Robust Structured Group Local Sparse Tracker Using Convolutional Neural Network Features

Student Research Symposium 2019

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Submitted to the Journal of Computer Vision and Image Understanding

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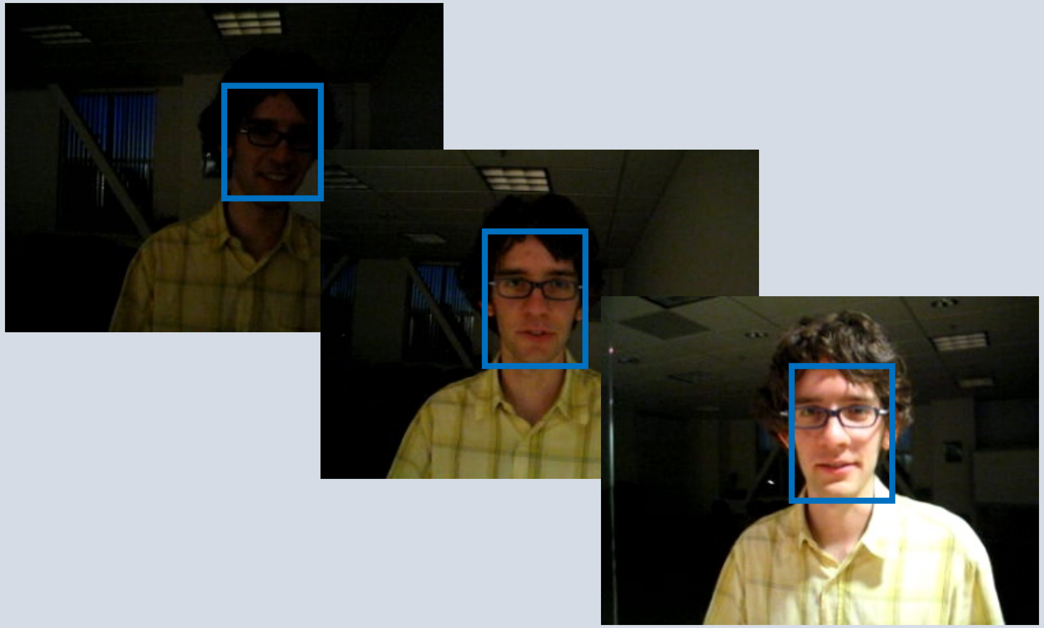
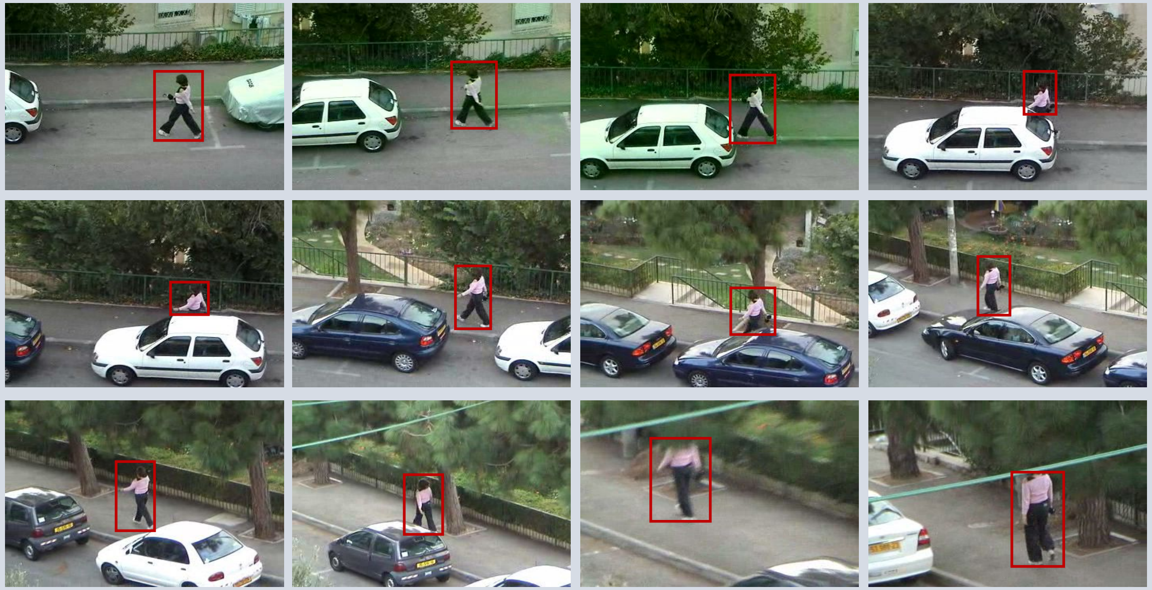
Introduction

- Tracking
  - Following a specific target throughout consecutive frames to determine its relative movement with respect to other objects.



Introduction

- Challenges
  - Deformation
  - Moving Camera
  - Scale Variation
  - Occlusion
  - Illumination Variation
  - Fast Motion
  - Background Clutters
  - …



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Introduction

➢ Challenges
  – Deformation
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Introduction

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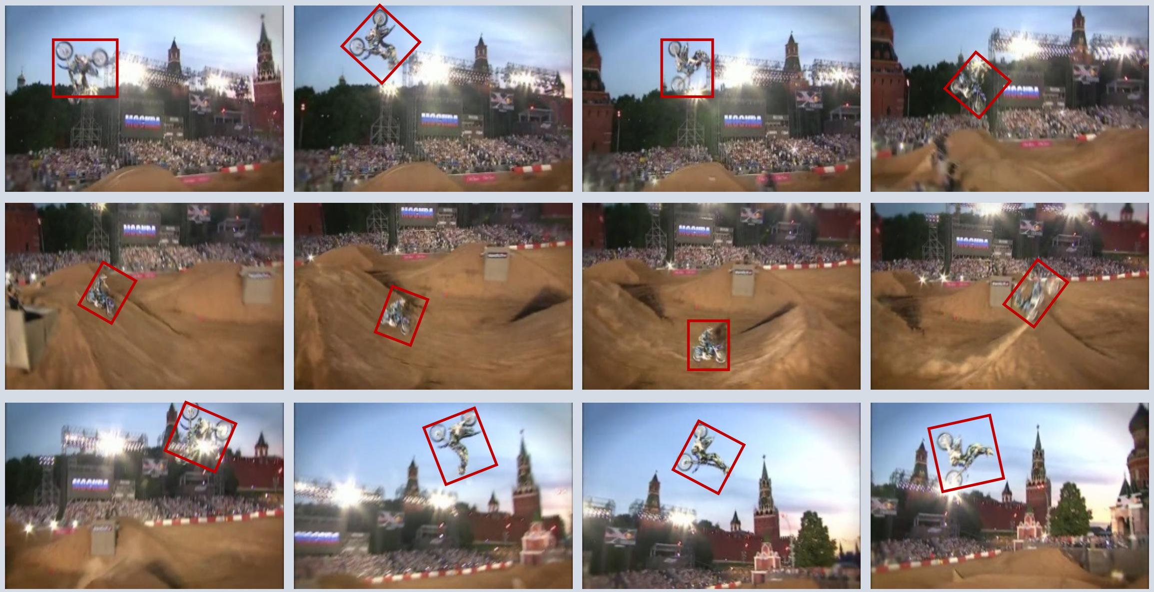

Introduction

- Applications:
  - Surveillance
  - Human motion analysis
  - Transportation
  - Navigation
  - …

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Introduction

- Literature Review
  - Tracking algorithms can be classified into following categories:
    - Discriminative :
      - They formulate a decision boundary to separate the target from the background [1,2]
    - Generative
      - They adopt a model to represent the target and formulate the tracking as a model-based searching procedure to find the most similar region to the target [4,5]
    - Correlation filter
      - They regress all the circular shift of the input features to a target Gaussian function in the Fourier domain [6,7]
    - Convolutional Neural Network
      - They use pre-trained network and/or train a new model for better feature representation [8,9].

Proposed method

- Motivation

  - We propose a robust deep features-based structured group local sparse tracker (DF-SGLST), which exploits the convolutional neural network (CNN) deep features of the local patches inside a target candidate and represent them in a novel convex optimization model.

Proposed method

➢Contributions of the proposed DF-SGLST

- Proposing a deep features-based structured local sparse tracker, which employs CNN deep features of the local patches within a target candidate and attains the spatial structure among the features of local patches inside a target candidate.
- Developing a convex optimization model, which introduces a group-sparsity regularization term to encourage the tracker to sparsely select the corresponding local patches of the same subset of templates to represent the CNN deep features of local patches of each target candidate.
- Designing a fast and parallel numerical algorithm based on the alternating direction method of multiplier (ADMM), which consists of two subproblems with closed-form solutions to efficiently and quickly solve the optimization model.

Proposed method

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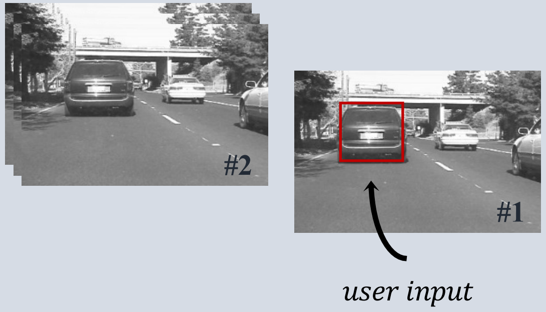

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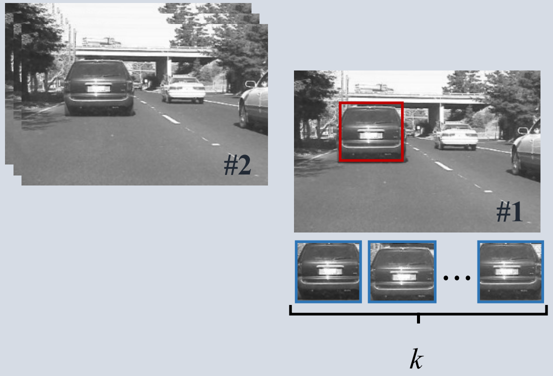

