Systematic Design of Attitude Control Systems for a Satellite in a Circular Orbit with Guaranteed Performance and Stability

Richard A. Hull†, Chan Ham‡, Roger W. Johnson‡

Abstract

In this paper two nonlinear control techniques are developed and compared for the satellite attitude control problem. The first technique is a robust recursive nonlinear method using Euler angle formulation. This method is related to integrator backstepping as well as feedback linearization techniques. However, in this paper a different formulation is presented which overcomes some of the previous difficulties in applying backstepping to this problem by treating the three axis satellite system as a fully coupled set of second order systems. The technique produces a robustly stable controller, which meets desired performance, accounts for system nonlinear behavior, and is easily implementable in a set of feedback equations that can be computed in real time.

The second technique is a learning control that updates the control input iteratively in order to enhance the transient performance of systems that are repeatedly executed over a fixed finite duration. It updates the control input by learning laws without the computation of system parameters and inverse dynamics of systems. The advantage of utilizing learning control schemes for attitude control systems is the enhancement of transient performance from trial to trial by taking advantage of the periodicity of the repeated system operation. By learning unknown parameters or time functions, the learning control can compensate nonlinear dynamics so that the desired performance can be achieved.

The performance of both techniques is demonstrated for a satellite attitude tracking maneuver which represents a satellite in a circular orbit maintaining one face toward the earth while tracking simultaneous sinusoidal pointing commands in the other two axes. It is shown that the recursive controller provides the desired tracking performance with reasonable control effort, and that the learning control technique can be used to compensate for periodic external disturbances.

†Corresponding author, Coleman Aerospace Company, 7675 Municipal Drive, Orlando, Florida 32819
  tel: (407) 354-0047, fax: (407) 354-1113,
  e-mail: Richard_Hull@mail.crc.com

‡Florida Space Institute / Department of Mechanical, Materials and Aerospace Engineering, University of Central Florida, Orlando, Florida
1. Introduction

Recursive control design methods have been expanded in the past decade to the domain of nonlinear controls to include adaptive and robust control design methods. In this paper, a recursive design method will be utilized to obtain output tracking performance for a satellite attitude control problem. The general problem is formulated as a coupled set of nonlinear equations to generate the controlling moments required for a simultaneous three axis tracking task. Previously, recursive control design methods have been used in systems with cascaded structure to generate Lyapunov based controllers with guaranteed stability. These methods were limited to providing tracking performance for derivatives of the desired output function up to the relative degree of the system. A dynamic recursive controller design was introduced by Hull to overcome the previous structural limitations and allow output tracking of derivative up to the order of the system, even for systems with less than full relative degree. The major requirement for implementation of this technique is stability of the zero-dynamics of the original system.

The fundamental idea of recursive controller design has been in use for long time, and is evident both implicitly and explicitly in control systems literature dating from the sixties. Integrator back stepping as a recursive design tool for cascaded systems was used by Saberi, Kokotovic and Sussmann, and further developed by Kanellakopoulos, Kokotovic and Morse.

A tutorial overview of backstepping was given in the 1991 Bode lecture by Kokotovic. Applying recursive design to systems with bounded uncertainty, robust recursive design techniques were introduced under the Generalized Matching Conditions (GMC's) by Qu. Recent work by Qu and Kaloust to extend the recursive design technique to systems which do not satisfy the cascaded structure, has motivated the generic “recursive-interlacing” design approach which consists of both forward and backward recursive design steps.

The recursive design approach is introduced in this paper to simplify a general sixth order nonlinear control problem, by considering it as second order matrix problem of coupled equations.

Learning control, a class of control systems that update the control input iteratively, can enhance the transient performance of systems that are repeatedly executed over a fixed finite duration. For the control process, a learning law adjusts the current control input based on error signals from previous operations. Learning control updates the control input by learning laws without computing system parameters and the inverse dynamics of systems, etc; therefore, it is relatively simple to implement compared to other schemes. Another advantage is that exact knowledge of the system dynamics is not required. Space systems are inherently nonlinear and cannot be modeled exactly, but the unmodelled dynamics effects and uncertainties are important considerations necessary to achieve desired control objectives. Learning control schemes are appropriate for satellites due to the periodicity of the repeated satellite mission and the perturbations from space environment such as solar pressure. Using learning control, the transient performance from orbit to orbit can be enhanced by taking advantage of this periodicity, and exact knowledge of the satellite dynamics is not required. Because its feedforward control compensates the state-dependent uncertainties using their bounding functions, the proposed learning control also provides robustness of the control system. This paper will demonstrate that the proposed learning control can compensate for external disturbances on the satellite without exact knowledge of the disturbing moments.

