FREQUENCY RESPONSE OF A GAS-FILLED TUBE WITH MINOR LOSSES

by

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Abstract

Frequency Response of a Gas-filled Tube with Minor Losses

by

Brian M. West, Master of Science
Utah State University, 2011

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A thesis on the study of the frequency response of a pneumatic system designed to provide pulsed flow for flow control applications is presented. The system consists of a high pressure air source, a high-frequency solenoid valve, a length of tube and a minor loss. The experiment mimics the pneumatic drive for our Coanda-Assisted Spray Manipulation actuator and applies to many flow control applications involving pulsed flow. Square wave signals of various frequency are fed to the solenoid valve. The flow at the exit of the system is measured with a single hot wire and compared to steady flow through the same geometry. The effect of the inlet pressure, tube length and the size of the minor loss is evaluated. These data are modeled using a Transmission Matrix Model.
Public Abstract

Frequency Response of a Gas-filled Tube with Minor Losses

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Major Professor: Dr. Barton L. Smith
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A thesis on the study of how quickly a pneumatic system responds to turning a valve on and off is presented. This has specific application to controlling gas flow systems. The system tested consists of a high pressure air source, an electronic valve, a length of tube, and a partial obstruction of the flow. The experiment mimics the pneumatic drive for our Coanda-Assisted Spray Manipulation device and applies to other devices as well. The valve is turned on and off at different frequencies. The flow at the exit of the system is measured and compared to steady flow through the same geometry. The effect of the inlet pressure, tube length and the size and position of the obstruction is evaluated. These data are modeled using a Transmission Matrix Model.
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Brian M. West
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Chapter 1

Introduction

Gases flowing in tubes driven by solenoid valves are used in many control systems. A solenoid valve is an electromechanical device that opens to allow fluid flow when an electrical signal is provided. Typically, a solenoid valve is “binary,” meaning that it is either open or closed. The frequency response of a solenoid system is a complex function of the components, fluid properties, and operating conditions. Various factors affect the response, such as supply pressure, tube length, and minor losses, but there has been little research on the specific effects of these parameters on the overall response.

Oscillating gas flow in a tube is known to act analogously to a spring-mass-damper or electrical system. This system can be modeled as an equivalent electrical circuit composed of resistors, an inductor, and capacitor in series provided the device dimensions are small compared to the acoustic wavelength. The air between the two ends behaves as a mass, moving back and forth in response to a forcing function. The air also acts as a spring as it compresses and expands due to changes in pressure. Viscous friction damps the system. At low frequencies, the system is well described by an electrical circuit consisting of an in line resistor, an inductor in line, and a capacitor and resistor in parallel going to ground [1]. The volume flow rate of the gas is analogous to current, and pressure is analogous to voltage. This circuit is shown in Fig. 1.1.

The compressibility of the gas is analogous to a capacitance, and is described by the equation for compliance, \( C = \frac{V}{\gamma p} \), where \( V \) is the volume of the gas, \( \gamma \) is the ratio of specific heats, and \( p \) is pressure. The inertia of the gas is analogous to inductance, described by \( L_I = \frac{\rho \Delta x}{A} \) where \( \rho \) is the gas density, \( \Delta x \) is the length of tube, and \( A \) is the cross sectional area [1]. The frequency response of this system is of particular interest in determining limits to control systems. In the experiment presented, however, the flow is not oscillatory. It
Fig. 1.1: Theoretical circuit for a gas filled tube. Compressibility is represented by the capacitor, momentum by the inductor, and losses by the resistances. \( R_C \) is associated with losses in the capacitor and \( R_L \) is associated with losses in the inductor. \( \dot{Q} \) is the mechanical equivalent to current and \( p \) is the equivalent to voltage.

is pulsed or steady flow, but the principles of the circuit in Fig. 1.1 still help predict the behavior of gas flowing through a tube.

The present experiment investigates the variation of magnitude response and wave shape for a gas filled tube as a function of frequency, mean input pressure, tube length, and minor loss

\[
K_L = \frac{\Delta p}{\rho_0 v^2/2},
\]

(1.1)

and will help provide limits for control systems.

An example of an application where this research would be helpful is the flame spray gun used in Coanda-Assisted Spray Manipulation (CSM) at the Experimental Fluid Dynamics Lab at USU, which is shown is Fig. 1.2. The main flow consists of particles that are melted, or partially melted, by a plasma. These particles are sprayed onto a surface to provide different coatings for durability, heat resistance, or other desired properties. The blue tubes are each fed by a solenoid valve and are connected to one of 16 control slots in the aluminum
A control slot is illustrated in green in Fig. 1.3. The flow comes through the back of the aluminum cylinder, is forced to turn 90° to flow perpendicular to the main flow, and is turned back 90° to flow parallel to the main flow and curved wall of the cylinder at the exit of the control slot. When a control slot is turned on, the main flow is influenced by the jet from the control slot to follow the curved surface of the cylinder. This is due to the coanda effect, which is the tendency of a fluid to follow a curved surface if it is near to it. By turning different slots on in sequence, the flame spray can be rotated at different rates or vectored at different angles. This is useful for spraying surfaces that are not flat, such as the inner surface of a pipe. How fast the flame can be rotated depends upon the response of the system consisting of the solenoid valve, tubing, and minor loss caused by the sharp turn in the flow. This CSM system is the motivation for the current study. A system of a solenoid valve, followed by a length of tube and a minor loss was chosen to model the system used in the CSM setup.
Fig. 1.3: Cross section of the flow in the CSM device.

After a review of literature, the experimental facility and measurements will be described. Results and modeling of the data will then be presented.
Chapter 2

Literature Review

Research has been done on the flow inside solenoid valves by Szente and Vad [2]. Their experiments used semi-empirical models to predict loss coefficients through a valve with a Borda-type orifice given the seat angle of the valve. A Borda-type orifice is an orifice with an entrance region that extends upstream in the flow. The seat angle interface between the Borda-type orifice and the valve seal varies. However, their interest was how the seat angle in the valve affected the magnitude of the minor loss through the valve, and the effect that this minor loss has on the frequency response at the exit was not presented.

Braud et al. [3] showed that velocities up to two times higher than the steady state velocity through the same tubes can be obtained when pressure pulses are applied. The effect of waves traveling in the tube were modeled, including their effect on the settling time. They presented a model for the flow at the exit of a gas filled tube in both the continuous and transient cases. The laws of conservation of mass and conservation of energy were used to develop a theoretical solution for the steady flow velocity based on temperature, pressure, ratio of specific heats, and geometry. For the transient case the velocity at the exit is related to a velocity signal traveling at the speed of sound. The reflection of this signal and the time it takes this signal to travel the length of the tube give a simplified solution of the velocity at the exit. This is similar to the Transmission Matrix model described in section 2.1, as it takes the influence of reflected waves into account, but is simpler and therefore less accurate. Also, the effects of pressure and frequency were not presented, and their tube did not have minor losses near its termination.