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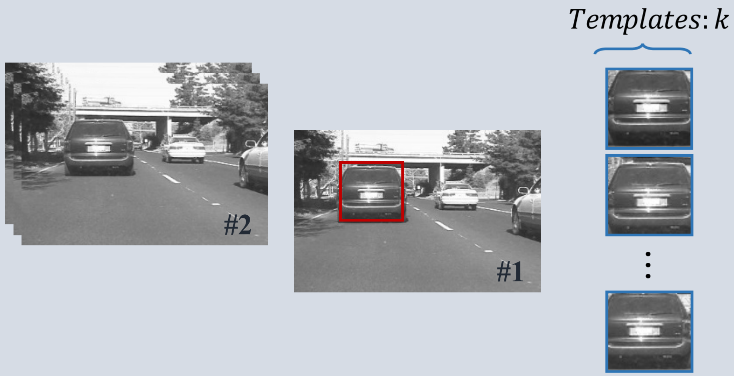

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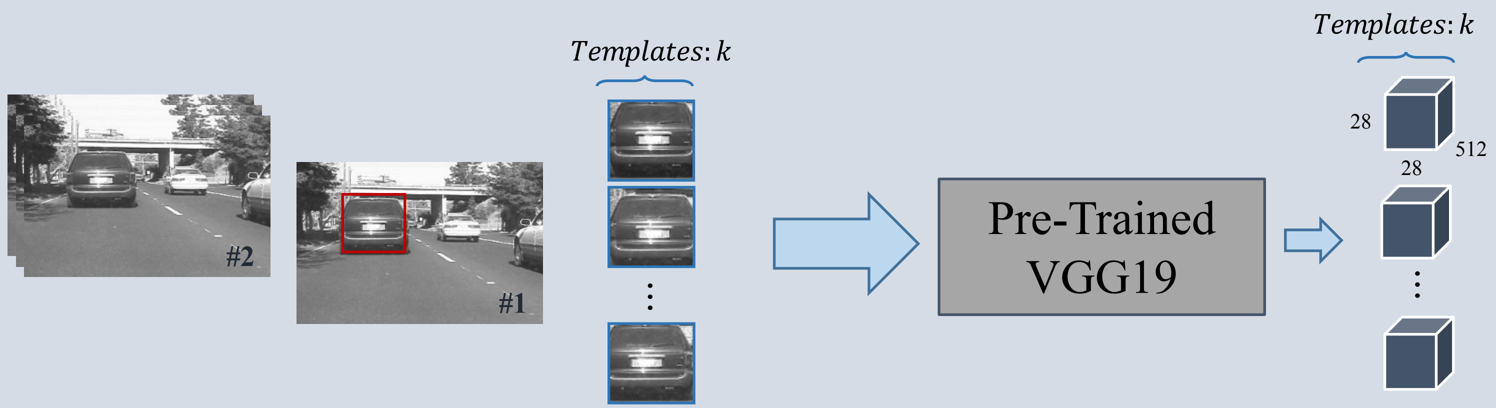

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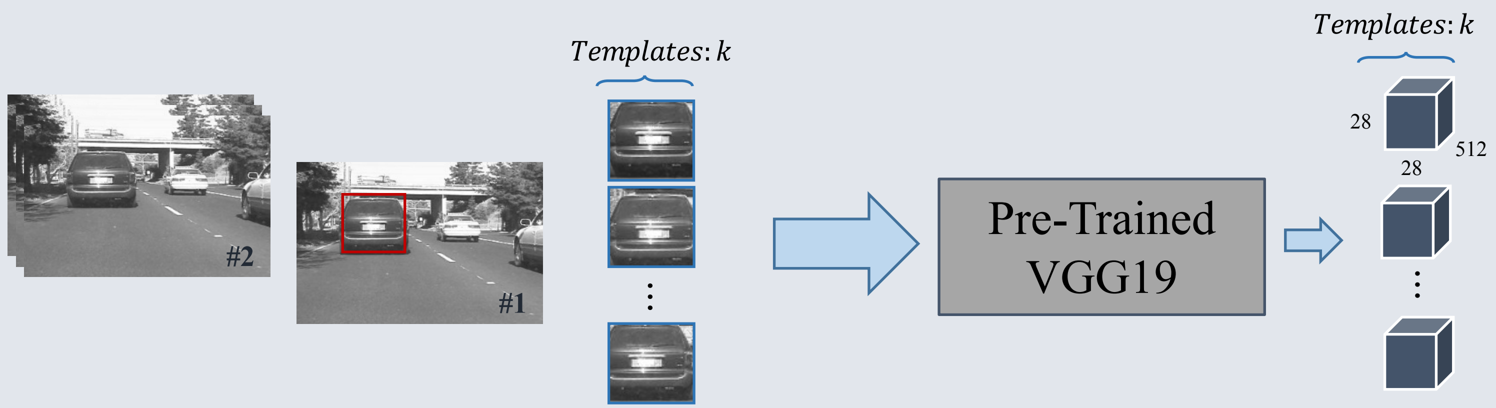

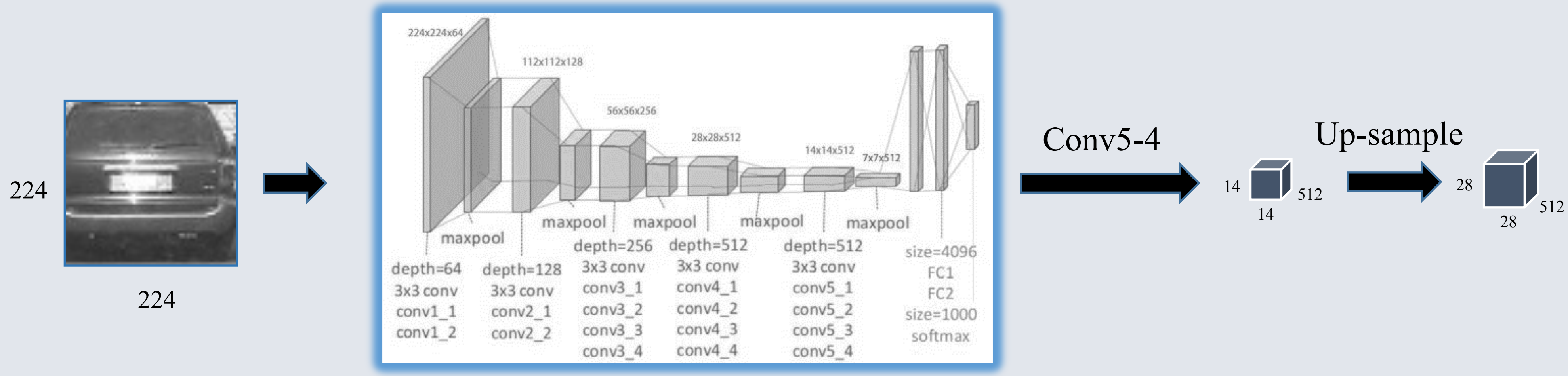

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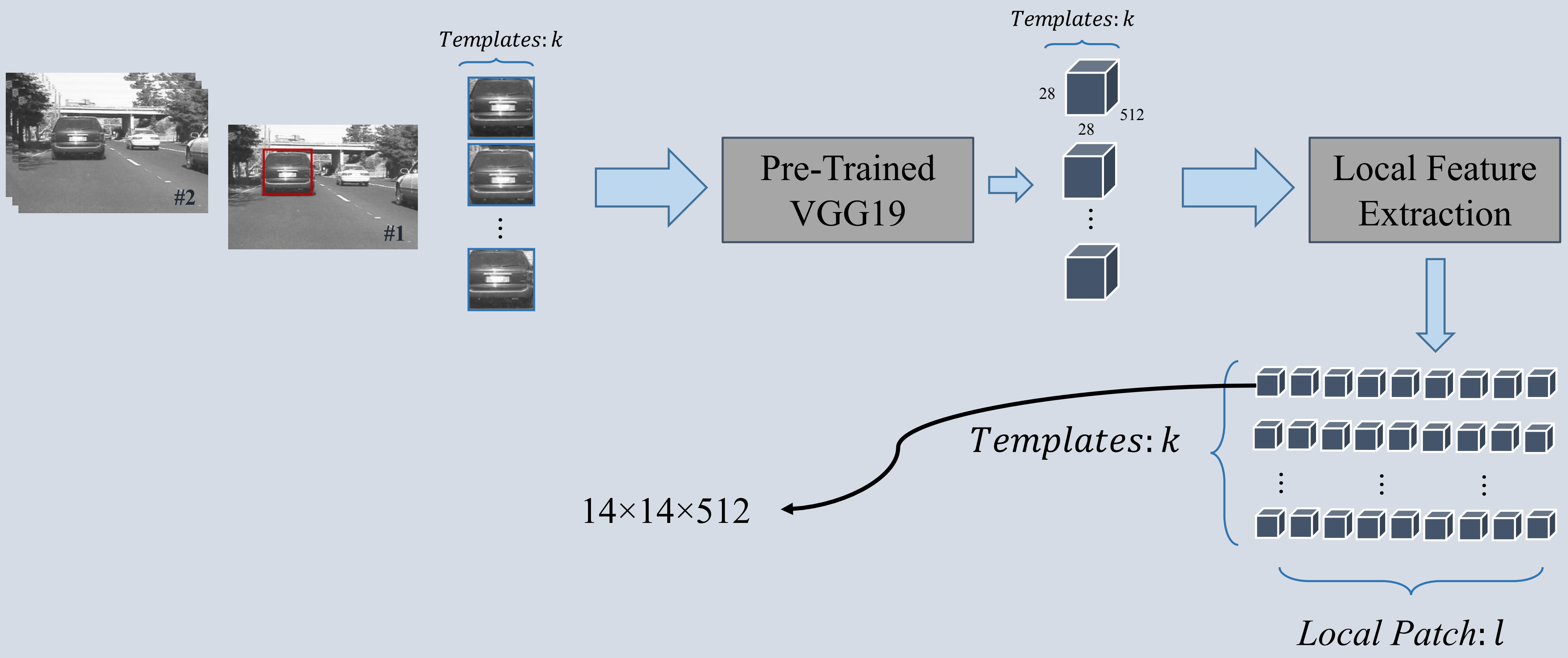

