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Pareto Optimality and Compromise for Environmental Water Management

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Key Points:

- The shape of the Pareto frontier influences whether decision-makers are more or less likely to cooperate over environmental water allocations
- Compromise is more likely where knees, or areas of maximum curvature, occur in concave tradeoff frontiers
- Management actions, like non-flow improvements to objectives can expand the Pareto curve and improve optima for one or both objectives

Supporting Information:

Supporting Information may be found in the online version of this article.

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Abstract

Water management usually considers economic and ecological objectives, and involves tradeoffs, conflicts, compromise, and cooperation among objectives. Pareto optimality often is championed in water management, but its relationships with the mathematical representation of objectives, and implications of tradeoffs for Pareto optimal decisions, are rarely examined. We evaluate the mathematical properties of optimized tradeoffs to identify promising regions for compromise, suggest strategies for reducing conflicts, and better understand whether decision-makers are more or less likely to cooperate on environmental water allocations. Cooperation and compromise among objectives can be easier when tradeoff curves are concave and more adversarial when tradeoff curves are convex. "Knees," or areas with maximum curvature, bulges, or breakpoints in concave Pareto frontiers, suggest more promising areas for compromise. Evaluating the shape of Pareto curves based on each objective's performance function can screen for the existence of knees amenable to compromise. We explore water management and restorations actions that improve and shift the location and prominence of knees in concave Pareto frontiers. Connecting river habitats and other non-flow management actions may add knees on locally concave regions of Pareto frontiers. Managing multiple streams regionally, rather than individually, can sometimes turn convex local tradeoffs into concave regional tradeoffs more amenable to compromise. Overall, this analysis provides a deep investigation of how the shape of tradeoffs influences the range and promise of decisions to improve performance, and illustrates that management actions may encourage cooperation and reduce conflict.

Plain Language Summary

Water management usually considers economic and ecological objectives, and involves tradeoffs, conflicts, compromise, and cooperation among objectives. We evaluate the mathematical properties of tradeoffs to identify promising regions for compromise, suggest strategies for reducing conflicts, and better understand whether decision-makers are more or less likely to cooperate on environmental water allocations. Cooperation and compromise among objectives can be easier when tradeoff curves are concave and more adversarial when tradeoff curves are convex. "Knees", or areas with maximum curvature, bulges, or breakpoints in concave tradeoffs, suggest more promising areas for compromise. We explore water management and restorations actions that improve and shift the location and prominence of knees in concave tradeoff curves. Connecting river habitats and other non-flow management actions may add knees on locally concave regions of tradeoffs curves. Managing multiple streams regionally, rather than individually, can sometimes turn convex local tradeoffs into concave regional tradeoffs more amenable to compromise. Overall, this analysis provides an investigation of how the shape of tradeoffs influences the range and promise of decisions to improve performance, and illustrates that management actions may encourage cooperation and reduce conflict.

1. Introduction

Environmental and human water management often involves conflicts and tradeoffs (Jager et al., 2015; Lund & Palmer, 1997). Pareto optimality is a core concept for efficient multi-objective management (Castelletti et al., 2013; Cohon, 1978/2004; Reed et al., 2013) and is often used to quantify and analyze tradeoffs among human and environmental objectives (Génova et al., 2019; Homa et al., 2005; Kraft et al., 2019; Null & Lund, 2012; Olivares et al., 2015; Shiau & Wu, 2007; Vogel et al., 2007). The mathematical characteristics of tradeoffs can help infer whether cooperation is likely, identify promising regions for compromise, establish the relative intractability of conflicts, and suggest strategies for reducing conflicts. In other words, the shape of Pareto frontiers helps in understanding and improving prospects for compromise among conflicting objectives. Yet, we rarely examine how the shapes of tradeoff curves might be managed to improve prospects for multi-objective solutions.

Pareto optimality was defined in 1906 by Vilfredo Pareto as the set of decision choices where performance on one objective could only be improved by worsening performance on other objectives (Ehrgott, 2012). Pareto optimal solutions, called non-dominated solutions in operations research, are now commonly used to screen the range of feasible solutions to a subset of efficient solutions for further consideration (Cohon, 1978/2004). The slope at any point on a Pareto frontier represents optimal and efficient tradeoffs among objectives. "Knees" in concave Pareto curves are areas of maximum bulge, defined as points that achieve maximum simultaneous improvement on multiple objectives (Das, 1999).

