

(4.27)

4.3.3. Effective Thermal Conductivity

The effective thermal resistance is solved for by summing the conduction resistance through the spheres in series with the contact resistance.

$$R_{eff} = 2R_{sp} + R_{contact}$$

(4.28)

This yields an expression for the effective thermal conductivity

$$k_{eff} = \frac{2 \sin(\theta_c)}{\pi R} \left[ \frac{1}{2R_{sp} + R_{contact}} \right].$$

(4.29)

To account for different packing configurations other than the assumed simple cubic packing configuration, particle shape and size distributions, and three dimensional heat flow paths, Equation (4.29) is modified to add the fit correction parameter $C$ yielding Equation (4.30). $C$ and $R''_{contact}$ can be found by linear regression to empirical data.
4.3.4. Model Discussion

The modified simple heat transfer model underestimates the thermal resistance through the spheres, $R_{sp}$. The model corrects for this underestimation by including the fit correction parameter $C$. Although the contribution to the effective thermal conductivity by conduction through the spheres is small, the value of the $C$ parameter shows how much the thermal resistance through the spheres differs due to three-dimensional heat flow compared with the assumed one-dimensional heat flow.

A comparison of the modified model and initial model predictions for effective vertical and horizontal thermal conductivity is shown in Figure 4-10. Properties for 0.5 mm diameter aluminum spheres from Table 4-1 were used except that the $B$ parameter was set to zero for the initial model prediction. The properties for 0.5 mm diameter aluminum spheres for the modified model prediction are shown in Table 4-5. The modified model was plotted using both the Hertz and JKR contact equations.

The modified model shows good agreement with the initial model when using the Hertz contact equation. Use of the JKR contact equation in the modified model has the effect of shifting the effective thermal conductivity up. Earlier it was noted that the initial model diverges from empirical data at low pressures. Use of the JKR contact model instead of the Hertz model decreases the divergence by computing a larger contact force, contact radius, and consequently a higher effective thermal conductivity at lower applied pressures. This effect is more important when the granular bed is in a vacuum environment than when it is immersed in a static fluid.

\[
k_{eff} = \frac{2 \sin(\theta_c)}{\pi R} \left[ \frac{1}{2CR_{sp} + R_{contact}} \right]
\]  

(4.30)
Figure 4-10. Comparison of initial derived model to modified vacuum model using Hertz and JKR contact equations.

Table 4-5. Properties Used for 0.5 mm Diameter Aluminum Spheres in Vacuum

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk Density</td>
<td>$\rho_{\text{bulk}}$</td>
<td>1665</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>$E$</td>
<td>70</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>$\nu$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Free Surface Energy</td>
<td>$\gamma$</td>
<td>35</td>
<td>mJ/m$^2$</td>
</tr>
<tr>
<td>Sphere Thermal Conductivity</td>
<td>$k_s$</td>
<td>237</td>
<td>W/m/K</td>
</tr>
<tr>
<td>At-Rest Earth Pressure Coefficient</td>
<td>$K_o$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Fit Parameter</td>
<td>$C$</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Thermal Contact Resistance</td>
<td>$R''_{\text{contact}}$</td>
<td>$10^{-7}$</td>
<td>m$^2$K/W</td>
</tr>
</tbody>
</table>
Heat transfer by radiation has been neglected by this model. Finite element simulations of the vacuum case performed for this study have shown that radiation contributes less than 5% of the total effective thermal conductivity.
CHAPTER 5
EXPERIMENTAL SETUP

Two experiments were designed to fulfill separate objectives. An experiment was designed and performed in air to show that effective thermal conductivity of granular beds is an anisotropic property. A second experiment was designed and performed inside a vacuum chamber to investigate the effects of varying physical properties of the granular particles.

5.1. Experiment in Air

The experimental setup consisted of a container, loading platform, and the KD2 Pro (needle probes and data logger) from Decagon Devices, Inc [26]. Figure 5-1 shows a diagram of the experimental setup.

![Diagram of experiment in air setup.](image-url)
Figure 5-2. Photograph of entire experiment in air system.

The container was made of acrylic with dimensions of 100 mm long, 40 mm wide, and 45 mm high. It was filled with granular material, covered by an acrylic plate, needle probes were inserted horizontally and vertically into the material, and a compressive pressure load was applied by increasing the load on the loading platform. The load was applied by filling a container with water, measuring its weight to achieve the desired applied pressure, and placing the container on the loading platform.

A dual needle probe was inserted vertically into the material. This probe measured the horizontal effective thermal conductivity directly. Dual needle probes allow for the measurement of thermal conductivity, thermal diffusivity, and volumetric heat capacity [27]. The determination of multiple thermal properties allow for the validation of the measurement through finite element simulation. One needle supplies a heat pulse and the temperature change is recorded by the other needle. The effective thermal conductivity and diffusivity were found by fitting Equation (5.1) using the curve fit tool in the Matlab software package to the heating data measured by the KD2 Pro system.
\[
\Delta T = \frac{q}{4\pi k} Ei\left(\frac{-r^2}{4at}\right)
\]

(5.1)

It has been shown that this method has an uncertainty below 10% of the measured values [28].

A single needle probe operates similar to the dual needle probe in that a heat pulse is supplied to the medium and the temperature at the center of the needle is recorded. The effective thermal conductivity is found by using the slope of the line fit through the log-linear portion of the temperature rise plot to Equation (5.2).

\[
k = \frac{q}{4\pi S}
\]

(5.2)

The uncertainty of this method has been shown to be less than 10% to 15% of the measured values in ASTM D5334 [5]. It is also discussed in ASTM D5334 that the values obtained using this method tend to be higher than the true values.

Two dimensional finite element simulations of a medium with different values for thermal conductivity in the horizontal and vertical directions were performed. These simulations showed that a single needle probe inserted in an anisotropic medium measures the average of the effective horizontal and vertical thermal conductivities. Finite element simulations were made to find correction coefficients for the effective thermal conductivities measured with the dual and single needle probes. Once the corrected effective horizontal and average thermal conductivities were calculated, the effective vertical thermal conductivity was calculated.