This remainder of this paper is organized as follows. In section 2, an attitude control problem for a satellite in circular orbit is formulated. In section 3, the proposed control design methods are developed. In section 4, simulation results are presented to show the performance of the proposed control schemes. Finally, conclusions are summarized in section 5.
2. Problem Formulation

Consider a satellite 3 axis attitude control problem, in which the goal is to track simultaneous motions in all three axes. Let the body attitudes be represented by the three Euler angle rotations:

\[ \Theta = [ \phi \ \theta \ \psi ]^T \] (1)

where \( \phi \) is the roll angle, \( \theta \) is the pitch angle, \( \psi \) is the yaw angle, and the assumed Euler rotation order is roll – pitch – yaw.

For illustration purposes, these desired output function be:

\[ \phi^d = \varpi \ t \] (2)

\[ \theta^d = b \cos \varpi \ t \] (3)

\[ \psi^d = c \sin \varpi \ t \] (4)

where \( \Theta^d = [ \phi^d \ \theta^d \ \psi^d ]^T \), represents the desired tracking vector of body rotation angles; \( b \) and \( c \) represent constant magnitudes, \( \varpi \) is the rotational frequency, and \( t \) represents the independent time variable. The desired pitch and yaw orientation angles are designed to track a sinusoidal path over the earth, while the constant rotation rate is synched to the orbital period in order to keep one face pointed toward the earth. This problem might be suitable for any number of earth, or other planetary, observation missions, in which a sensing device such as a radar, or laser is used to map features over a wide swath of terrain.

Let the body rotation angular rates be represented as:

\[ \Omega = [ p \ q \ r ]^T \] (5)

where: \( p \) is the roll rate, \( q \) is the pitch rate and \( r \) is the yaw rate. Then, it is well known that the time derivatives of the Euler angles can be expressed as nonlinear functions of the Euler angles and body rates as follows:

\[ \ddot{\Theta} = F(\Theta, \Omega) \]

Assuming constant mass and constant inertia, the time derivative of the rigid body momentum equation can be written as:

\[ J \ddot{\Omega} = - (\Omega \times J \Omega) + M + U \] (7)

where: \( J \) is the inertia matrix (3x3), \( M \) is a vector (3x1) of the external disturbing moments, \( U \) is a vector (3x1) of the applied control moments, and \( x \) represents the vector cross product.

Solving equation (7) we have:

\[ \ddot{\Omega} = - J^{-1} [ (\Omega \times J \Omega) - M - U ] \] (8)

3. Control Design Methods

3.1 Recursive Control Design

Combining the sets of equations represented in (6) and (8), we have a set of 6 state equations in:

\[ [ \phi \ \theta \ \psi \ p \ q \ r ]^T \] representing the satellite body Euler angles orientations and body axis rotational rates. It is assumed that all six states can be accurately measured by the on-board Inertial Measurement Unit (IMU).

At this point it is useful to observe that in matrix form, the state equations are:

\[ \ddot{\Theta} = F(\Theta, \Omega) \] (9)

\[ \ddot{\Omega} = G(\Omega, M, U) \] (10)

so that the input control vector, \( U \), is really only two vector integrations away from the states we wish to track, namely, \( \Theta \). In this sense, the coupled set of vector equations (9) and (10) are of full relative degree. Using the recursive
design approach, we introduce a new state variable vector, $Z_1$, defined as:

$$Z_1 = \Theta - \Theta^d$$  \hspace{1cm} (11)

which represents the tracking error vector. Differentiating and adding a fictitious control term and a common set of constant gains represented by a positive scalar multiplier $k_1$, we construct:

$$\ddot{Z}_1 = - k_1 Z_1 + k_1 Z_1 + \dot{Z}_1$$  \hspace{1cm} (12)

Introducing a second new state variable vector, $Z_2$, defined as:

$$Z_2 = k_1 Z_1 + \dot{Z}_1$$  \hspace{1cm} (13)

Differentiating, the second set of state equations in the transformed system, we want a stable system in $Z_2$, so we write another fictitious control equation with constant gains represented by a positive scalar $k_2$.

$$\ddot{Z}_2 = - k_2 Z_2 + k_2 Z_2 + \dot{Z}_2$$  \hspace{1cm} (14)