Knees in concave Pareto frontiers suggest areas for compromise where acceptable tradeoffs are more likely, subject to constraints (Rachmawati & Srinivasan, 2009). Breakpoints are common along Pareto frontiers where constraints bound the feasible solution space and may form knees. An extreme example of a knee is a utopia point, or ideal solution without conflict, that has a single Pareto-optimal solution at the maximum point for all objectives. The more of a knee that exists on a concave Pareto frontier, the more likely the knee is to embody an acceptable tradeoff rate and compromise solution. Since knees have outsized importance for decision-makers (Deb & Gupta, 2011), understanding management strategies that add or enlarge knees on concave regions of Pareto frontiers is also needed. Exploring management actions where one objective has a large improvement per unit degradation in another, could be beneficial to overcome decision biases and move decision-makers from established decisions to innovative decisions (Tversky & Kahneman, 1974).

Previous research has explored techniques for identifying knees along Pareto frontiers and to concentrate searches for Pareto optimal solutions around such points (Deb & Gupta, 2011; Liang et al., 2018; Yu et al., 2019). For example, Das (1999) used normal-boundary intersection to find points spread evenly on the Pareto frontier; Rachmawati and Srinivasan (2009) used preference-based selection pressure to identify solutions near knees; and multiple researchers have used multi-objective evolutionary algorithms to selectively search knee regions for optima relevant to decision-makers (Branke et al., 2004; Zhang et al., 2014). However, evaluating the shape of the Pareto frontier and water management actions that add, improve, or remove knee regions has not been studied. This is important to understand and improve potential for compromise in challenging and seemingly intractable water problems.

Water system modeling often assumes centralized decision-making to attain optimal solutions (Giuliani & Castelletti, 2013). In reality, decision-makers seek to maximize their own benefits which may result in conflicts. Cooperation among decision-makers is usually required to reach better outcomes. Cooperation is advantageous in many complex social and biological interactions (Cohen, 1998; Hui et al., 2016; Madani & Lund, 2012). Game theory connects Pareto optimality with cooperation in water resources problems, where decision-makers that work together achieve better outcomes for all than decision-makers that work unilaterally and consider only their own objectives (Madani, 2010). Alternative levels of cooperation among independent decision-makers with concave tradeoff curves also have been modeled (Anghileri et al., 2013; Giuliani & Castelletti, 2013). Most water management optimization research focuses on reservoir re-operation as the primary management action (Shiau & Wu, 2007; Steinschneider et al., 2014; Suen & Eheart, 2006; Yang & Cai, 2011; Yin et al., 2011) and no water management research has evaluated differences in cooperation for alternative Pareto tradeoff curve shapes. Tradeoff curve shapes have been explored in other resource allocation problems, primarily in ecological literature to understand relationships between traits and fitness. Farahpour et al. (2018) show that reproduction versus competition shapes species diversity and eco-evolutionary dynamics with both convex and concave tradeoffs. Similarly, Ehrlich et al. (2017) use convex, concave, and linear functional trait tradeoff shapes to predict species invasions or coexistence.

This study evaluates the mathematical properties of Pareto optimal frontiers to highlight zones of easier compromise or more difficult conflict. The problem of river basin water allocation for ecosystem protection versus economic benefit provides an illustrative example. We explore management actions that add or broaden knees in concave tradeoff curves and that enlarge decision space in convex tradeoff curves to add knees favorable to compromise. We first describe shapes of bi-objective tradeoff curves when underlying performance (benefit) functions for human and environmental water allocations are convex, linear,

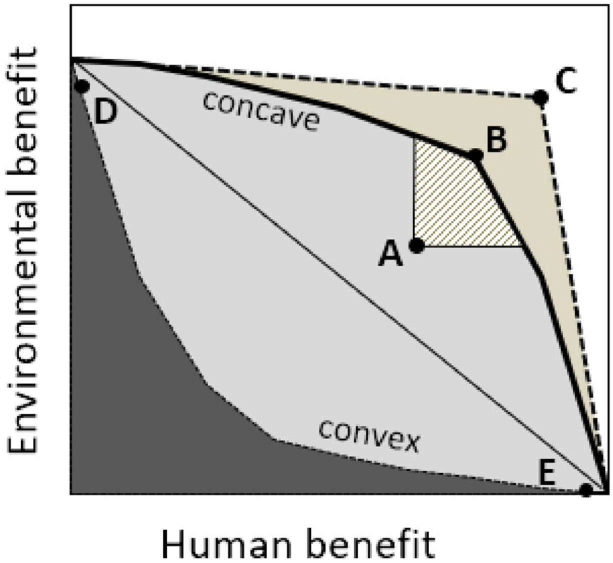

Figure 1. Pareto optimal frontiers of performance tradeoff between environmental and economic objectives. Feasible solutions lie below each Pareto frontier (shaded). Point A shows an inferior, or dominated, point on the bold, solid concave tradeoff frontier, with a slight knee at point B. The bold, dashed line shows a concave tradeoff frontier with a clear knee of likely compromise between objectives at point C. The linear solid line indicates a constant tradeoff and the dotted line shows a convex tradeoff curve, for which compromise between objectives will be challenging because knees exist at endpoints D and E.