### 5.2. Experiment in Vacuum

The experimental setup consisted of a container, loading platform, vacuum chamber, vacuum pumps, Hot Disk TPS 500 Thermal Constants Analyser [29], and computer. Figure 5-3 and Figure 5-4 show schematics of the experimental setup.
Figure 5-3. Overall experiment in vacuum system diagram.

Figure 5-4. Experimental Section (inside of vacuum chamber) diagram.
Figure 5-5. Photographs showing (1) the vacuum chamber (top left) and (2) the vacuum pump system consisting of mechanical and diffusion pumps (top right).

Figure 5-6. Photographs showing (1) the loading system (top left) and (2) the entire experimental setup.
The container was made of acrylic and had dimensions of 50 mm long, 50 mm wide, and 75 mm high. It was filled with granular material, covered by an acrylic plate, Hot Disk sensors inserted horizontally and vertically into the material, and placed inside of a vacuum chamber. The vacuum chamber was connected to the vacuum pump system consisting of a mechanical and a diffusion pump. The mechanical pump is used first to achieve low pressures and the diffusion pump is used to generate and maintain a high vacuum environment. The compressive load was applied by filling a container with sand, measuring its weight to achieve the desired applied pressure, and placing the container on the loading platform.

Hot Disk sensors were inserted horizontally and vertically into the material. Unlike the needle probe configuration, the Hot Disk sensors measure the effective horizontal and vertical thermal conductivities directly. The sensor inserted horizontally measures the effective vertical thermal conductivity while the sensor inserted vertically measures the effective horizontal thermal conductivity.

The Hot Disk measurement system uses a transient plane source technique to measure thermal properties [30]. The sensor heats the sample through resistive heating and the temperature change of the sensor is found through the change in resistance of the sensor. The typical temperature change by this method is about 5°C. Thermal properties are determined from the temperature change data taken during the measurement time. A measurement time of 80 seconds was determined to be appropriate for all measurements made with this setup.

Residual temperature gradients within the granular bed from previous thermal conductivity measurements result in incorrect thermal property determination. Care was taken to ensure that no temperature gradients existed within the granular bed during measurement. In order to cool the sample more rapidly to its initial temperature, convective heat transfer was used. The vacuum chamber was repressurized by allowing air into the chamber. The vacuum pumps were then used to evacuate the air in the chamber, generating a vacuum environment. This
process was repeated until the Hot Disk system showed no temperature drift in the granular bed before a measurement was taken, effectively flushing the residual heat from the granular bed and restoring the granular bed to an equilibrium room temperature. Generally this required the process of pressurizing-depressurizing to be repeated three to five times.

The Hot Disk system measures thermal conductivity, thermal diffusivity, and volumetric heat capacity. The determination of multiple thermal properties allow for the validation of the measurement through finite element simulation. The Hot Disk TPS 500 has a measurable range of 0.03 W/m/K to 100 W/m/K for thermal conductivity. The system is reported to be better than 5% accurate for thermal conductivity measurements with a reproducibility of 2% [31].

Figure 5-7. Photograph of Hot Disk sensor.
CHAPTER 6

EXPERIMENTAL RESULTS AND DISCUSSION

The results from experiments performed in air and vacuum environments and finite element simulations are presented and discussed in this section.

6.1. Materials

Two materials were used in the experiment performed in air, commercial pure powdered titanium and spherical copper shot purchased from Sigma-Aldrich [32]. The particles were chosen because they had high thermal conductivities and the resulting effective thermal conductivity of the particle bed was within the measurable range of the needle probes used in this study. Figure 6-1 and Figure 6-2 show photographs of the two materials. The titanium powder particles are irregular in shape and the copper shot particles are near spherical with angular surface features.

Three types of particles were used in the experiment performed in the vacuum chamber, 0.2 mm and 0.5 mm diameter alumina spheres, and 0.2 mm diameter stainless steel spheres. The alumina spheres were purchased from Union Process [33]. The stainless steel spheres were purchased from Next Advance [34]. The particles were chosen because the resulting change in effective thermal conductivity of the particle bed was within the measurable range of the Hot Disk TPS 500 measurement system used in this study. Figure 6-3, Figure 6-4 and Figure 6-5 show photographs of all three spheres. All three particle types appear to be fairly uniformly distributed and spherical.
Figure 6-1. Photograph of powdered titanium (shown with 0.127 mm diameter wire).

Figure 6-2. Photograph of spherical copper shot (shown with 0.254 mm diameter wire).
Figure 6-3. Photograph of 0.2 mm diameter alumina spheres (shown with 0.254 mm wire).

Figure 6-4. Photograph of 0.5 mm diameter alumina spheres (shown with 0.254 mm wire).
Figure 6-5. Photograph of 0.2 mm diameter stainless steel spheres (shown with 0.254 mm wire).

6.2. Experiment in Air

The objective of the experiment performed in air is to show that effective thermal conductivity is an anisotropic property of granular beds.

6.2.1. Effective Thermal Conductivity Results

Measurements were begun at atmospheric conditions in air. Subsequent measurements were made at 2 kPa intervals of applied vertical compressive pressure. Each data point represents the average of six measurements in Figure 6-6 and Figure 6-7. The error bars represent a 95% confidence interval on the measurements. The lines fit through the data represent the fitted results from the theoretical model using the mechanical and thermal properties from Table 6-1.
Table 6-1. Properties Used to Fit the Theoretical Model to the Experimental Results Obtained in Air

<table>
<thead>
<tr>
<th>Name</th>
<th>Property</th>
<th>Powdered Titanium Value</th>
<th>Copper Spherical Shot Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle Diameter</td>
<td>D_{particle}</td>
<td>45-150</td>
<td>595-841</td>
<td>μm</td>
</tr>
<tr>
<td>Average Diameter</td>
<td>D_{average}</td>
<td>97.5</td>
<td>715</td>
<td>μm</td>
</tr>
<tr>
<td>Bulk Density</td>
<td>\rho_{bulk}</td>
<td>2000</td>
<td>5500</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Young's Modulus</td>
<td>E</td>
<td>114</td>
<td>121</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>\nu</td>
<td>0.34</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>Sphere Thermal Conductivity</td>
<td>k_s</td>
<td>21.9</td>
<td>401</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Fluid Thermal Conductivity</td>
<td>k_f</td>
<td>0.025</td>
<td>0.025</td>
<td>W/m/K</td>
</tr>
</tbody>
</table>