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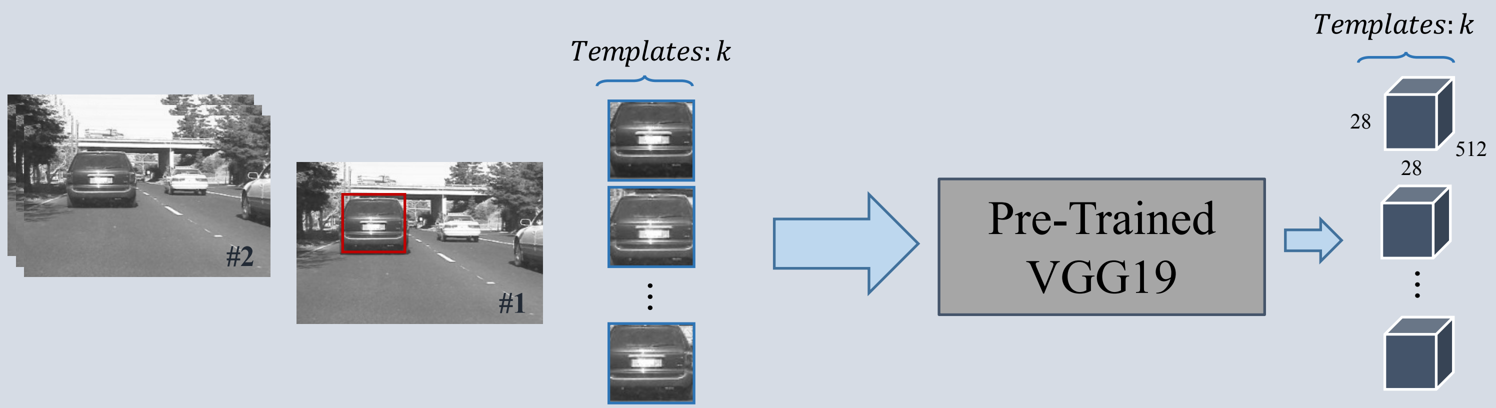

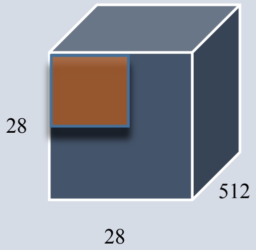

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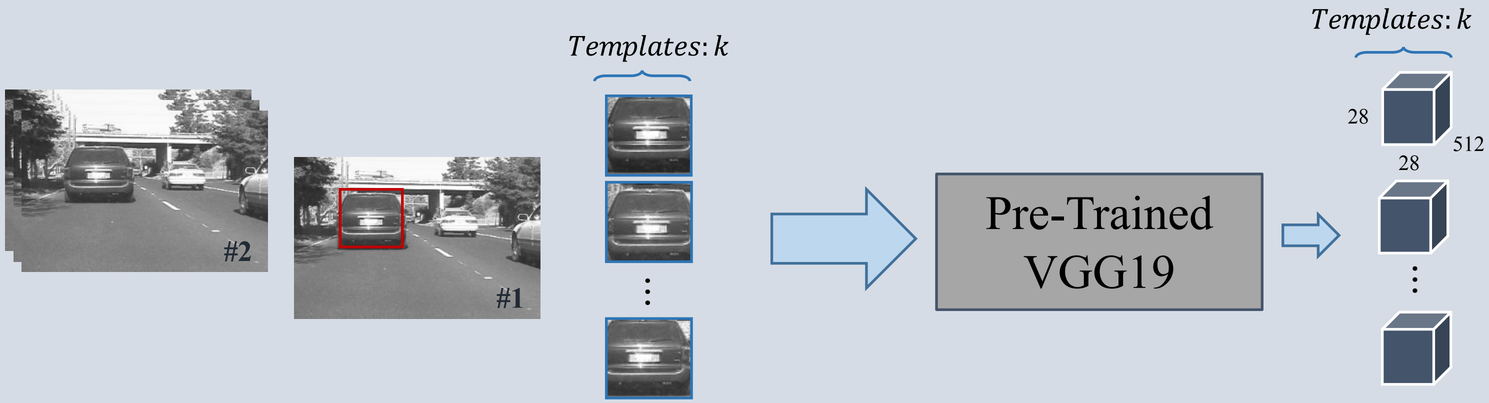

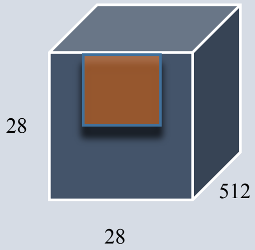

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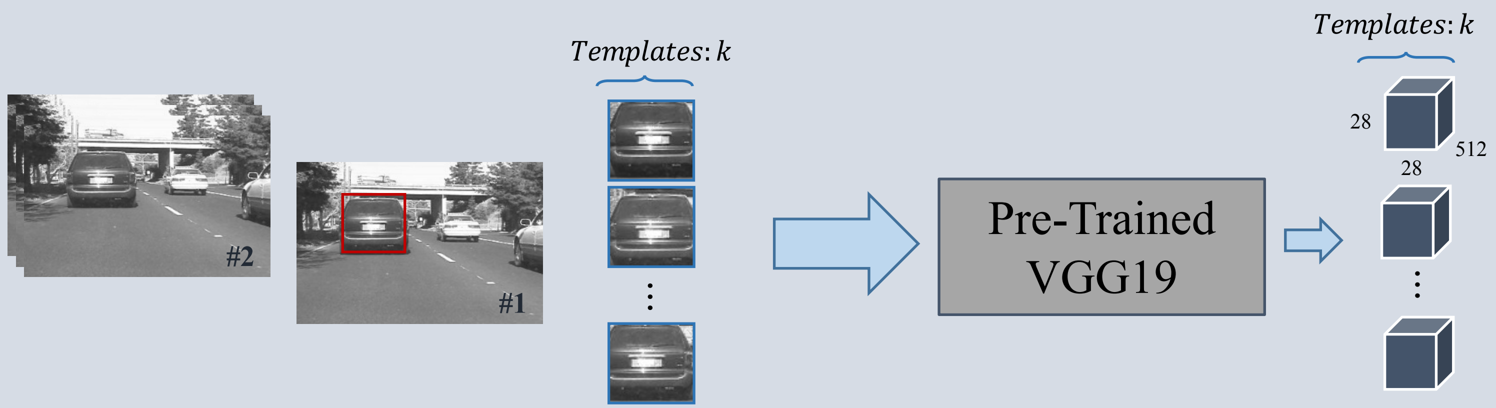

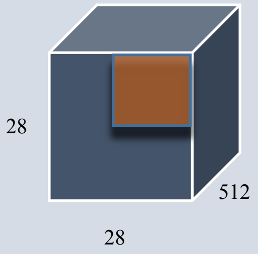

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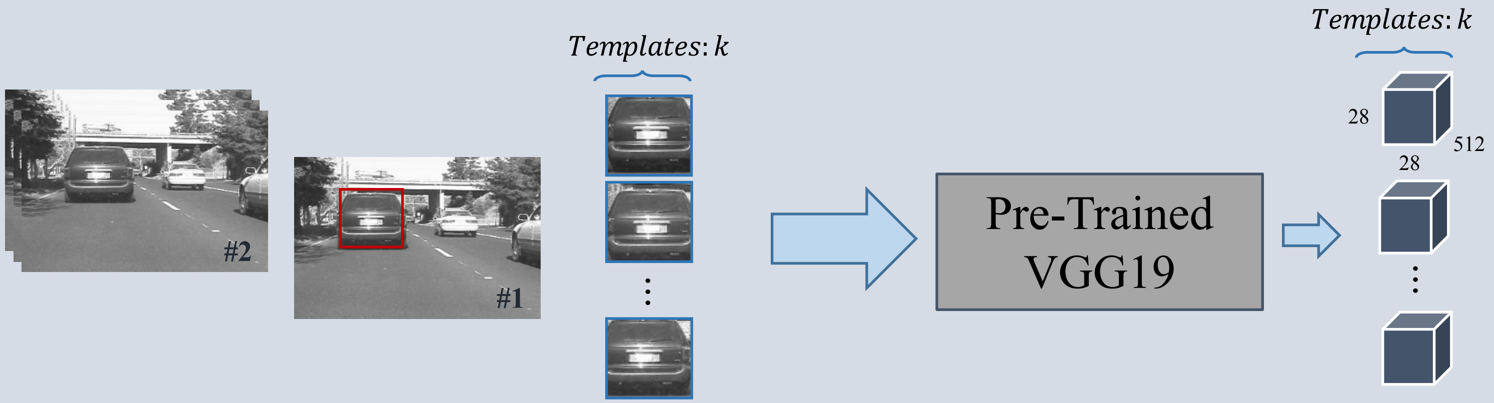

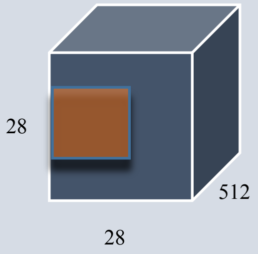

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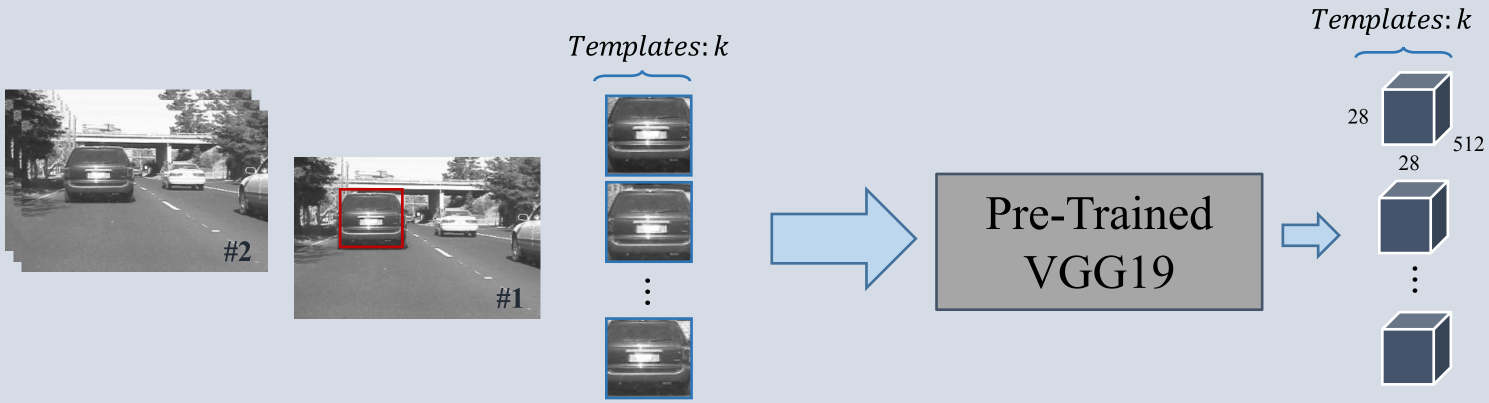