concave, or mixed, noting the location of knees which suggest opportunities for compromise between competing water uses. We then evaluate the tradeoff curve shape and presence of knees for three restoration examples including (a) connecting a river with its floodplain to increase environmental or economic performance, (b) non-flow environmental improvement to expand the Pareto frontier with locally concave knee regions, and (c) specializing rivers for either environmental or human objectives to enlarge the solution space, capitalize on antagonistic knees at the endpoints of a Pareto curve, and reduce conflict-prone tradeoffs from convex Pareto frontiers. From this analysis, we extend the concept of knees to cases where the Pareto frontier is locally concave, but not concave over the entire domain. These different situations offer lessons for governments, stakeholders, and citizens interested in seeing conflicts resolved.

2. Pareto Optimality and Knees of the Pareto Front

Figure 1 shows alternative Pareto optimal curves for a common problem of allocating available streamflow between economic objectives, measured in monetary units, and environmental objectives, conceptualized here as fish population. As more water is allocated to the environment, fish population increases but then less water is diverted for economic objectives (such as, agricultural irrigation) and economic performance diminishes. Each Pareto-efficient water allocation is a point on the tradeoff curve showing water volume to competing users in some time of interest (season, year, etc.). The timing and quality of streamflows are critical for fish populations and other environmental objectives, and extensive literature exists on functional environmental flows and replicating the natural flow regime (Poff et al., 1997; Yarnell et al., 2020). However, quantifying economic and environmental water allocation tradeoffs using water volume has an established basis in the literature (Vogel et al., 2007). Interior points, like point A in Figure 1, are dominated by solutions on the concave tradeoff curve (such as B) because solutions on the frontier can improve performance on one or both objectives (over the diagonally striped area) without harming another objective.

When the feasible set is convex, meaning a line between any two points is contained in the set (all but the dark shaded area in Figure 1), the Pareto frontier is concave. The bold, solid curve in Figure 1 has a concave tradeoff frontier, shown by the shaded gray and diagonally striped areas. The slope of the curve at any differentiable point is the efficient tradeoff of the two objectives at that point (e.g., fish per dollar). Appendix A provides mathematical proof of the convexity/concavity of Pareto frontiers with performance function convexity/concavity for a competitive resource.

A wide range of acceptable tradeoffs between the two objectives cluster about point C in Figure 1, where compromise is easiest because both parties give up little value while preserving most benefits around this point. Compromise is harder for the solid bold curve, but still relatively easy around point B, a less distinct knee. Under regulatory pressure, irrigators will more easily surrender water that can be made up through inconvenient operations or fallowing of low-value crops, before considering fallowing high-value crops on their most productive lands. Such situations also make it financially easier for governments or environmental groups to compensate farmers for these smaller losses. Thus, most wetland restoration tends to occur on lower-valued agricultural lands (Suddeth Grimm & Lund, 2016).

Tradeoffs without a concave knee will be harder for compromise, or more antagonistic, because they lack of an identifiable focus (Schelling, 1980). The solid-line Pareto curve implies a linear tradeoff, with no outstanding compromise points. The convex dashed curve lacks promising compromise points. Preferences would favor strongly antagonistic single-objective optima near points D and E, which lose performance quickly on one objective but gain slowly on the other.

Knees at breakpoints of Pareto frontiers can be defined by the subdifferential, or set of slopes that support a function at that point, where there will be a discrete change in slopes of adjacent Pareto frontier segments. We call this the "Degree of Kneediness," which we define as the magnitude of curvature (the second derivative) at any point or the discrete change in slope at a breakpoint with an undefined second derivative.

3. Concavity and Cooperation Versus Convexity and Conflict

Evaluating the shape of Pareto curves based on mathematical properties of each objective's performance (benefit) function can screen for the existence of knees. Specifically, if performance functions are both linear or convex, then we can anticipate the Pareto front will lack knees amenable to compromise. Concave performance functions or concave regions in performance functions are promising for compromise and negotiation, although bulges or breakpoints in concave performance functions may be infeasible due to constraints.