The contact force used in the model was calculated using a method similar to the method of Tehranian and Abdou [15]. The number of unit cells, N, in the cross section of the container is given by the ratio of the container cross-sectional area, A_c, to the cylindrical control volume cross-sectional area, A_v,

\[
N = \frac{A_c}{A_v}
\]  

where A_v is given by

\[
A_v = \pi R^2
\]

The contact force, F_c, used in Equation (4.5) is found by

\[
F_c = \frac{P \cdot A_c}{N}
\]

where P is the compressive pressure applied.

The experimental results for titanium powder are presented in Figure 6-6. Although any of the models found in the literature may be plotted with the experimental data, only the model derived is shown on the plot to emphasize the anisotropic effective thermal conductivity. The dashed line in Figure 6-6 represents the model fit by linear regression for the effective vertical
thermal conductivity in powdered titanium. The fit accounted for a 31.8 mm layer of material above the measurement location. The solid line in Figure 6-6 represents the model fit by linear regression for the effective horizontal thermal conductivity. This fit accounted for a 15 mm layer of material above the location of measurement. The layer of material above the measurement location for the effective vertical and horizontal thermal conductivities creates a slight load that is not included in the applied pressure and causes the effective vertical thermal conductivity to be slightly higher than the effective horizontal thermal conductivity at zero applied load.

The model curve fit for effective vertical thermal conductivity (dashed line) shown in Figure 6-6 had a correlation coefficient ($r^2$) of 0.794. The A and B parameters were found to be 25.3 and 0.710, respectively. The model curve fit for effective horizontal thermal conductivity

![Graph showing effective vertical and horizontal thermal conductivity for titanium powder.](image-url)

Figure 6-6. Effective vertical and horizontal thermal conductivity for titanium powder.
(solid line) shown in Figure 6-3 had an $r^2$ value of 0.969. The A and B parameters were 25.3 and 0.677, respectively. The fact that parameter A is much larger than unity indicates that the contact resistance of the sphere-sphere interface is much larger than the perfect contact modeled by the Hertzian contact equation. The parameter A effectively accounts for the contact resistance caused by factors such as surface roughness. For different types of materials or materials with different surface characteristics, parameter A is expected to be different. If the measurement bias is removed by subtracting the initial difference between the measured effective vertical and horizontal thermal conductivities from the effective vertical thermal conductivities, the A, B, and $r^2$ parameters become 40.1, 0.684, and 0.794 for the vertical and 40.1, 0.669, and 0.969 for the horizontal. There is no difference between A, B and $r^2$ parameters if the effective horizontal thermal conductivities are shifted up by the difference between the initial effective vertical and horizontal thermal conductivities.

The experimental results for spherical copper shot are presented in Figure 6-7. Once again, only the derived model is shown on the plot to emphasize the anisotropic effective thermal conductivity. The dashed line in Figure 6-7 also represents the model fit by linear regression through the effective vertical thermal conductivity data of the spherical copper shot. Like the model fit in Figure 6-6, this model fit also accounted for the 31.8 mm layer of material above the measurement location. The solid line in Figure 6-7 represents the model fit by linear regression through the effective horizontal thermal conductivity data of the spherical copper shot. Like the model fit in Figure 6-6, the fit also accounted for a 15 mm layer above the measurement location. Just as in Figure 6-6, this load due to the weight of the layer of particles is not included in the applied pressure and causes the effective vertical thermal conductivity to be slightly higher than the effective horizontal thermal conductivity at zero applied pressure.
Figure 6-7. Effective vertical and horizontal thermal conductivity for spherical copper shot.

The model curve fit for effective vertical thermal conductivity (dashed line) in Figure 6-7 had an $r^2$ value of 0.995. The A and B parameters were determined to be 32.8 and 0.888, respectively. The model curve fit for effective horizontal thermal conductivity (solid line) shown in Figure 6-7 had an $r^2$ value of 0.993. The A and B parameters were determined to be 32.8 and 0.699, respectively. If the measurement bias is removed by subtracting the initial difference between the measured effective vertical and horizontal thermal conductivities from the effective vertical thermal conductivities, the A, B, and $r^2$ parameters become 51.9, 0.486, and 0.995 for the vertical and 51.9, 0.724, and 0.993 for the horizontal. The A and $r^2$ parameters are not affected if the measurement bias is removed by adding the initial difference between the two measured conductivities to the effective horizontal thermal conductivities. The B parameter becomes 0.920...
for the effective vertical thermal conductivity and 1.15 for the effective horizontal thermal conductivity.

Figure 6-7 shows that statistically significant results were obtained for the near spherical copper shot, however it is not readily apparent from Figure 6-6 if statistically significant results were obtained in titanium powder. One tailed t-tests were performed for each applied pressure increment to verify that statistically significant results were achieved, meaning that the values for the effective horizontal and vertical thermal conductivities are not equivalent. The t-test performed on the data at applied pressure of 0 kPa showed no statistical significance, however the t-tests performed on the data from the rest of the applied pressure increments showed statistical significance with a significance level of 0.05. The t-tests performed for the data contained in Figure 6-6 and the data shown in Figure 6-7 show that the effective vertical and horizontal conductivities do not have the same value. The same trend is shown for both the different materials and particle sizes. Effective horizontal and vertical conductivity increase with increasing applied pressure. The rate of increase in effective thermal conductivity is different for each. This is likely due to the differing stress distributions in the horizontal and vertical direction within the granular bed because of the $K_o$ values.