Whitmore and Fox [4] derived a second-order model for the response of a pressure sensing system with sinusoidal pressure inputs. Their model is based on the Berg and Tijdeman (BT) method, which is a standard for predicting frequency response in pneumatic
systems. The BT method is complex, however, and is fixed in the frequency domain. Often, a simpler method in the time domain is desired. The model developed by Whitmore and Fox is more accurate than previous simplifications of the Berg and Tijdeman method. The principles used in determining the acoustic influences, reflections produced by components, and frequency response of pneumatic systems are helpful in predicting the response of a pulsed system. This analysis differs from the current experiment in that it dealt with no net flow.

2.1 Transmission Matrix Model

Flow through a solenoid valve introduces oscillations to the mean flow. Such steady-oscillatory flow in a piping system can be analyzed either in the time domain or in the frequency domain. A Transmission Matrix model has been developed by Dr. Fei Liu and Dr. Lou Cattafesta at the University of Florida. The Transmission Matrix method has been an effective tool in determining the frequency response of piping systems [5]. A Transmission Matrix (TM) model can be created to relate the input and output pressures and velocities across an element of a flow system. If any two of these quantities are known, the TM can be used to find the other two. An advantage of the TM model is that the pressures and velocities of multiple components in series can be modeled by multiplying the input pressure and velocity by a TM for each component. Instead of developing a new model for each new system, the TMs for each component can be easily combined.

The present piping system, illustrated at Fig. 2.1, consists of duct (pipe) and orifice components. The TM representation for the total system can be deduced once the TM of each component is derived. This model developed by Liu and Cattafesta will be used to model the experimental data to make the results more useful in the design of pneumatic systems.
Fig. 2.1: Illustration of a piping system used for the TM model.

2.1.1 Duct

The TM of duct component, such as pipe 1, 2, 3 and 4 in Fig. 2.1, is given by [6] as

\[
\begin{bmatrix}
    p' \\
    u'
\end{bmatrix}
_{\text{out}} = e^{jM_kcL} \cdot 
\begin{bmatrix}
    \cos(kcL) & -j\rho_0c_0\eta\sin(kcL) \\
    \frac{-j}{\rho_0c_0\eta}\sin(kcL) & \cos(kcL)
\end{bmatrix}
\begin{bmatrix}
    p' \\
    u'
\end{bmatrix}
_{\text{in}}
\]

(2.1)

where \( L \) is the length of the pipe, \( M = u_m/c_0 \ll 1 \) is the Mach number of mean flow, 
\((\cdot)'\) denotes the perturbation of the mean flow at the entrance and exit of the pipe, which
is assumed to be small in comparison with its mean flow counterparts. The air density
and isentropic speed of sound in air are denoted \( \rho_0 \) and \( c_0 \), respectively, and \( \eta \) and \( kc \) are
denoted as

\[
\eta = 1 + \frac{\alpha + \zeta M}{k_0} - j\frac{\alpha + \zeta M}{k_0}
\]

(2.2)

\[
k_c = [k_0 - j(\alpha + \zeta M)]/(1 - M^2)
\]

(2.3)
where $k_0$ is the wave number. The coefficients $\alpha$ and $\zeta$ account for the visco-thermal loss, and are given by

$$\alpha = \frac{1}{a} \sqrt{\frac{\omega \mu}{2 \rho_0 c_0^2}} \left(1 + \frac{\gamma - 1}{\sqrt{Pr}}\right)$$

(2.4)

and

$$\zeta = \frac{\psi}{2D}$$

(2.5)

where Pr is the Prandtl number, $2a = D$ is the hydraulic diameter of the pipe, $\mu$ is the dynamic viscosity, $\gamma$ is the ratio of specific heats, $\psi = 4\tau_w / (1/2\rho_0 U^2)$ is the pipe friction factor (assumed to have a value of 0.0072), $\tau_w$ is the average wall shear stress, and $\omega = k_0 C$ is the angular frequency of the oscillation. The solenoid valve is excited by a non-sinusoidal periodic signal, in this case a pulse. For this reason, the periodic oscillation is decomposed into Fourier series (harmonics), and the system response is the summation of the response to each harmonic.

2.1.2 Orifice

The TM of an orifice is given by [7] as

$$\begin{bmatrix} p' \\ u' \end{bmatrix}_{\text{out}} = \begin{bmatrix} 1 & -\rho_0 c_0 M K_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p' \\ u' \end{bmatrix}_{\text{in}}$$

(2.6)

where the coefficient $K_L$ represents the combination of the effect of the geometry and the vena contracta of the orifice. The coefficient $K$ depends on the orifice geometry and orifice Reynolds number. Typical values of coefficient $K$ as function of Reynolds number are published in [8].

Hence, the TM of the piping system shown in Fig. 2.1 is
\[
\begin{bmatrix}
p'_2 \\
u'_2
\end{bmatrix} = [\text{TM}_{\text{pipe}4}] [\text{TM}_{\text{pipe}3}] [\text{TM}_{\text{orifice}}] [\text{TM}_{\text{pipe}2}] [\text{TM}_{\text{pipe}1}] \begin{bmatrix}
p'_1 \\
u'_1
\end{bmatrix}.
\] (2.7)

Moreover, because the exit of the piping system is an unflanged open-end of a duct of radius \( r_0 \) with a flow of Mach number \( M \), the radiation flow impedance of the orifice is given by [9–11],

\[
\frac{p'_2}{u'_2} = Z_r(M) = R_r(M) + jX_r(M)
\]

\[
R_r(M) \approx R_r(0) - 1.1M \rho_0 c_0
\]

\[
X_r(M) \approx X_r(0)
\]

\[
Z_r(0) = R_r(0) + jX_r(0) = \rho_0 c_0 \frac{1 + |\Re|}{1 - |\Re|}
\] (2.8)

where the reflection coefficient \( \Re \) at the orifice is

\[
\Re = |\Re| e^{j(\pi - 2k_0\delta)}
\]

\[
|\Re| \approx 1 + 0.01336k_0r_0 - 0.59079(k_0r_0)^2 + 0.33756(k_0r_0)^3 - 0.06432(k_0r_0)^4
\] (2.9)

for \( k_0r_0 \leq 1.5 \), where \( \delta \) is the end correction which is given by

\[
\delta = \left[ 0.6133 - 0.1168(k_0r_0)^2 \right] r_0 \quad k_0r_0 \leq 0.5
\]

\[
= \left[ 0.6393 - 0.1104k_0r_0 \right] r_0 \quad 0.5 < k_0r_0 < 2.
\] (2.10)

Thus, from Eq. (2.7) and (2.8), once one of the variables \( (p'_1, u'_1, p'_2, \text{ and } u'_2) \) is known, other variables can be resolved. For all cases, the pressure after the solenoid valve and mean flow velocity at the exit are known.