Performance on an objective usually is a function of resources allocated. Where both objectives have monotonically increasing performance with competitive resource allocation (i.e., both performance functions have a positive first derivative with respect to water use over the entire domain, and the resource is consumptively used), the tradeoff curve will have a negative slope and there will be a tradeoff over the entire range of water allocations (see all cases in Table 1). Linear economic and environmental performance functions occur when the rate of objective performance is constant as more resources are allocated. Linear economic performance comes, for example, from irrigation of a single-crop, with homogenous yield in all the irrigated area. Increasing water allocation will always result in the same increment of revenues. Linear performance functions lead to a linear Pareto frontier (Table 1 Row A). Concave economic and environmental performance means decreasing marginal returns, meaning that the first resource units allocated add the greatest value to the objective, followed by continued increasing or constant value, but at a decreasing incremental rate. Concave economic performance occurs, for example, when water is allocated for irrigation of multiple crops with different prices and/or different yields over the irrigated land. When both objectives have concave performance functions for a resource, the allocation of a fixed common resource leads to a concave tradeoff curve (Table 1 Row B). Convex increasing performance for both economic and environmental water allocation means that each objective has increasing incremental benefits from greater resource allocation, leading to a convex tradeoff curve (Table 1 Row C). Pareto frontiers are shaped by the functions defining performance on objectives. Pareto frontiers can be compound, with both concave and convex segments (Table 1 Row D) or flatten with compound performance functions (Table 1 Row E).

We focus on cases where competing uses for water involve tradeoffs among objectives (negative-sloping Pareto curve) where knees in Pareto curves create incentives for compromise (Table 1 Rows B and C). Convex Pareto curves imply more difficult conflicts as described above. Physical, ecological, economic, and societal processes mix to affect objective performance, sometimes with multiple knees and often with discontinuities that create a partially non-differentiable Pareto frontier.

4. Tradeoffs and Actions for Compromise

4.1. Knees at Thresholds in Tradeoffs

Flat or dead zones can occur when performance thresholds exist for one objective, but not the other (Figure 2a, performance curves in Figure S1 in Supporting Information S1). This can arise where some water can be diverted without harming the fish population, but also where some streamflow threshold must be surpassed before any fish survive or where some level of economic activity generates by-products that create toxic conditions for biota. Once all fish are dead, there is little reason to dedicate remaining water to instream flows (Mar, 1981). A similar economic dead zone might occur where a minimum diversion volume is needed to fill an irrigation system before any water can be delivered to farms. Such thresholds on performance and tradeoff functions create non-concavity and can make negotiation of solutions more difficult. In Figure 2a, there are two knees in locally concave regions of the Pareto frontier, one is broad and smooth at the highest fish population performance level of the curve and the other at the extreme of economic performance with zero fish. The economic interest in this case continues to derive economic benefit after all fish have been eliminated, creating some incentive for eliminating fish entirely. The horizontal portions of the

Table 1
Pareto Frontier Shapes Resulting From Varying Economic and Environmental Performance Functions of Water Allocation (Q)

Economic(Q)	Environment(Q)	$\frac{\partial^2 Env}{\partial Econ^2}$	Pareto frontier
A: Linear	Linear	zero	Linear
B: Concave	Concave	negative	Concave
C: Convex	Convex	positive	Convex
D: Concave	Convex	a: negative b: positive	Compound
E: Logistic (compound)	Logistic (compound)	Negative	Flatter Concave

Note. The sign of $\partial^2 Env/\partial Econ^2$ determines the shapes of the frontier, which are shown here for examples when $\partial Env/\partial Econ$ is negative.

tradeoff curve are shown to illustrate the dead zone, although the tradeoff curve has a discontinuity where fish population becomes zero and the knee.

4.2. Additional Knees From Improved Pareto Frontier Equilibrium

Sometimes small increases in water quantity or quality, additional resources, or a technology change can shift the Pareto curve to a more productive equilibrium (Figure 2b). Where a fixed amount of water is to be allocated among water users, the Pareto frontier could shift outward with small environmental water

