An Empirical Comparison of Confidence Interval for Relative Potency

Catherine H. Lung
Utah State University

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AN EMPIRICAL COMPARISON OF CONFIDENCE INTERVAL FOR RELATIVE POTENCY

by

Catherine H. Lung

A report submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Applied Statistics

Plan B

UTAH STATE UNIVERSITY
Logan, Utah

1976
ACKNOWLEDGEMENTS

I wish to express my sincere appreciation to Professor Donald V. Sisson, my major professor, for the enormous amount of time and guidance extended toward the organization of this paper. His assistance and encouragement were instrumental toward the completion of this paper.

Thanks are also extended to Dr. Rex L. Hurst and Professor Frederick J. Post, members of my graduate committee, for their assistance and serving on my advisory committee.

I would like to extend my sincere thanks to my parents of their patience and understanding as I completed my school years.

Appreciation is also extended to my fiance Ti Mai Wang for his encouragement and patience during my graduate study at Utah State University.

Catherine H. Lung
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</tr>
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<th>Description</th>
<th>Page</th>
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</tr>
</tbody>
</table>
Biological assays are essentially biological experiments. To compare the potencies of treatments on an agreed scale is generally of more interest than to compare the magnitude of effects of different treatments.

The relative potency, \( R = \frac{a}{b} \), is defined as the ratio of the means of two equally effective doses where \( a \) is the mean of \( A \) and \( b \) is the mean of \( B \). It is an estimate of the potency of one preparation, \( A \), relative to that of the other, \( B \).

Different procedures have been proposed to obtain the values of \( R \) and its confidence interval. Three of these methods are compared and discussed here. They utilize the coefficient of variation, Fieller's theorem, and logarithms.

1.1 Coefficient of variation method

In this method, the means, \( a \) and \( b \), of two preparations, are assumed to be two independent quantities. The square of the coefficient of variation of the ratio of \( a \) and \( b \) can be expressed as the sum of the squares of the coefficients of variation of \( a \) and \( b \) (Finny, 1964)

\[
(\text{C.V. of } a/b)^2 = (\text{C.V. of } a)^2 + (\text{C.V. of } b)^2 \tag{1.1.1}
\]

The application of this equation should be limited to data for which the coefficient of variation of \( b \) is small. This method usually is used
in normally distributed data, and the sampled population should not be assumed normal if the coefficient of variation of \( b \) is too big. Because of its common use, people still use it in many practical applications. Since in (1.1.1), \( a/b \) is equal to \( R \), the variance of \( R \) can be derived as

\[
V(R) = \frac{1}{b^2} \left[ V(a) + R^2 V(b) \right] \tag{1.1.2}
\]

Using the standard error of \( R \), the confidence interval can be defined as follows:

\[
R_u, R_L = R \pm t_{(a/2, u)} \sqrt{\frac{V(a) + R^2 V(b)}{b^2}} \tag{1.1.3}
\]

where \( u \) is the degrees of freedom

\( \alpha \) is the probability that the given interval does not cover the true relative potency

\( t \) is the ordinary t-deviate with \( u \) d.f. at \( 1-\alpha \) probability level

1-2 Fieller's theorem

This theorem is frequently used in biological assays, as a method of assigning confidence intervals for a ratio (Finny, 1964). The formula for the confidence interval is

\[
R_u, R_L = \frac{R-(gV_{12}/V_{22}) \pm (ts/b) \sqrt{V_{11}-2RV_{12}+R^2V_{22}}-g(V_{11}V_{12}^2/V_{22})}{1-g} \tag{1.2.1}
\]
where $V_{ij}$ is the appropriate element from the inverse matrix required to obtain the variance of a coefficient, ie. $V(b) = s^2 V_{11}$, etc.

$s$ is pooled standard error when two preparations are assumed to have the same variances.

$t$ is the ordinary $t$-deviate with $n_a + n_b - 2$ degrees of freedom at $1-\alpha$ confidence level, if the variances are assumed equal.

$g$ is referred to $t^2 s^2 V_{22}/b^2$ ($s^2$ is pooled variance when $A, B$ have same variances).

