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Examining Quadratic Relationships Between Traits and Methods in Two Multitrait-Multimethod Models

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EXAMINING QUADRATIC RELATIONSHIPS BETWEEN TRAITS AND METHODS IN TWO MULTITRAIT-MULTIMETHOD MODELS.

By

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Abstract

Psychological researchers are interested in the validity of the measures they use, and the multitrait-multimethod design is one of the most frequently employed methods to examine validity. Confirmatory factor analysis is now a commonly used analytic tool for examining multitrait-multimethod data, where an underlying mathematical model is fit to data and the amount of variance due to the trait and method factors is estimated. While most contemporary confirmatory factor analysis methods for examining multi-trait multi-method data do not allow relationships between the trait and method factors, a few recently proposed models allow for the examination of linear relationships between traits and methods. While these models provide a great advantage in analyzing multitrait-multimethod data, there is no theoretical reason why trait and method relationships should always be linear, and quadratic relationships are frequently proposed in the social sciences. Therefore, this paper proposes and describes in detail two models for examining quadratic relations between traits and methods. An application to a multi-rater study of children with attention deficit and hyperactivity disorder (N=800) and results of a Monte Carlo study to test the applicability of the method under a variety of data conditions are described.
Introduction

In psychology, researchers frequently want to be able to examine the validity of the tests and measurements they use. Convergent validity is one aspect of validity that researchers typically study, and it is defined as the extent to which different measures (or “methods of measurement”) of the same psychological construct are related (Campbell & Fiske, 1959; Cronbach & Meehl, 1955). Another important aspect of validity is discriminant validity, which is defined as the extent to which measures of different constructs measured with the same method are sufficiently different from each other (Campbell & Fiske, 1959). For example, Cole, Martin, Powers, & Truglio (1996) examined the extent to which children’s self-reports of their depression, were concordant with the reports of their parents, their teachers, and their peers, thus testing convergent validity. In the same study, the authors also examined discriminant validity by looking at the extent to which the relations between the ratings of child depression, academic competence, and social competence were inflated due to the reporter providing the measurement.

A common approach for examining convergent and discriminant validity is called the multitrait-multimethod (MTMM) design. To this end, researchers gather data on multiple traits (e.g., depression, self-esteem, competence) gathered with multiple methods (Campbell & Fiske, 1959). Each component of the study, which is referred to as a trait-method unit (TMU), consists of one trait measured by one method (e.g., self-reports of social competence; Campbell & Fiske, 1959). Convergent and discriminant validity are evaluated using specific criteria based on the correlation matrices between TMUs (Campbell & Fiske, 1959). Specifically, convergent validity is supported by strong correlations between ratings of the same trait by different methods. Whereas, discriminant validity is supported by weak correlations between different traits measured by the same method. Although the article is widely cited and the method heavily used,
several shortcomings of the analytic method have been noted (Widaman, 1985). These shortcomings include differing reliabilities of the measures included in each correlation, a lack of a formal test of statistical significance for the criteria, and very complex analysis required for large numbers of traits and methods (Marsh & Hocevar, 1988; Widaman, 1985).

Confirmatory factor analysis (CFA) models have been applied to remedy the shortcomings of MTMM designs. CFA explains the complex MTMM correlation matrix as a function of trait-related variance and method-related variance. This is accomplished by fitting a factor model to the MTMM correlation matrix and testing whether the model is a good fit to the data (Browne, 1985; Eid, 2000; Kenny, 1976; Marsh & Hocevar, 1983; Widaman, 1985). There are several different models of the MTMM matrix within the larger category of CFA for MTMM. Each one represents method and trait effects in different ways.

For example, the correlated-traits correlated-methods model (CTCM) includes as many trait factors and method factors as there are traits and methods in the study, and allows correlations between the trait factors, and correlations between the method factors, but no correlations between the trait and method factors (Widaman, 1985). The correlated-traits-uncorrelated-methods model (CTUM), like the CTCM model, specifies as many trait variables and method variables as are included in the study, and allows correlations between the trait variables but restricts the method variables to be correlated with each other (Marsh & Hocevar, 1983). The correlated-traits-correlated-uniqueness (CTCU) model includes as many trait factors as there are traits in the study, and instead of method factors allows correlations between the error variables for the same method (Kenny, 1976).

Most CFA-MTMM models posit that trait factors can be correlated with other trait factors, and method factors can be correlated with other method factors. However, correlations
between trait and method factors are typically restricted to zero, either for ease of interpretation, statistical reasons, or based on the definition of method factors as regression residuals (e.g., Eid, 2000). Nonetheless, such trait-method correlations could be present and meaningful in practical applications of the MTMM approach. Specifically, trait-method correlations indicate that method effects are larger or smaller depending on the level of the trait.

An example of a potential trait-method relationship is that peer and self report effects could be related to an individual’s level of extraversion. When an individual is low in extraversion, it may be more difficult for peers to judge the extent to which they prefer to be alone rather than with others, and the discrepancy between peer and self report will be higher. Method effects (discrepancies between peer and self reports) would therefore be larger at low levels of the trait. When an individual is high in extraversion, it may be easier for the peer to see that an individual prefers to be around others and the peer and self reports might agree more highly. Method effects would thus be smaller at high levels of the trait. This would be reflected in a negative relationship between the trait (extraversion) and the method effect (peer report vs. self-report): the higher the trait score, the smaller the method effect.

Effects like these are ignored in most frequently used MTMM models (Podsakoff, Mackenzie, Lee, & Podsakoff, 2003). In most MTMM models, it is implicitly assumed that trait levels are unrelated to method effects. This is either because the method effects are defined as uncorrelated with the trait factors (Eid, 2000; Kenny, 1976), or because of concerns with identification, convergence, overfitting, or interpretability of model parameters (Marsh & Grayson, 1995; Widaman, 1985). These restrictions are typically chosen for statistical expediency rather than for substantive reasons. In fact, the creators of the MTMM approach later acknowledged that “method and trait or content are highly interactive and interdependent” (Fiske
& Campbell, 1992). In addition, Marsh and Grayson (1995), in a review of CFA-MTMM models, wrote that in models in which trait-method correlations are allowed in principle, they are typically constrained to 0 “to avoid technical estimation problems and to facilitate decomposition of variance into trait and method effects, not because of substantive likelihood or empirical reasonableness” (p.181). In summary, it is plausible that trait and method factors could be correlated, yet most commonly used statistical models for MTMM data do not allow the estimation of correlations between trait and method factors.

Early applications of CFA-MTMM models found large trait-method correlations when using correlated-trait correlated-methods models (Boruch & Wolins, 1970; Kalleberg & Kluegel, 1975; Schmitt, 1975). However, the models used in these studies (correlated-trait correlated-methods models) have conceptual problems and show frequent convergence, admissibility, and interpretation problems (Geiser, Eid, & Nussbeck, 2008; Marsh, 1989). The addition of correlations between trait and method factors compounded these problems (Kenny & Kashy, 1992; Marsh, 1989). New methods were therefore needed to estimate trait-method relationships.

Recently, two CFA-MTMM models have been proposed that allow for the estimation of trait-method correlations and do not show the same estimation and identification problems as the CT-CM approach: the latent difference (LD) and the latent means (LM) model (Pohl, Steyer, & Kraus, 2008; Pohl & Steyer, 2010). These models are based on true score theory and can be used to estimate linear relationships between traits and methods. Linear relationships between traits and methods indicate that method effects increase or decrease as a function of the trait level. One shortcoming of the LD and LM models in their current form is that quadratic relationships between traits and methods cannot be tested. A quadratic relationship could indicate that, for example, parent reports of their children’s attention deficit and hyperactivity (ADHD) are less
discrepant each other at low levels of symptoms. In addition, they could be more discrepant at intermediate levels of symptoms, possibly because some ADHD symptoms may not be visible to the parents. Lastly, they could be less discrepant at the highest levels of symptoms because high levels of ADHD symptoms may be more easily observed by the parents than lower levels.