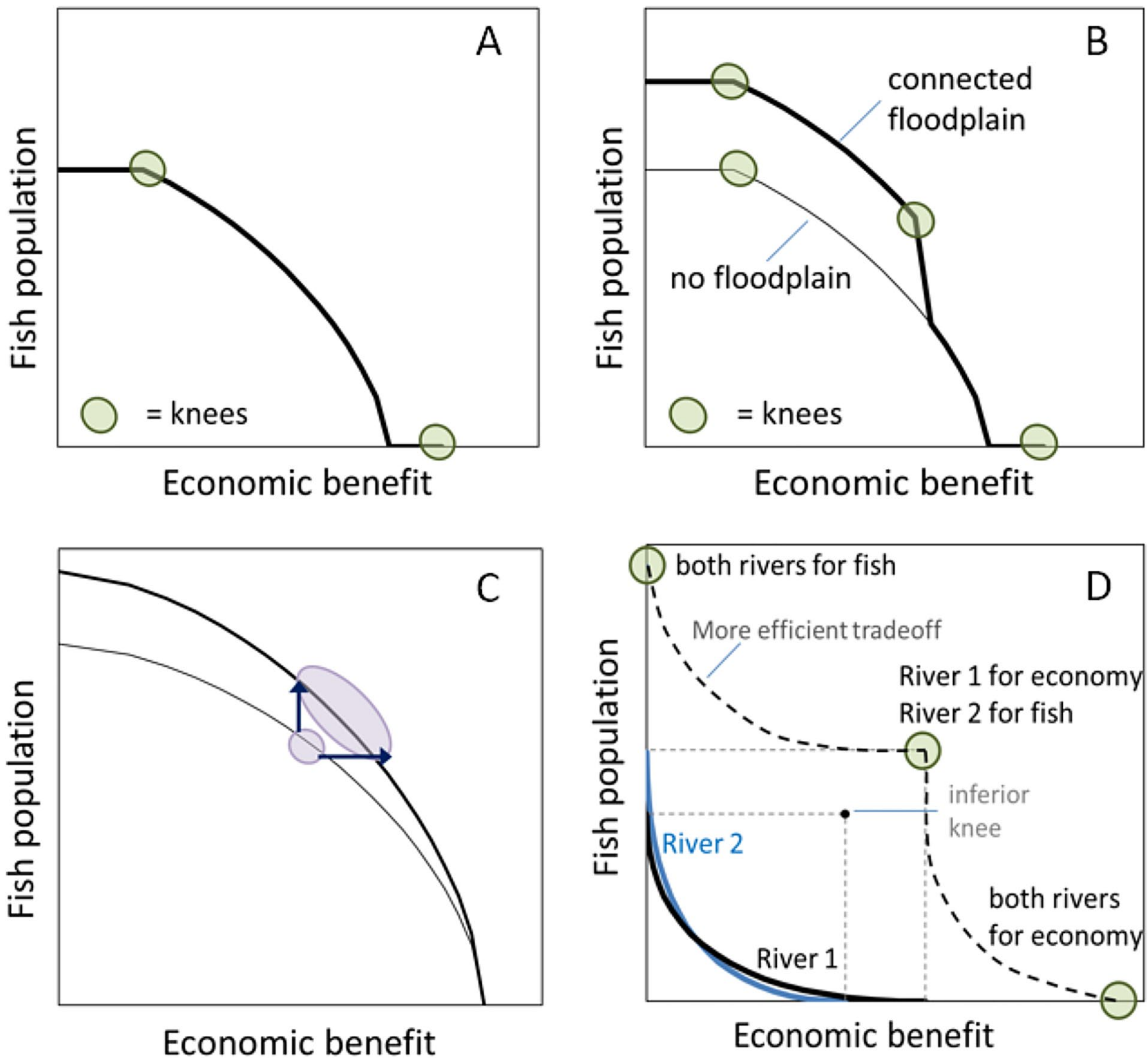

Figure 2. (a) Tradeoff curve with two knees, where some water can be diverted before fish population declines and where fish population falls to zero before all water is diverted. (b) Tradeoff curve with a connected floodplain, which expands habitat for some flow ranges, creates a new performance threshold, and shifts tradeoff curve outward to a new Pareto frontier with additional knees. (c) Non-flow habitat improvement shifts the tradeoff curve outward along the fish population axis. Zones of potential compromise (shown in purple) expand along the new Pareto frontier. (d) Regionally specializing local rivers for either economic or environmental objectives can make compromise easier. Fish production can be maximized in river 2, while economic benefits are maximized in river 1, resulting in optimal specialization and forming a regional composite attractive compromise knee.

transfers that provide overbank flows and connect rivers to floodplains (Figure S1 in Supporting Information S1) (Jeffres et al., 2008; Zeug & Winemiller, 2008) or through a change in management policies (Homa et al., 2005). Additional examples include managed aquifer recharge that increases summer baseflows for downstream fish or improves stream temperatures (water quality) for fish (Van Kirk et al., 2020), streamflow thresholds that connect stream reach habitats (Kraft et al., 2019), or reaches with favorable conditions for instream recreation (Génova et al., 2019). In some cases, the resulting Pareto frontier will be compound (Figure 2b), concave at the large scale, but with local convex regions. Such performance thresholds can add knees to Pareto tradeoffs to increase the likelihood of attractive compromise solutions.

Equity among competing objectives is not a goal of Pareto optimality, where all points along curves are equally efficient, but are unlikely to be seen as equally fair to competing parties. However, competing water users, governments, or other groups often seek to balance across objectives and may prefer knees in a central location on the front. Additional knees toward the middle of the tradeoff curve can be created if a government or environmental group offers financial compensation for some reduction in water use by an economic use. The purchase of water, land, a seasonal flooding easement, or other forms of compensation can create a knee in the effective tradeoff curve. Such a knee can be further accentuated when government regulation or environmental litigation are likely if agreement does not occur (Vissers, 2017).

Negotiations and the context of negotiations may be manipulated by interested parties to create or shift the location of knees in the tradeoff curves perceived by decision-makers. An unacceptable knee location for irrigators in this case, might become acceptable with the purchase of water by an environmental group, government agency, or a downstream irrigator who might reuse the environmental flow after it has passed this reach (Ugai, 2017).