### 2.2 Fanno Flow in Ducts

The TM described above does not take compressible effects into account. Properties are assumed to be constant. However, friction in the tubes causes a change in properties.
For longer tubes this becomes more significant. Fanno flow theory can be used to obtain a more accurate prediction of the flow characteristics in a tube.

A derivation of Fanno flow is found in [12]. Fanno flow deals with the effects of viscous friction between the fluid and walls of a tube. Modeling friction as wall shear stress and including this with the 1D momentum equation yields the differential equation

$$\frac{4fdx}{D} = \frac{2}{\gamma M^2} (1 - M^2) \left[ 1 + \frac{1}{2} (\gamma - 1) M^2 \right]^{-1} \frac{dM}{M}. \quad (2.11)$$

Defining the location at which $M = 1$ as $x = L^*$ and integrating from $x = 0$ to $x = L^*$ results in the equation

$$\frac{4f_{ave}L^*}{D} = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma + 1}{2\gamma} \ln \left[ \frac{(\gamma + 1) M^2}{2 + (\gamma - 1) M^2} \right]. \quad (2.12)$$

Ratios of pressure, $p$, density, $\rho$, temperature, $T$, and total pressure, $p_o$, are given as

$$\frac{T}{T^*} = \frac{\gamma + 1}{2 + (\gamma - 1) M^2} \quad (2.13)$$

$$\frac{p}{p^*} = \frac{1}{M} \left[ \frac{\gamma + 1}{2 + (\gamma - 1) M^2} \right]^{1/2} \quad (2.14)$$

$$\frac{\rho}{\rho^*} = \frac{1}{M} \left[ \frac{2 + (\gamma - 1) M^2}{\gamma + 1} \right]^{1/2} \quad (2.15)$$

$$\frac{p_o}{p_o^*} = \frac{1}{M} \left[ \frac{2 + (\gamma - 1) M^2}{\gamma + 1} \right]^{(\gamma + 1)/[2(\gamma - 1)]}. \quad (2.16)$$

Starred quantities refer to properties for sonic conditions. Equation 2.12 can be used to solve for Mach number further down the tube. From Fanno flow theory we learn that Mach number is always driven towards unity by friction. For supersonic inlet flow friction causes the Mach number to decrease, the pressure to increase, the temperature to increase, and velocity to decrease. For subsonic flow the Mach number increases, pressure decreases, temperature decreases, and velocity increases due to friction. For both cases, the total
pressure always decreases. These effects are due to the fact that the point of maximum entropy occurs for sonic flow, where $M = 1$. This illustrated by a Fanno curve, such as the one depicted in Fig. 2.2.

2.3 Scattering Matrix for an Orifice

Liu and Cattafesta have proposed a quasi-steady compressible model for an orifice. The 2D geometry consists of uniform inflow, a jet region with a vena contracta caused by the orifice, a turbulent mixing region, and uniform outflow. Flow perturbations are represented by acoustic waves $p^i_1$ and $p^i_2$ ($i = 1, 2, j$), which are the incident and reflected waves, respectively. The index $i = 1$ corresponds to the region upstream of the orifice, $i = 2$ to the region downstream of the orifice, and $i = j$ to the jet region. It is assumed that the perturbations are harmonic with radial frequency $\omega$. The pressure perturbations near the orifice are $p^i_1 = p^i_1 + p^i_2$. Velocity perturbations are given by $u^i_1 = \frac{p^i_1 + p^i_2}{\rho_1c_i}$, and density perturbations by $\rho^i_1 = \frac{p^i_1 + s^i_1}{c_i^2}$, where $c$ is the local speed of sound and $s$ is entropy. Introducing a linear perturbation to the continuity equation, $\rho_1u_1 = \rho_2u_2$, yields
\[
\frac{1}{c_1} \left[ p_{1t}^{l+} (1 + M_1) - p_{1t}^{l-} (1 - M_1) \right] = \frac{1}{c_2} \left[ p_{2t}^{l+} (1 + M_2) - p_{2t}^{l-} (1 - M_2) + s^t M_2 \right]
\] (2.17)

and
\[
\frac{1}{c_1} \left[ p_{1t}^{l+} (1 + M_1) - p_{1t}^{l-} (1 - M_1) \right] = \frac{A_i}{A_j} \frac{1}{c_j} \left[ p_{jt}^{l+} M_j + \rho_j u_j c_j \right]
\] (2.18)

where \( A_i \) is the cross sectional area. The linear perturbation of the momentum equation yields
\[
\left( 1 + \frac{A_j}{A_2} M_j^2 \right) p_{jt}^{l+} + 2 u_j \rho_j c_j M_j \left( \frac{A_j}{A_2} \right) = (1 + M_2)^2 p_{2t}^{l+} + (1 - M_2)^2 p_{2t}^{l-} + M_2 s^t. \]

(2.19)

The linear perturbation of the conservation of total enthalpy yields
\[
\frac{1}{\rho_1} \left[ (1 + M_1) p_{1t}^{l+} + (1 - M_1) p_{1t}^{l-} \right] = \frac{1}{\rho_2} \left[ (1 + M_2) p_{2t}^{l+} + (1 + M_2) p_{2t}^{l-} - \frac{s^t}{\gamma - 1} \right]
\] (2.20)

and
\[
\frac{1}{\rho_1} \left[ (1 + M_1) p_{1t}^{l+} + (1 - M_1) p_{1t}^{l-} \right] = \frac{1}{\rho_j} \left( \rho_j c_j M_j u_j + p_j \right).
\] (2.21)

From equations 2.17 through 2.21 a scattering matrix could be developed for an orifice using theory presented by Hofmans et al. [7]. Modeling of an orifice is expanded from previous models by Hofmans to include compressibility effects and a dependence on Mach number for the vena contracta factor. This factor is the ratio of the cross-sectional area of the jet to that of the orifice.