6.2.2. $K_o$ Determination

$K_o$ values could be measured using available experimental methods. The simple model for effective thermal conductivity developed in this study, although approximate in nature, can also be used to estimate $K_o$ values from experimental data. $K_o$ values were calculated from the model fit first to effective horizontal thermal conductivity data and then extrapolated to effective vertical thermal conductivity data. The slopes of the effective vertical thermal conductivity curve extrapolated from the effective horizontal thermal conductivity data were compared to the slopes
of the model fit to the effective vertical thermal conductivity. The \( K_o \) value was adjusted until the sum of the square of the differences of the slopes was minimized.

The \( K_o \) value for the titanium powder and the spherical copper shot were calculated to be 0.736 and 0.221, respectively. The difference in \( K_o \) values between the different particles is due mostly to differences in particle size and shape. For example, if the particles were all cubes and perfectly aligned, the \( K_o \) value should be close to 0. The spherical copper shot is cube-like and therefore has a smaller \( K_o \) value than the more angular titanium powder. For comparison, \( K_o \) for silt is typically between 0.2 and 0.3, sand is about 0.4, and clay varies between 0.3 and 0.6 [35].

6.2.3. Path Contributions

The contributions of the parallel conduction paths to the effective thermal conductivities in titanium powder calculated by the model curve fits (including A and B parameters) in Figure 6-6 are shown in Figure 6-8. The contributions of the parallel conduction paths to the effective thermal conductivities in spherical copper shot are shown in Figure 6-9.

Figure 6-8 shows similar trends to Figure 4-4. The greatest contribution to the effective thermal conductivity comes through the sphere-fluid-sphere conduction path. The contribution of the sphere-contact-sphere conduction path increases and the contribution of the sphere-fluid-sphere conduction path decreases with increasing applied vertical compressive pressure as predicted in Figure 4-4. The relative contribution from the sphere-fluid-sphere conduction path is lower in titanium powder than the contribution predicted in aluminum. The contribution to the effective vertical and horizontal thermal conductivities by the sphere-contact-sphere conduction path calculated by the model curve fit in titanium powder shown in Figure 6-8 is lower than that shown in Figure 4-4 because titanium has a much lower thermal conductivity than aluminum. As was shown in Figure 4-4, the weight of the particles above the measurement location was not
included in the applied pressure for Figure 6-8. This leads to a slight contribution due to the sphere-contact-sphere path at zero applied pressure.

Figure 6-9 shows comparable trends to Figure 6-8 and Figure 4-4. The relative contribution to the effective thermal conductivities from the sphere-fluid-sphere conduction path in spherical copper shot is lower than that in titanium powder or aluminum. The contribution to the effective thermal conductivities from the sphere-contact-sphere conduction path in copper spherical shot is much greater than that seen in titanium powder or aluminum. This greater contribution from the sphere-contact-sphere conduction path in the spherical copper shot is mostly a result of the thermal conductivity of the spherical copper shot particles being much greater than the thermal conductivity of the titanium powder and aluminum particles. The weight of the particles above the measurement location was not included in the applied pressure for Figure 6-9. This leads to the contribution due to the sphere-contact-sphere path at zero applied pressure also seen in Figure 4-4 and Figure 6-8.

![Graph showing percent contribution to effective thermal conductivity of sphere-fluid-sphere and sphere-contact-sphere conduction paths in titanium powder.](image-url)

Figure 6-8. Percent contribution to effective thermal conductivity of the sphere-fluid-sphere and sphere-contact-sphere conduction paths in titanium powder.
The enhancement of the A parameter in the model fit and the difference in the contribution to effective thermal conductivity of the sphere-fluid-sphere and sphere-contact-sphere (Figure 6-8 in titanium and Figure 6-9 in copper) conduction paths from those predicted in Figure 4-4 are probably due to the random packing arrangement and shape irregularities of poly-sized titanium and copper particles shown in Figure 6-1 and Figure 6-2. The particles in each experiment contact multiple particles and heat is transferred three-dimensionally through these multiple contact areas resulting in an enhancement of the A parameter and an increase in contribution to the effective thermal conductivity through the multiple sphere-contact-sphere conduction paths. The model developed for this research was a quasi-one-dimensional resistance network heat transfer model based on a simple cubic packing arrangement of mono-sized spheres and as such compensates for the multiple contact areas by increasing the A coefficient in Equation (4.19). With the large difference in particle size, shape, and thermal conductivity of the
two materials used in experiments, the fact that coefficients A and B are in a relatively close range for both types of particles indicates that the current model is applicable for a wide range of particles.

6.2.4. Deviations from Expected Results

The separation of the effective vertical and horizontal thermal conductivities at the lowest pressure was measured to be greater than expected by the model curve fit to the data. Figure 6-10 shows the model fit prediction of the effective vertical thermal conductivity extrapolated from the effective horizontal thermal conductivity fit with the data fits for effective horizontal and vertical thermal conductivity for titanium powder. Figure 6-11 compares the model fit prediction of the effective vertical thermal conductivity extrapolated from the effective horizontal thermal conductivity fit for spherical copper shot. Just as in Figure 6-6 and Figure 6-7, the effective vertical thermal conductivity should be slightly greater than the effective horizontal thermal conductivity because the weight of the layer above the measurement location was not included in the applied pressure.

The difference between the model fit to the effective vertical thermal conductivity and the extrapolated effective vertical thermal conductivity from the effective horizontal thermal conductivity for the titanium powder is a constant value of 0.0096 W/m/K over the applied pressure range 0 to 20 kPa. This discrepancy is most likely caused by the bias measurement uncertainty of the two probes.

The difference between the model fit to the effective vertical thermal conductivity and the extrapolated effective vertical thermal conductivity from the effective horizontal thermal conductivity for the spherical copper shot is a constant value of 0.0848 W/m/K over the applied pressure range from 0 to 20 kPa. Again, the bias uncertainty of the probes may be the reason for the difference. The anisotropy of effective thermal conductivity is clearly shown by the differing
Figure 6-10. Effective thermal conductivity comparison of vertical, horizontal, and vertical extrapolated from horizontal model fits for titanium powder.