This leads to the controller design equation:

$$k_1 k_2 Z_1 + (k_1 + k_2) \dot{Z}_1 + \ddot{Z}_1 = 0$$  \hspace{1cm} (15)

In term of the original variables, we have:

$$\ddot{Z}_1 = \ddot{\Theta} - \ddot{\Theta}^d = F(\Theta, \Omega) - \ddot{\Theta}^d$$  \hspace{1cm} (16)

$$\ddot{Z}_1 = (\partial F/\partial \Theta) \dot{\Theta} + (\partial F/\partial \Omega) \dot{\Omega} - \ddot{\Theta}^d$$  \hspace{1cm} (17)

Substituting (11), (16) and (17) into the control design equation (15) and expanding:

$$k_1 k_2 (\Theta - \Theta^d) + (k_1 + k_2) [F(\Theta, \Omega) - \ddot{\Theta}^d]$$

$$+ (\partial F/\partial \Theta) \dot{\Theta} + (\partial F/\partial \Omega) \dot{\Omega} - \ddot{\Theta}^d = 0$$  \hspace{1cm} (18)

Substituting (6) and (8) into equation (18) and solving for $U$, we derive the recursive design control equation to be:

$$U = - J (\partial F/\partial \Omega)^{-1} \{ k_1 k_2 (\Theta - \Theta^d) + (k_1 + k_2) [F(\Theta, \Omega) - \ddot{\Theta}^d]$$

$$+ (\partial F/\partial \Theta) \dot{\Theta} + (\partial F/\partial \Omega) \dot{\Omega} - \ddot{\Theta}^d \} + \Omega x J \Omega - M$$  \hspace{1cm} (19)

The fader gain $k_f$ is a time dependent gain term which has been introduced to reduce large initial control moments that would be caused by the initial errors in tracking the derivatives of $\Theta^d$. Completing the other terms in equation (19), we can compute:

$$\partial F/\partial \Theta =$$  \hspace{1cm} (20)

$$\partial F/\partial \Omega =$$  \hspace{1cm} (21)

$$(\partial F/\partial \Omega)^{-1} =$$  \hspace{1cm} (22)

$$\ddot{\Theta}^d =$$  \hspace{1cm} (23)
\[ \Theta^d = \begin{bmatrix} 0 \\ b \Theta^2 \cos \Theta t \\ -c \Theta^2 \sin \Theta t \end{bmatrix} \]  

Finally, if the inertia matrix is diagonal:

\[ J = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix} \]  

the cross product term in equation (19) can be written:

\[ \Omega \times J \Omega = \begin{bmatrix} (J_3 - J_2) q r \\ (J_1 - J_3) p r \\ (J_2 - J_1) p q \end{bmatrix} \]  

Normally, however, \( J \) is a positive definite symmetric matrix, and so a somewhat more complicated expression for the Coriolis acceleration term is required. In that case we shall assume that this term is computed numerically.

### 3.2 Learning Control Design

In this section, a learning control design is introduced to control the attitude of satellites. The main motivation of the proposed control is to demonstrate an effective compensation scheme especially for the unmodelled external disturbing moments, \( M \). The objective of the proposed learning control can improve the system performance over repeated orbit operations by compensating for the unknown external disturbances. The proposed learning control design is based on the following assumption of the disturbances.

**Assumption:** Disturbances, \( M(\Theta, t) \), are unknown but periodic time functions as:

\[ \| M(\Theta_1, t) - M(\Theta_2, t) \| \leq c \| \Theta_1 - \Theta_2 \| \]  

where \( c \) is a positive constant.

To proceed with the learning control design, rewrite (13) using a new state variable:

\[ X_j = Z_{2,j} \Rightarrow X_j = k_p Z_{4,j} + \ddot{Z}_{1,j} \]  

where \( k_p \) is a positive constant control gain and subscript \( j \) is the index of learning trials, or the sequence of orbit periods. It is obvious that the objective of the proposed control system is to make \( X_j \rightarrow 0 \) as \( j \rightarrow \infty \) so that the Euler angles, \( \Theta \), can track the desired angles, \( \Theta^d \), as \( \dot{\Theta}^- = -k_1 \dot{Z}_i \) in (12). From orbit operation to orbit operation, the proposed control enhances the overall control performance as it learns and compensates for the unknown disturbance dynamics.