### 2.4 Wave Propagation

The scattering matrix relates the waves traveling upstream and downstream from the orifice. Oscillations in velocity are coupled to pressure gradients, and oscillations in pressure are coupled to velocity gradients, as explained by Swift [1]. Wave propagation is tied to these oscillations, and depends primarily on inertia and compressibility. Viscosity causes the attenuation of waves at solid boundaries. Attenuation can also be caused by thermal
relaxation. Thermal relaxation refers to the fact that temperature oscillations are caused by pressure oscillations in a gas, and these oscillations are reduced in magnitude near a solid boundary. This is because solids generally have higher heat capacities and maintain a constant temperature. Fluid near a solid boundary is maintained close to this temperature, so oscillations decrease. It is important to note that when the flow in the jet region becomes sonic, no information can travel in either direction across this region. Depending on the ratio of the tube and orifice areas, a relatively low Mach number in the tube can result in sonic flow in the jet, and choking occurs. Even if higher pressures are applied upstream, the velocity can not increase beyond choked conditions.

In the current experiment, the effects of the characteristics of the tube are studied further with varying minor losses, tube lengths, and pressures. Fanno flow theory will be applied to the TM for a duct. The TM model will then be compared to the experimental results.
Chapter 3

Purpose and Objectives

The main purpose of the current study is to determine how fast a control system responds under various conditions. Of specific interest is the Coanda-Assisted Spray Manipulation system. Objectives of the thesis are:

- Present a universal criteria for determining the usefulness of a velocity signal for pulsed flow
- Predict Standard deviation of the velocity at the exit of a specified system
- Validate predictions with experimental data
- Determine the effects of input pressure, frequency, tube length, and minor loss on the response of a system.
Chapter 4

Data Acquisition

The methods used to gather data are described, along with some discussion of concerns and uncertainty.

4.1 Experimental Setup

A picture of the test setup is shown in Fig. 4.1. Compressed air is supplied from a volume booster and pressure regulator controlled by a DAQ board. A pressure sensor is used along with Labview to set a desired pressure output. This air is fed through tubes to a solenoid valve, which is controlled by the DAQ board through a solid state relay. Another tube connects the valve to a block containing a high-speed pressure sensor. This is followed by more tubing connecting this block to a minor loss element. After the minor loss, tubing connects it to an exit block that creates a stationary exit plane at which a hotwire probe is placed to measure the velocity of the air. The instruments used will be described, followed by a description of the flow system.

4.1.1 Instruments

Compressed air is supplied to a volume booster from Control Air Inc, type 600, which has a sensitivity of .25 inches of water and an accuracy of 1% of output span. Pressure is regulated through an electropneumatic transducer from Control Air Inc, type 500X, upstream of the volume booster. The range for input voltage to the transducer is 0 to 10 Volts. It has a sensitivity of 0.1% of span per 1-psig and a response time of less than 0.25 seconds. The control voltage to the transducer is controlled using an analog signal from a 16-bit DAQ board from National Instruments, model NI PCI - 6221, a junction box, model number BNC-2120, and LabVIEW software version 8.2. The DAQ board has a maximum
Fig. 4.1: Experimental setup for a system with 3.5 inches of tube length. Dimensions are given in inches. The probe is positioned at the exit plane using a camera with microscope optics above the exit. The 2-inch exit block holds the tube to allow a hotwire probe to be safely placed at the exit. The minor loss holder is located at 3.5 inches, and the pressure sensor is at 5.8 inches. The solenoid valve is at the right hand side of the image. For longer tube lengths, the tube between the minor loss and the pressure sensor is changed.
sampling rate of 250,000 samples per second. A pressure transducer from Omega, model number PX309-150 G5V, is placed at the output of the volume booster and its analog output is sampled using the DAQ board. The Omega pressure transducer has a range of 0 to 150-psig and an accuracy of 0.25% of full scale. The factory calibration was used. The Labview software sets the time-averaged pressure from the volume booster to the desired value by reading the pressure from the Omega sensor and adjusting the voltage to the electropneumatic transducer.

The hotwire probe is custom made by Auspex Co., model AHWU-100-USU. It is a 1-mm long sensor that is 5-µm in diameter, and is shown in Fig. 4.2. The probe is constructed by soldering a coaxial cable between the two prongs. An acid is then applied on the coaxial cable. This acid eats the outer layer of the cable, leaving the very fine tungsten strand. The probe is connected to a constant temperature anemometer from TSI, model number IFA-300, and its output is read by the DAQ board. The resistance of the cable connecting the probe to the anemometer and the resistance of the probe are measured first. The operating resistance is set to the probe resistance times 1.4. When the anemometer is set to run, the tungsten wire heats up due to an electrical current. As air passes over the wire, heat is transferred away from the wire. The change in temperature causes a change in the resistance of the probe. The probe is connected to a resistance bridge with a known voltage across the bridge. When the resistance of the probe changes, the voltage across the bridge changes. The anemometer works by adjusting the voltage to the hotwire probe to maintain the same voltage across the bridge. This voltage can be recorded and used to determine the velocity of the air by calibrating the hotwire probe.

The hotwire probe was calibrated using a calibration unit on site. Compressed air is supplied to a chamber with a small hole for air to escape from. The volume of the chamber is sufficiently large compared to the diameter of the hole that the flow in the chamber can reasonably be approximated as having zero velocity. At the time of calibration, ambient temperature, barometric pressure, and relative humidity are measured and recorded. From these properties, the density of the ambient air is calculated. A baratron from MKS Instru-
ments, model number 698A12TRA, is connected to the calibration chamber to read the the gague pressure. This baratron has a range of 0 to 100-mmHg. The resolution is $10^{-6}$ of full scale, and the accuracy is listed as 0.12% of the reading. A signal conditioner from MKS Instruments, model number 270D-4, displays the output from the baratron. The pressure from the baratron and voltages from the hotwire probe are recorded for values between 0 and 100 mmHg. The pressure values are converted to Pascals. Velocities in meters per second are then calculated using Bernoulli’s equation:

$$v_2 = \sqrt{\frac{2\Delta p}{\rho}}. \quad (4.1)$$

A third order polynomial curve fit is applied to the calibration data in Matlab to compute the air velocity from the hotwire voltage. An example calibration is shown in Fig. 4.3

An Endevco piezo-resistive pressure sensor, model 8510B-5, is placed in a block after the solenoid valve. Although a factory calibration was supplied, a new calibration was done to extend the the manufacturer’s calibration from the linear range for 0 to 5-psi, to 0 to
Fig. 4.3: Plot comparing the data collected from calibration and the curve fit from Matlab. The data from calibration is represented by (○), the polynomial curve fit for the higher velocity region by the solid black line, and the curve fit for the lower velocity region by the dashed red line.
15-psi. It was calibrated using a manometer, pvc pressure vessel, and a G3006 differential amplifier from AA Lab Systems LTD in a G-3000 series mainframe. The pressure vessel was constructed out of a coupler and two end caps. The end caps were glued in, and three holes were drilled and tapped. One hole was threaded to fit the Endevco sensor, a second fit a valve for a bike pump, and the third fit a barbed tube fitting. A tube was connected between the pressure vessel and the manometer. An excitation voltage of 0.592 Volts was supplied from the amp to the Endevco sensor, and the output from the sensor was read and displayed by the amp. The gain was set to 100, which is the minimum for the amp. The pressure vessel was pressurized using a bicycle pump. The fluid level of the manometer and the voltage output by the sensor were recorded for manometer levels between 0 and 29-inches of mercury. A curve fit, shown in Fig. 4.4, was fit to the data, and used to calculate pressure in Pascals as a function of voltage from the sensor.