4.3. Expanding Knees From Improved Pareto Frontier Equilibrium

Streamflow alone does not always limit fish population. Where stream temperature, dissolved oxygen, food availability, turbidity, cover, predation, or other factors limit habitat suitability and fish population, those conditions must be improved to increase environmental performance (Figure 2c). Restoration that improves such non-flow conditions could push the Pareto frontier outward along the fish habitat axis, perhaps at some financial cost for the non-flow actions. As a result, either or both economic and environmental performance improve and zones of equitable compromise expand on the new, more efficient Pareto frontier (shown in the purple circle in Figure 2c).

The Pareto curve also could shift outward from improved efficiency that increases economic benefits. Water reuse or improved water supply operations might increase economic benefits, although they do not increase total water availability for competing water uses, like environmental flows (Grafton et al., 2018; Ward & Pulido-Velazquez, 2008).

4.4. Enlarging Conflict-Prone Problems to Encourage Compromise

Where fish habitat and economic production are both convex functions, initial reductions in water allocation cause the largest declines in both fish habitat and economic benefits. It is better to concentrate water allocations to one use in a basin, rather than less efficient compromise allocations in each of multiple basins (Mar, 1981). It can be more efficient to manage one river solely for economic benefit and another river for fish habitat to improve total benefit over poor compromises in both rivers (Figure 2d). This idea parallels international trade, where two countries produce different goods for which they each have a comparative advantage, then trade to consume outside their individual Pareto tradeoff frontiers. In Figure 2d, fish habitat and economic benefit compete for water from river 1 and 2, with increasing returns to scale. Antagonistic knees occur at endpoints where both rivers are managed for economic benefit or fish population. However, if river 2 is managed solely for fish population, and river 1 managed for economic benefit, a new aggregate knee occurs where specialized rivers can create a regional compromise. The tradeoff curve moves to a new Pareto frontier at that point, with zones of difficult compromise between knees. An inferior specialization can lead to inferior knees if river 1 is managed for fish population and river 2 is managed for economic benefit (Figure 2d). For a system of repeated convex tradeoffs, multiple rivers or tributaries for example, there is incentive to preserve some unregulated rivers that maintain natural flow characteristics for native species (Poff et al., 1997; Yarnell et al., 2020) with other rivers devoid of native habitat, managed solely for human water uses (Null, 2016; Pittock & Hartmann, 2011). As more rivers are included in analysis, additional knees of potential compromise increase, though many may be inferior knees.

5. Discussion

Water management often produces severe conflicts and unintended consequences for stakeholders, as a classic example of a “wicked” problem (Liebman, 1976; Lund, 2012; Rittel & Webber, 1973). Widespread or prolonged environmental degradation from water resources management is one such unintended consequence (Vörösmarty et al., 2010). Since early water resource system models were developed more than 50 years ago, environmental objectives have become more important and increasingly drive water management (Reuss, 2003). Multi-objective optimization highlights the best system-wide performance, although individual stakeholders seek to optimize their own interest's performance rather than overall performance (Madani, 2010). Understanding how to encourage cooperation and avoid conflict are high-level needs when management objectives conflict (Raiffa et al., 2002).

Evaluating the shape of Pareto frontier solutions to identify and develop promising compromises is a straight-forward approach to encourage cooperation, contain conflict, structure environmental water allocations, and inform habitat restoration decisions and incentives. Knowing the performance of each objective as a function of water allocation (or other management decisions) can give a good qualitative idea of the general shape of efficient tradeoff frontiers, whether convex, concave, linear, or compound.

Knees in concave Pareto frontiers indicate promising regions for compromise, and management actions that create locally concave regions of Pareto curves can add knees. Non-flow improvements to environmental or economic objectives can expand the Pareto curve and lead to new equilibria better for one or both objectives. When interests directly conflict, specializing water management in multiple rivers for economic or environmental production may improve overall performance. Enlarging problems in this manner can improve prospects for negotiation and solution (Raiffa et al., 2002).

5.1. Streamflow—Environmental Response Relationships

Commonly the first units of a resource are the most highly valued, with decreasing marginal benefits. Economic performance of resource allocation is a classic case, with declining marginal economic value as consumption increases. Environmental performance also often has declining marginal value with water or other inputs, as illustrated by the Endangered Species Act, which extends to threatened and endangered species and places a very high value on retaining the last remnants of a species, but is not particular about abundant native species populations.

For fluvial processes that depend on streamflows, the first units of water diverted upstream reduce baseflows when diverted in summer for irrigation water supply, or reduce flood magnitudes when winter water is stored in upstream reservoirs. Lower summer baseflows are likely to discourage riparian vegetation, increase stream temperatures, and concentrate fish and other organisms in pools. Flooding also drives some geomorphic processes and cues fish species, whereas reduced flooding often disconnects rivers from floodplains (Poff et al., 1997).

