Fair Resource Allocation in an MEC-Enabled Ultra-Dense IoT Network with NOMA

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**Motivation and Objective**

**Challenges**
- Ultra-dense Internet of Things (IoT) networks need massive IoT devices to perform computation-intensive and delay-sensitive tasks, it is challenging for IoT devices to execute those tasks due to their low computation capability and limited resource.
- It is of crucial importance to design efficient resource allocation schemes for improving computation performance or for decreasing computation cost.
- Unfairness may occur when there exist massive IoT devices all with very limited computation capability.

**Potential Solutions**
- Mobile edge computing (MEC) can significantly improve the computation capability of IoT devices by deploying MEC servers in proximity to IoT users.
- An MEC-enabled ultra-dense IoT network with NOMA can effectively balance the computation efficiency and energy efficiency.

In order to achieve a better trade-off between computation efficiency and energy efficiency, we propose a new method for the resource allocation in an MEC-enabled ultra-dense IoT network with NOMA. Moreover, in order to address fairness among massive IoT users, we introduce fairness index into the utility function of the proposed scheme.
An MEC-enabled ultra-dense IoT network as shown is considered, $N$ IoT devices that need to execute computation-intensive yet delay-sensitive tasks and one MEC server that can provide MEC service for those IoT devices.

- Partial computation offloading mode is supported, part of tasks for local computing and remaining part for offloading to the MEC server for computing.
- NOMA is applied so that multiple devices can offload their tasks simultaneously by using the same physical radio resource.
- Each IoT device can perform local computing and computation task offloading at the same time.
Let $\mathcal{N} = \{1, 2, \ldots, N\}$ denote the set of UEs. Let $g_n$ and $p_n$ represent the channel gain and the transmission power between the MEC server and UE $n$.

MEC server ranks the UEs by their channel quality, i.e., $g_1^2 \geq g_2^2 \geq \cdots \geq g_N^2$. All the $N$ users can send the offloading data to the MEC simultaneously on the same radio resource.

Successive interference cancellation (SIC) technique is applied at the receiving side to decode the signal for each user, i.e., for decoding $n$th user’s signal, the signals from user $i$, $1 \leq i \leq n-1$, are treated as interference and the signals from user $m$, $n \leq m \leq N$, are all removed from the composite received signal.

The offloading rate and power consumption for the $n$th user can be expressed as

$$r_n^{\text{off}} = B \log_2 \left( 1 + \frac{p_n g_n^2}{\sum_{i=1}^{n-1} p_i g_i^2 + \sigma^2} \right), \quad (1)$$

and

$$p_n^{\text{off}} = \zeta p_n + p_r, \quad (2)$$

where $\sigma^2$ is the power of the noise, $B$ is the bandwidth. $\zeta$ is the amplifier coefficient, and $p_r$ is the constant circuit power consumed for signal processing.
Let $C_n$ be the number of computation cycles required to process one bit of data for UE $n$ locally. Let $f_n$ denote the computing speed of the processor (cycles per second). The local computing rate of the $n$th UE can be expressed as

$$r^{\text{local}}_n = \frac{f_n}{C_n}$$

(3)

The power consumption of local computing is modeled as a function of processor speed $f_n$. It can be given as

$$p^{\text{local}}_n = \epsilon f_n^3,$$

(4)

where $\epsilon$ is the effective capacitance coefficient of the processor’s chip.
Combining data offloading and local computing, the total computation rate $R_n$ for UE $n$ can be expressed as $R_n = r_n^{\text{off}} + r_n^{\text{local}}$. The total power consumption $P_n$ of UE $n$ is $P_n = p_n^{\text{off}} + p_n^{\text{local}}$.

In order to consider the computation rate fairness among users, the following utility $U_\alpha(R_n)$ is defined, where $\alpha$ is the fairness index.

$$U_\alpha(R_n) = \begin{cases} \frac{R_n^{1-\alpha}}{1-\alpha} & \text{if } \alpha \geq 0, \; \alpha \neq 1, \\ \ln(R_n) & \alpha = 1. \end{cases}$$ (5)