The voltage to the solenoid valve is controlled by a solid state relay, model number HFS33, driven by the DAQ board. The relay has four labeled connection terminals. The positive voltage from a power source was connected to terminal 2+ on the relay. Terminal −1 is connected to a wire to the solenoid valve. This wire passes the voltage from the power source to the solenoid. A digital control voltage from the NI junction box was connected to terminal 3+, and the digital ground was connected to terminal −4. Square waves of various frequencies are sent through the relay to create pulses in the gas-filled tube.

The solid state relay was checked to determine its influence on the response. Fig. 4.5 shows the signal sent to the relay from the DAQ board and the output from the relay. These signals were viewed on an oscilloscope and sampled with the DAQ board. The output from the relay follows the input very closely. The difference in magnitudes is not important. It is merely passing a digital on or off signal to the solenoid. As can be seen in Table 4.1, the relay did contribute to a phase lag for the response of the system, but the magnitude was not attenuated. The phase angle delay for turning on was less than for turning off.

4.1.2 Flow System

The air from the volume booster is split into parallel using brass pipe parts and fed
Fig. 4.4: Calibration curve for the pressure sensor from Endevco.

Table 4.1: Relay Response

<table>
<thead>
<tr>
<th>f [Hz]</th>
<th>$\phi_{on}$ [Deg]</th>
<th>$\phi_{off}$ [Deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td>100</td>
<td>18</td>
<td>28.8</td>
</tr>
<tr>
<td>200</td>
<td>21.6</td>
<td>50.4</td>
</tr>
</tbody>
</table>
Fig. 4.5: Plot of the input from the DAQ board, (□), and the output from the relay, (○)
through two 5/32-inch inner diameter tubes into a block from FESTO, part number 197446, via push-in fittings, part number 183740. The other two ports into the block were plugged. The solenoid valve from FESTO, part number 196123, is mounted on this block, and the other slots for valves were closed with blanking plates from FESTO. An excitation voltage of 24-V was supplied from the solid state relay through a cable from FESTO, part number 193691, with one wire connected to terminal 1- on the solid state relay, and the other connected to -V on the power source. A push-in fitting, part number 192808, connects a tube to the solenoid valve, and this tube connects the valve to a block containing a high-speed pressure sensor from Endevco and a flow passage of the same ID as the tube. Another tube connects this block to the variable minor loss. The length of the tube between the minor loss and pressure sensor is the length that was varied in experiments. After the minor loss element, a short section of tubing connects the minor loss to an exit block, shown in Fig. 4.6. Two concentric holes were drilled horizontally in this block. A hole the with a diameter equal to the inner diameter of the tubing used was drilled through the entire block. A second hole has a diameter equal to the outer diameter of the tubing, and was drilled partway through the block. The tubing is inserted in this second hole. The bottom of this hole was made square so that the end of the tube sits flush against it so there is as little gap as possible between the tubing and the passage in the exit block.

After Calibration, the hotwire probe is placed on a traverse from velmex, model MA2521K2-S4, and positioned at the exit plane. The traverse moves the probe horizontally 2-mm for every revolution of the screw in the traverse. A PULNiX camera, model TM-9701, and microscope were used to make placing the probe at the exit plane more accurate. Fig. 4.7 is a picture of the camera and the monitor used to view the position of the probe. Labview is used to set the desired source pressure. A Labview program is run that generates a square wave to drive the solenoid and then records the time, voltage from the hotwire probe, and voltage from the endevco sensor, and writes these values to a text file.

The minor loss is varied by placing disks with different sized sharp-edged holes drilled through their center inside an orifice holder. Losses are modeled as a sudden contraction and
Fig. 4.6: Block that holds tubing in place so a hotwire probe can be safely placed at the exit plane.

A sudden expansion. Losses were computed for an expansion using \( K_L = \alpha (1 - d^2/D^2) \) [13]. Values for \( K_L \) for a sudden contraction were taken from [13]. A sketch of the orifice holder is shown in Fig. 4.8.

The solenoid valve is driven by a 50% duty cycle square wave at various frequencies. The driving wave is the output from the relay in Fig. 4.5. Sampling rates are varied to provide independent samples and to capture a fixed number of periods for each non-steady case. In this experiment, 250 periods were captured for each case. All cases capture 10,000 data points. Rates and sampling times are given in Table 4.2. Parameters varied in the experiment are shown in Table 4.3. Pressure data were not collected for cases with a 30 psi source because the Endevco sensor is only rated to 15 psi.

Matlab code was used to process the data for all 320 cases. The data from the text files produced by Labview were read in, and subroutines were used to convert the voltages from the hotwire probe and endevco sensor into velocity and pressure, respectively. General parameters were calculated, such as \( U \) and \( U^* \), and the velocity data was phase averaged over all 250 waves to get an average waveform for each case. This waveform could then be
Fig. 4.7: Monitor and microscope used to view the position of the probe. This setup is used to allow more accuracy in placing the probe in the center of the flow at the exit plane of the tubing system.

Fig. 4.8: Minor loss element. Discs with varying center holes are held between the two pieces to create a sudden contraction and expansion.
Table 4.2: Sampling Parameters

<table>
<thead>
<tr>
<th>Signal Frequency $f$ [Hz]</th>
<th>Sampling Frequency $f_s$ [Hz]</th>
<th>Sampling Period $T$ [sec]</th>
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<td>4000</td>
<td>2.5</td>
</tr>
<tr>
<td>200</td>
<td>8000</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Table 4.3: Experiment Variables

<table>
<thead>
<tr>
<th>$f$ [Hz]</th>
<th>$K_L$</th>
<th>Tube Length [in]</th>
<th>Pressure [psi]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.346</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>50</td>
<td>0.815</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>100</td>
<td>1.135</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>200</td>
<td>1.454</td>
<td>100</td>
<td>30</td>
</tr>
</tbody>
</table>

plotted to observe the shape of the wave in each case.