Currently, such quadratic relationships cannot be tested within the CFA-MTMM framework. Some methodologists have previously noted theoretical models involving quadratic effects are frequently posited in psychology, but methodological understanding of how to test those models is lacking (Aiken & West, 1991). Ignoring quadratic trait-method relationships could result in either disregarding a true threat to convergent validity, or inappropriately concluding that convergent validity is lower than it truly is. To remedy this issue and avoid these problems, this paper will propose an approach for examining quadratic trait-method relations with MTMM data.

The remainder of the paper is organized as follows: First, the LD and LM models are reviewed. Second, the interpretation of linear trait-method relationships in the latent difference and latent means models is compared. Third, two new models are described which represent extensions to the original LD and LM models to modeling quadratic relationships between traits and methods. Fourth, interpretation of quadratic trait-method relationships in the two models is described. Lastly, an application of the linear and quadratic LD and LM models to real data is presented.

The Latent Difference and Latent Means Models

In this section, the mathematical definitions and assumptions characterizing the LD and LM models are presented. For simplicity, only one trait that is measured by two methods is considered, as this is the simplest possible way to examine convergent validity (Geiser, Eid,
West, Lischetzke, & Nussbeck, 2012). Even this minimal design allows for the estimation of trait-method relationships, and the extension to additional traits and methods is straightforward.

In addition, I consider models with multiple indicators, in line with the basic TMU model proposed by Marsh & Hocevar (1988). This approach is preferable because it allows for the examination of latent TMUs that are corrected for measurement error (Marsh & Hocevar, 1988). Before introducing the LD and LM models, I discuss the basic TMU model that serves as the basis for formulating both the LD and LM models.

**The Basic TMU Model**

Throughout the present paper, I consider one trait that is measured with two methods ($m$). Each method uses three observed variables (indicators) $Y_{im}$, where $i$ indicates the observed variable. Note that no index for the trait is required, given that only one trait is considered. In the basic TMU model, all variables $Y_{im}$ that share the same method (index $m$) measure a common method-specific latent factor (true score variable) $T_m$ as well as a measurement error variable. The measurement equation for each variable is

$$Y_{im} = \alpha_{im} + \lambda_{im} T_m + \epsilon_{im},$$

(1)

where $\alpha_{im}$ is a constant intercept parameter, $\lambda_{im}$ is a constant factor loading parameter, $T_m$ is a latent variable representing the TMU after correcting for random measurement error, and $\epsilon_{im}$ is an indicator-specific measurement error variable. The means and variances of the $T_m$ factors are freely estimated, and the $\epsilon_{im}$ variables have variances estimated, but have means of 0 by definition as measurement error variables (Steyer, 1989). For model identification purposes, researchers typically fix the intercept ($\alpha_{im}$) and factor loading ($\lambda_{im}$) of one indicator to 0 and 1, respectively. In line with classical test theory, it is assumed that the covariance between the $T_m$
factors and the error variables $\varepsilon_{im}$ are zero, and that the covariances between all $\varepsilon_{im}$ factors are zero.

All $T_m$ factors are allowed to be correlated. High correlations indicate strong convergent validity across methods. Strong method effects would be indicated by weak correlations between $T_m$ factors. One limitation of the basic TMU model is that it does not contain latent variables representing method effects (method factors). Therefore, method effects cannot be related to external variables. The LD and LM models are equivalent to the baseline TMU model, but address this limitation by including method factors. Furthermore, the basic TMU model uses “method-specific” trait factors (i.e., the $T_m$ factors do not reflect “pure” trait effects, but contain method-specific effects as well). The LM model also addresses this second limitation.

Measurement invariance of the loadings and intercepts of the observed indicators across the two methods (also called “strict” or “scalar” invariance) is a prerequisite for meaningful interpretations of the structural parameters in the baseline TMU model (Pohl, Steyer, & Kraus, 2008; Pohl & Steyer, 2010). Allowing non-invariance for some indicators would mean that changes in the latent variables are due to changes in the measurement structure of the indicators rather than meaningful changes due to the change in method (Geiser, Burns, & Servera, 2014; Pohl, Steyer, & Kraus, 2008; Pohl & Steyer, 2010). Prior to fitting either the LD or the LM model, invariance of the model loadings and intercepts should be tested in the basic TMU model using model difference tests (Cheung & Rensvold, 2002). Because measurement invariance is a prerequisite for interpretation, Figure 1 refers to the $\lambda_{im}$ and $\alpha_{im}$ parameters with only a subscript $i$, because they are assumed to be invariant to the method. After establishing measurement invariance in the TMU model, researchers can move on to examine method effects in the more-complex LD and LM models.
The LD Model

The LD model defines a method factor as the difference between the $T_m$ variable and the TMU variable pertaining to a chosen reference method $m = 1$ (Geiser et al., 2012; Pohl et al., 2008). The LD approach thus requires the selection of a reference method, against which another method is contrasted. It has been suggested that in MTMM models requiring the selection of a reference method that a “gold standard” method or a method with a clear structural difference from the others should be chosen as the reference method (Geiser, Eid, & Nussbeck, 2008). For example, when self- and other-reports are used in a study, it would be natural to select self-report as the reference method unless there are theoretical or practical reasons to prefer a different method (e.g., in studies of very young children, parent reports might be seen as more dependable than the children’s self-reports).

In the LD model, the latent TMU variables are decomposed into the trait as measured by the reference method ($T_1$) and the difference between the reference and non-reference methods ($T_m - T_1$), that is:

\[ T_m = T_1 + (T_m - T_1) , \]  

\( (2) \)

Where $T_m$ refers to the trait as measured by method $m$, $T_1$ refers to the trait as measured by reference method 1, and $(T_m - T_1)$ is a variable representing the difference between $T_m$ and $T_1$. A method factor $M_m$ can then be defined as the difference between the two TMU variables:

\[ M_m = (T_m - T_1) . \]  

\( (3) \)

The mean and covariance structure of the trait and method variables are unrestricted, such that means, variances, and covariances can be estimated for all trait and method factors.
Individual scores on the method factor in the latent difference model indicate the difference between the non-reference method and the reference method for a given individual. For example, if both a mother and a father rate a child’s inattention level with mother-report used as the reference method, and the scores of the latent TMU variables are 1 for the mother and 1.5 for the father, then the value of the method variable for that child is 0.5. This would indicate that the child’s score is overestimated by 0.5 by the teacher relative to his or her mother.

The model is interpreted by examining the mean and variance of the latent method and latent trait variables along with the regression of the method variable on the trait variable. The mean of the method factor indicates the average difference between the non-reference method as compared to the reference method. For example, if the mean of the method factor is 0.5, then, on average, all fathers in the sample could be expected to rate children 0.5 points higher than all mothers in the sample. The variance of the method factor indicates the spread of the individual method effect scores. For example, if the variance of the method factor is 0.81, then the standard deviation is $\sqrt{0.81} = 0.9$, which means that each discrepancy is, on average, 0.9 points away from the mean difference score. Therefore, even though fathers rate 0.5 points higher than mothers on average, it is likely that if the mother rates the child as a 1, a substantial number of father ratings for that child (68%) were anywhere between 0.6 and 2.4. High convergent validity would be supported if the mean of the method factor was close to zero, and the variance of the method factor was relatively low, because this would indicate that the scores from both methods tend to show strong agreement.

The covariance between the trait and method factors can be represented either as a covariance parameter or, equivalently, as a linear regression of the method factor on the trait factor, as follows:
\[ M_m = \beta_1 T_i + \beta_0 + \zeta_{M_m}, \]  

(4)

Where \( \beta_1 \) represents a constant regression weight parameter, \( \beta_0 \) represents a constant intercept parameter, and \( \zeta_{M_m} \) represents a regression residual variable that reflects variance in \( M_m \) that is not accounted for by \( T_i \). The \( \beta_1 \) parameter represents the strength of the relationship between the trait factor and the method factor. Figure 1B shows a path diagram of this model, with the mean structure included.