When $\alpha = 0$, the utility function is the sum computation rate for all UEs; when $\alpha = 1$, the utility function is the sum logarithmic function of UE rates, which normally provides proportional fairness; when $\alpha = \infty$, the utility function is the minimum rate among all UEs, which corresponds to the max-min fairness.
The optimization problem aims to minimize the total power consumption as well as maximize the achievable data rate utility. The problem is formulated as

\[
P_1 : \max_{f_n, p_n} \Phi_{SE} \sum_{n \in N} U_\alpha(R_n) - \Phi_{EE} \sum_{n \in N} P_n \tag{6a}
\]

s.t. \( C1 : \ p_n \geq 0, \ \forall n \in N, \) \tag{6b}
\( C2 : \ R_n \geq R_n^{th}, \ \forall n \in N, \) \tag{6c}
\( C3 : \ P_n \leq P_n^{th}, \ \forall n \in N, \) \tag{6d}
\( C4 : \ f_n^{min} \leq f_n \leq f_n^{max}, \ \forall n \in N. \) \tag{6e}

\( P_1 \) is a resource allocation problem that optimizes the offloading power \( p_n, \) local computing chip frequency \( f_n. \) \( \Phi_{SE} \) and \( \Phi_{EE} \) are the weighting factors that can be used to prioritize different computation service requirements of UEs.
Proportional Fairness $\alpha = 1$

When $\alpha = 1$, the system utility is the sum logarithmic function of UE’s computation rate. In this case, the proportional fairness can be achieved. The original problem $P_1$ under this case can be expressed as

$$P_2 : \max_{f_n, p_n} \Phi_{SE} \sum_{n \in \mathcal{N}} \ln(B \log_2(1 + \frac{p_n g_n^2}{\sum_{i=1}^{n-1} p_i g_i^2 + \sigma^2}) + \frac{f_n}{C_n}) - \Phi_{EE} \sum_{n \in \mathcal{N}} (\zeta p_n + p_r + \epsilon f_n^3)$$

s.t. $C1 - C4$.  \hspace{1cm} (7)

$P_2$ is a non-convex optimization problem.
Let \( a = [a_1, \cdots, a_N]^T \), \( P_2 \) can be transformed into

\[
P_3 : \quad \max_{f_n, \rho_n, a_n} \Phi_{SE} \sum_{n \in \mathcal{N}} a_n - \Phi_{EE} \sum_{n \in \mathcal{N}} (\zeta \exp(\rho_n) + p_r + \epsilon f_n^3)
\]

subject to

\[
a_n \geq \ln(R_n^{th}), \quad \forall n \in \mathcal{N}, \quad (8a)
\]

\[
\zeta \exp(\rho_n) + p_r + \epsilon f_n^3 \leq P_n^{th}, \quad \forall n \in \mathcal{N}, \quad (8b)
\]

\[
f_n^{min} \leq f_n \leq f_n^{max}, \quad \forall n \in \mathcal{N}, \quad (8c)
\]

\[
\ln(2^{\exp(a_n) - \frac{f_n}{c_n} B} - 1) + \ln(\sum_{i=1}^{n-1} \frac{g_i^2}{g_n^2} e^{\rho_i - \rho_n} + \frac{\sigma^2}{g_n^2} e^{-\rho_n}) \leq 0, \quad \forall n \in \mathcal{N}. \quad (8d)
\]

The objective function is jointly concave with respect to \( a_n \) and \( \rho_n \) due to the subtraction of a linear term and convex term. However, the first part of the last constraint (8d) is non-convex. Thus, the problem is non-convex.
In order to tackle it, the SCA method is applied. By introducing the auxiliary variables $x_n$, we have $\exp(a_n) - \frac{f_n}{C_n} \leq \exp(x_n)$ and $\exp(x_n) \leq B \log_2(1 + \frac{\exp(\rho_n)g_n^2}{\sum_{i=1}^{n-1} \exp(\rho_i)g_i^2 + \sigma^2})$. By using the SCA technique, the first-order Taylor expansion is used to approximate the right part. Thus, $P_3$ can be solved by iteratively solving the following approximate problem, given as

\[
P_4 : \max_{f_n, \rho_n, a_n, x_n} \Phi_{SE} \sum_{n \in \mathcal{N}} a_n - \Phi_{EE} \sum_{n \in \mathcal{N}} (\zeta \exp(\rho_n) + p_r + \epsilon f_n^3) \tag{9a}
\]

s.t. $a_n \geq \ln(R_n^{th})$, $\forall n \in \mathcal{N}$, \tag{9b}

$\zeta \exp(\rho_n) + p_r + \epsilon f_n^3 \leq P_n^{th}$, $\forall n \in \mathcal{N}$, \tag{9c}