Figure 6-11. Effective thermal conductivity comparison of vertical, horizontal, and vertical extrapolated from horizontal model fits for spherical copper shot.
rates of increase in effective thermal conductivity with increasing compressive pressure regardless of the constant bias uncertainty in the measurement.

The fitting of the current model to the experimental data appears to validate the utility of the model for applications in different granular beds. The model under-predicted the contribution of the sphere-contact-sphere conduction path and overpredicted the contribution of the sphere-fluid-sphere conduction path for random packings of poly-sized particles. When calibrated with empirical data for coefficients A and B, the model accurately predicted the trend of effective vertical thermal conductivity from the effective horizontal thermal conductivity over the range of vertical applied pressure from 0 to 20 kPa for both types of particles.

6.2.5. **Lunar Anisotropic Effective Thermal Conductivity Prediction**

The model developed in this study can give an estimation of the anisotropic behavior of effective thermal conductivity with increasing depth in the lunar surface. Figure 6-12 shows such a prediction using particles of radius 45.8 μm and properties of 10.9 GPa for Young’s modulus [36], and 11.7 W/m/K for thermal conductivity (combined contribution of compounds in lunar soil from [37]). Poisson’s ratio was assumed to be 0.5, which is typical for dry lunar soils. The lunar surface has an angle of repose reported to be 35°. The equation for K_o given by Michalowski [6] gives a K_o of 0.426 for the lunar surface. The particle radius is similar to that of the titanium powder, so the A parameter is set to the same value as the model fit to the titanium powder data. The lunar surface is in a vacuum environment, meaning there is no conduction.
through an interstitial fluid. The heat flow from the sphere-fluid-sphere conduction path in the model can be eliminated by setting B to 0.

Figure 6-12 shows the prediction of effective vertical and horizontal thermal conductivity of the regolith on the lunar surface. For comparison, the effective horizontal thermal conductivities measured at the Apollo 15 and 17 sites were 0.015 W/m/K and 0.017 W/m/K [38]. Using a bulk density of 1890 kg/m³ and a lunar gravity constant of 1.62 m/s², these effective horizontal thermal conductivities were measured under vertical applied pressures between 3 and 7 kPa, respectively. The model predicts effective horizontal thermal conductivities of 0.010 W/m/K and 0.018 W/m/K showing a general agreement in the thermal conductivity range with the measurements taken in situ (33% smaller and 5% larger in prediction). The desired effective thermal conductivity value for heat flow calculations is the vertical, which, according to this...
prediction, is much larger than the effective horizontal thermal conductivity. The model predicts values of effective vertical thermal conductivity of 0.0185 and 0.0325 W/m/K for 3 and 7 kPa applied vertical pressure. These values are about 77% larger than the predicted values for effective horizontal thermal conductivity. It should be noted that this calculation is based on a simplified heat transfer model with loose, noncohesive lunar regolith, while the Apollo measurements were conducted in tightly packed, dense lunar regolith. Because the difference in vertical and horizontal thermal conductivity is a function of the ratio of vertical and horizontal stress of soil, it is likely that the $K_v$ of 0.426 is an over-estimation for the real regolith. Figure 6-12 gives an estimation of the maximum range of thermal conductivity anisotropy for lunar soils. It will be prudent to consider regolith property anisotropy in the use of in situ measurement data.

6.3. Experiment in Vacuum

6.3.1. Effective Thermal Conductivity Results

Measurements were performed in the vacuum chamber. The mechanical pump was used first to get low pressure within the chamber. The diffusion pump was then used to achieve a high vacuum environment (diffusion pumps are capable of achieving pressures on the order of $10^{-6}$ torr). Between measurements, air was pumped into and out of the chamber to allow the material to cool by convective heat transfer in order to speed the data collection process.

Measurements were made at 2 kPa intervals of applied vertical compressive pressure. Each data point shown in the figures throughout this section represents the average of ten measurements. A heating power of 0.1 W and measurement time of 80 seconds were used by the Hot Disk software to calculate thermal properties. The error bars in the figures throughout this section represent a 95% confidence interval on the measurements taken. The lines fit through the data represent the fitted results from the modified theoretical model using the properties in Table 6-2.
Table 6-2. Properties Used to Fit the Modified Theoretical Model to Experimental Results Obtained in Vacuum

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>0.2 mm Alumina</th>
<th>0.5 mm Alumina</th>
<th>0.2 mm Stainless Steel</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk Density</td>
<td>$\rho_{\text{bulk}}$</td>
<td>2320</td>
<td>2350</td>
<td>4500</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>$E$</td>
<td>375</td>
<td>375</td>
<td>200</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>$\nu$</td>
<td>0.22</td>
<td>0.22</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Sphere Thermal Conductivity</td>
<td>$k_s$</td>
<td>35</td>
<td>35</td>
<td>16</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Free Surface Energy</td>
<td>$\gamma$</td>
<td>40</td>
<td>40</td>
<td>36</td>
<td>mJ/m$^2$</td>
</tr>
</tbody>
</table>

The experimental results for the 0.2 mm alumina spheres are presented in Figure 6-13. The dashed line in Figure 6-13 represents the modified model fit by linear regression for the effective vertical thermal conductivity in powdered titanium. The fit accounted for a 25 mm layer of material above the effective vertical thermal conductivity measurement location. The solid line in Figure 6-13 represents the model fit by linear regression for the effective horizontal thermal conductivity. This fit accounted for a 50 mm layer of material above the location of effective horizontal thermal conductivity measurement. The layer of material above the measurement location for the effective vertical and horizontal thermal conductivities creates a slight load that is not included in the applied pressure and causes the effective thermal conductivity to be slightly higher than the effective thermal conductivity at zero applied load.

The modified model fit for effective vertical thermal conductivity (dashed line) shown in Figure 6-13 had a correlation coefficient ($r^2$) of 0.975. The $C$ parameter was found to be 452 and the thermal contact resistance, $R_{\text{contact}}^\prime$, was found to be $9.48 \times 10^{-9}$ m$^2$K/W. The model curve fit for effective horizontal thermal conductivity (solid line) shown in Figure 6-13 had an $r^2$ value of
Figure 6-13. Effective vertical and horizontal thermal conductivity for 0.2 mm diameter alumina spheres.