When the trait-method relationship is parameterized as a linear regression, the mean and variance of the method factor and the covariance of the method factor with the trait factor are instead represented by the intercept \( (\beta_0) \), residual variance \( (\zeta_{M_m}) \), and regression weight \( (\beta_1) \) parameters. The intercept reflects the expected value of the method factor when the reference-method trait level is zero. The residual variance indicates the variance of the method factor not accounted for by a linear relationship with the reference-method trait factor. The regression weight parameter indicates the degree to which the method factor is linearly related to the reference trait factor. For example, a positive regression weight parameter would show that when mothers rate inattention more highly, fathers tend to overestimate hyperactivity more highly than for low mother scores. A negative regression weight parameter indicates that, as the levels of the trait as measured by the reference method increase, the method scores become smaller. That is, when mothers rate children’s inattention more highly, fathers might tend to either overestimate inattention less strongly (difference scores get closer to zero), or underestimate inattention compared to mothers more strongly (difference scores get more negative). The size of the regression weight parameter is dependent on the variance of the reference-method trait variable and the residual variance of the method variable, and should therefore be considered in
conjunction with the variance of the trait factor. The standardized regression weight indicates the correlation between reference trait and method factor.

**The Latent Means Model**

An alternative to the LD model is the latent means (LM) model, which does not require the selection of a reference method. In the LM model, a common trait factor is defined as the mean of the trait factors for both methods, and the method effect factor is defined as the deviation of either method from the mean (Pohl & Steyer, 2010). The LM model may therefore be more meaningful when a clear reference method is not available, or the methods are not clearly distinguishable (e.g., interchangeable judges), since the common trait factor is defined as the average of scores from both methods. The trait variable for a LM model in which one trait is measured by two methods is defined as:

$$ T = \frac{T_1 + T_2}{2} , $$

where $T_2$ is the trait factor measured by method 2, and $T$ is the common trait factor. The method effect variables are defined as:

$$ M_1^* = T_1 - T $$
$$ M_2^* = T_2 - T $$

Where $M_2^*$ is the method variable for method 2. (The method factors in the LM model are denoted as $M_2^*$ to differentiate them from the method factors in the LD model, which are defined as differences from a reference method rather than differences from an average.) Since $T$ is defined as the mean of the trait variables for both methods, the deviations from the trait factor
add up to 0 by definition, and are therefore equal in magnitude but oppositely signed\(^1\), as shown in equation 7:

\[
M_1^* + M_2^* = 0 \\
M_1^* = -M_2^*
\] (7)

Given the deterministic relationship between method factors as shown in equation 7, for designs with two methods, only one method factor is needed and the subscript \(m\) can be dropped. Therefore, the structural model for the TMU factors is given by:

\[
T_1 = T - M^* \\
T_2 = T + M^*
\] (8)

Although a reference method is not chosen, one method must be selected to have positive loadings onto the method factor and the other must have negative loadings onto the method factor. In the LM model, both \(T_1\) and \(T_2\) are completely determined by the \(T\) and \(M\) factors. Similarly to the LD model, the means, variances, and covariances of all latent variables are freely estimated. The covariance between the trait and method factors can also be parameterized as a linear regression of the method factor on the trait factor:

\[
M^* = \beta_1^*T + \beta_0^* + \zeta_M^* ,
\] (9)

where \(\beta_1^*\) is a constant regression weight parameter, \(\beta_0^*\) is a constant intercept parameter, and \(\zeta_M^*\) is a latent residual variable representing variability in \(M^*\) that is not accounted for by \(T\).

\(^1\) This is only true for the two method case in the latent means model. Three or more methods require that \(m-1\) method effects be estimated and the value of one method effect is a deterministic function of the others (Pohl & Steyer, 2010).
Figure 1C shows a path diagram of the LM model with the mean structure and latent regression included.

Interpretation of the latent means model primarily focuses on the examination and comparison of the mean and variance of the method factor relative to the trait factor. The mean of the method factor indicates the average deviation from the common trait for the method that has positive loadings on the $M^*$ factor. The individual scores on the method factor indicate $\frac{1}{2}$ times the difference between each method’s scores for that individual. For example, if a mother assigns an inattention value of 2 for her child, and a father rates that child as a 3, the score for that individual on the $M^*$ factor would be 0.5. The mean of the $M^*$ factor is the mean of these individual scores, and therefore the mean difference between the two methods is 2 times the mean of the $M^*$ factor. The variance of the method factor indicates the spread of the method factor scores. Since the $M^*$ factor represents half the distance between the TMU scores, the variance of $M^*$ is $\frac{1}{4}$ the size of the variance of the method factor in the LD model. For this reason, it may be easier to interpret the standard deviations of the $M^*$ factor instead of the variance. The standard deviation of the $M^*$ factor indicates how far away the typical individual $M^*$ score is from the mean of the $M^*$ variable, and can be compared to the standard deviation of the $T$ variable to judge the size of the effect. Convergent validity is supported by a relatively low mean and standard deviation of the method factor.

Lastly, a potential linear relationship between the $M^*$ factor and the trait factor can be examined by analyzing the $\beta_0^*$, $\text{Var}(\zeta_{M^*})$, and $\beta_1^*$ parameters. The intercept parameter indicates the expected value of the $M^*$ variable when the trait factor is 0. The variance of the residual variable [$\text{Var}(\zeta_{M^*})$] indicates how much of the variance in the $M^*$ variable is linearly unrelated to the trait variable. The regression slope parameter, ($\beta_1^*$) represents the size of the relationship.
between the trait variable and the $M^*$ variable. Higher values of the regression slope parameter indicate that values of the $M^*$ factor increase more steeply as values of the trait increase.

**Comparison of the LD and LM Models for Analyzing Trait-Method Relations**

Although the LD and LM models employ a similar strategy to define and analyze method effects, each provides a different perspective on the definition of the trait and method factors. The two models are similar in that they both allow trait-method relationships. In both models, the covariance between the trait and method factor can be modeled as a linear regression of the method factor on the trait factor. The trait factors, however, are defined differently in each model, which means that the trait-method relationship has a different meaning. Although model fit is identical for the linear LD and LM models, the variance of the trait and method factors, and the size and direction of the trait-method relationship can be very different in each model.

In the LD model, the trait factor is defined to be the trait as measured by a reference method, and the method factor is the difference between the reference method TMU and the non-reference method TMU. Therefore, the trait-method relationship is the relationship between the level of the reference method TMU and the difference between the two TMU variables. Consider a mother and father rating a child’s level of inattention on a 1 to 5 continuous scale, where 5 indicates higher inattention symptoms. After correcting for measurement error, the mother rated the child’s inattention as 2.5, and the father rated the child’s inattention as 1.5. If I were to use the LD model for this data and select the mother’s rating as the reference method, the trait factor score for that child is 2.5, and the method factor score for the father is -1, because the father rated the child 1 point lower than the mother did. If in a study of multiple children using their mothers’ and fathers’ ratings of inattention, I find that in general, when mother ratings of inattention are higher, that the difference between father and mother ratings increases, this would indicate a
positive trait-method relationship. That is, the father method effect is larger when mothers rate children as higher on inattention. In this way the trait-method relationship in the LD model refers to the relationship between the reference method TMU variable and the difference between the TMUs.

In the LM model, the trait factor is defined as the mean of the trait as measured by both methods, and the method factor \( M^* \) is the deviation of either TMU from the trait factor. In the LM model, the trait factor has a completely different meaning than in the LD model, as it takes information from both TMUs. \( M^* \), however, has a similar meaning to \( M \) in the LD model, since it represents \( \frac{1}{2} \) of the difference between the two TMUs. If I consider the example in the previous paragraph using the LM model instead, a mother rating of 2.5 and a father rating of 1.5 will result in a \( T \) value of 2, method values of 0.5 for the mother rating and \(-0.5\) for the father rating. If in a study of multiple children rated on their inattention by mothers and fathers, we find that, when the value of the mean of mother and father rating increases, the differences between father and mother ratings increase, this would indicate a positive trait-method relationship in the LM model.