From (6), (8), (16), and (17), it follows that:

\[ \ddot{X}_j = k_p (F_j - \ddot{\Theta}^d) + (\partial F_j/\partial \Theta) \ddot{\Theta}_j \]

\[ + (\partial F_j/\partial \Omega) \dot{\Omega}_j - \ddot{\Theta}^d \]

\[ = k_p (F_j - \ddot{\Theta}^d) + (\partial F_j/\partial \Theta) F_j \]

\[ + J^{-1} (\partial F_j/\partial \Omega)(M_j - \Omega_j x J \Omega_j) \]

\[ - \ddot{\Theta}^d + J^{-1} (\partial F_j/\partial \Omega) U_j \]  

\[ (29) \]
It follows from assumption (27) that:

\[
\begin{align*}
\ddot{X}_j &= J^{-1} \left( \partial F_j / \partial \Omega_j \right) M (\Theta^d) - (k_p \ddot{\Theta}^d + \dot{\Theta}^d) \\
&+ \left[ (\partial F_j / \partial \Theta_j) + k_p \right] F_j \\
&- J^{-1} \left( \partial F_j / \partial \Omega_j \right) (\Omega_j \times J \Omega_j) \\
&+ J^{-1} \left( \partial F_j / \partial \Omega_j \right) U_{k,j} \\
&+ J^{-1} \left( \partial F_j / \partial \Omega_j \right) U_{u,j}
\end{align*}
\]  

(30)

Consequently, \( U_{k,j} \) is given by:

\[
U_{k,j} = - J \left( \partial F_j / \partial \Omega_j \right)^{-1} \left\{ \left[ (\partial F_j / \partial \Theta_j) + k_p \right] F_j \\
- (k_p \ddot{\Theta}^d + \dot{\Theta}^d) \right\} + (\Omega_j \times J \Omega_j)
\]  

(33)

And, the second control part \( U_{u,j} \) is proposed to have the formulation:

\[
U_{u,j} = R_j + \Delta_j
\]  

(34)

where \( R_j \) is the robust control and \( \Delta_j \) is the learning contribution to compensate the unknown periodic time function of disturbances.

Then the proposed control is in the form:

\[
U_{u,j} = - \left[ (1+\alpha/2) X_j + Z_{i,j} + \frac{1}{4} c^2 X_{i,j} / k_p \right] - \Delta_j
\]  

(35)

and the learning law is designed as:

\[
\Delta_j = \Delta_{j-1} + \alpha X_j
\]  

(36)

with the control gain \( \alpha > 0 \).

Remark (Stability and convergence analysis): The proposed learning control is designed using Lyapunov’s second approach. As shown previously, readers can find that the system in (12) and (27) under controls (33) and (35) is globally and asymptotically stable with respect to the sequence of orbit periods.

4. Simulation Results

4.1 Recursive Controller Simulation

A computer simulation program was written in MATLAB to provide simulation analysis of the three-axis satellite attitude control problem. The spacecraft was assumed to be in orbit, with an initial attitude vector of:

\[
\Theta(0) = [0^\circ \ 60^\circ \ 0^\circ]^T
\]

The spacecraft was assumed to have an inertia vector (kg m²) of:

\[
J =
\begin{bmatrix}
3000 & -300 & -500 \\
-300 & 3000 & -400 \\
-500 & -400 & 3000
\end{bmatrix}
\]

The desired tracking vector parameters from equations (2) – (4) are arbitrarily chosen to be:

\( \bar{\upsilon} = 2 \pi / 400 \) (rad/min)

\( b = 60^\circ \)

\( c = -60^\circ \)
where 400 minutes represents the orbital and rotational periods.

The recursive design controller equations were programmed from equation (19), assuming that there are no disturbing moments, that is:

\[ M = 0 \]

In order to reduce large initial control moments, an additional fader gain term was applied to the \( \dot{\Theta}^d \) and \( \Theta^d \) terms in equation (19). The fader gain was computed as:

\[ k_i = 1 - e^{-0.1t} \]

Finally, the controller gains \( k_1 \) and \( k_2 \) were chosen to make equation (15) behave as a classical second order system:

\[
(s^2 + 2\alpha \dot{s} + s^2)Z_i = 0
\]

where \( s \) is the Laplace operator. By equating the coefficients:

\[ k_1 + k_2 = 2\alpha \dot{s} \]
\[ k_1 \cdot k_2 = \ddot{s}^2 \]

Solving for \( k_1 \) and \( k_2 \):

\[ k_1 = \frac{\ddot{s} + (\alpha^2 - 1)^{0.5}}{k_1} \]
\[ k_2 = \frac{\ddot{s}^2}{k_1} \]

Using natural frequency, \( \ddot{s} = 0.5 \text{ rad/min} \), and damping factor, \( \alpha = 0.5 \), we derive the controller gains to be: \( k_1 = 1.8666 \), \( k_2 = 0.134 \).