0.961. The C parameter was 442 and the thermal contact resistance, $R_{\text{contact}}$, was $8.97 \times 10^{-9}$ m²K/W. The parameter C effectively accounts for the three-dimensional heat flow through the spheres, which is essentially a small constant value.

The experimental results for 0.5 mm alumina spheres are presented in Figure 6-14. The dashed line in Figure 6-14 also represents the modified model fit by linear regression through the effective vertical thermal conductivity data of the spherical copper shot. Like the model fit in Figure 6-13, this model fit also accounted for the 25 mm layer of material above the effective vertical thermal conductivity measurement location. The solid line in Figure 6-14 represents the model fit by linear regression through the effective horizontal thermal conductivity data. Like the model fit in Figure 6-13, the fit also accounted for a 50 mm layer above the effective horizontal
Figure 6-14. Effective vertical and horizontal thermal conductivity for 0.5 mm diameter alumina spheres.

thermal conductivity measurement location.

The modified model fit for effective vertical thermal conductivity (dashed line) shown in Figure 6-14 had a correlation coefficient \( r^2 \) of 0.977. The C parameter was found to be 371 and the thermal contact resistance, \( R''_{\text{contact}} \), was found to be \( 7.67 \times 10^{-9} \) m\(^2\)K/W. The model curve fit for effective horizontal thermal conductivity (solid line) shown in Figure 6-17 had an \( r^2 \) value of 0.920. The C parameter was 358 and the thermal contact resistance, \( R''_{\text{contact}} \), was \( 7.13 \times 10^{-9} \) m\(^2\)K/W.

The experimental results for 0.2 mm stainless steel spheres are presented in Figure 6-15. The dashed line in Figure 6-15 also represents the modified model fit by linear regression through the effective vertical thermal conductivity data of the spherical copper shot. Like the model fit in
Figure 6-13, this model fit also accounted for the 25 mm layer of material above the effective vertical thermal conductivity measurement location. The solid line in Figure 6-15 represents the model fit by linear regression through the effective horizontal thermal conductivity data. Like the model fit in Figure 6-13, the fit also accounted for a 50 mm layer above the effective horizontal thermal conductivity measurement location.

The modified model fit for effective vertical thermal conductivity (dashed line) shown in Figure 6-15 had a correlation coefficient ($r^2$) of 0.960. The $C$ parameter was found to be 182 and the thermal contact resistance, $R''_{\text{contact}}$, was found to be $1.60 \times 10^{-8}$ m$^2$K/W. The model curve fit for effective horizontal thermal conductivity (solid line) shown in Figure 6-15 had an $r^2$ value

![Figure 6-15. Effective vertical and horizontal thermal conductivity for 0.2 mm diameter stainless steel spheres.](image-url)
of 0.958. The C parameter was 177 and the thermal contact resistance, $R''_{\text{contact}}$, was $1.48 \times 10^8 \text{m}^2\text{K/W}$.

6.3.2. **Modified Model Fit Discussion**

The experimental data obtained in vacuum does not show the large separation at zero applied pressure the experimental data in air showed. This can be attributed to an experimental setup that introduces fewer uncertainties in the measurement of effective vertical and horizontal thermal conductivity. The bias uncertainty in the measurement of effective thermal conductivity is assumed to be the cause of the discrepancy between the measured and predicted effective thermal conductivities as discussed in section 6.2.4. The experiments performed in air relied on the use of needle probes which required a combination of measurements from the two probes to determine the effective thermal conductivities. The experiments performed in vacuum were able to measure the effective vertical and horizontal thermal conductivities independently. The bias uncertainty in the effective vertical thermal conductivity is much less for the vacuum case than the case in air.

The thermal contact resistances determined by the modified model fit to the experimental data are smaller than typical values. Typical values of thermal contact resistances range from $10^{-6}$ to $10^{-4} \text{m}^2\text{K/W}$ [21]. The thermal contact resistances calculated by the modified model are on the order of $10^{-8} \text{m}^2\text{K/W}$. It should be remembered that the modified model is attempting to model three-dimensional heat transfer through a one-dimensional model. The modified model considers only one contact point. In reality, there are multiple contact points through which heat is conducted. Multiple contact points have the effect of lowering the thermal contact resistance when modeled as a single contact point.

A comparison of the calculated thermal contact resistances for the three particles studied in the vacuum chamber are shown in Table 6-3. As is expected, the thermal contact resistances
Table 6-3. Comparison of Thermal Contact Resistance, $R''_{\text{contact}}$, Values for Spheres Measured in Vacuum

<table>
<thead>
<tr>
<th>Sphere Diameter</th>
<th>Material</th>
<th>$R''_{\text{contact}}$ (m²K/W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 mm</td>
<td>Alumina</td>
<td>9.48x10⁻⁹</td>
</tr>
<tr>
<td>0.5 mm</td>
<td>Alumina</td>
<td>7.67x10⁻⁹</td>
</tr>
<tr>
<td>0.2 mm</td>
<td>Stainless Steel</td>
<td>1.48x10⁻⁸</td>
</tr>
</tbody>
</table>

for the same material are very similar. The thermal contact resistance calculated for the 0.2 mm diameter alumina spheres was on the same order of magnitude as the 0.5 mm alumina spheres. The difference between these two values is most likely due to the surface roughness from manufacturing the two sizes of spheres being different. The difference in thermal contact resistance between the 0.2 mm diameter alumina and stainless steel spheres is due primarily to the two different materials.

6.3.3. $K_o$ Determination

The modified model for effective thermal conductivity developed in this study, although approximate in nature, can also be used to estimate $K_o$ values from experimental data. $K_o$ values were calculated from the modified model fit first to effective horizontal thermal conductivity data and then extrapolated to effective vertical thermal conductivity data. The slopes of the effective vertical thermal conductivity curve extrapolated from the effective horizontal thermal conductivity data were compared to the slopes of the model fit to the effective vertical thermal conductivity. The $K_o$ value was adjusted until the sum of the square of the differences of the slopes was minimized.