4.2 Uncertainty Analysis

The hotwire probe was aligned in the calibrator manually. Care was taken to place the probe as closely in the center of the opening and at the exit plane as possible. To assure that the human error in placing the probe was negligible, the probe was removed and replaced in the calibrator facility several times for a low and high pressure. The voltage from the hotwire probe was recorded each time. At a set pressure there was no significant difference between trials. The pressure sensor used has a resolution of 0.0001 mm Hg. Atmospheric pressure was read from a barometer with a resolution of 0.1 mm Hg. The calibration data were fit to a 3rd order polynomial and the error between the calibration points and the curve was computed. This error was kept below 5% of the velocity. The curve fit was often split in two sections, as shown in Fig. 4.3, to keep error small across the broad range of velocities needed. For high pressure, very large velocities were experienced, but the ability to accurately report low velocities was still required because the valve was shut for part of the cycle.

To collect data the probe was again aligned manually, but a traverse was used to make
placing the probe right at the exit plane much easier and more accurate. The velocities were then phase averaged over 250 cycles to produce one characteristic period. The precision uncertainty for each point on the wave is given by $1.96 \times \frac{\sigma}{\sqrt{N}}$ with 95% confidence, where $\sigma$ is the standard deviation of the phase-averaged data. Precision uncertainty was kept within 5% of the mean by increasing $N$ to 250 for all cases.

4.3 TM Code

Matlab code was provided by Dr Fei Liu and Dr Lou Cattafesta of the University of Florida. This code decomposes the pressure signal directly after the solenoid valve into the frequency domain. A TM is then calculated for each component and a velocity component at the exit is calculated. The code was modified from what was supplied by Liu and Cattafesta to reconstruct these components back into a waveform. A subroutine was also added to discretize the variable tube length and calculate the Mach number in each section using fanno flow theory. This was done to better model the compressible effects in the tube. Results will be compared to experimental data.
Chapter 5

Results

5.1 System Behavior

For most flow control applications, pneumatic control relies on both the largest and smallest velocity. In other words, achieving a large velocity at the exit is of little use if the velocity does not also reduce to near zero. After trying several other measures of the pulse “robustness,” we have settled on using the standard deviation of the velocity signal as a measure, since it is sensitive to both the maximum and minimum velocity. The standard deviation of the velocity signal is used to help measure how close to a square wave the velocity signal became due to the flow system. If the velocity fails to return to a zero velocity between pulses, or if the peak velocity is attenuated, the standard deviation decreases.

The standard deviations of the velocities are normalized by half the steady flow (0 Hz, corresponding to when the valve is open) value since the standard deviation for a square wave with a duty cycle of 50% that goes from 0 to $A$ is $A/2$. By normalizing the standard deviation of the phase-averaged velocity ($U$) by half the steady flow velocity in the same system ($U^*$), a perfect square wave output will result in a value of unity while steady flow will result in a value of 0. Therefore, in general, $0 < U/U^* < 1$.

In all cases, the system response as a function of frequency is compared to steady flow through the same system. Sonic flow is suspected at points in the tube for some operating conditions, and is probable in the minor loss, especially for cases with the highest loss, since this corresponds to the smallest flow area. Steady flow was measured at various source pressures to observe general trends in the behavior of the average velocity. In Fig. 5.1, the variation of velocity with the supply pressure for steady flow is shown. The trend in the data appears to follow a power law and looks similar to the data of [3]. This was not the
Fig. 5.1: Steady flow velocity (divided by 2) as a function of source pressure for a minor loss value of $K_L = 1.135$. $L = 1 \text{ in} (\circ)$, $L = 10 \text{ in} (\square)$, $L = 30 \text{ in} (\diamond)$, $L = 100 \text{ in} (\triangle)$

case for the largest $K_L$ value, which is shown in Fig. 5.2. It is believed that the change in behavior is due to compressible effects. The pressure does not scale with $\rho v^2$ in most cases, which is likely due to compressible effects. With the pressure regulator available to us, the lowest supply pressure we could maintain was 2 psig, and even for lower pressures the steady state velocities did not scale with $\rho v^2$. In Fig. 5.1, data for all lengths of tube follow the same trend, but contrary to what was expected, the case for the shortest tube produced the lowest velocities. It is theorized that friction in the tubes is causing the flow to accelerate towards sonic flow, as explained in Fanno flow theory.

Once the steady-flow system behavior was known, the impact of pulsation frequency
Fig. 5.2: Steady flow velocity (divided by 2) as a function of source pressure for a minor loss value of $K_L = 1.454$. $L = 1\text{in} (\circ), L = 10\text{in} (\square), L = 30\text{in} (\Diamond), L = 100\text{in} (\triangle)$
was determined for a variety of combinations of tube lengths and minor losses. Examples of how the signal frequency and tube length influence the output velocity waveform are shown in Fig. 5.3 and Fig. 5.4. The effect of the magnitude of the minor loss is shown in Fig. 5.5. In each of the plots, \( v/v^* \) is the phase-averaged velocity normalized by the mean steady velocity (note that \( U^* = v^*/2 \)) and \( t/T \) is time normalized by the cycle period.

In Fig. 5.3, waveforms from four different cases are shown. All experimental parameters except frequency were the same for all four cases. As expected, the phase angle between the excitation of the relay and the rise in velocity at the exit plane increased with frequency. This is likely due mainly to the time it takes the velocity wave to travel from the opening in the valve to the exit plane. The distance did not change for higher frequencies, but the proportion of the period it took for the velocity to travel the length of the tube increased because for a higher frequency the period is shorter. The increase in phase angle due to an increase in frequency appears to be linear, which supports this theory. The general shape of the waveforms was similar to those recorded by Braud et al. [3], especially for the 10 Hz case. The velocity rises to a sharp peak, and then oscillates around a steady state value. In the current study, velocities close to twice that of the steady state value were not observed. However, there were no minor losses in the study by Braud. The velocities attenuated slightly with an increase in frequency, as was expected, except for the case driven at 200 Hz. This may be close to a resonant frequency. The falling edges of the signals appear to fall more slowly than the rising edges rise. This is probably not due to the solenoid, as the manufacturer specifies a slightly faster time for closing the valve compared to opening.

The only parameter varied in Fig. 5.4 was the length of tube between the solenoid and the minor loss. The general trend of signal degradation with increasing length is expected. The signals from the longer tube length were attenuated, but it appears that a steady velocity is approached instead of a zero velocity. The momentum in a system increases with increasing tube length, and is analogous to inductance. An inductor allows a DC voltage to pass, but resists fluctuations in the voltage. Similarly, the steady value (DC) of velocity for an oscillating pneumatic system passes more readily through a long tube than
Fig. 5.3: Phase-averaged dimensionless velocity vs. \( t/T \) for several frequencies. As frequency increases the phase angle increases and the signal approaches a flat line. The dashed lines are the signal from the solid state relay. A tube length of 3 inches, \( K_L = 0.815 \), and an input pressure of 2 psig was used. The data for each case has been phase shifted to align the beginning of the excitation of the relay to allow for easy comparison of the phase angles between the relay response and the exit plane velocity.
Fig. 5.4: Phase-averaged dimensionless velocity vs. $t/T$ for several lengths. An increase in tube length increases the capacitance of the system, which causes the signal to flatten out. A minor loss of 1.135, an input pressure of 10 psi, and frequency of 10 Hz was used. The data from each case has been phase shifted to make comparison of the signals easier.