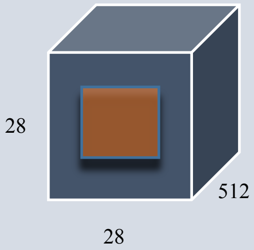

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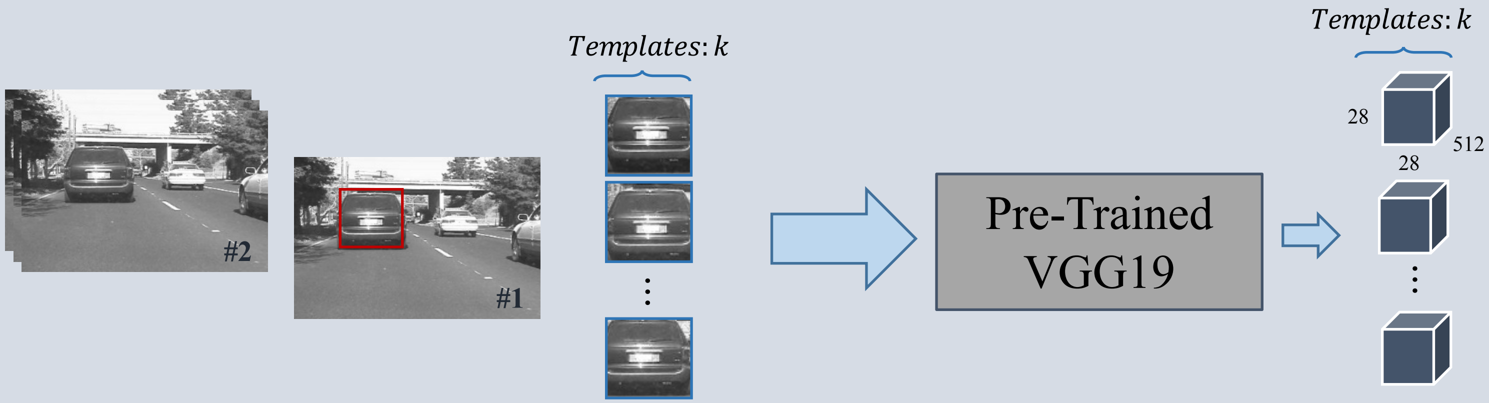

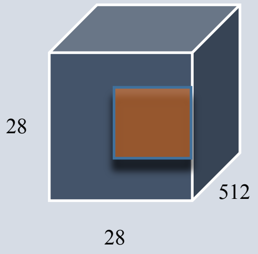

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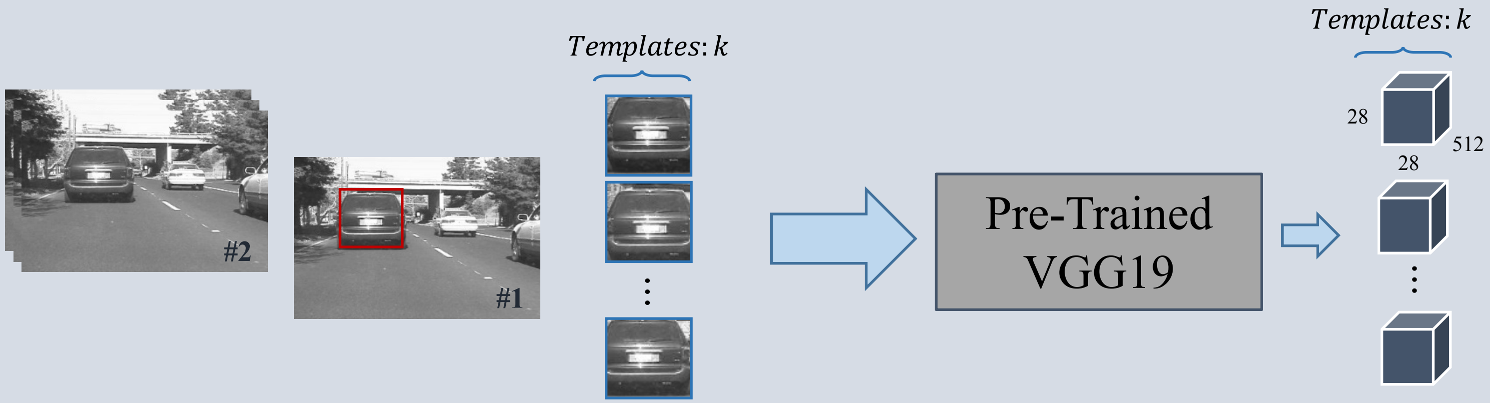

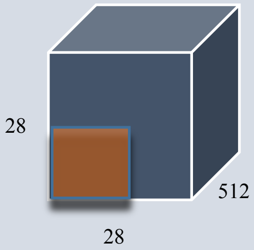

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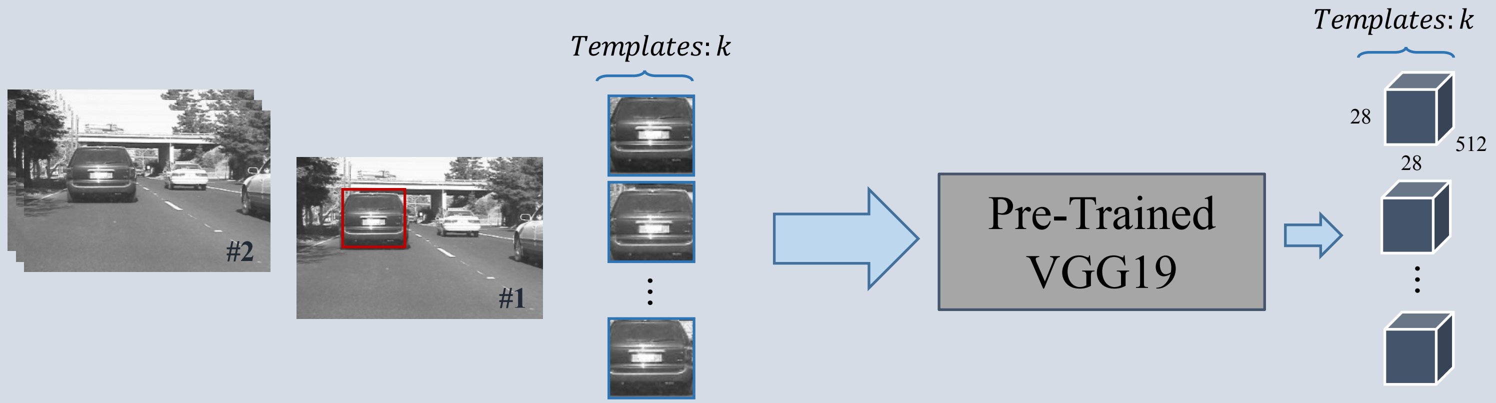

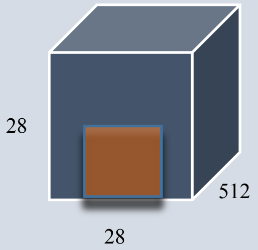

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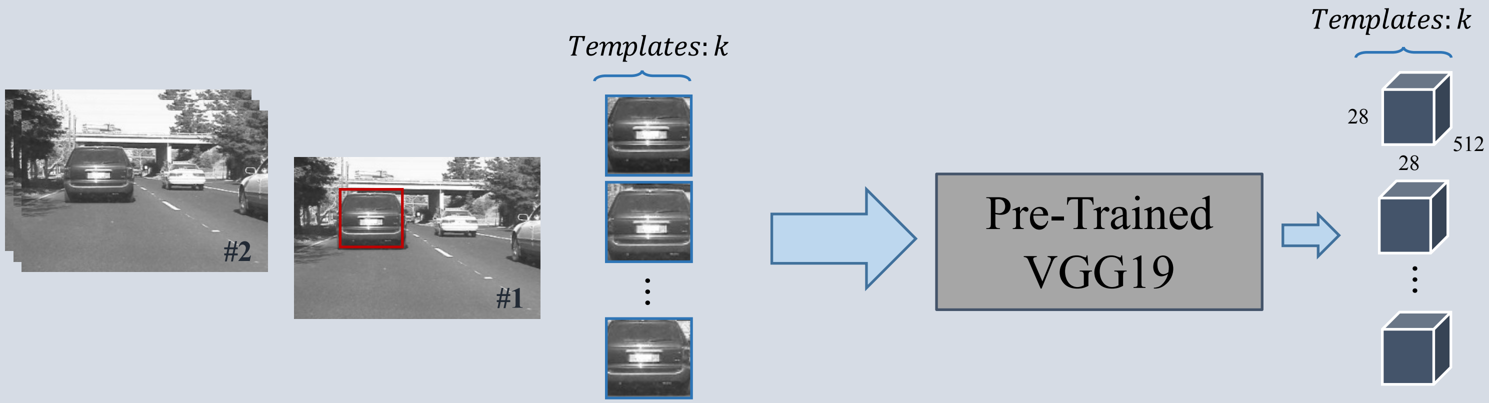