Since the data used in this report were two random samples, the means can be assumed to be independent and $V_{12} = 0$. (1.2.1) can be simplified as followed:

$$R_{u, R_L} = \frac{R \pm (ts/b) \sqrt{V_{11} + R^2 V_{22} - gV_{11}}}{1-g}$$

(1.2.2)

The value of $g$ varies inversely as $b$ is large compared with its standard error. If $g$ is neglected, (1.2.2) turns out to be

$$R_{u, R_L} = R \pm \frac{ts}{b} \sqrt{V_{11} + R^2 V_{22}}$$

(1.2.3)

In this case, (1.2.3) is the same as (1.1.3).

In the case of $\alpha$ is equal to 0.05, when $g$ is less than 0.05, that $b$ is at least nine times its standard error is required, it is customary to be ignored. When $g$ exceeds 0.2, the width of the confidence interval will be much understated by use of coefficient of variation formula. Difficulties arise if $g$ exceeds 1.0, as $b$ is then not significantly different from zero (Finny, 1964).
1-3 The method of logarithms

This method involves taking logarithm of the original observations. The antilog of the difference of two means is the estimate of the relative potency. Confidence intervals are calculated from simple mean comparison methods. Normality of the distribution is often found in logarithm units. The pooled variance should give the best possible estimate of the population variance, if the variances are equal. The method of logarithms has the major advantage of simplicity.

1-4 General procedure

A computer program was written to evaluate the most efficient method to get the confidence interval of relative potency by the three methods.

Chapter two of this study introduces the computer application. It also describes how the program was established, how the data were read in, and what kind of output the program provided.

Chapter three contains the results of the computer runs, and discusses them empirically.

Chapter four contains the conclusions of this study.
Chapter II

COMPUTER APPLICATION

2-1 Computer program

The program to be described here was written to save the tremendous amount of work required for a Monte Carlo comparison of the three methods. With the use of this program, more conditions could be evaluated than if the work was done by hand. The program was written in Fortran IV, and was run on a Burroughs Model 6700 computer.

The program was broken into two segments. The first segment used a subroutine, random normal variable (RNOR) (Hurst, Knop, 1971), to generate two sets of data each containing ten observations. One represented a test preparation and the other a standard. The logarithm of each value was obtained to be used in the method of logarithms.

The second segment computed the means, relative potency, confidence interval for the relative potency, and length of this confidence interval for each of the three methods. Each computer run generated one thousand data sets.

Eight computer runs were made, one for each of the possible situations arising from two states of each of three different conditions. The first condition was the magnitude of the g value. For large g, the data were generated with b as twice its standard error. For small g, b was ten times its standard error. The second condition was magnitude of the means (large vs small), and the third condition was equal variances vs unequal variances. The eight categories, along with the summarization of the results, are indicated in Table 1.
Table 1. Summarization of computer output

<table>
<thead>
<tr>
<th>categories</th>
<th>( \bar{X} )</th>
<th>( S_X )</th>
<th>( \bar{Y} )</th>
<th>( S_Y )</th>
<th>N</th>
<th>range of ( g )</th>
<th>method's name</th>
<th># of shortest c.i.</th>
<th>ave. length of c.i.</th>
<th># of type I error</th>
</tr>
</thead>
<tbody>
<tr>
<td>small mean</td>
<td>50</td>
<td>5</td>
<td>40</td>
<td>5</td>
<td>1000</td>
<td>0.011</td>
<td>coef. of var.</td>
<td>616</td>
<td>0.167901</td>
<td>54</td>
</tr>
<tr>
<td>equal variance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Filler's</td>
<td>0.163828</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>logarithms</td>
<td>383</td>
<td></td>
<td>0.170452</td>
<td>56</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>138</td>
<td></td>
<td>0.236003</td>
<td>45</td>
<td></td>
<td></td>
<td>Filler's</td>
<td>0.237203</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>logarithms</td>
<td>862</td>
<td></td>
<td>0.225605</td>
<td>70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>small mean</td>
<td>20</td>
<td>2</td>
<td>15</td>
<td>3</td>
<td>1000</td>
<td>0.017</td>
<td>coef. of var.</td>
<td>555</td>
<td>0.152921</td>
<td>56</td>
</tr>
<tr>
<td>unequal variance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Filler's</td>
<td>0.153613</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>logarithms</td>
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<td></td>
<td>0.155835</td>
<td>61</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>140</td>
<td></td>
<td>0.250369</td>
<td>52</td>
<td></td>
<td></td>
<td>Filler's</td>
<td>0.251675</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>logarithms</td>
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<td></td>
<td>0.239295</td>
<td>81</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>small mean</td>
<td>250</td>
<td>25</td>
<td>150</td>
<td>25</td>
<td>1000</td>
<td>0.011</td>
<td>coef. of var.</td>
<td>555</td>
<td>0.152921</td>
<td>56</td>
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<tr>
<td>equal variance</td>
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<td></td>
<td></td>
<td>Filler's</td>
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<td>55</td>
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<td>436</td>
<td></td>
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<td>61</td>
<td></td>
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<td>140</td>
<td></td>
<td>0.250369</td>
<td>52</td>
<td></td>
<td></td>
<td>Filler's</td>
<td>0.251675</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>logarithms</td>
<td>860</td>
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<td>0.239295</td>
<td>81</td>
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<tr>
<td>large mean</td>
<td>250</td>
<td>25</td>
<td>200</td>
<td>40</td>
<td>1000</td>
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<td>coef. of var.</td>
<td>140</td>
<td>0.250369</td>
<td>52</td>
</tr>
<tr>
<td>unequal variance</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td>Filler's</td>
<td>0.251675</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>logarithms</td>
<td>860</td>
<td></td>
<td>0.239295</td>
<td>81</td>
<td></td>
<td></td>
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</tbody>
</table>
Table 1. (continued)