$f_n^{min} \leq f_n \leq f_n^{max}$, $\forall n \in \mathcal{N}$, \tag{9d}

$\exp(a_n) - \frac{f_n}{C_n} \leq \exp(\bar{x}_n^k) + \exp(\bar{x}_n^k)(x_n - \bar{x}_n^k)$, \tag{9e}

$\exp(x_n) \leq B \log_2(1 + \frac{\exp(\rho_n)g_n^2}{\sum_{i=1}^{n-1} \exp(\rho_i)g_i^2 + \sigma^2})$, \tag{9f}

where $\bar{x}_n^k$, $n \in \mathcal{N}$ are the given local points at the $k$th iteration. The above problem is convex and can be readily solved by using the existing convex optimization tool.
Max-Min Fairness $\alpha = \infty$

In this case, the objective of the system is to maximize the minimum computation rate among all the users. The original optimization problem $P_1$ becomes

$$P_5 : \max_{f_n,p_n} \Phi_{SE} \min_{n \in \mathcal{N}} B \log_2(1 + \frac{p_n g_n^2}{\sum_{i=1}^{n-1} p_i g_i^2 + \sigma^2}) + \frac{f_n}{C_n} - \Phi_{EE} \sum_{n \in \mathcal{N}} \zeta p_n + p_r + \epsilon f_n^3$$

s.t. $C1 - C4$. (10)
It is difficult to directly solve the max-min problem $P_5$. By introducing a new variable $l$, auxiliary variables $\rho_n = \log(p_n)$, the auxiliary variable $z_n$,

$$\exp(z_n) \leq B \log_2\left(1 + \frac{\exp(\rho_n)g_n^2}{\sum_{i=1}^{n-1} \exp(\rho_i)g_i^2 + \sigma^2}\right), \quad \text{(11)}$$

and apply the SCA method to solve the problem $P_6$ by iteratively solving the approximate problem, given as

$$P_6 : \max_{f_n, \rho_n, l, z_n} f_n, \rho_n, l, z_n \in \mathcal{N}$$

$$-\Phi_{EE} \sum_{n \in \mathcal{N}} (\zeta \exp(\rho_n) + p_r + \epsilon f_n^3) + \Phi_{SE} * l \quad \text{(12a)}$$

$$s.t. \quad l \geq R_n^\text{th}, \forall n \in \mathcal{N}, \quad \text{(12b)}$$

$$\zeta \exp(\rho_n) + p_r + \epsilon f_n^3 \leq P_n^\text{th}, \forall n \in \mathcal{N}, \quad \text{(12c)}$$

$$f_n^\text{min} \leq f_n \leq f_n^\text{max}, \forall n \in \mathcal{N}, \quad \text{(12d)}$$

$$l \leq \exp(\bar{z}_n^k) + \exp(\bar{z}_n^k)(z_n - \bar{z}_n^k) + \frac{f_n}{C_n} \quad \text{(12e)}$$

$$\exp(z_n) \leq B \log_2\left(1 + \frac{\exp(\rho_n)g_n^2}{\sum_{i=1}^{n-1} \exp(\rho_i)g_i^2 + \sigma^2}\right) \quad \text{(12f)}$$

where $\bar{z}_n^k, n \in \mathcal{N}$ are the given local points at the $k$th iteration.
Algorithm 1: The SCA iterative algorithm for $P_4$ and $P_6$

1) Input settings:
   the error tolerance $\xi > 0$, $R_n^{th} > 0$ and $P_n^{th} > 0$,
   the maximum iteration number $K$.

2) Initialization:
   $k = 0$, $f_n(0)$, $\rho_n(0), a_n(0)$ $\bar{x}_n^0$;

3) Optimization:
   $\triangleright$ for k=1:K
       solve $P_4/P_6$ by using the interior-point method;
       obtain the solution $\{f_n^k, \rho_n^k, a_n^k, x_n^k\}$ and the system efficiency $H_k^*$;
       if $\|H_k^* - H_{k-1}^*\| \leq \xi$;
           the maximum system efficiency $H^*$ is obtained;
           break;
       else
           update $\bar{x}_n^{k+1} = x_n^k$ and $k = k + 1$.
   $\triangleright$ end