We reviewed literature on the shape of environmental water performance to assess if environmental benefits remain until the final units of water were removed (concave environmental performance with knees amenable to compromise) or whether initial reductions in streamflow cause the greatest degradation to the environment (convex environmental performance with antagonistic knees). Most of this literature indicates concave environmental performance functions, where environmental harm increases with more heavily altered systems when most water is appropriated away from environmental uses (Gleick & Palaniappan, 2010; Jowett, 1997). Some studies with measured or modeled data from specific systems had similar relationships (Giuliani & Castelletti, 2013; Kraft et al., 2019; Parasiewicz, 2001; Poff et al., 2010; Schmitt et al., 2018; Steinschneider et al., 2014). In these systems, compromise may be possible, especially at knees in the Pareto curve, and compromise is particularly beneficial to environmental interests that often have less negotiating power than economic interests (Giuliani & Castelletti, 2013). However, another study showed convex environmental performance, where initial reductions or alterations in streamflow cause the greatest degradation to the environment. For fish species with small range sizes, the probability of imperilment increases most rapidly with initial hydropower manipulation, defined as the ratio of water through turbines divided by available water (McDonald et al., 2012).

We also sampled published literature to validate our bi-objective examples and incorporate human-environmental tradeoffs and complexities in real-world cases. Knees are occasionally highlighted for environmental water management to identify zones of potential compromise (Castelletti et al., 2011; Homa et al., 2005; Vogel et al., 2007). Yang and Cai (2011) found that adding an ecological objective to maximize fish diversity could improve downstream fish habitat without jeopardizing the primary flood control objective. However, Suen and Eheart (2006) found that allocating water to meet an ecological objective representing desired intermediate disturbance led to conflicts with domestic water delivery, but had little conflict with hydropower generation and minor conflict with irrigation water delivery. The overwhelming majority of studies evaluated human and environmental water allocation tradeoffs in the context of reservoir re-operation (for example Shiau & Wu, 2007; Steinschneider et al., 2014; Suen & Eheart, 2006; Yang & Cai, 2011; Yin et al., 2011).

Although reservoirs are instrumental in managing water for people and ecosystems, our research highlights under-studied approaches to expand problems for non-flow ecological improvements and habitat connection. These topics are studied, but are typically disconnected from water allocation decisions. For example, several studies indicate large non-linear reductions in habitat connectivity from construction of the first few barriers in river systems (McKay et al., 2013; Null et al., 2014; O'Hanley et al., 2013). This results in convex environmental performance and a tradeoff curve that is convex in all or some regions of the Pareto frontier, so knees on a tradeoff curve are likely to be adversarial. Kneed Pareto optimal tradeoffs also exist for allocations of land among competing wetland ecosystem objectives (for different species) and economic activities (Suddeth Grimm & Lund, 2016). Enlarging the solution space by specializing systems for either environmental or economic performance, and thus creating a new knee in the tradeoff curve, can be promising for fisheries objectives that conflict with hydropower or water supply (Silva et al., 2018).

5.2. Uncertainty and Limitations of Pareto Tradeoffs

This study focuses on identifying regions of the Pareto frontier where competing water users may be more amenable to compromise or negotiation based on mathematical properties of Pareto tradeoffs. Bayesian uncertainty or robust decision approaches often quantify uncertainty (Herman et al., 2019; Lempert, 2003; Matrosov et al., 2013). These methods highlight solutions that perform satisfactorily over a wide range of future conditions. Additional research is needed to understand water allocation relationships with uncertainty that fundamentally change the shape of tradeoff curves. Where uncertainty is high, knees may be missed (Ghosh, 2019). Further research might examine management actions to add or enlarge fuzzy or probabilistic knees.

Knees will be unattractive if they occur in places unacceptable to decision-makers. Well-formulated constraints or external incentives could address this problem. Also, water dedicated to the environment may fail to meet legal objectives or be mis-used if supplied at inappropriate times. The analysis presented here, which summarizes benefit to competing water uses over any time period of interest (month, season, year), does not address flow timing.

Finally, decision biases are common in multi-objective management where decision-makers gravitate toward established solutions that impede innovation (Giuliani et al., 2014; Tversky & Kahneman, 1974). Highlighting knees as regions to improve performance and reduce conflict may motivate decision-makers to consider new solutions that encourage cooperation. A broad array of management alternatives, including reservoir re-operation and river restoration, may add or enlarge knees on locally concave regions of the Pareto frontier. Where performance thresholds are surpassed from increased water quantity, improved water quality, or other resources such as pairing environmental water allocations with physical habitat restoration, the Pareto curve could be shifted to a more favorable frontier with additional or more prominent knees. However, knees cannot be identified without knowing the shape of the Pareto front. Such an analysis requires, in general, a relatively complete knowledge of the Pareto front, so that knees can be visually or mathematically identified.