<table>
<thead>
<tr>
<th>categories</th>
<th>$\bar{X}$</th>
<th>$S_x$</th>
<th>$\bar{Y}$</th>
<th>$S_y$</th>
<th>N</th>
<th>range of g</th>
<th>method's name</th>
<th># of shortest c.i.</th>
<th>ave. length of c.i.</th>
<th># of type I error</th>
</tr>
</thead>
<tbody>
<tr>
<td>equal variance</td>
<td>4</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>1000</td>
<td>0.333</td>
<td>Fieller's</td>
<td>871</td>
<td>1.421835</td>
<td>62</td>
</tr>
<tr>
<td>small variance</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>0.027</td>
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<td></td>
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</tr>
<tr>
<td>small means</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>unequal variance</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0.4</td>
<td>1000</td>
<td>0.365</td>
<td>Fieller's</td>
<td>872</td>
<td>0.357325</td>
<td>59</td>
</tr>
<tr>
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<td></td>
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<td></td>
<td></td>
<td>0.010</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>large means</td>
<td>100</td>
<td>50</td>
<td>300</td>
<td>50</td>
<td>1000</td>
<td>0.319</td>
<td>Fieller's</td>
<td>807</td>
<td>2.013227</td>
<td>65</td>
</tr>
<tr>
<td>unequal variance</td>
<td>100</td>
<td>50</td>
<td>200</td>
<td>60</td>
<td>1000</td>
<td>0.332</td>
<td>Fieller's</td>
<td>892</td>
<td>1.557741</td>
<td>44</td>
</tr>
<tr>
<td>large means</td>
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<td></td>
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<td></td>
<td></td>
<td>0.025</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>unequal variance</td>
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<td>50</td>
<td>200</td>
<td>60</td>
<td>1000</td>
<td>0.332</td>
<td>Fieller's</td>
<td>108</td>
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</tr>
<tr>
<td>large g</td>
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<td></td>
<td>0.015</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2 contains a sample of the computer output. The first heading identifies the preparation. Then follows the list of the data points, ten for each preparation, and the logarithm corresponding to each of them. The next set of heading shows the relative potency, standard error of the relative potency, confidence interval from using the method of coefficient of variation, value of g, confidence interval from using Fieller's theorem, relative potency from logarithm data without taking its antilog, standard error of this relative potency, and the confidence interval from the method of logarithms without taking antilog. The sample value corresponding to each category follows. Then, the antilog value of confidence interval for the method of logarithms is listed. The length of the confidence interval for each method is indicated below them. For each computer run, there were one thousand sets of output listing the above information. At the end of one thousand sets, the range of g, average length of confidence interval for each method, number of shortest confidence intervals among three methods out of the one thousand sets data, and number of type I errors committed in each method were then printed out.
Table 2. Sample of the computer output