Because they define the trait and method factors differently, the LD and LM models will also have different values of their structural parameters (i.e., variance of the trait factor, variance of the method factor, and the covariance between the trait and method factors). This is because each of these is identified by a different function of the variances and covariance between the two TMUs used to make up the model. In appendix B, I derive each of the structural parameters of the LD and LM models from the baseline TMU model. From the identification of the variables, it is clear how even though the models will fit a given dataset equivalently, each model will provide different values for the variance of the trait and method factors and the covariance
between trait and method factors. The two models therefore examine different aspects of the underlying relationship between the TMU variables.

**Quadratic Relationships Between Traits and Methods**

It is plausible that, in addition to linear relationships, quadratic relationships between the trait and method effect occur in practice. For example, the relationship between trait and method effects may be quadratic and nonmonotonic, such that at extreme levels of a trait, method effects tend to be lower, and at central levels of a trait, method effects tend to be higher. Using the same example of mother and father ratings of child inattention, it is plausible that mothers and fathers may agree on levels of symptoms when they are low or when they are high. However, mothers and fathers may disagree when children show intermediate levels of symptoms.

Although plausible in many applications, quadratic trait-method relations have not yet been tested and models have not yet been proposed to examine them. To remedy this problem, in this paper I present new extensions to the LD and LM models that allow for quadratic trait-method relationships.

**The Latent Difference Model with Quadratic Trait-Method Relationships**

Equation 10 shows an extension of the LD model that allows for quadratic relationships between trait and method factors in the structural part of the model. In this extended model, the method factor is regressed on the $T_1$ factor as in the original LD approach. In addition, the method factor is also regressed on a quadratic term ($T_1^2$) constructed from the $T_1$ factor:

$$M_m = \beta_0 + \beta_1 T_1 + \beta_2 T_1^2 + \zeta_{M_m},$$

(10)

where $\beta_0$ is a real constant intercept parameter, $\beta_1$ and $\beta_2$ are real constant regression slope parameters, and $\zeta_{M_m}$ is a latent residual variable that represents method variance not accounted
for by $T_1$ or $T_1^2$. Figure 2A shows a path diagram of this model. Examination of the parameters from the regression equation will give the shape of the relationship between the trait and latent difference factors. If the $\beta_2$ value is significantly different from zero, then there is a significant quadratic effect present in the data. A positive $\beta_2$ value indicates an upward u-shaped curve, and a negative $\beta_2$ value indicates a downward-facing inverted-u-shaped curve.

To determine the points at which the values of the method factor are implied to be zero and the maximum level of discrepancy, researchers should calculate the roots and the inflection point of the quadratic function. Note that they are only substantively meaningful if they are within the observed range of the measures. The inflection point is the value of the trait level at which the relationship between the two variables changes direction, and thus reflects the trait level that corresponds to the minimum or maximum level of the method effect. The formula for the inflection point is given by

$$-\frac{\beta_1}{2\beta_2}.$$  

For example, for the quadratic function: $.1T_1^2 - .7T_1 + .6 + \zeta_M$, the inflection point is $\frac{.1}{.7} = 3.5$. This means that below 3.5, values of the method factor decrease as the value of the trait factor increases. Above 3.5, values of the method factor increases the value of the trait factor increase. The model-predicted minimum or maximum value of the method factor can be obtained by plugging the trait-factor value into the model equation. For the previous example, the predicted value of $M_m$ is thus $1*(-3.5)^2 - .7*-3.5 + .6 = -0.63$. This means that for the previously given model-estimated formula, the maximum predicted value of $M_m$ occurs when $T$ is at 3.5 is $-0.63$. 
The size of this maximum predicted value should be interpreted in the units of the measure being examined.

The values of $T$ for which the predicted value of $M_m$ is zero are called the roots of the function. The roots are given by the quadratic formula, which is shown here:

$$T_i = \frac{-\beta_1 \pm \sqrt{\beta_1^2 - 4\beta_2\beta_0}}{2\beta_2}.$$  \hspace{1cm} (A12).

There may be zero, one, or two roots. The roots of the method factor regression are only of substantive interest if they are within the range of the observed data. In the example given, the roots are $T_1=1$ and $T_1=6$. This means that method effects are predicted to be lowest at those levels of the trait.

In addition, a good way to interpret the quadratic function is to plot the trend in the range of the observed data and compare it with the linear model. In this way, the substantive significance (or lack thereof) of the quadratic trend will become more easily apparent.

**The Latent Means Model with Quadratic Trait-Method Relationships**

The proposed LM structural model with a quadratic trait-method relationship is analogous to the previously described extension of the LD model and is given by:

$$M^* = \beta_2^* T^2 + \beta_1^* T + \beta_0^* + \zeta_{M^*}.$$  \hspace{1cm} (A13)

where $\beta_0^*$ is a real constant intercept parameter, $\beta_1^*$ and $\beta_2^*$ are real constant regression slope parameters, and $\zeta_{M^*}$ is a latent residual variable that represents variance not accounted for by $T$ or $T^2$. Figure 2B shows a path diagram of this model. The inflection point and roots of this function can be also be calculated using equation 11 and equation 12, and carry the same
meaning. Note that when using the latent means parameterization, the values on the method factor reflect \( \frac{1}{2} \) of the total difference between the TMU variables.

**Comparison of Quadratic Trait-Method Relationships In The LD And LM Models**

An important difference between the linear and quadratic LD and LM models is that when the quadratic extension is added to the LD and LM models, it is no longer true that model fit will be the same for both types of models.

It is well known that the distribution of quadratic and interaction terms is not a normal distribution (Moosbrugger, Schermelleh-Engel, & Klein, 1997). In latent variable models with quadratic structural relationships, the endogenous latent variables and their indicators are therefore implied to be non-normally distributed (Klein & Moosbrugger, 2000).

The implied distribution of the TMU variables is multivariate normal for the linear LD and LM models. For the quadratic LD model, the observed variables for the reference TMU variable \( T_1 \) are implied to be multivariate normal, and the observed variables for the non-reference TMU variable \( T_m \) are implied to be non-normal. For the quadratic LM model, all observed variables for both TMU variables are implied to be non-normal. This is because the latent \( M^* \) variable is implied to have a non-normal distribution, and both latent TMU variables are dependent on the \( M^* \) variable. In this way, each model has a different implication for the distribution of the observed variables.

Although the fit of the models will be different, the nature of the construct being measured and the validity of the methods should be the primary criteria for choosing the LD or LM model in analysis as opposed to model fit. For example, the LD model should be chosen when there is a clear structural difference between methods, or when the reference method is a
“gold standard” method that is widely accepted as a valid measure. The LM model should be chosen when the construct is best defined as the mean of the scores between the two methods.

**Estimation of Quadratic Relationships In The LD and LM Models**

A variety of estimation methods have been developed to analyze quadratic relationships in structural equation models (for an overview, see Harring, Weiss, & Hsu, 2012). In the present paper, I focus on the latent moderated structural equations approach (LMS), which uses numerical integration to approximate the probability density of the quadratic term, and models the probability density of the dependent variables as a mixture of the normal and nonnormal distributions in the independent variable part of the model (Klein & Moosbrugger, 2000). The expectation-maximization (EM) algorithm is then used to estimate the mixing proportions of the normal and nonnormal densities in the dependent variable portion of the model (Klein & Moosbrugger, 2000). The technique has been shown to be unbiased and efficient compared with other techniques and is robust to moderate nonnormality, although other estimation methods may be more robust to nonnormality (Harring, Weiss, & Hsu, 2012). Unlike some other methods, the LMS method does not provide an absolute measure of model fit, but likelihood ratio tests can be used to perform nested model tests against linear models without a quadratic term (Kelava et al., 2011; Klein & Moosbrugger, 2000). In addition, the LMS technique is readily available in the software package Mplus (Muthén & Muthén, 1998-2015).