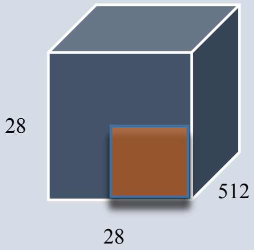

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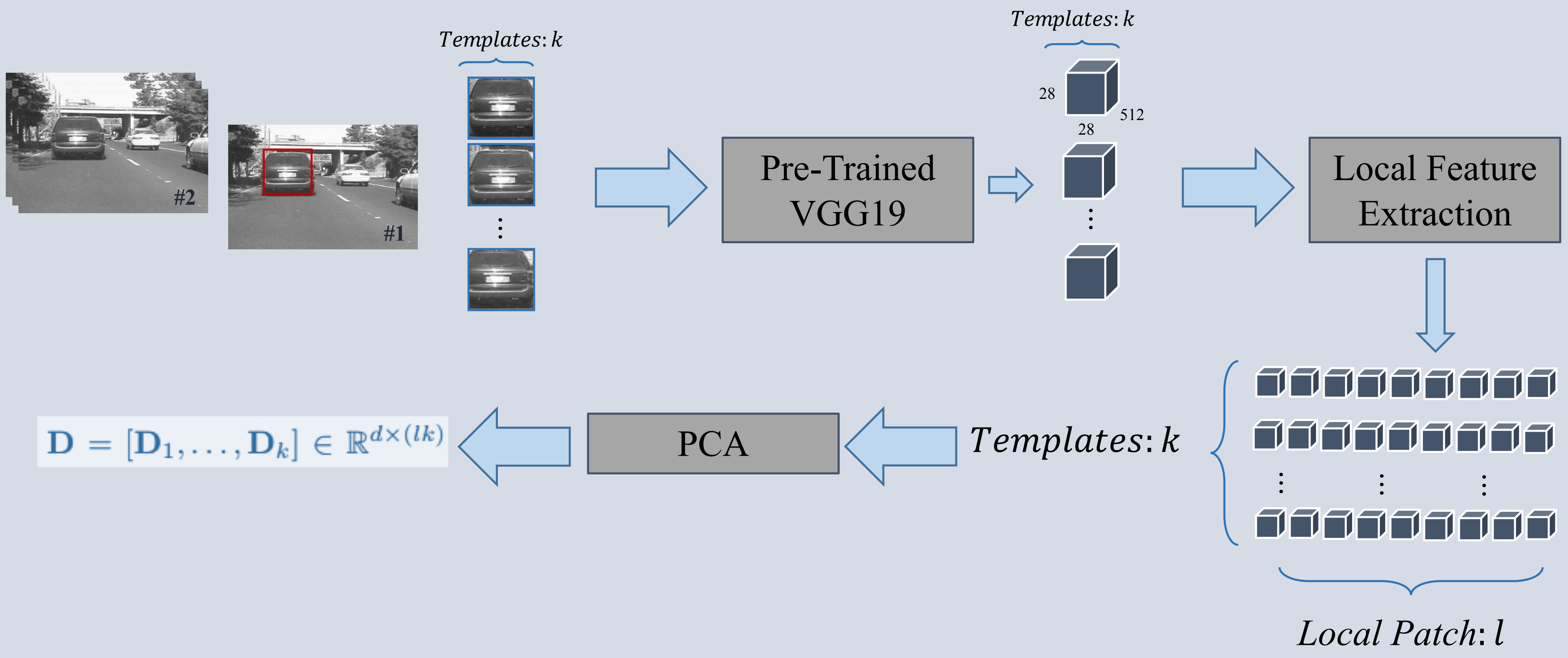

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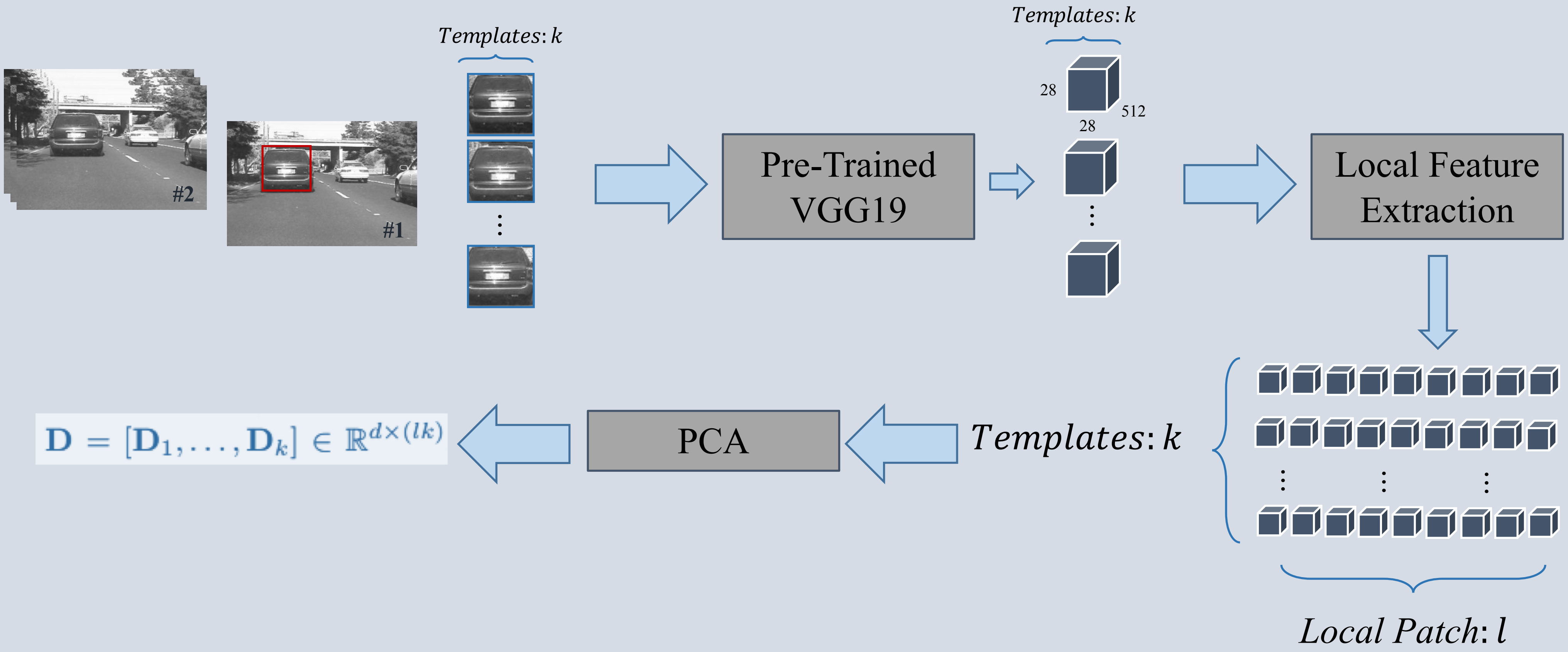

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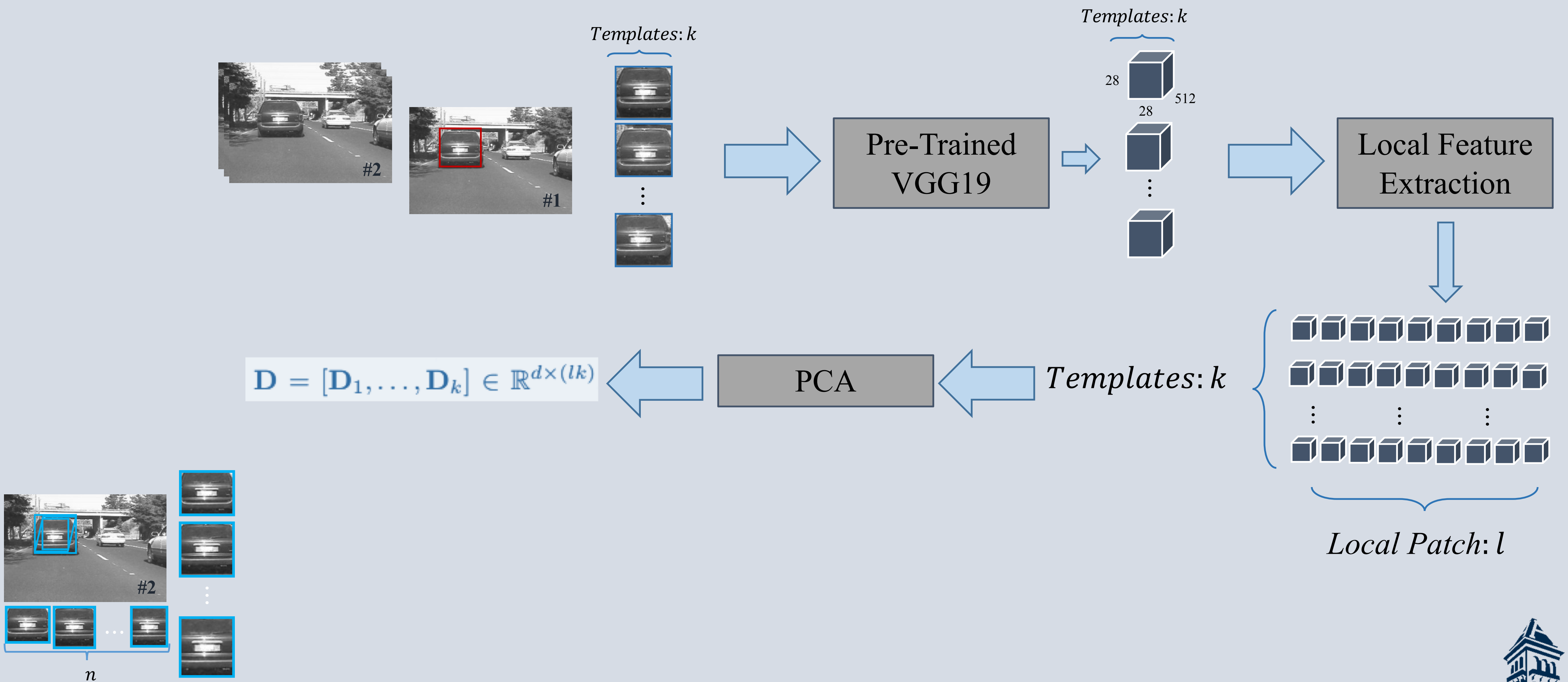