The simulation results for the recursive controller design with no external disturbances are shown in figures 1 through 4. Figure 1 shows the achieved Euler angles over one orbital period, and figure 2 shows the tracking errors between the achieved and the desired Euler angles. Figure 3 shows the body angular rates, and figure 4 shows the control effort required. These results demonstrate that the recursive controller provides the desired tracking performance within one orbital period, and with reasonable control effort. (Refer to similar problem by Stansbery and Cloutier^17.)
4.2 Learning Controller Simulation

To show the effectiveness of the learning control, external disturbing moments to approximate exoatmospheric aerodynamic effects are assumed to be:

\[ M = \begin{bmatrix} 0.5 \cos \varpi t \cdot (0.1 \theta + \psi) \\ 1.0 \sin \varpi t \cdot (0.1 \theta) \\ 0.8 \cos \varpi t \cdot (0.1 \psi) \end{bmatrix} \]

The proposed control \( U_{\alpha} \) is implemented with control gain \( \alpha = 5 \) and bounding constant \( c = 1 \).

The simulation results in figures 5 and 6 show that the proposed learning control is effective for compensating uncertain disturbances, maintaining the same order of tracking accuracy as in the previous case without unmodelled disturbances.

5. Conclusions

A satellite attitude control problem was formulated which is representative of a satellite in a circular orbit maintaining one face toward the earth while tracking a sinusoidal pointing command in the other two axes.

First a recursive backstepping method was used to design a fully coupled nonlinear controller for this task. The innovative approach was to consider the fully coupled dynamics of the matrix system when employing the recursive technique. This leads to a controller design for a
multivariable system that is of full relative degree. Simulation results were presented for the recursive design for the tracking task with no external disturbances. These results demonstrate that the recursive controller provides the desired tracking performance within one orbital period, and with reasonable control effort.

A learning control was then designed to compensate for periodic external dynamics. The proposed technique is applied in the standard backward recursive design. It is guaranteed to achieve asymptotically stable with respect to the number of orbit periods in performing repeated missions. In addition, the proposed design is robust since it does not require exact knowledge of their dynamics. Simulation results were then presented to demonstrate that the learning controller maintains the same tracking accuracy achieved with the recursive design, even in the presence of unmodelled external disturbances.

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References


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**Biographical Information**

**Richard A. Hull** received his B.S. in Engineering Science and Mechanics from the University of Florida in 1972, and his M.S. and Ph.D. degrees in Electrical Engineering from the University of Central Florida in 1993, and 1996. He has worked as a guidance and control systems engineer in the aerospace industry for over 20 years for Martin Marietta, McDonnell Douglas and Boeing companies. He has also worked as a research engineer for the Navigation and Controls Branch of the U.S. Air Force Research Laboratory, at Eglin Air Force Base, Florida. Currently he is a systems engineer for Coleman Aerospace Company in Orlando, Florida, and serves as an Associate Editor for the IEEE Control Systems Society.

**Chan Ho Ham** received his Ph.D from the Department of Electrical and Computer Engineering at the University of Central Florida in 1995. From 1996 to 1998, he worked as a lead engineer with the Satellite Business Division at the Hyundai Electronics Industries. He is currently working as an assistant professor with the Florida Space Institute and the Department of Mechanical, Materials, and Aerospace Engineering at the University of Central Florida. His main research interests include learning control of nonlinear uncertain systems, fuzzy control, robust control and their applications to space systems and mechatronic systems.

**Roger W. Johnson** received his BS degree from the US Naval Academy, Annapolis in 1952, his MS degree from MIT in 1958 and his Ph.D., from the University of California at Los Angeles (UCLA) in 1966. He has approximately twelve years of experience in directing engineering and exploratory development of new technical engineering concepts while serving in the USAF. His experience with industry includes directing operations and maintenance of launch processing and instrumentation systems for the U.S. Space Shuttle for two years. He also has nineteen years of experience in teaching and directing graduate and undergraduate research projects at the US Air Force Academy (USAFA), the US Air Force Institute of Technology (AFIT), and University of Central Florida (UCF).