**Application**

I now present an application of both the quadratic LM and quadratic LD models to real data of multi-rater reports of childhood attention deficit and hyperactivity disorder (ADHD) to demonstrate the estimation and interpretation of non-linear trait-method relationships within the model extensions.
Sample

The data for this application come from a longitudinal study of childhood symptoms of ADHD (Burns, Servera, Bernad, Carrillo, & Geiser, 2014). The sample consisted of children from 30 schools in Madrid and the Balearic Islands in Spain. Children were rated by their teachers, mothers, and fathers regarding several ADHD symptoms and academic performance. For this demonstration, I analyzed mother and father reports of child inattention at the first time point \(N = 752\).

Measures

The measures used were two subscales of the Child and Adolescent Disruptive Behavior Inventory (Burns & Lee, 2010a; Burns & Lee, 2010b). The inattention (ADHD-IN) subscale consisted of nine symptoms. Symptoms of inattention were rated on a 6-point scale ranging from 0 (nearly occurs none of the time [e.g., 2 or fewer times per month]) to 5 (nearly occurs all the time [e.g., many times per day]). ADHD-IN symptoms were split into three parcels of three items each, using a parceling technique designed to create homogeneous parcels (Little et al., 2013). The parcels were treated as continuous measures. Skewness and kurtosis of the parcels was moderate (Skewness was between 1.12 and 2.08 for all parcels; kurtosis was between 0.79 and 3.92 for all parcels; skewness and kurtosis were significantly different from 0 for all observed variables). Previous simulations have shown that LMS is robust to this level of nonnormality (Harring, Weiss, & Hsu, 2012).

Analysis Strategy

The standard latent means and latent difference models with linear trait-method relationships were fit to both datasets to illustrate the differences in interpretation between the two models. Then, the new quadratic extensions were fit to each of the four models. The fit of
the quadratic models was compared against the fit of the linear models using a likelihood ratio test. Statistical significance of the quadratic term was determined using the likelihood ratio test as opposed to a Wald test based on the model-estimated standard errors, as recommended by Klein and Moosbrugger (2000).

Results

Table 1 shows descriptive statistics for the mother and father ratings of child inattention. First, a TMU model was fit with strong invariance constraints (Equation 1). The model also included correlated residual variances between identical parcels across methods to account for relationships between the indicators that are not accounted for by the trait factor (Figure 1A with correlated residuals added; Marsh & Hocevar, 1988). The TMU model showed a good fit to the data when parcel-specific effects were accounted for, \( \chi^2(9, N = 752) = 17.42, p = .04 \), RMSEA = .04, CFI = 0.998. Table 2 shows the model parameters of the underlying TMU model.

**Measurement Model.** Standardized factor loadings for the observed inattention indicators are presented in Table 2, and represent the correlation between the observed variable and the latent TMU factor. Reliability is indicated by the standardized loadings squared. The lowest standardized loading is .9, and the highest is .95. The reliabilities of the observed indicators thus ranged from .81 to .9. The correlations between same parcels ranged between .32 and .44, meaning that moderate parcel-specific effects were present.

**LD model.** In the LD analyses, mother report of child inattention was selected as the reference method \( (m = 1) \), against which father report of child inattention \( (m = 2) \) was contrasted. Mother report was selected as the reference method because previous research has suggested that mental health professionals tend to view mothers as most knowledgeable about children’s mental health symptoms (Loeber, Green, & Lahey, 1990). As a consequence, the trait factor \( T_1 \) in the
model represented inattention as measured by mother reports and the method factor \( M_2 \) represented the difference between father and mother reports.

The model was parameterized with the trait-method relationship as a linear regression of the method factor on the trait factor (Figure 1B). The key parameters in the linear LD model are the mean and variance of the trait factor \([ E(T_i) \) and \( Var(T_i) ]\), and the parameters in the regression of the method factor on the trait factor \([ \beta_0, \beta_1 \) and \( Var(\zeta_{M_2}) ]\). \( E(T_i) \) was estimated to be 0.95 (\( t=27.67 \)), meaning that the average mother rating of inattention symptoms was relatively low (given the 0 to 5 response scale). Variance of mother reports was estimated as \( Var(T_i) =0.69 \).

The relationship between mother-rated inattention and the method effect of father rating was parameterized as a linear regression, with the following equation:  
\[
M_2 = -0.16T_i + 0.18 + \zeta_{M_2} 
\]
. The overall mean of the method factor can be derived from the regression equation as \( \beta_0 + \beta_1E(T_i) \), which implies that \( E(M_2) = 0.02 \). The \( \beta_1 \) parameter was significant (\( t=-5.98 \)) and negative, indicating that fathers will underestimate (i.e., fathers rate lower than mothers) when mothers rate inattention more highly. Specifically, based on the estimated negative slope coefficient, the father-mother rating differences were expected to decrease by 0.16 points for every one point increase in mother-rated inattention symptoms. The residual variance \( Var(\zeta_{M_2}) \) was estimated to be 0.23, showing that the method factor variance was only slightly reduced by taking into account the regression on the trait factor \( (R^2 = .08) \). The model-implied variance of the method factor is therefore \( Var(M_2) = \beta_1^2Var(T_i) + Var(\zeta_{M_2}) =0.25 \), which is substantially smaller than the variance of the trait factor. The linear model suggests that the mother and father
ratings show the best agreement at the low end of the scale, and father ratings become increasingly discrepant when mothers rate more highly.

A likelihood ratio test comparing the linear LD model to an LD model with a quadratic term in the regression of the method factor on the trait factor (Equation 10; Figure 1A) fit better than the linear model $\chi^2(df = 1, N = 752) = 6.682, p = .009$, indicating that the quadratic term was statistically significant. The equation for the difference factor was

$$M_2 = 0.06T_i^2 - 0.33T_i + 0.24 + \zeta_{M_2}.$$  

The quadratic coefficient was significant ($t = 2.93$) and positive, indicating a u-shaped curve. Figure 3A shows this model in the range of the observed scores compared to the linear model. The roots of the equation are 0.87 and 4.74, meaning that the model-estimated points at which the estimated value of the $M_2$ is 0 are at 0.87 and 4.74. The first root is very close to the mean of the trait scores, meaning that when mothers rated inattention at the average level, fathers tended to agree on the level of inattention. The second root is at the extreme end of the possible range of observed scores, indicating that when mothers rated inattention very highly, fathers tended to agree on the level of inattention as well. The inflection point of the function was 3.1, meaning that the slope of the line was decreasing when mother-rated inattention symptoms were below 3.1, and increasing when mother-rated inattention symptoms were above 3.1. The minimum value of $M_2$ when mothers rated inattention at 3.1 is $-0.21$, indicating that the highest discrepancy between mothers and fathers was at an intermediate level of inattention symptoms, and the level of discrepancy was relatively small.

The quadratic function showed a substantial difference from the linear model. The quadratic model suggests that fathers were not very discrepant from mothers at low levels of mother-rated inattention and high levels of mother-rated inattention. However, at intermediate levels of mother-rated inattention (between mother ratings of 1 and 4), father ratings tended to be
lower than mother ratings. The quadratic model therefore shows that discrepancy between the two methods at higher levels of the trait (above $T=3.1$), is just as small as at lower levels of the trait, which is a characteristic that would have been overlooked had we only fit the linear model.