6. Conclusions

Fostering resolutions of environmental conflicts is an ongoing need in water resources management (Vörösmarty et al., 2010). The general shape of the Pareto frontier depends directly on mathematical properties of its component objective performance functions and affects the ability to negotiate a solution among competing interests. The Pareto frontier can be shaped by a range of environmental and water management decisions. Estimating performance on environmental and economic objectives as a function of water allocation, and studying its properties, can help identify problems more susceptible to compromise and suggest how to modify or enlarge the solution space to improve prospects for negotiation.

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Appendix A: Convexity/Concavity of Pareto Frontier With Performance Function Convexity/Concavity for a Competitive Resource

Consider a system with n objectives competing for a limited amount of water. We assume that each objective i is characterized by an increasing function on the amount of water $f_i : \mathbb{R}_+ \to \mathbb{R}$, which may represent economic or environmental benefit, or any performance metric as long as it is increasing with water allocation. We denote by $\left(x_1,\ldots,x_n\right) \in \mathbb{R}_+^n$ an allocation of water. Let b be the amount of water available to distribute competitively among objectives, limited by $\sum_i x_i \le b$ on the allocation.

Let $S = \left\{y \in \mathbb{R}^n : y_i = f_i\left(x_i\right), x \in \mathbb{R}_+^n, \sum_i x_i \le b\right\}$ be the set of feasible solutions, where y_i denotes the value of objective i under the feasible allocation x. The non-dominated points in S form the Pareto frontier.

Proposition 1: *If f_i is concave for all i, then the set S is convex.*

Proof: Let $y^1, y^2 \in S$ and $\lambda \in [0,1]$. We must prove that $y := \lambda y^1 + (1-\lambda) y^2 \in S$, since is the definition of a convex set. For $i = 1,\ldots,n$, we have:

$$\begin{aligned} y_i &= \lambda y_i^1 + (1-\lambda) y_i^2 \\ &= \lambda f_i\left(x_i^1\right) + (1-\lambda) f_i\left(x_i^2\right) \\ &\le f_i\left(\lambda x_i^1 + (1-\lambda) x_i^2\right) \text{concavity of } f_i \end{aligned}$$

We denote $\lambda x_i^1 + (1-\lambda) x_i^2$ simply as x_i. Note that:

$$\begin{aligned} \sum_i x_i &= \sum_i \lambda x_i^1 + (1-\lambda) x_i^2 \\ &= \lambda \sum_i x_i^1 + (1-\lambda) \sum_i x_i^2 \\ &\le \lambda b + (1-\lambda) b \text{ constraint on } x_i^{'} s \text{ in } S \\ &= b \end{aligned}$$

Therefore, $x = \left(x_1,\ldots,x_n\right)$ is a feasible allocation.

Up to this point $y_i \le f_i\left(x_i\right)$, but we need the equality to hold. As f_i is increasing, we can reduce each x_i by the amount that is required to attain the equality. It is easy to see that the resulting allocation is still feasible.

If set S is convex, then the Pareto frontier must be concave (in the sense stated in Section 2—Pareto Optimality and Knees of the Pareto Front).

Analogously, to show that a Pareto frontier is convex we must prove that the complement of the set of feasible solutions $S^c = \left\{y \in \mathbb{R}^n : y_i = f_i\left(x_i\right), x \in \mathbb{R}_+^n, \sum_i x_i > b\right\}$ is convex. The procedure is similar to the above, under the hypothesis that f_i are convex instead.

Proposition 2: *If f_i is concave for $i \in I$ with $|I| \le n$, and $\left\{x_i\right\}_{i \notin I}$ are fixed at any feasible value. Then, the set S in convex.*

Proof: Let $y^1, y^2 \in S$ and $\lambda \in [0,1]$, and we proceed the same way as before. Notice, however, that $y_i^1 = y_i^2$ for $i \notin I$, since the respective x_i is fixed, and therefore, for each i:

$$\begin{aligned} y_i &= \lambda y_i^1 + (1-\lambda) y_i^2 \\ &= y_i^1 = f_i\left(x_i\right) \end{aligned}$$

So, we obtain the desired equalities without assuming every f_i is concave, but only for $i \in I$.

More simply, Proposition 2 states that for any feasible partial allocation, if involved objectives are concave, then the resulting set S is convex and the Pareto frontier is concave. Thus, any subgroup of concave objectives could have incentives to compromise.

Data Availability Statement

The data supporting the findings of this study are available within the article and on Hydroshare.org (Null et al., 2021).

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