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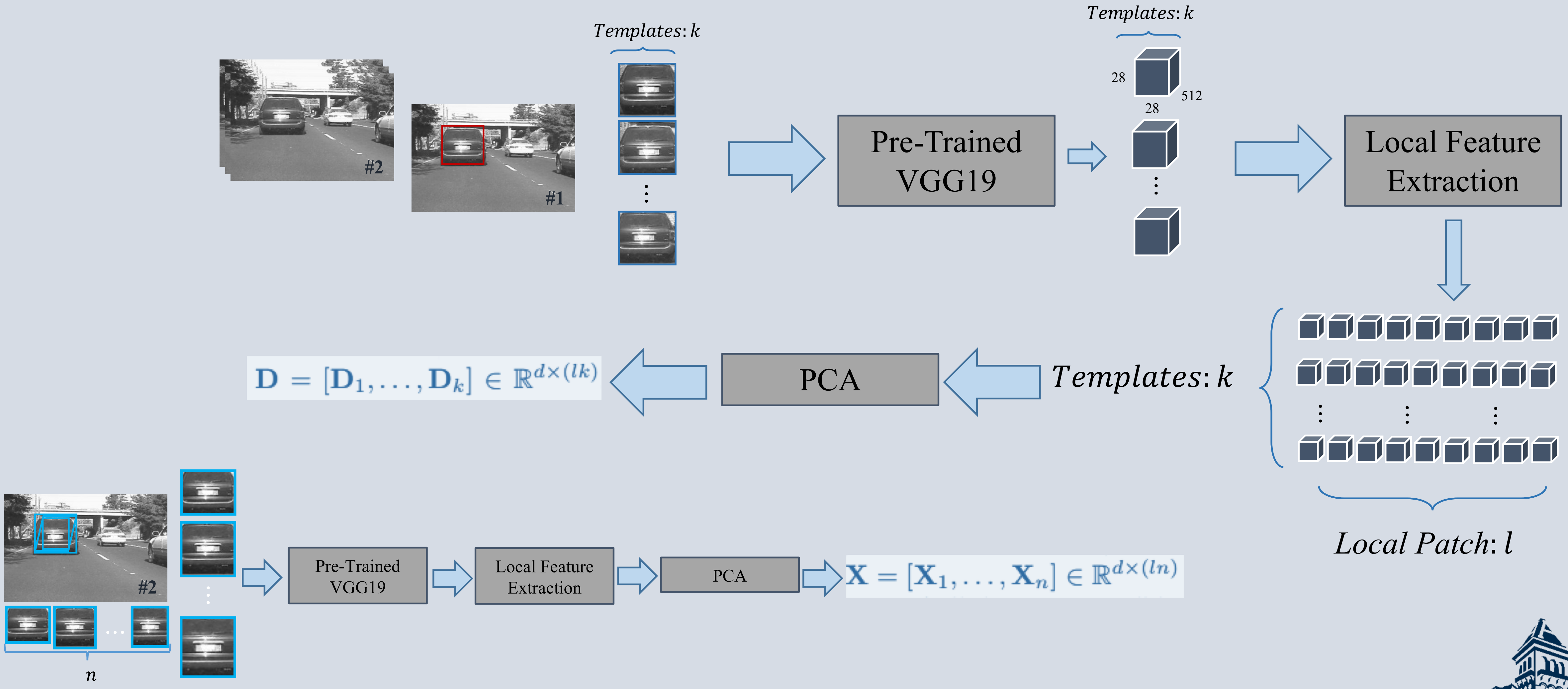

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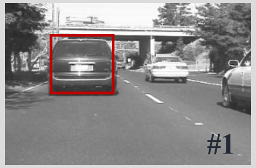
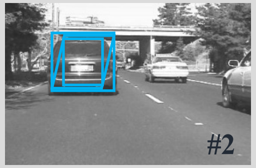


$$\mathbf{D} = [\mathbf{D}_1, \ldots, \mathbf{D}_k] \in \mathbb{R}^{d \times (lk)}$$



$$\mathbf{X} = [\mathbf{X}_1, \ldots, \mathbf{X}_n] \in \mathbb{R}^{d \times (ln)}$$

$$\underset{\mathbf{C} \in \mathbb{R}^{(lk) \times l}}{\text{minimize}} \quad \|\mathbf{X}_j - \mathbf{D}\mathbf{C}\|_{\mathrm{F}}^2 + \lambda \left\| \left[\mathbf{C}_1(:) \ldots \mathbf{C}_k(:)\right]^\top \right\|_{1,\infty} \tag{1a}$$

$$\text{subject to} \quad \mathbf{C} \geq 0, \tag{1b}$$

$$\mathbf{1}_{lk}^\top \mathbf{C} = \mathbf{1}_l^\top, \tag{1c}$$

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Proposed method-Numerical Method

$$\underset{\mathbf{C}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad \|\mathbf{X}_j - \mathbf{DC}\|_F^2 + \lambda \left\| \left[\mathbf{C}_1(:) \ldots \mathbf{C}_k(:)\right]^\top \right\|_{1,\infty} \tag{1a}$$

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$$\mathbf{1}_{lk}^\top\mathbf{C}=\mathbf{1}_l^\top, \tag{1c}$$

$$\underset{\substack{\mathbf{C}\in\mathbb{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbb{R}^k}}{\text{minimize}} \quad \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\mathbf{1}_k^\top\mathbf{m} \tag{2a}$$

$$\text{subject to} \quad \mathbf{C}\geq 0, \tag{2b}$$

$$\mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, \tag{2c}$$

$$\mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top\geq\mathbf{C}. \tag{2d}$$

Proposed method-Numerical Method

$$
\begin{aligned}
\underset{\mathbf{C}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{D}\mathbf{C}\|_{\mathrm{F}}^2+\lambda\left\|\left[\mathbf{C}_1(:)\ldots\mathbf{C}_k(:)\right]^\top\right\|_{1,\infty} && \text{(1a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(1b)}\\
& \mathbf{1}_{lk}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(1c)}
\end{aligned}
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\begin{aligned}
\underset{\substack{\mathbf{C}\in\mathbb{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbb{R}^k}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{D}\mathbf{C}\|_{\mathrm{F}}^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(2a)}\\
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\underset{\substack{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbb{R}^k}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{D}\mathbf{C}\|_{\mathrm{F}}^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(3a)}\\
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& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(3c)}\\
& \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top=\mathbf{C}+\mathbf{U}, && \text{(3d)}\\
& \mathbf{U}\geq 0. && \text{(3e)}
\end{aligned}
$$

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Proposed method-Numerical Method

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\begin{aligned}
&\underset{\mathbf{C}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} && \|\mathbf{X}_j-\mathbf{DC}\|_\mathrm{F}^2+\lambda\left\|\left[\mathbf{C}_1(:)\ldots\mathbf{C}_k(:)\right]^\top\right\|_{1,\infty} && \text{(1a)}\\
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\end{aligned}
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\begin{aligned}
&\underset{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} && \|\mathbf{X}_j-\mathbf{DC}\|_\mathrm{F}^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l && \text{(4a)}\\
&\text{subject to} && \mathbf{C}\geq 0, && \text{(4b)}\\
& && \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(4c)}\\
& && \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(4d)}\\
& && \mathbf{U}\geq 0, && \text{(4e)}
\end{aligned}
$$

Proposed method-Numerical Method

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\begin{aligned}
\underset{\mathbf{C}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_\mathrm{F}^2+\lambda\left\|\left[\mathbf{C}_1(:)\ldots\mathbf{C}_k(:)\right]^\top\right\|_{1,\infty} && \text{(1a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(1b)}\\
& \mathbf{1}_{lk}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(1c)}
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& \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top\geq\mathbf{C}. && \text{(2d)}
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\underset{\substack{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbb{R}^k}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_\mathrm{F}^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(3a)}\\
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& \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top=\mathbf{C}+\mathbf{U}, && \text{(3d)}\\
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$$

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\begin{aligned}
\underset{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_\mathrm{F}^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l && \text{(4a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(4b)}\\
& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(4c)}\\
& \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(4d)}\\
& \mathbf{U}\geq 0, && \text{(4e)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\mathbf{C},\hat{\mathbf{C}},\mathbf{U},\hat{\mathbf{U}}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_\mathrm{F}^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l\\
& +\frac{\mu_1}{2}\left\|\mathbf{C}-\hat{\mathbf{C}}\right\|_\mathrm{F}^2+\frac{\mu_2}{2}\left\|\mathbf{U}-\hat{\mathbf{U}}\right\|_\mathrm{F}^2 && \text{(5a)}\\
\text{subject to} \quad & \hat{\mathbf{C}}\geq 0, && \text{(5b)}\\
& \mathbf{1}_{(lk)}^\top\hat{\mathbf{C}}=\mathbf{1}_l^\top && \text{(5c)}\\
& \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(5d)}\\
& \hat{\mathbf{U}}\geq 0, && \text{(5e)}\\
& \mathbf{C}=\hat{\mathbf{C}},\ \mathbf{U}=\hat{\mathbf{U}}. && \text{(5f)}
\end{aligned}
$$

Proposed method-Numerical Method

$$
\begin{aligned}
\underset{\mathbf{C}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\left\|\left[\mathbf{C}_1(:)\dots\mathbf{C}_k(:)\right]^\top\right\|_{1,\infty} && \text{(1a)}\\
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\begin{aligned}
\underset{\substack{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbb{R}^k}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(3a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(3b)}\\
& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(3c)}\\
& \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top=\mathbf{C}+\mathbf{U}, && \text{(3d)}\\
& \mathbf{U}\geq 0. && \text{(3e)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l && \text{(4a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(4b)}\\
& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(4c)}\\
& \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(4d)}\\
& \mathbf{U}\geq 0, && \text{(4e)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\mathbf{C},\hat{\mathbf{C}},\mathbf{U},\hat{\mathbf{U}}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l\\
& +\frac{\mu_1}{2}\left\|\mathbf{C}-\hat{\mathbf{C}}\right\|_F^2+\frac{\mu_2}{2}\left\|\mathbf{U}-\hat{\mathbf{U}}\right\|_F^2 && \text{(5a)}\\
\text{subject to} \quad & \hat{\mathbf{C}}\geq 0, && \text{(5b)}\\
& \mathbf{1}_{(lk)}^\top\hat{\mathbf{C}}=\mathbf{1}_l^\top && \text{(5c)}\\
& \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(5d)}\\
& \hat{\mathbf{U}}\geq 0, && \text{(5e)}\\
& \mathbf{C}=\hat{\mathbf{C}},\ \ \mathbf{U}=\hat{\mathbf{U}}. && \text{(5f)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_\mu(\mathbf{C},\mathbf{U},\hat{\mathbf{C}},\hat{\mathbf{U}},\mathbf{\Lambda}_1,\mathbf{\Lambda}_2)=&\ \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l\\
&+\frac{\mu}{2}\left\|\mathbf{C}-\hat{\mathbf{C}}+\frac{\mathbf{\Lambda}_1}{\mu}\right\|_F^2+\frac{\mu}{2}\left\|\mathbf{U}-\hat{\mathbf{U}}+\frac{\mathbf{\Lambda}_2}{\mu}\right\|_F^2 && \text{(6)}
\end{aligned}
$$