The $K_o$ value calculated for the 0.2 mm alumina spheres, 0.5 mm alumina spheres, and 0.2 mm stainless steel spheres are shown in Table 6-4. Again, for comparison, $K_o$ for silt is
Table 6-4. Comparison of At-Rest Earth Pressure Coefficient, \( K_o \), Values for Spheres Measured in Vacuum

<table>
<thead>
<tr>
<th>Sphere Diameter</th>
<th>Material</th>
<th>( K_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 mm</td>
<td>Alumina</td>
<td>0.228</td>
</tr>
<tr>
<td>0.5 mm</td>
<td>Alumina</td>
<td>0.085</td>
</tr>
<tr>
<td>0.2 mm</td>
<td>Stainless Steel</td>
<td>0.222</td>
</tr>
</tbody>
</table>

typically between 0.2 and 0.3, sand is about 0.4, and clay varies between 0.3 and 0.6 [35]. When comparing the \( K_o \) values for the two sphere materials of the same size, the values of \( K_o \) for 0.2 mm alumina and stainless steel are very similar. A comparison of the two sphere sizes of the same material shows that the values for \( K_o \) are much different for 0.2 mm and 0.5 mm alumina. Particle size appears to play a larger role than material composition in the value of \( K_o \).

6.3.4. Physical Property Effects

The experimental data obtained in vacuum support the findings of the finite element simulations. A comparison of the effective vertical thermal conductivity curves of the 0.2 mm alumina and stainless steel spheres show the effects of Young’s modulus on effective thermal conductivity. A comparison of the effective vertical thermal conductivity curves of the 0.2 mm and 0.5 mm alumina spheres shows the effects of size on effective thermal conductivity. Although the physical property effects are investigated only on effective vertical thermal conductivity, the same findings apply to the effective horizontal thermal conductivity as the difference between the two effective thermal conductivities is a difference in pressure.

The modified model fits to the effective vertical thermal conductivity of 0.2 mm stainless steel and alumina spheres have been plotted together in Figure 6-16 to show the effect of Young’s modulus on effective thermal conductivity.

Although stainless steel has a lower bulk thermal conductivity than alumina, it has a
Figure 6-16. Comparison of effective vertical thermal conductivities of 0.2 mm diameter stainless steel and alumina spheres.

higher effective thermal conductivity than alumina spheres of the same size. As seen in Equation (4.25), Young’s modulus is inversely proportional to the cube of the contact radius. As Young’s modulus increases, the contact radius decreases. Effective thermal conductivity is inversely proportional to the contact radius squared as seen in Equation (4.27). As the contact radius increases, the effective thermal conductivity increases. As Young’s modulus increases, the contact radius decreases which causes the effective thermal conductivity to decrease. Stainless steel has a Young’s modulus of 200 GPa and alumina has a Young’s modulus of 375 GPa. The lower Young’s modulus in stainless steel yields a higher effective vertical thermal conductivity than alumina because of the larger resulting contact radius. The contact radius as a function of pressure for 0.2 mm diameter stainless steel and alumina spheres is presented in Figure 6-17.
Figure 6-17. Comparison of contact radii for 0.2 mm diameter stainless steel and alumina spheres.

These results agree with the finite element simulations and apply in a similar manner to the effective horizontal thermal conductivity.

The modified model fits to the effective vertical thermal conductivity of 0.2 mm and 0.5 mm diameter alumina spheres have been plotted together in Figure 6-18 to show the effect of particle size on effective thermal conductivity.

Larger spheres have higher effective thermal conductivity than smaller spheres of the same material. As seen in Equation (4.25), sphere radius is proportional to the cube of the contact radius. As sphere radius increases, the contact radius increases. Effective thermal conductivity is inversely proportional to the contact radius squared as seen in Equation (4.30). As the contact radius increases, the effective thermal conductivity increases. As sphere radius increases, the
Contact radius increases which causes the effective thermal conductivity to increase. The higher sphere radius for 0.5 mm diameter alumina yields higher effective vertical thermal conductivity than 0.2 mm diameter alumina because of the larger resulting contact radius. The contact radius as a function of pressure for 0.5 mm and 0.2 mm diameter alumina spheres is presented in Figure 6-19. These results agree with the finite element simulations and apply in a similar manner to the effective horizontal thermal conductivity.

Figure 6-18. Comparison of effective vertical thermal conductivities of 0.2 mm and 0.5 mm diameter alumina spheres.
Figure 6-19. Comparison of contact radii for 0.2 mm and 0.5 mm diameter alumina spheres.
CHAPTER 7

SUMMARY AND CONCLUSIONS

A literature review determined that no work had been done previously on the anisotropic effective thermal conductivity of granular beds. Simple models based on experimental data and models from the literature were derived to predict anisotropic compressive pressure-dependent effective thermal conductivity. Experiments were designed and performed in air and vacuum environments. The data from these experiments show that effective thermal conductivity is an anisotropic property of granular materials for the first time. The main conclusions of this project are as follows:

- Effective thermal conductivity is an anisotropic property of granular beds due to the anisotropic stress distribution resulting from gravity or other applied loads to a granular bed. This was shown by experimental data obtained from experiments performed in air and vacuum environments.

- The derived theoretical models fit experimentally obtained data well and can be used to understand and predict anisotropic effective thermal conductivity when calibrated to experimental data.

- The interstitial fluid makes a large contribution to effective thermal conductivity in the experiment for granular beds immersed in a static fluid. The contribution due to heat transfer by radiation is much smaller.

- Decreasing Young’s modulus and increasing granular particle size increase anisotropic effective thermal conductivity of granular beds. This was shown by experimental data obtained by experiments performed in vacuum and finite element simulation.
• Thermal contact resistance is the most important parameter in determining the effective thermal conductivity of a granular bed in a vacuum environment. This was shown by finite element simulation.