The minor loss had a large impact on the response, as can be seen in Fig. 5.5. Forcing the flow through an orifice causes a pressure drop and also a decrease in velocity, as was to be expected. It was not expected that the signals would tend to flatten out at a non-zero velocity. It is noted again in this figure that the velocity initially rises more quickly than it falls. This is very pronounced for the highest minor loss. It is not known why the signal tends to be more rounded for $K_L = 0.815$ and $K_L = 1.135$, but is sharper again for
Fig. 5.5: Phase-averaged dimensionless velocity vs. $t/T$ for several minor loss values. The minor loss has a large impact on the magnitude of the response. The loss also contributes to a slow discharge time, which flattens the signal. A tube length of 10 inches, an input pressure of 30 psi, and frequency of 10 Hz was used.

$K_L = 1.454$, which has a peak followed by an oscillation in velocity.

Data were acquired with no minor loss and a 1-inch tube to examine the response of the valve. The data are shown in Fig. 5.6. The manufacturer specifies the nominal time for the valve to open and close as 1.9 ms and 1.7 ms, respectively. Given these values and backed by earlier experiments, the cutoff frequency for the valve is about 278 Hz. The on and off times do produce phase lag, but it is about the same on both ends. For low pressure, the system was more sensitive to frequency. In the circuit shown in Fig. 1.1 the equivalent capacitance is described by $C = \frac{V}{\gamma p}$. As pressure is reduced, the capacitance is increased,
Fig. 5.6: The frequency response for a system with no minor loss and minimal tubing, which causes frequencies to be attenuated more.

Measurements were made for all 320 combinations of variables. The ratios of $U/U^*$ for all cases are shown in Fig. 5.7, Fig. 5.8, Fig. 5.9, and Fig. 5.10. The standard deviation of the velocity normalized by the steady flow value divided by 2, $U/U^*$, is graphed along the y-axis. This represents the standard deviation of the velocity normalized by the standard deviation of a square wave with an amplitude equal to the steady flow velocity through the same system. The response generally drops off with frequency as expected, although, in some cases, it increases. This is especially true for the longer tubes which may have resonant frequencies in the range of the excitation frequencies and reflecting waves. Generally, as the frequency increases, the pulses become less distinct until they deteriorate to a steady state.
velocity. Any value below about 0.6 is deemed not useful for control. A solid black line was added at $U/U^* = 0.6$ for reference. This value was chosen as rough guideline by observing the shape of the waveforms of many of the cases.

Some other trends are noted:

1. While a larger input pressure results in larger velocity at the exit for steady flow, larger pressures result in reduced performance for pulsed flow.

2. This becomes more true for longer tubes. For the longest tube in this study, a 30 psig supply case could not be successfully pulsed even at 10 Hz.

3. The minor loss has an effect similar to the tube length.

The general downward trend with increasing frequency in Fig. 5.7 was anticipated. However, two curves increased for a frequency of 200 Hz. Since the other parameters stay the same for a given curve, this behavior is probably due to resonance and reflected waves. It can be observed that higher losses result in lower values of the normalized standard deviation. For $K_L = 1.454$ it would be very difficult to produce a signal usable for control above just 10 Hz. It is noted that no parameter was completely consistent in its effect on the response of the system, indicating that the parameters are coupled to each other. For example, the curve for 30-psi and $K_L = 0.815$ crosses the curve for 10-psi and $K_L = 1.135$ twice. It is unexpected that a higher minor loss would perform better for any case. A lower source pressure usually performed better than higher source pressures. However, for the lowest minor loss, $K_L = 0.346$, the trend was the opposite.

For the cases with a 10 inch length of tube, shown in Fig. 5.8, a pressure value of 10 psi gave better results than 2 or 30 psi for a minor loss of $K_L = 0.346$. It is possible that 2 psi is insufficient to rapidly charge the system. On the opposite side of the spectrum, 30 psi may flood the system with pressure above a value that can produce any further increase in velocity. For the other minor loss values, a source pressure of 2 psi produced a more distinct signal, similar to cases with a 1-inch length of tube. For these higher minor loss values, $U/U^*$ drops off very quickly with increasing frequency. As is expected, more cases
Fig. 5.7: Normalized standard deviation for 3-inch tube length. The curves become less ordered as frequency increases because the signal is no longer a distinct pulse and random noise is more of an influence. In this paper a value of $U/U^*$ below 0.6 is defined as no longer being a distinct pulse.
Fig. 5.8: Normalized standard deviation for a 10-inch tube length. One value rises above a value of 1 due to spikes in the transient phase [3]. The spikes act as outliers, increasing the standard deviation for the period. Legend shown in Fig. 5.7.

fall below $U/U^* = 0.6$ for cases with a 10-inch length of tube than for a 1-inch length. This makes sense, as the longer tube length creates more entropy and a higher drop in total pressure [12]. It is once again noted that the response of the system is a complex relation of all the parameters tested, as many of the lines cross each other at different frequencies.

For the 30-inch cases, shown in Fig. 5.9, several of the curves appear to have a peak at 100 Hz, and multiple others seem to increase for 200 Hz. However, the 200 Hz cases are probably unusable for control. Again, 10 psi gave the best results for the lowest minor loss, and 2 psi generally gave better results for the higher loss values. Also, the values of $U/U^*$ were lower than those for the 10-inch cases, as is expected.
Fig. 5.9: Normalized standard deviation for a 30-inch tube length. Legend shown in Fig. 5.7.
For most of the cases with a 100-inch tube, shown in Fig. 5.10, the value of $U/U^*$ is below 0.6. This makes sense, as the inertance of the longer tubes is higher, and this attenuates the signal from the solenoid [1]. Differing from the other lengths, a source pressure of 10 psi with the lowest minor loss only produced better results than a 2-psi source for 100 and 200 Hz. No definite peaks in the curves are observed for cases with 100-inch tubes. Many of the curves drop quickly and then remain fairly flat, indicating that even 10 Hz is probably beyond the cutoff frequency for the pneumatic system. These curves rise slightly with frequency, but not to significantly better values of $U/U^*$.