Proposed method-Numerical Method

$$
\begin{aligned}
\underset{\mathbf{C}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\left\|\left[\mathbf{C}_1(:)\ldots\mathbf{C}_k(:)\right]^\top\right\|_{1,\infty} && \text{(1a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(1b)}\\
& \mathbf{1}_{lk}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(1c)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\substack{\mathbf{C}\in\mathbb{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbb{R}^k}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(2a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(2b)}\\
& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(2c)}\\
& \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top\geq\mathbf{C}. && \text{(2d)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\substack{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbb{R}^k}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(3a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(3b)}\\
& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(3c)}\\
& \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top=\mathbf{C}+\mathbf{U}, && \text{(3d)}\\
& \mathbf{U}\geq 0. && \text{(3e)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l && \text{(4a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(4b)}\\
& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(4c)}\\
& \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(4d)}\\
& \mathbf{U}\geq 0, && \text{(4e)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\mathbf{C},\hat{\mathbf{C}},\mathbf{U},\hat{\mathbf{U}}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l\\
& +\frac{\mu_1}{2}\left\|\mathbf{C}-\hat{\mathbf{C}}\right\|_F^2+\frac{\mu_2}{2}\left\|\mathbf{U}-\hat{\mathbf{U}}\right\|_F^2 && \text{(5a)}\\
\text{subject to} \quad & \hat{\mathbf{C}}\geq 0, && \text{(5b)}\\
& \mathbf{1}_{(lk)}^\top\hat{\mathbf{C}}=\mathbf{1}_l^\top && \text{(5c)}\\
& \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(5d)}\\
& \hat{\mathbf{U}}\geq 0, && \text{(5e)}\\
& \mathbf{C}=\hat{\mathbf{C}},\ \ \mathbf{U}=\hat{\mathbf{U}}. && \text{(5f)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_\mu(\mathbf{C},\mathbf{U},\hat{\mathbf{C}},\hat{\mathbf{U}},\mathbf{\Lambda}_1,\mathbf{\Lambda}_2)=\ & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l\\
& +\frac{\mu}{2}\left\|\mathbf{C}-\hat{\mathbf{C}}+\frac{\mathbf{\Lambda}_1}{\mu}\right\|_F^2+\frac{\mu}{2}\left\|\mathbf{U}-\hat{\mathbf{U}}+\frac{\mathbf{\Lambda}_2}{\mu}\right\|_F^2 && \text{(6)}
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{C}^{t+1},\mathbf{U}^{t+1}) := \ & \underset{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}}{\arg\min}\ \mathcal{L}_\mu(\mathbf{C},\mathbf{U},\hat{\mathbf{C}}^t,\hat{\mathbf{U}}^t,\mathbf{\Lambda}_1^t,\mathbf{\Lambda}_2^t)\\
& \text{subject to (5d)} && \text{(7)}\\
(\hat{\mathbf{C}}^{t+1},\hat{\mathbf{U}}^{t+1}) := \ & \underset{\mathbf{C},\mathbf{U}\in\mathbb{R}^{(lk)\times l}}{\arg\min}\ \mathcal{L}_\mu(\mathbf{C}^{t+1},\mathbf{U}^{t+1},\hat{\mathbf{C}},\hat{\mathbf{U}},\mathbf{\Lambda}_1^t,\mathbf{\Lambda}_2^t)\\
& \text{subject to (5b), (5c), (5e).} && \text{(8)}\\
\mathbf{\Lambda}_1^{t+1} = \ & \mathbf{\Lambda}_1^t+\mu(\mathbf{C}^{t+1}-\hat{\mathbf{C}}^{t+1})\\
\mathbf{\Lambda}_2^{t+1} = \ & \mathbf{\Lambda}_2^t+\mu(\mathbf{U}^{t+1}-\hat{\mathbf{U}}^{t+1}) && \text{(9)}
\end{aligned}
$$

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Proposed method-Numerical Method

$$\begin{aligned} \underset{\mathbf{C}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\left\|[\mathbf{C}_1(:)\dots\mathbf{C}_k(:)]^\top\right\|_{1,\infty} && \text{(1a)} \\ \text{subject to} \quad & \mathbf{C}\geq 0, && \text{(1b)} \\ & \mathbf{1}_{lk}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(1c)} \end{aligned}$$

$$\begin{aligned} \underset{\substack{\mathbf{C}\in\mathbf{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbf{R}^k}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(2a)} \\ \text{subject to} \quad & \mathbf{C}\geq 0, && \text{(2b)} \\ & \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(2c)} \\ & \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top\geq\mathbf{C}. && \text{(2d)} \end{aligned}$$

$$\begin{aligned} \underset{\substack{\mathbf{C},\mathbf{U}\in\mathbf{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbf{R}^k}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(3a)} \\ \text{subject to} \quad & \mathbf{C}\geq 0, && \text{(3b)} \\ & \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(3c)} \\ & \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top=\mathbf{C}+\mathbf{U}, && \text{(3d)} \\ & \mathbf{U}\geq 0. && \text{(3e)} \end{aligned}$$

$$\begin{aligned} \underset{\mathbf{C},\mathbf{U}\in\mathbf{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l && \text{(4a)} \\ \text{subject to} \quad & \mathbf{C}\geq 0, && \text{(4b)} \\ & \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(4c)} \\ & \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(4d)} \\ & \mathbf{U}\geq 0, && \text{(4e)} \end{aligned}$$

$$\begin{aligned} \underset{\mathbf{C},\hat{\mathbf{C}},\mathbf{U},\hat{\mathbf{U}}\in\mathbf{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l \\ & +\frac{\mu_1}{2}\left\|\mathbf{C}-\hat{\mathbf{C}}\right\|_F^2+\frac{\mu_2}{2}\left\|\mathbf{U}-\hat{\mathbf{U}}\right\|_F^2 && \text{(5a)} \\ \text{subject to} \quad & \hat{\mathbf{C}}\geq 0, && \text{(5b)} \\ & \mathbf{1}_{(lk)}^\top\hat{\mathbf{C}}=\mathbf{1}_l^\top && \text{(5c)} \\ & \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(5d)} \\ & \hat{\mathbf{U}}\geq 0, && \text{(5e)} \\ & \mathbf{C}=\hat{\mathbf{C}},\ \ \mathbf{U}=\hat{\mathbf{U}}. && \text{(5f)} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_\mu(\mathbf{C},\mathbf{U},\hat{\mathbf{C}},\hat{\mathbf{U}},\mathbf{\Lambda}_1,\mathbf{\Lambda}_2)=\|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l \\ +\frac{\mu}{2}\left\|\mathbf{C}-\hat{\mathbf{C}}+\frac{\mathbf{\Lambda}_1}{\mu}\right\|_F^2+\frac{\mu}{2}\left\|\mathbf{U}-\hat{\mathbf{U}}+\frac{\mathbf{\Lambda}_2}{\mu}\right\|_F^2 \end{aligned} \quad \text{(6)}$$

$$\begin{aligned} (\mathbf{C}^{t+1},\mathbf{U}^{t+1}) := & \underset{\mathbf{C},\mathbf{U}\in\mathbf{R}^{(lk)\times l}}{\arg\min}\ \mathcal{L}_\mu(\mathbf{C},\mathbf{U},\hat{\mathbf{C}}^t,\hat{\mathbf{U}}^t,\mathbf{\Lambda}_1^t,\mathbf{\Lambda}_2^t) \\ & \text{subject to (5d)} \end{aligned} \quad \text{(7)}$$

$$\begin{aligned} (\hat{\mathbf{C}}^{t+1},\hat{\mathbf{U}}^{t+1}) := & \underset{\mathbf{C},\mathbf{U}\in\mathbf{R}^{(lk)\times l}}{\arg\min}\ \mathcal{L}_\mu(\mathbf{C}^{t+1},\mathbf{U}^{t+1},\hat{\mathbf{C}},\hat{\mathbf{U}},\mathbf{\Lambda}_1^t,\mathbf{\Lambda}_2^t) \\ & \text{subject to (5b), (5c), (5e).} \end{aligned} \quad \text{(8)}$$

$$\begin{aligned} \mathbf{\Lambda}_1^{t+1} &= \mathbf{\Lambda}_1^t+\mu(\mathbf{C}^{t+1}-\hat{\mathbf{C}}^{t+1}) \\ \mathbf{\Lambda}_2^{t+1} &= \mathbf{\Lambda}_2^t+\mu(\mathbf{U}^{t+1}-\hat{\mathbf{U}}^{t+1}) \end{aligned} \quad \text{(9)}$$

For 7

$$\begin{aligned} \underset{\mathbf{z}_i\in\mathbf{R}^{2l}}{\text{minimize}} \quad & \frac{1}{2}\mathbf{z}_i^\top\mathbf{Q}\mathbf{z}_i+\mathbf{z}_i^\top\mathbf{q}_i && \text{(10a)} \\ \text{subject to} \quad & \mathbf{A}\mathbf{z}_i=\mathbf{0} && \text{(10b)} \end{aligned}$$