**LM model.** The LM model is an alternative parameterization of the basic TMU model, and thus showed an equivalent fit to the LD model for this data. I again fit the regression parameterization to the data. The mother factor was assigned negative loadings on the method factor, which means that the structural equations were set up as follows: $T_1 = T - M^*$ ; $T_2 = T + M^*$ , where $T_1$ is the TMU factor for the mother ratings of child inattention, and $T_2$ is the TMU factor for the father ratings of child inattention. The key parameters in the LM model are the means and variances of the common trait and common method factors, and the relation between them $[E(T), Var(T), \beta^*_0, \beta^*_1 \text{ and } \zeta^*_M]$. In the linear LM model (Equation 9) fit to the inattention data, $E(T)$ was estimated to be 0.93. $Var(T)$ was estimated to be 0.64. Note that although this is the same data, the trait factor in the LM model reflects the average of both variables, instead of only the reference variable.

The model-estimated linear equation for $M^*$ was $M^* = 0.001T + 0.011 + \zeta^*_M$. $\beta^*_1$ and $\beta^*_0$ was not significant, ($t=0.04$; $t=0.621$, respectively). This means that no linear relationship between $T$ and $M^*$ exists in the data. The model-implied mean of the method factor is not significantly different from zero, and it is constant across all values of the trait. The residual variance was estimated as $Var(\zeta^*_M) = 0.06$. The residual variance was significant ($t=13.08$), meaning that the model-implied variance of $M^*$ is also significant.

The quadratic extension to the latent means model (Equation 12) fit the data better than the linear model. A likelihood ratio test comparing the two models showed a significant
improvement in fit for the quadratic model \[ \chi^2(df = 1, N = 752) = 9.614, p = .002 \]. The model-estimated structural equation was \[ M^* = -0.036T^2 + 0.11T - 0.034 + \zeta_{M^*} \]. The quadratic coefficient was significant \( t = -2.92 \) and negative, indicating an inverse u-shaped curve. Figure 3B displays both the linear and quadratic functions. The roots of the quadratic function are 0.35 and 2.7, meaning that at these inattention trait values the model-implied value of the method factor is 0. The inflection point of the model-estimated function was 1.78, meaning that for mean inattention factor scores below \( T=1.53 \), the overall slope of the line was positive, and for trait scores above \( T=1.53 \), the overall slope of the line was negative. The model-predicted maximum value of \( M^* \) was 0.05, which means that when the mean level of mother and father ratings is 1.53, the average difference between mother and father scores is 0.1. Above trait values of 2.7, the model-estimated value of the method effect becomes increasingly negative. At the value of the highest estimated trait factor score in the data (\( T=4.2 \)), the model-estimated value of \( M^* \) is \(-0.21\), meaning that the difference between mother and father ratings is estimated to be 0.42, with mothers rating higher than fathers. At this level of the scale, this is almost half the difference between a rating of 4 (anchor is “Very often occurs [several times per day]”) and 5 (anchor is “Nearly occurs all the time [e.g., many times per day]”). Because the mother report’s method factor loadings fixed to -1, this means that mothers rated children higher on inattention than fathers at this level of the common inattention trait.

Note that the LD and LM models uncovered different aspects of the underlying data. The quadratic LD model showed that when mothers rated inattention at intermediate levels, mothers rated higher than fathers, and the discrepancy decreased at higher levels of mother-rated inattention. The quadratic LM model, however, showed that when the average rating of inattention is intermediate, fathers tended to rate inattention slightly higher than mothers, and
this discrepancy was relatively minimal. The quadratic LM model also showed that mothers rate higher than fathers as the average trait level was above 2.7.

Each quadratic model also provided substantively different information than the linear model, showing that the quadratic model may be useful in identifying trait levels at which convergent validity is stronger or weaker. In summary, both models found that convergent validity was strongest at low levels of the inattention, and marginally weaker at higher levels of inattention. When comparing discrepancy to mother ratings of inattention, discrepancies are highest at intermediate levels of inattention. When comparing discrepancy to the mean of mother and father-rated inattention, discrepancies are highest at high levels of inattention.

Simulation Study

In addition to the practical application, I examined the performance of the quadratic LD and LM models under a larger set of sample conditions using a Monte Carlo simulation. Although the LMS estimation procedure is well-studied (Harring, Weiss & Hsu, 2012; Kelava et al., 2011; Klein & Moosbrugger, 2000), it is unknown how well-powered the method is for common sample sizes in psychological research when using the LD and LM models. To test power, the simulation used parameter estimates from the LD and LM models presented in the application as population models and varied the sample size, indicator reliability, and effect size of the quadratic term in both the LM and LD models. Effect size values were based on Cohen’s recommendations for small, medium, and large partial regression coefficients (Cohen, 1992). To test the Type 1 error rate, the simulation used parameter estimates from the LD and LM models as population models, but changed the value of the quadratic term to zero. The estimator then searched for a quadratic effect where none was present, and the proportion of replications with a significant quadratic effect was regarded as the Type I error rate. The Type 1 error models were
varied with respect to sample size and indicator reliability. Further details of the simulation and software code for the simulation are provided in Appendix A and the online supplemental material.

The power simulation found that for medium and large effect sizes, with indicator reliability of .8, power of 0.8 was achieved at sample sizes above $N=250$. Small effect sizes had low power at all sample sizes and reliabilities. Similar results were found for the LM model, although the LM model had slightly reduced power compared to the LD model. The Type 1 error rates were within the acceptable range (between 2.5% and 7.5%) for all conditions except $N=100$, where there was slight inflation of the type I error rate ($\alpha =10\%$). The findings are in line with previous simulations with the LMS estimator (Cham, West, Ma, & Aiken, 2012; Harring, Weiss, & Hsu, 2012; Kelava et al., 2012; Klein & Moosbrugger, 2000), which found that it performs better with larger effect sizes, and that power to detect latent interactions is frequently lower than would be expected in an observed variable model.

**Discussion**

The LD and LM models of CFA-MTMM analysis both allow examining linear relationships between trait and method factors. In the present paper, I proposed new extensions of both models to incorporate potential non-linear relationships between traits and methods. The extensions were shown to provide useful insights on convergent validity for real data and to perform well based on an empirical application and a Monte Carlo simulation study.

The LD model requires the choice of a reference method, and the method factor represents the differences between the trait as measured by the reference method and another method. The LM model does not require the choice of a reference method, and the method factor instead represents the difference between the common trait factor and the method in question.
The quadratic LD model represents the effect of the different levels of the trait as measured by the reference method on the discrepancy between the two TMUs. The quadratic LM model represents the effect of the level of the average trait level (as measured by both methods) on the discrepancy between the two TMUs. The LD model is more appropriate when researchers have a clear reference method that is a gold standard method or has a clear structural difference from the other methods. The LD model is more appropriate when absolute discrepancy of the non-reference method as compared to the reference method is the research question of interest.

The LM model is more appropriate when researchers do not have a clear reference method available. The LM model is also more appropriate when the research question regards discrepancy between the two methods when there is no clear method to compare against.

Regarding the statistical performance of the LD and LM quadratic models, the LD model both appear to work equivalently to detect quadratic effects, and that small effects are difficult to detect in both models.

This work extends and complements Koch, Kelava & Eid (2017), who examined LD models and CTC(M-1) models where the method effect variables are regressed on explanatory variables that interact with the level of the trait variable. These models were also estimated using the LMS method.

**Advantages of the quadratic LD and LM models**

The LD and LM models provide researchers with straightforward and easily interpretable models that are based in clear mathematical decompositions of the latent variables, and allow researchers to examine the relationships between method effects and trait levels.

Both the linear and quadratic LD and LM models allow researchers to examine the extent to which method effects are dependent on trait levels. The new quadratic models allow for less
restrictive assumptions about the trait-method relationship. These models allow researchers to examine more complex questions about method effects than were possible with previous latent variable MTMM modeling techniques. In addition, substantial trait-method effects may be ignored, and convergent validity may be assumed to be constant at all levels of the trait when in fact there are certain levels of the trait where convergent validity is higher or lower than others.

In addition, the simulation shows that estimation of quadratic effects in the LD and LM models using the LMS method is well-powered at sample sizes above 250 with medium to large effect sizes. The method also has acceptably low Type-1 error rates for both types of models.