4) Output:
   $\{f_n^*, \rho_n^*\}$ and system efficiency $H^*$
In this case, the tradeoff between the weighted sum computation rate and the power consumption cost is considered. The original problem $P_1$ can be expressed as

$$
\begin{align*}
\mathbf{P}_7 : \quad & \max_{f_n, p_n} \Phi_{SE} \sum_{n \in \mathcal{N}} \left( B \log_2 \left( 1 + \frac{p_n g_n^2}{\sum_{i=1}^{n-1} p_i g_i^2 + \sigma^2} \right) + \frac{f_n}{C_n} \right) - \Phi_{EE} \sum_{n \in \mathcal{N}} (\zeta p_n + p_r + \epsilon f_n^3) \\
\text{s.t.} \quad & C1 - C4.
\end{align*}
$$

(13)

The problem (13) is NP-hard. In order to solve it, we apply a SCALE method, by introduce the equation:

$$
a \log z + b \leq \log_2 (1 + z).$$

(14)

That is tight at $z = z_0$ when the approximation constants are given as

$$
\begin{align*}
a &= \frac{z_0}{1 + z_0}, \\
b &= \log_2 (1 + z_0) - \frac{z_0}{1 + z_0} \log_2 (z_0).
\end{align*}
$$

(15a)

(15b)
By applying the SCALE method to the problem (13), and the logarithmic change of variables \( \rho_n = \log(p_n) \), we can obtain the following problem as

\[
P_8 \quad \max_{f_n, \rho_n} \Phi_{SE} \sum_{n \in \mathcal{N}} (B \overline{R}_n(\rho_n; a_n, b_n) + \frac{f_n}{C_n}) - \Phi_{EE} \sum_{n \in \mathcal{N}} (\zeta \exp(\rho_n) + p_r + \epsilon f_n^3),
\]

\[
\text{s.t.} \quad B \overline{R}_n(\rho_n; a_n, b_n) + \frac{f_n}{C_n} \geq R_n^{th}, \forall n \in \mathcal{N},
\]

\[
\zeta \exp(\rho_n) + p_r + \epsilon f_n^3 \leq P_n^{th}, \forall n \in \mathcal{N},
\]

\[
f_n^{\min} \leq f_n \leq f_n^{\max}, \forall n \in \mathcal{N},
\]

where

\[
z_n = \frac{\exp(\rho_n)g_n^2}{\sum_{i=1}^{n-1} \exp(\rho_i)g_i^2 + \sigma^2},
\]

\[
a_n = \frac{z_n}{1 + z_n}, \quad b_n = \log_2(1 + z_n) - \frac{z_n}{1 + z_n} \log_2(z_n),
\]

and

\[
\overline{R}_n(\rho_n; a_n, b_n) = a_n[\log_2(g_n^2) + \ln 2 \rho_n - \log(\sum_{i=1}^{n-1} \exp(\rho_i)g_i^2 + \sigma^2)] + b_n.
\]

The problem (16a) is a standard concave maximization problem.
Algorithm 2: The SCALE iterative algorithm for $P_8$

1) **Input settings:**
   - the error tolerance $\xi > 0$, $R_{th}^n > 0$ and $P_{th}^n > 0$, $a_n^1 = 1$, $b_n^1 = 0$,
   - the maximum iteration number $K$.

2) **Initialization:**
   - $k = 0$, $f_n(0)$, $\rho_n(0)$, $a_n(0)$ and $b_n(0)$;

3) **Optimization:**
   - $\triangleright$ for $k=1:K$
     - solve $P_9$ by using the interior-point method;
     - obtain the solution $\{f_n^*, \rho_n^*\}$ and the rate approximation $\overline{R}_n^{k*}$;
     - if $\|\overline{R}_n^{k*} - \overline{R}_{n-1}^{k-1*}\| \leq \xi$;
       - the maximum system efficiency $H^*$ is obtained;
       - break;
     - else
       - update $a_{n+1}^k$, $b_{n+1}^k$ by (21) and $k = k + 1$.
     - end
   - $\triangleright$ end

4) **Output:**
   - $\{f_n^*, \rho_n^*\}$ and system efficiency $H^*$
Figure 2: System Efficiency for $\alpha = 1$ ($\Phi_{SE} = 0.3$, $\Phi_{EE} = 0.7$).
Figure 3: System Efficiency for $\alpha = \infty$ ($\Phi_{SE} = 10^{-6}$, $\Phi_{EE} = 0.5$).
Figure 4: System Efficiency for $\alpha = 0$ ($\Phi_{SE} = 0.3$, $\Phi_{EE} = 0.7$).
Thank You!