• Further work can be done to gain a greater understanding of the anisotropic effective thermal conductivity of granular beds. Topics of further study could include:
  - Finite element simulations of other packing arrangements (BCC and FCC)
  - Effect of particle shape
  - Effect of atmosphere (composition or pressure)
  - Extending experiments to greater applied loads
CHAPTER 8

CONTINUED WORK

Much work will continue in the area of effective thermal conductivity measurement and finite element simulation. Improvements in the experimental setup and directions of further study are suggested. Much work can yet be done to gain a greater understanding of the compressive pressure-dependent anisotropic effective thermal conductivity of granular beds.

8.1. Experimental Setup Improvements

In this section, two improvements to the overall experimental setup are discussed, improvements in the application of the load and $K_0$ measurement.

8.1.1. Load Application

The current experimental setup for applying the load is not ideal. Care must be taken to ensure that the loading platform is level when the load is applied. If the loading platform is not level when the load is applied, the load is not applied uniformly to the granular bed. The current experimental setup has no means of determining the pressure distribution within the granular bed, measurements are assumed to be taken under uniform compressive applied load, so it is of utmost importance to ensure that the load be applied uniformly to the bed. Once the load has been placed on the level loading platform, the person making effective thermal conductivity measurements must ensure that the loading platform remains level during the measurement time. Depending on the size and shape of the container and material used to fill the container that applies the load, a slight disturbance (an inadvertent bump as another person walks by, pressure being reintroduced to the vacuum chamber, etc.) can cause the loading platform to tip and spill the contents of the container.
The design of a new means of applying the load would be desirable. Criteria for the design would include the ease of implementation, current size of the vacuum chamber, and uniformity of the resulting load. Two new loading systems are currently under consideration, spring loading and clamp loading.

Spring loading would offer the benefit of knowing the applied load. The load is proportional to the spring constant and displacement of the spring. The applied load could then be controlled by controlling the spring displacement via a screw-like device. Springs are available in many shapes and sizes and would pose little problem fitting inside of the current vacuum chamber. The uniformity of the applied load would be controlled by assuring that the load is applied normal to the surface of the granular bed. This can be achieved by constructing a setup that locks the container with the granular bed and measurement sensors into place during measurement and allows the granular bed container to be removed after measurement to switch out materials.

Clamp loading would not offer the same benefit of knowing the applied load without measurement. The load could be measured by purchasing load cells or other force measurement transducers. Clamps come in various shapes and sizes and one could be selected that would fit inside of the current vacuum chamber. A setup could be constructed to ensure that the compressive load is applied uniformly in the desired direction.

8.1.2. \( K_o \) Measurement

Many methods exist for the measurement of \( K_o \). Soil stress transducers, as mentioned by Harris [38] and Pytka [39] could be used to determine \( K_o \) from the measured stress distribution in the granular bed. Jaky [40] showed that \( K_o \) can be determined from the angle of repose of granular media. The angle of repose of granular materials is the angle that will naturally form when poured on a flat surface and form a conical pile. The angle of repose can be determined in
many ways. For example, Zhou et al. [41] removed the sides from a uniform height of granular material to find the angle of repose. Carrigy [42] designed a “rotatable-drum apparatus” to measure the angle of repose.

The simplest and most cost effective method to measure $K_o$ would likely be to acquire two load cells or other force measurement transducers. One force measurement would need to be made in both the vertical and horizontal directions. This could be achieved by attaching a transducer to the bottom of the container filled with granular material or the plate used to apply the load. The other transducer could be attached to the inner side of the container filled with granular material. $K_o$ would then be computed by taking the ratio of the measured horizontal and vertical stresses within the granular bed.

8.2. Areas of Further Study

Four areas of further study are mentioned in this section, finite element simulation, the effect of particle shape, atmosphere effects, and greater loads than studied here.

8.2.1. Finite Element Simulation

Two-dimensional axisymmetric finite element simulations have been used in this study of effective thermal conductivity of granular beds because of their simple construction and quick solution times. This axisymmetric condition, however, allows only the simulation of SC granular particle packing arrangement. Simulations of BCC and FCC packing arrangements would require three-dimensional models. Three-dimensional finite element simulations could give insight into how closely the random packing of granular particles in experiments match SC, BCC, and FCC packing arrangements. The information gained from finite element simulations of BCC and FCC packing arrangements could lead to improved models of effective thermal conductivity by including a consideration of the packing arrangement.
8.2.2. Particle Shape Effects

The experimental setup presented in section 5.3 could be used to investigate the effect of particle shape on effective thermal conductivity. In order to perform this study, similar size particles of the same material should be studied. The results presented in sections 6.1.3 and 6.3.4 show that $K_o$ values are largely dependent on size and may be dependent on shape. Spherical and angular particles could be studied to determine the effects of shape on $K_o$ and anisotropic effective thermal conductivity.

8.2.3. Atmosphere Effects

Two aspects of the effect of atmosphere on effective thermal conductivity can be studied, atmospheric composition and atmospheric pressure. The current vacuum chamber and system would need to be modified or a new chamber designed in order to investigate either of these conditions. Experiments could be performed in a vacuum chamber in an atmosphere of helium, nitrogen, argon, or other obtainable gas. Reducing heat transfer by convection through the granular bed in an atmosphere other than air would require further research on convection in porous media. Effective thermal conductivity measurements made while adjusting the atmospheric pressure of any gas the granular particles are immersed in could give insight into heat transfer through planetary bodies such as Mars.

8.2.4. Greater Applied Loads

All of the contact models discussed in this study approach the Hertz contact equation at greater applied loads. The choice of contact model would then become trivial. The applied pressure range in this study was limited by the experimental design, and as such is much smaller than those found in the literature. The contact radii for each material in this study are much smaller than intuitively expected. Greater loads would yield larger contact radii more in line with what would be expected. The theoretical models derived in this study perform better at higher
loads. If a new loading system as discussed in section 8.1.1. were used in future effective thermal conductivity measurements, a better model fit would be achieved to experimental data.
REFERENCES