**Limitations of the quadratic LD and LM models**

Limitations of the LD and LM models are that they require strong measurement invariance of the indicators used for the formation of the latent variables. This may be a limiting factor for many research situations.

A limitation of the quadratic extensions to the LD and LM models is that power to detect small quadratic effects is very low for all sample sizes. An additional limitation of this research is that only the LMS method was used for estimation. Additional estimation methods should be tested to compare, since power is somewhat diminished with the LMS method compared to the traditional quadratic SEM model.

An additional limitation of the quadratic LD and LM models is that it assumes that discrepancy between methods will be such that one method consistently rates higher or lower at a given level of the trait. However, it may be the case that the absolute value of the method factor scores is dependent on the level of the underlying trait or some other explanatory variable. In this case, the absolute value of the method factor should be the subject of study, disregarding the
direction of the discrepancy. Further research should examine ways to model the absolute value of the method factor in addition to the observed values of the method factor.

**Conclusions**

The LD and LM models represent an important advancement in the modeling of multitrait-multimethod data. Along with other new models for examining trait method interactions (e.g., Litson, Geiser, Burns, & Servera, 2017), they represent a new avenue for multitrait-multimethod research that explicitly examines the relationship between traits and methods and asks questions about convergent validity that are innovative and important. The quadratic extensions to the LD and LM models expand the toolbox of multimethod researchers for examining convergent validity to complex trait method relationships that may be overlooked if researchers only use linear models.
References


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Table 1  
*Correlation matrix and descriptive statistics of mother and father ratings of child inattention from Burns et al. study*

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<th>Source Parcel</th>
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<th>Mother 2</th>
<th>Mother 3</th>
<th>Father 1</th>
<th>Father 2</th>
<th>Father 3</th>
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Table 2

*Model estimates from TMU model fit to mother and father ratings of child inattention.*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate/Standardized Estimate</th>
<th>Standard Error</th>
<th>p</th>
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<td>$E(\text{Inattention}_{\text{Father}})$</td>
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<td>$\text{Var}(\text{Inattention}_{\text{Mother}})$</td>
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</table>

*Note. $\lambda_i$ = Factor loadings of indicators onto latent TMU factors; $\alpha_i$ = Intercept parameters of indicator variables; $\varepsilon_{im}$ = Measurement error variable for indicator variables; $i=$Indicator; $m=$Method (1=Mother report, 2=Father report)*
Figure 1. Path diagrams of baseline latent TMU model (TMU; A), Latent difference model (LD; B), and Latent means model (LM; C). $Y_{it}$ = observed variable ($i=$indicator, $m=$method); $T_m$ = Trait as measured by method $m$; $M_2$ = Method factor in LD model; $M^*$ = Method factor in LM model; $E(T_m)$ = Mean of trait factor as measured by method $m$; $\beta_0$ = Regression intercept coefficient for method factor in LD and LM models; $\beta_1$ = Latent regression slope coefficient of method factor on trait factor in LD and LM models; $T_1$ = Trait factor in LM model; $M^*$ = Method factor in LM model; $\beta_1$ = Regression slope parameter for linear term ($T_1$ or $T$); $\beta_2$ = Regression slope parameter for quadratic term ($T_1^2$ or $T^2$); $\zeta_{M_2}$ = Residual variance of method effect in LD model; $\zeta_{M^*}$ = Residual variance of method effect in LM model; $\lambda_i$ = Factor loading; $\alpha_i$ = Intercept; $\epsilon_{im}$ = error variable.
Figure 2. Path diagrams of quadratic extensions to latent difference models (LD; A), and latent means model (LM; B). \( Y_{it} \) = observed variable \( (i=\text{indicator}, m=\text{method}); \) \( T_m \) = Trait as measured by method \( m; \) \( M_2 \) = Method factor in LD model; \( M^* \) = Method factor in LM model; \( E(T_m) \) = Mean of trait factor as measured by method \( m; \) \( \beta_0 \) = Regression intercept coefficient for method factor in LD and LM models; \( \beta_i \) = Latent regression slope coefficient of method factor on trait factor in LD and LM models; \( \beta_i \) = Latent regression slope coefficient of method factor on quadratic term in LD and LM models; \( T = \) Trait factor in latent means model; \( M^* = \) Method factor in latent means model; \( \beta_i = \) Regression slope parameter for linear term \( (T_1 \text{ or } T); \) \( \beta_z = \) Regression slope parameter for quadratic term \( (T_1^2 \text{ or } T^2); \) \( \zeta_{M_2} = \) Residual variance of method effect in latent difference model; \( \zeta_{M^*} = \) Residual variance of method effect; \( \lambda_i = \) Factor loading; \( \alpha_i = \) Intercept; \( \epsilon_{im} = \) error variable.
Figure 3. Plots of the estimated linear and quadratic trait-method relationships in LD and LM models for parent ratings of children’s inattention. Relationships estimated with linear models are represented as solid lines. Relationships estimated with quadratic models are represented as dashed lines. A. Mother-rated inattention and method effect of father ratings (LD model). B. Common inattention factor score and method effect factor of mothers and teachers (LM Model).
Appendix A

Simulation Study Of Power And Type 1 Error For LMS With Quadratic LD And LM Models

A Monte Carlo study examining the power and type 1 error of the LMS method to detect quadratic effects in LD and LM models was performed. I attempted to address the following specific questions: 1) Under which conditions does the LMS method for the quadratic LD and LM methods provide sufficient statistical power for the effect sizes observed in the applications? 2) Do Type-1 errors occur only 5% of the time in the quadratic LD or LM models?

Data Generation.

The simulation used parameter values taken from the two sets of applications that showed significant quadratic effects. Separate population models were simulated for each type of model (LD and LM), because quadratic trait-method relationships in one model do not necessarily imply similar quadratic trait method relationships in the other. Three different factors were varied in the simulation: the sample size (14 levels), reliability of indicators (4 levels), and the amount of residual variance in the structural part of the model \([ \text{Var} (\zeta_{M_2}) \text{ and } \text{Var} (\zeta_{M_4}) ]\) (5 levels). The sample sizes ranged from \(N = 100\) to \(N = 750\), in increments of 50. The reliability of indicators ranged from .6 to .9, representing typical values of indicator reliability in the social sciences. The size of the quadratic parameter was selected to represent small, medium, and large effect sizes for partial correlation coefficients (.02, .15, .35) according to Cohen (1980). The implied residual variance was calculated using formulas for the variance of product terms from Goodman (1960). To examine type 1 error, I also simulated an population model with no quadratic effect present for each of the sample size and reliability conditions. Using all
combinations of the data conditions resulted in 224 population models for the latent difference and latent means models. The R package MplusAutomation was used to generate the models (Hallquist & Wiley, 2017; see the appendix for code used to generate models).

**Analysis Strategy.**

Each simulated dataset was fit with both a misspecified linear model and a quadratic model. I recorded instances of non-convergence and improper solutions. The log-likelihood values of the misspecified linear models were then compared to the log-likelihood values of the appropriately specified quadratic models to test the statistical significance of the quadratic effect with the likelihood ratio test. To answer the proposed questions, power, and type I error were analyzed and summarized for each cell of the simulation design, and compared against typical cutoffs of acceptability.

**Power.**

Both power and type 1 error for the quadratic effects were examined using the chi-square likelihood ratio test suggested by Klein and Moosbrugger (2000) for the LMS method. For models that contained a true quadratic effect, power was analyzed as the percentage of replications that showed a significant value of the likelihood ratio test statistic or the wald test. Power was considered acceptable when 80% of replications with a true effect in the population correctly rejected the null hypothesis that there was no quadratic effect (Cohen, 1980).