<table>
<thead>
<tr>
<th>TEST</th>
<th>STANDARD</th>
<th>(logarithms)</th>
<th>TEST</th>
<th>STANDARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>109.2709</td>
<td>66.9807</td>
<td>2.0385</td>
<td>1.8259</td>
<td></td>
</tr>
<tr>
<td>53.1406</td>
<td>85.2869</td>
<td>1.7254</td>
<td>1.9309</td>
<td></td>
</tr>
<tr>
<td>44.3946</td>
<td>93.1306</td>
<td>1.6473</td>
<td>1.9691</td>
<td></td>
</tr>
<tr>
<td>97.5339</td>
<td>75.0064</td>
<td>1.9891</td>
<td>1.8751</td>
<td></td>
</tr>
<tr>
<td>194.8005</td>
<td>86.4155</td>
<td>2.2896</td>
<td>1.9366</td>
<td></td>
</tr>
<tr>
<td>150.6867</td>
<td>86.5651</td>
<td>2.1781</td>
<td>1.9473</td>
<td></td>
</tr>
<tr>
<td>250.2525</td>
<td>79.1504</td>
<td>2.3984</td>
<td>1.8985</td>
<td></td>
</tr>
<tr>
<td>105.6379</td>
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<td>1.8971</td>
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</tr>
<tr>
<td>77.3805</td>
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<td>1.8886</td>
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</tr>
<tr>
<td>76.1352</td>
<td>91.3591</td>
<td>1.8816</td>
<td>1.9608</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R</th>
<th>S.D. R</th>
<th>CONF. INT. FOR R</th>
<th>G</th>
<th>FIELLERS THM.</th>
<th>LOGR</th>
<th>S.D. LR</th>
<th>CONF. INT. FOR LR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.709</td>
<td>0.128</td>
<td>0.4197</td>
<td>0.9975</td>
<td>0.161</td>
<td>0.5004</td>
<td>-0.093</td>
<td>0.076</td>
</tr>
<tr>
<td>0.5582</td>
<td>0.5582</td>
<td>0.5582</td>
<td>0.5582</td>
<td>0.5582</td>
<td>0.5582</td>
<td>0.5582</td>
<td>0.5582</td>
</tr>
</tbody>
</table>

0.5778 0.6887 0.6065

(there are nine hundred and ninety-nine sets data after this, then ..... )

THE RANGE OF G IS FROM 0.011 TO 0.001

AVE LENGTH OF C.I. 0.167901 0.168828 0.170452

# OF SHORTEST C.I. 616 0 383

# OF TYPE I ERROR 54 53 56
3-1 General discussion

Assays may widely vary in precision, so that each estimation of potency should be accompanied by an estimate of its reliability. This may be expressed as its standard error or its confidence interval. The smaller the standard error or the shorter the confidence interval is, the better the precision is. Although a shorter confidence interval is more desirable, it is also important that the true value of the parameter is contained within the confidence interval. Counting the number of type I errors (those confidence intervals not containing the true value of the parameter) occurring in each method becomes necessary, because this will also be used to evaluate the desirability of each method. The method which has the shortest average confidence interval length or has the largest number of shortest confidence intervals is not necessarily the best method among the three. This is an important point which should be remembered. Therefore, the average length of confidence interval, the number of shortest confidence intervals, and the number of type I errors in one thousand sets of generalized data are all given consideration for the purposes of this study.

It has been mentioned before that in data with a small value of $g$, the $b$ value was set at ten times its standard error. Also, if $g$ is very small and can be neglected, the formula for obtaining the confidence interval can be reduced to the same as coefficient of variation formula.
in the upper half (the small g portion) of Table 1, the value of g never exceeded 0.02. It would be small enough to be neglected, but for the study point of view, even though g is very small, it is still used in the formula of Fieller's theorem. Generally speaking, small g had shorter average length of the confidence interval than those with large g. The range of large g in this study is from 0.01 to 0.365, the value of g greater than 0.365 would be virtually invalid to the study. The value of the means, either large or small, had very little effect on the range of g. For example, in small g, equal variances and small means had the same range as equal variances and large means. It also has been mentioned before that when the value of g is greater than 0.2, the width of the confidence interval will be much understated by use of coefficient of variation formula. There are a number of samples of output listed on Table 3 which demonstrated the previous statement. The 95% confidence level is used through the study, hence the values of t used were 2.101 with 18 degrees of freedom, and 2.262 with 9 degrees of freedom. For the number of shortest confidence interval, some sets of data did not add to one thousand, because there are some with confidence intervals of equal length.