Proposed method-Numerical Method

$$
\begin{aligned}
\underset{\mathbf{C}\in\mathbb{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\left\|\left[\mathbf{C}_1(:)\ldots\mathbf{C}_k(:)\right]^\top\right\|_{1,\infty} && \text{(1a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(1b)}\\
& \mathbf{1}_{lk}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(1c)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\substack{\mathbf{C}\in\mathbf{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbf{R}^{k}}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(2a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(2b)}\\
& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(2c)}\\
& \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top\geq\mathbf{C}. && \text{(2d)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\substack{\mathbf{C},\mathbf{U}\in\mathbf{R}^{(lk)\times l}\\ \mathbf{m}\in\mathbf{R}^{k}}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\lambda\mathbf{1}_k^\top\mathbf{m} && \text{(3a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(3b)}\\
& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(3c)}\\
& \mathbf{m}\otimes\mathbf{1}_l\mathbf{1}_l^\top=\mathbf{C}+\mathbf{U}, && \text{(3d)}\\
& \mathbf{U}\geq 0. && \text{(3e)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\mathbf{C},\mathbf{U}\in\mathbf{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l && \text{(4a)}\\
\text{subject to} \quad & \mathbf{C}\geq 0, && \text{(4b)}\\
& \mathbf{1}_{(lk)}^\top\mathbf{C}=\mathbf{1}_l^\top, && \text{(4c)}\\
& \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(4d)}\\
& \mathbf{U}\geq 0, && \text{(4e)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\mathbf{C},\hat{\mathbf{C}},\mathbf{U},\hat{\mathbf{U}}\in\mathbf{R}^{(lk)\times l}}{\text{minimize}} \quad & \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l\\
& +\frac{\mu_1}{2}\left\|\mathbf{C}-\hat{\mathbf{C}}\right\|_F^2+\frac{\mu_2}{2}\left\|\mathbf{U}-\hat{\mathbf{U}}\right\|_F^2 && \text{(5a)}\\
\text{subject to} \quad & \hat{\mathbf{C}}\geq 0, && \text{(5b)}\\
& \mathbf{1}_{(lk)}^\top\hat{\mathbf{C}}=\mathbf{1}_l^\top && \text{(5c)}\\
& \mathbf{E}(\mathbf{C}+\mathbf{U})=\frac{\mathbf{I}_k\otimes\mathbf{1}_l\mathbf{1}_l^\top}{l}(\mathbf{C}+\mathbf{U}), && \text{(5d)}\\
& \hat{\mathbf{U}}\geq 0, && \text{(5e)}\\
& \mathbf{C}=\hat{\mathbf{C}},\quad \mathbf{U}=\hat{\mathbf{U}}. && \text{(5f)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_\mu(\mathbf{C},\mathbf{U},\hat{\mathbf{C}},\hat{\mathbf{U}},\mathbf{\Lambda}_1,\mathbf{\Lambda}_2)=&\ \|\mathbf{X}_j-\mathbf{DC}\|_F^2+\frac{\lambda}{l^2}\mathbf{1}_{(lk)}^\top(\mathbf{C}+\mathbf{U})\mathbf{1}_l\\
&+\frac{\mu}{2}\left\|\mathbf{C}-\hat{\mathbf{C}}+\frac{\mathbf{\Lambda}_1}{\mu}\right\|_F^2+\frac{\mu}{2}\left\|\mathbf{U}-\hat{\mathbf{U}}+\frac{\mathbf{\Lambda}_2}{\mu}\right\|_F^2 \qquad \text{(6)}
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{C}^{t+1},\mathbf{U}^{t+1}) &:= \underset{\mathbf{C},\mathbf{U}\in\mathbf{R}^{(lk)\times l}}{\arg\min}\ \mathcal{L}_\mu(\mathbf{C},\mathbf{U},\hat{\mathbf{C}}^t,\hat{\mathbf{U}}^t,\mathbf{\Lambda}_1^t,\mathbf{\Lambda}_2^t)\\
&\qquad \text{subject to (5d)} \qquad \text{(7)}\\
(\hat{\mathbf{C}}^{t+1},\hat{\mathbf{U}}^{t+1}) &:= \underset{\mathbf{C},\mathbf{U}\in\mathbf{R}^{(lk)\times l}}{\arg\min}\ \mathcal{L}_\mu(\mathbf{C}^{t+1},\mathbf{U}^{t+1},\hat{\mathbf{C}},\hat{\mathbf{U}},\mathbf{\Lambda}_1^t,\mathbf{\Lambda}_2^t)\\
&\qquad \text{subject to (5b), (5c), (5e).} \qquad \text{(8)}\\
\mathbf{\Lambda}_1^{t+1} &= \mathbf{\Lambda}_1^t+\mu(\mathbf{C}^{t+1}-\hat{\mathbf{C}}^{t+1}) \qquad \text{(9)}\\
\mathbf{\Lambda}_2^{t+1} &= \mathbf{\Lambda}_2^t+\mu(\mathbf{U}^{t+1}-\hat{\mathbf{U}}^{t+1})
\end{aligned}
$$

For 8 & 9

$$
\begin{aligned}
\underset{\mathbf{z}_i\in\mathbf{R}^{2l}}{\text{minimize}} \quad & \left\|\hat{\mathbf{C}}-(\mathbf{C}+\frac{\mathbf{\Lambda}_1}{\mu})\right\|_F^2 && \text{(11a)}\\
\text{subject to} \quad & \hat{\mathbf{C}}\geq 0, && \text{(11b)}\\
& \mathbf{1}_{(lk)}^\top\hat{\mathbf{C}}=\mathbf{1}_l^\top && \text{(11c)}
\end{aligned}
$$

$$
\begin{aligned}
\underset{\mathbf{z}_i\in\mathbf{R}^{2l}}{\text{minimize}} \quad & \left\|\hat{\mathbf{U}}-(\mathbf{U}+\frac{\mathbf{\Lambda}_2}{\mu})\right\|_F^2 && \text{(12a)}\\
\text{subject to} \quad & \hat{\mathbf{U}}\geq 0 && \text{(12b)}
\end{aligned}
$$

Experimental results

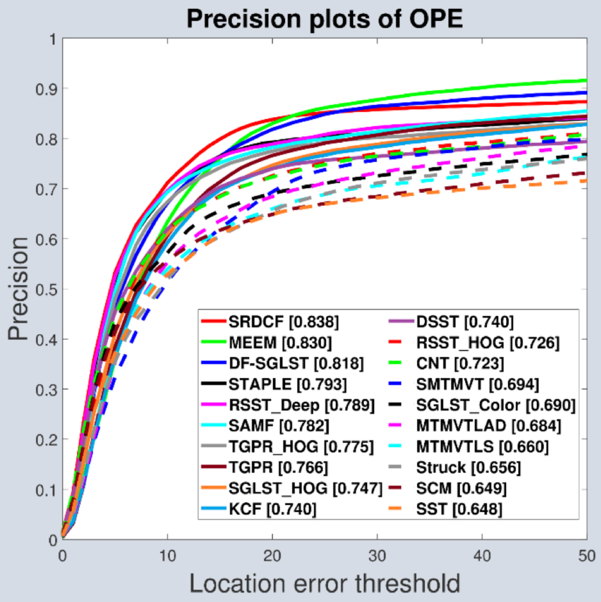
- We evaluate the performance of the proposed DF-SGLST and its two variants (i.e. SGLST_Color and SGLST_HOG) on the object tracking benchmark (OTB), which contains fully annotated videos with substantial variations. We evaluate these three trackers on both OTB50 and OTB100 benchmarks for fair comparison since not all the trackers provide the results on both benchmarks.

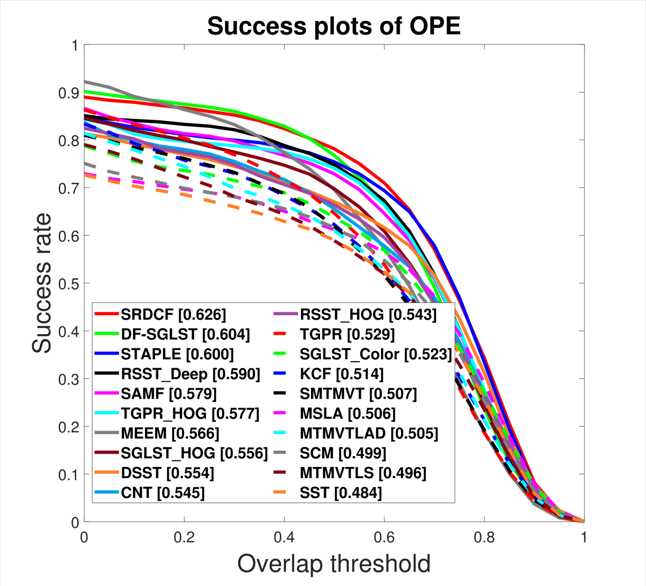
- We utilize two metrics, namely, bounding box overlap ratio and center location error.

- Using these two matrics, we plot success plots and precision plots for all trackers.

Experimental results

- OTB50:
  - This benchmark consists of 50 annotated sequences, where 49 sequences has one annotated target and one sequence *jogging* has two annotated targets.





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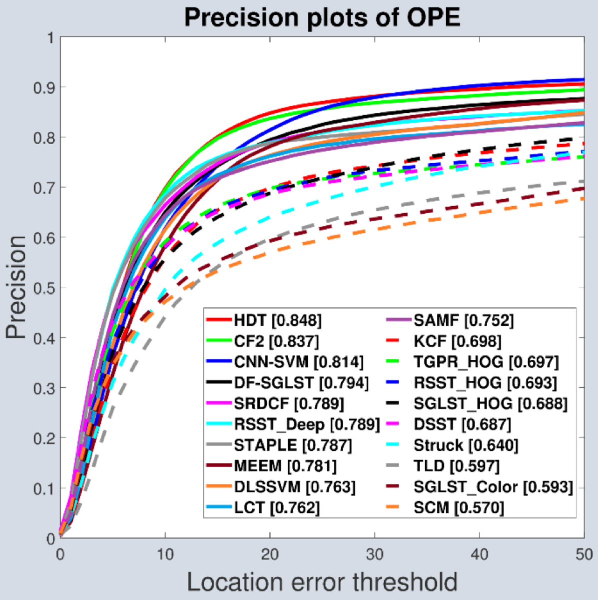
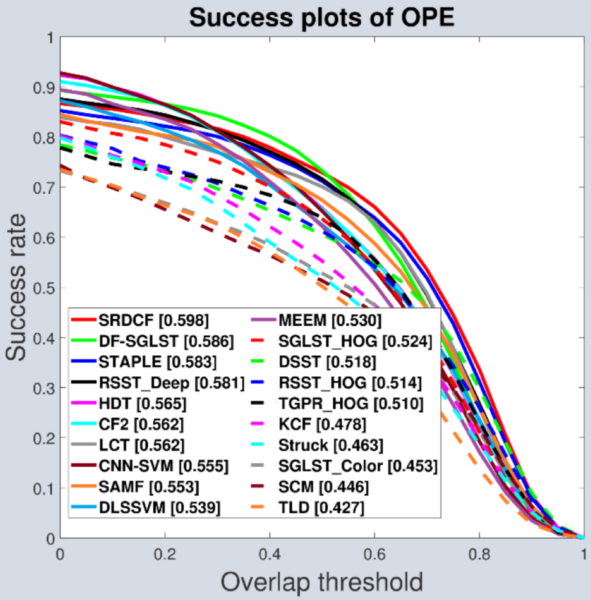
Experimental results

- ➢OTB100:
  - ➢This benchmark extends OTB50 by adding 50 additional annotated sequences.





Results on Sequences

Car4

Results on Sequences

Faceocc1

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Results on Sequences

walking2

Results on Sequences

football

Thank you