The reason that a steady value is approached as opposed to a zero velocity is not fully
Fig. 5.11: Velocity trace showing the difference in slope between a rising and falling edge. The case having a tube length of 100 inches, minor loss of 0.346, pressure of 30 psi, and frequency of 50 Hz is shown in the figure.

understood. It is hypothesized that this is due to the capacitance of the gas, and that the “capacitor” can be charged more quickly than it can be discharged. An example of this is shown in Fig. 5.11, which is a plot of one full phase averaged cycle for one data set. The x-axis is time divided by the cycle period, and the y-axis is the normalized velocity. A sharp rise can be seen in the plot as well as a slow fall. Because of this slower fall time, the waveform does not return to a zero value before rising again.

The losses between the pressure source and the variable length of tube are less than the losses from the tube to the exit. This slows the discharge of the air in the tube and contributes to the slower fall time. The setup was reversed with a 10-inch tube to observe
Fig. 5.12: Phase averaged period for a reversed setup. The minor loss is placed immediately after the solenoid valve and is followed by a length of tube. In this setup it appears to have a slower rise and a quicker fall. A 10-inch tube length, minor loss of 0.815, pressure of 10 psi, and frequency of 10 Hz is shown in the figure.

The minor loss was placed directly after the solenoid valve, followed by a length of tube. Even with the shorter length a slower rise time can be seen in Fig. 5.12.

5.2 TM Comparison

The TM appears to work well for cases with a low source pressure and low minor loss. Fig. 5.13 shows the waveform predicted by the TM model and the waveform obtained from the hotwire. The TM model predicts the shape of the waveform well. The model doesn’t do
as well predicting the response for higher minor loss elements, where compressible effects are more important. This can be seen in Fig. 5.14.

In Fig. 5.13 the TM model does a good job of predicting the general shape the waveform. It has a sharp peak in velocity, followed by an oscillation about a steady value. It does overestimate the magnitude of the peak, and oscillate more than the experimental data shows. The TM prediction follows the falling edge of the experimental data, but once again overestimates the magnitudes, and does not follow the slight rise in velocity immediately after the falling edge. However, since a hotwire probe cannot distinguish between a positive and a negative velocity, it is possible that the velocity really does dip below zero at that
Fig. 5.14: Comparison of the waveform predicted by the TM model and the one obtained from experiment. The tube length was 3 inches, $K_L = 1.1356$, source pressure was 2 psi, and the frequency was 50 Hz. Experimental Data (○), TM Prediction (□)

point. Part of the extra oscillation after the rise or fall is probably due to the truncated Fourier series used to decompose the pressure data before applying the TM. Also, the TM for an orifice assumes incompressible flow, so compressibility effects there are neglected.

In Fig. 5.14 the TM model does much worse than in Fig. 5.13. The difference was the higher minor loss element. The TM grossly overestimates the velocity fluctuations in the system, and it also doesn’t predict the shape well. The data from the hotwire probe produce a fairly square signal, while the TM prediction is closer to a sine wave. It is obvious that compressible effects can not be neglected in the modeling of flow through an orifice.

The TM model was used to estimate the standard deviation of the velocity for several
cases. Fig. 5.15 shows a comparison of TM estimates and experimental results for a 3-inch length of tube. The model does better for lower minor losses. This was expected since the orifice TM assumes incompressible flow. The TM prediction is also better for lower pressures. This makes sense because higher pressures would correspond to larger compressible effects. It is surprising that the TM is so much worse at predicting the response for a case at 10 Hz than for higher frequencies.

Fig. 5.16 show a comparison of TM estimates and experimental results for a tube length of 100 inches. The TM estimates deviate from the experimental results as pressure is increased, and more drastically as the minor loss coefficient increases. It is interesting that the TM does a better job at predicting the response of the system with the lower minor loss with a longer tube. The prediction of the response for 10 psi and a 100-inch tube is markedly better than the 3-inch tube prediction. With the longer tube and lower minor loss, it is possible that the tube is the more dominant factor on the response of the system. The TM prediction assumes compressible flow in the tube, but not in the orifice, so it makes sense that it would do better in cases where the tube effects are more important than those of the orifice. As in the cases with a 3-inch tube, the TM predictions are way off for the higher minor loss value, but the best prediction is for 100 Hz in the 100-inch case, and at 200 Hz in the 3-inch case.
Fig. 5.15: Comparison of $U/U^*$ predicted by the TM model and the one obtained from experiment. The tube length was 3 inches.
Fig. 5.16: Comparison of $U/U^*$ predicted by the TM model and the one obtained from experiment. The tube length was 100 inches. Legend shown in Fig. 5.15.
Chapter 6

Conclusion and Future Work

Experiments on the response of a pneumatic system consisting of a pressure source, a solenoid valve, a length of tube, and a minor loss, have been presented. The velocity of the air at the exit of the system was measured with a hotwire probe. The pressure immediately after the solenoid valve was measured with a piezo-resistive pressure transducer. The standard deviation of velocity at the exit, $U$, divided by $U^*$, corresponding perfect square wave, was presented as a universal criteria for determining the usefulness of a velocity signal for pulsed flow control, as it indicates whether the wave fluctuates between both a high and a low velocity with equal weight. In general, $0 < U/U^* < 1$, though for some cases $U/U^* > 1$ due to wave interactions.

The general shape of the waveform measured by the hotwire probe was similar to the results presented by Braud [3] for cases with lower minor loss values. As expected, it was found that the response of the system decays with an increase in the tube length, an increase in the minor loss, or an increase in the driving frequency. These results support the gas system model presented by Swift [1]. The tube length appears to primarily increase the inertance of the system. Pressure seems to primarily affect the compliance of the system. For longer tubes and higher pressures, Fanno flow theory becomes more important in modeling the response of the system. The minor loss appears to primarily make compressible effects more important.

The system was found to respond more rapidly to the opening of the solenoid valve than to its closing. This was seen as a sharp increase in the velocity to a higher value and a slower decline back to zero. Often, the system did not return to a zero velocity value. Intuition can be used to guess the general trend in the response of the system, but the response of the system is a complex function of the elements in the system.
A transmission matrix model was compared with the data for several cases, and it was found that the model predicts the same trends with frequency and similar velocity fluctuations as found in the experiments for cases that can be considered incompressible.

Future work could be done to examine which values of $U/U^*$ translate to a usable signal for control applications. Different cases could be run in a flow control application to see which cases influenced the flow in the desired manner. Another area for study is the effect of varying the size of tubing, and different minor losses, such as sharp bends or leaks. Further work could also be done to implement a TM for an orifice that takes compressible effects into account to improve the accuracy of TM predictions.
References