3-2 The coefficient of variation method

This method always had the shortest average length of confidence interval, and in 6 of the 8 situations it had the largest number of shortest confidence intervals among the three, except different variances in small g. It always had several more type I errors than that of the Fieller's theorem method, but usually several less than that of the method of logarithms. At most, only about 6.5% of data was found having type I errors in each set of data. The formula to get the confidence interval is
Table 3. Samples of output with value of g greater than 0.2

<table>
<thead>
<tr>
<th>value of R</th>
<th>value of g</th>
<th>the coef. of var. method</th>
<th>Fieller's theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>conf. int.</td>
<td>length of c.i.</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.2913</td>
<td>0.8286</td>
</tr>
<tr>
<td>0.5</td>
<td>0.21</td>
<td>0.285</td>
<td>0.8012</td>
</tr>
<tr>
<td>2</td>
<td>0.22</td>
<td>1.0808</td>
<td>3.3732</td>
</tr>
<tr>
<td>2</td>
<td>0.23</td>
<td>1.2736</td>
<td>3.9626</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>1.1056</td>
<td>3.7123</td>
</tr>
<tr>
<td>0.5</td>
<td>0.279</td>
<td>0.2526</td>
<td>0.8666</td>
</tr>
<tr>
<td>0.5</td>
<td>0.298</td>
<td>0.3515</td>
<td>1.2391</td>
</tr>
<tr>
<td>0.5</td>
<td>0.313</td>
<td>0.2219</td>
<td>0.8452</td>
</tr>
<tr>
<td>2</td>
<td>0.319</td>
<td>1.3102</td>
<td>5.1066</td>
</tr>
<tr>
<td>0.5</td>
<td>0.32</td>
<td>0.244</td>
<td>0.8961</td>
</tr>
<tr>
<td>2</td>
<td>0.333</td>
<td>0.8711</td>
<td>3.8076</td>
</tr>
<tr>
<td>0.5</td>
<td>0.365</td>
<td>0.2826</td>
<td>1.2250</td>
</tr>
</tbody>
</table>
much simpler than Fieller's theorem, and this is an advantage of this method. Also, the shortest average length of confidence interval is another of its advantages.

3-3 Fieller's theorem

This method always had the least type I errors. The average length of confidence interval is next to the coefficient of variation method. Although, it never has the shortest confidence interval among three methods, the average length of confidence interval is still shorter than the method of logarithms, except for the situation with different variances and small $g$.

According to the length of confidence interval, it can be said that this method is better than the method of logarithms. From the number of type I errors record, this method will be the "best" of all. But the formula to get the confidence interval is more complicated than the other two. From the economic point of view, this method may be not as good.

3-4 The method of logarithms

This method is better than the other two when the data shows a small $g$ and different variances. It has the shortest average length of confidence interval and largest number of shortest confidence intervals under these conditions. But the number of type I errors is about 3% more than the other two methods.

In general, when this method is used, the variance within treatments is more homogeneous. People will use them as same variances, therefore, in this method, equal variances of the two preparations is assumed, and the pooled variance is used to estimate the population variance.
When the range of the set of generalized data is not too wide (small \( g \) portion of Table 1), the length of the confidence interval with this method is very good in comparison with the other two methods. However, for the data in the large \( g \) portion of Table 1, the standard error is big compared with its mean. In this case, the data spreads wider than those with small standard errors. There is a higher probability of getting a very small observation to a very large one. In other words, the range of the generalized data increases and the length of the confidence interval also increases. It seems that this method increased the length of confidence interval faster than the other two methods.

The formula for getting the confidence interval with this method is the simplest one, because it does not have the problem of a variance of a ratio. When it is used to estimate \( R \), it only uses the difference of two means instead of the ratio. It is easy to calculate as a straightforward group comparison between means is used. This is an advantage of this method.
Chapter IV

CONCLUSION

The suggested method to be used for each category is indicated in Table 4. The following guidelines might be considered in the selection of the appropriate method.

From the standpoint of simplicity and economy, the method of logarithms could be used with equal variances and small $g$. Since under those conditions the average length of the confidence interval and the number of type I errors differed very little from the coefficient of variation method.

Fieller's theorem is a more conservative method than the other two. It is also the most exact procedure. Although the average length of confidence interval is a little larger than the method of coefficient of variation, the number of type I errors is small making it the most accurate.

The coefficient of variation method appears to be the most robust. It could be used in all categories. It is not too complicated to calculate, and under most conditions, it will provide good results.
Table 4. Suggested method to be used in different categories

<table>
<thead>
<tr>
<th>categories</th>
<th>$\bar{X}$</th>
<th>$S_X$</th>
<th>$\bar{Y}$</th>
<th>$S_Y$</th>
<th>$N$</th>
<th>range of $g$</th>
<th>suggested method</th>
</tr>
</thead>
<tbody>
<tr>
<td>small variance</td>
<td>50</td>
<td>5</td>
<td>40</td>
<td>5</td>
<td>1000</td>
<td>0.011 - 0.001</td>
<td>coefficient of variation</td>
</tr>
<tr>
<td>unequal variance</td>
<td>20</td>
<td>2</td>
<td>15</td>
<td>3</td>
<td>1000</td>
<td>0.017 - 0.001</td>
<td>logarithms</td>
</tr>
<tr>
<td>equal variance</td>
<td>250</td>
<td>25</td>
<td>150</td>
<td>25</td>
<td>1000</td>
<td>0.011 - 0.001</td>
<td>coefficient of variation</td>
</tr>
<tr>
<td>unequal variance</td>
<td>250</td>
<td>25</td>
<td>200</td>
<td>40</td>
<td>1000</td>
<td>0.017 - 0.001</td>
<td>logarithms</td>
</tr>
<tr>
<td>equal variance</td>
<td>4</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>1000</td>
<td>0.333 - 0.027</td>
<td>coefficient of variation</td>
</tr>
<tr>
<td>unequal variance</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>.4</td>
<td>1000</td>
<td>0.365 - 0.01</td>
<td>coefficient of variation</td>
</tr>
<tr>
<td>equal variance</td>
<td>100</td>
<td>50</td>
<td>300</td>
<td>50</td>
<td>1000</td>
<td>0.319 - 0.025</td>
<td>coefficient of variation</td>
</tr>
<tr>
<td>unequal variance</td>
<td>100</td>
<td>50</td>
<td>200</td>
<td>60</td>
<td>1000</td>
<td>0.332 - 0.015</td>
<td>coefficient of variation</td>
</tr>
</tbody>
</table>
REFERENCES


The program is for two preparations having the same variances. From this program, it can be extended to the program for the preparations with unequal variances. The differences of these two programs are indicated as following:

1. In the formula of getting g, the pooled variance (indicated as SPX) is used in equal variances, but the variance of b (SIGX) is used in unequal variances.

2. As to the t value, for the equal variances is 2.101 (t-deviate with 18 degrees of freedom) and for the unequal variances is 2.262 (t-deviate with 9 degrees of freedom).

3. VX (VY) is indicated as the variance over the sample mean. In equal variances, SPX/number of observations of A (or B) is equal to VX and VY. In unequal variances, VX is equal to SIGX/number of observations of B, VY is equal to SIGY/number of observations of A, where SIGX, SIGY are variances of B and A respectively.
DIMENSION X(10), Y(10), A(10), B(10)
IS=6749157
AA=1.
BB=0
DO 99 IK=1,1000
WRITE(6,10)
10 FORMAT(///,14X,'TEST',9X,'STANDARD',25X,'TEST',9X,'STANDARD')
DO 1 I=1,10
13 CA=RNOR(IS)
   IF(CA.LE.-2.) GO TO 13
   X(I)=CA*2.+4.
14 CB=RNOR(IS)
   IF(CB.LE.-4.) GO TO 14
   Y(I)=CB*2.+8.
   A(I)=ALOG10(X(I))
   B(I)=ALOG10(Y(I))
   WRITE(6,15) X(I),Y(I),A(I),B(I)
15 FORMAT(1OX,Fl0.4,5X,Fl0.4,21X,Fl0.4,5X,Fl0.4)
1 CONTINUE
WRITE(6,40)
40 FORMAT(///,SX,' R ',SX,'S.D. R',4X,'CONF. INT. FOR R',10X,'G',6X,*'FIELLERS THM. ',10X,'LOGR',5X,'S.D. LR',5X,'CONF. INT. FOR LR')
XB=0
YB=0
Z1B=0
Z2B=0
SIGX=0
SIGY=0
SIGZ1=0
SIGZ2=0
I=10
DO 5 K=1,I
   XB=XB+X(K)/FLOAT(I)
   YB=YB+Y(K)/FLOAT(I)
   Z1B=Z1B+A(K)/FLOAT(I)
   Z2B=Z2B+B(K)/FLOAT(I)
5 CONTINUE
R=YB/XB
RL=Z2B-Z1B
DO 6 J=1,I
   SIGX=SIGX+(X(J)-XB)**2
   SIGY=SIGY+(Y(J)-YB)**2
   SIGZ1=SIGZ1+(A(J)-Z1B)**2
   SIGZ2=SIGZ2+(B(J)-Z2B)**2
6 CONTINUE
SPX=(SIGX+SIGY)/(2*FLOAT(I)-2)
SPY=SQRT(SPX)
SPZ=(SIGZ1+SIGZ2)/(2*FLOAT(I)-2)
G=(2.101)**2*SPX/(XB**2*FLOAT(I))
IF(G.LT.AA) AA=G
IF(G.GE.BB) BB=G
VX=SPX/FLOAT(I)
VY=SPX/FLOAT(I)
VR = SQRT((1./XB**2)*(VY+R**2*VX))
VM = SQRT(SPZ/FLOAT(I)+SPZ/FLOAT(I))
ANS1 = R - 2.101*VR
ANS2 = R + 2.101*VR
IF (ANS1.GT.2.0R.ANS2.LT.2) JJ = JJ + 1
PP = ANS2 - ANS1
PPP = PPP + PP
ANS3 = (R - (2.101*SPY/XB)*(1/FLOAT(I)+R**2/FLOAT(I)-G/FLOAT(I)**2)**(1/2)) / (1-G)
ANS4 = (R + (2.101*SPY/XB)*(1/FLOAT(I)+R**2/FLOAT(I)-G/FLOAT(I)**2)**(1/2)) / (1-G)
IF (ANS3.GT.2.0R.ANS4.LT.2) KK = KK + 1
PQ = ANS4 - ANS3
PPQ = PPQ + PQ
ANS5 = RL - 2.101*VM
ANS6 = RL + 2.101*VM
WRITE (6, 20) R, VR, ANS1, ANS2, G, ANS3, ANS4, RL, VM, ANS5, ANS6
20 FORMAT (3X, F6.3, 4X, F6.3, 3X, F7.4, 3X, F7.4, 8X, F5.3, 3X, F7.4, 4X, F6.4, 4X, F8.4)
IF (ANS5.LT.0) GO TO 21
ANSX = 10**ANS5
GO TO 23
21 ANS5 = ANS5 + 1
ANSX = 10**ANS5 / 10
23 IF (ANS6.LT.0) GO TO 22
ANSY = 10**ANS6
GO TO 24
22 ANS6 = ANS6 + 1
ANSY = 10**ANS6 / 10
24 PR = ANSYS - ANSX
IF (ANSX.GT.2.0R.ANSY.LT.2) II = II + 1
PPR = PPR + PR
WRITE (6, 11) ANSX, ANSY, PP, PQ, PR
IF (PP.LT.PR.AND.MM.PLT.PQ) JA = JA + 1
IF (PP.LT.MM.AND.MM.MM.PLT.PR) KA = KA + 1
IF (PR.LT.MM.AND.MM.MM.PLT.PR) IA = IA + 1
99 CONTINUE
PPP = PPP / 1000
PPQ = PPQ / 1000
PPR = PPR / 1000
WRITE (6, 12) AA, BB, PPP, PPQ, PPR, JA, KA, IA, JJ, KK, II
STOP
END
FUNCTION TNOR(IR)
C* GENERATES A RANDOM NORMAL NUMBER (0,1)
C* IARG IS A LARGE ODD INTEGER FOR A BEGINNING ARGUMENT
C* REQUIRES FUNCTION RN WHICH GENERATES A UNIFORM RANDOM NUMBER 0-1
$SET OWN
   DATA I/0/
$RESET OWN
   IF(I.GT.0) GO TO 30
10  X=2.0*RANDOM(IR)-1.0
    Y=2.0*RANDOM(IR)-1.0
    S=X*X+Y*Y
    IF(S.GE.(1.0)) GO TO 10
    RNOR=X*S
$SET OWN
   GO2=Y*S
$RESET OWN
   I=1
   GO TO 40
30  RNOR=GO2
    I=0
   40  RETURN
END
VITA

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