**Type I error.**

For models that contained no true quadratic effect, Type-1 error was analyzed as the percentage of replications that showed a significant value of the likelihood ratio test statistic despite a true value of zero in the population. Bradley (1978) suggested a liberal criterion for
Type-I error, in which Type-I error rates between 2.5% and 7.5% are considered acceptable for an $\alpha$ level of 5%. This criterion was adopted in the present study.

**Results.**

**Convergence and Improper solutions.**

All replications converged on a solution. Improper solutions were a problem in less than 0.01% of replications. A detailed analysis showed that improper solutions only occurred for the smallest sample size of $N=100$ and reliability of 0.6 in the LD model with small and medium effect sizes (.06%, and .02% of replications, respectively). In the LM model, improper solutions also only occurred for $N=100$ and reliability of 0.6, with small, medium, and large effect sizes (.04%, .06%, and .01%, respectively). In both cases, the improper estimate was a latent variable correlation greater than 1. These replications were included in the data, since the occurrence of improper solutions was so small as not to influence the overall conclusions of the study.

**Power.**

Power was evaluated for the likelihood ratio test and the wald test. Power for the wald test and likelihood ratio tests were approximately equal. A $t$-test comparing obtained power values for each cell showed that the difference between the two tests was not significant, $t(165)=-.32, p=.74$. Power was substantially affected by effect size, sample size, and indicator reliability. Figure A1 shows the power for the LM and LD models by sample size, effect size and indicator reliability. Power of .8 was attained for large effects and indicator reliability with sample sizes greater than 250. For medium-sized effects, power of .8 was attained with sample sizes greater than 450. For small effects, power was below .5 for all sample sizes with low and high indicator reliability. Power was slightly lower for the LM models overall with both tests, and low reliability more strongly affected power in the LM model.
Type I error.

For the LD and LM model, the type I error rates for the LRT were between 2.5% and 7.5% for all data conditions. For the wald test, the type I error rate for both the LM and LD models were 10% only when indicator reliability was .8 and sample size was N=100. In all other conditions, type I error rates were between 2.5% and 7.5%.

Discussion

The simulation found that estimating the quadratic LD and LM models works well with the LMS method. The power for medium and large effect sizes is good when sample sizes are in the range typically seen in psychological research (between $N=250$ and $N=500$). The power for small effect sizes is low to moderate with all sample sizes. The LM model has slightly lower power to detect quadratic effects than the LD model. This is likely because the method factor in the LM Models were much smaller, and the quadratic effects were also substantially smaller in absolute value, although the standardized effect sizes were equivalent to the LD model.

Type I error was appropriate for all conditions, suggesting that the quadratic models do not simply fit extraneous noise in the models, and a significant result usually indicate a true effect in the population.

Models examining interaction and higher order effects that correct for measurement error typically suffer from low statistical power, and previous findings of simulations examining the LMS estimator show low power for latent interaction effects (Cham et al., 2012; Kelava et al., 2012). An unfortunate consequence of this is that in multimethod research, quadratic relations between trait and method may be inappropriately ignored. Further research should examine alternative estimation strategies and approaches (e.g., latent variable mixture modeling).
estimation techniques for small samples) to be able to detect quadratic trait-method relationships more often.
Figure A1. Power to detect quadratic trait-method correlations with the LD and LM models in simulated data sets. LRT=Likelihood ratio test; Wald=Wald test; LD=Latent Difference Model; LM=Latent Means model. Power of .8 and above is shaded.
Appendix B

Identification Of Structural Parameters For The LD And LM Models

In this appendix, I derive the identification formulas for each structural variance parameter of both the LD and LM models and compare them to each other. I use the rules of covariance algebra as described by Kenny (1970) to derive the variance of the trait and method factors and the covariance between the trait and method factors in both the LD and LM models.

Identification Of LD Model Parameters

The variance of the trait factor for the LD model is the variance of $T_1$, because $T_1$ is defined as the trait factor. The variance of the method factor is identified as a function of the two underlying TMU variables. I can start by examining the definition of the method factor in the LD model, which is given by:

$$ M = T_2 - T_1 $$  \hspace{1cm} \text{(B1)}

Following from equation B1, the variance of the method factor can be written as follows:

$$ Var(M) = Var(T_2 - T_1) $$  \hspace{1cm} \text{(B2)}

Using the variance definition and sum rules of covariance algebra (Kenny, 1979, p.21), equation B2 becomes:

$$ Var(M) = Cov(T_2 - T_1, T_2 - T_1) $$
$$ Var(M) = Cov(T_2, T_2) + Cov(T_2, -T_1) + Cov(-T_1, T_2) + Cov(-T_1, -T_1) $$
$$ Var(M) = Var(T_1) + Var(T_2) + 2 Cov(-T_1, T_2) $$ \hspace{1cm} \text{(B3)}
Finally, following the constant rule of covariance algebra (Kenny, 1979, p.21), equation B3 becomes:

\[
Var(M) = Var(T_1) + Var(T_2) - 2Cov(T_2, T_1). \quad (B4)
\]

The variance of the method factor in the LD model is therefore identified by the sum of the variances of the two TMU factors and the covariance between the TMU factors.

In the LD model the covariance between the trait and method factors is identified by a function of the covariance of the two TMU factors and the variance of the reference factor. This follows from substituting the definition of the trait and method factors into the covariance formula:

\[
Cov(T_1, M) = Cov[T_1, (T_2 - T_1)] . \quad (B5)
\]

Following the sum rule, equation B5 becomes:

\[
Cov(T_1, M) = Cov(T_1, T_2) + Cov(T_1, -T_1) \quad (B6)
\]

Following the constant rule and variance definition rules, equation B6 become:

\[
Cov(T_1, M) = Cov(T_1, T_2) - Var(T_1) \quad (B7)
\]

Therefore the covariance of \( T_1 \) and \( M \) in the LD model is a function of the covariance between \( T_1 \) and \( T_2 \) and the variance of \( T_1 \).

**Identification of LM model parameters.**

Note that the following identification formulas are only true when the number of methods compared is two. More complex identification formulas are required when using three or more methods with the LM model.
The identification for the variance of the trait factor follows from the definition of the trait factor for the LM model when there are two methods:

\[ T = \frac{T_1 + T_2}{2} \]

\[ T = \frac{1}{2}T_1 + \frac{1}{2}T_2 \]  \hspace{1cm} (B8)

\[ Var(T) = Var(\frac{1}{2}T_1 + \frac{1}{2}T_2) \]

Following from the variance definition and sum rules of covariance algebra, equation B8 becomes:

\[ Var(T) = \text{Cov}(\frac{1}{2}T_1, \frac{1}{2}T_2) + \text{Cov}(\frac{1}{2}T_1 + \frac{1}{2}T_2) + \text{Cov}(\frac{1}{2}T_2, \frac{1}{2}T_1) + \text{Cov}(\frac{1}{2}T_2, \frac{1}{2}T_2) \]  \hspace{1cm} (B9)

\[ Var(T) = Var(\frac{1}{2}T_1) + Var(\frac{1}{2}T_2) + 2\text{Cov}(\frac{1}{2}T_1, \frac{1}{2}T_2) \]

And finally, following the Constant rule of covariance algebra, equation B9 becomes:

\[ Var(T) = \frac{1}{2} \cdot \frac{1}{2} Var(T_1) + \frac{1}{2} \cdot \frac{1}{2} Var(T_2) + 2 \cdot \frac{1}{2} \cdot \frac{1}{2} \text{Cov}(T_1, T_2) \]

\[ Var(T) = \frac{1}{4} Var(T_1) + \frac{1}{4} Var(T_2) + \frac{2}{4} \text{Cov}(T_1, T_2) \]  \hspace{1cm} (B10)

\[ Var(T) = \frac{Var(T_1) + Var(T_2) + 2\text{Cov}(T_1, T_2)}{4} \]

Equation B10 shows that the variance of the LM model is a function of both TMU variances and their covariance.

In the LM model, the variance of the method factor can be derived from the definition of the method factor:
Following the sum rule, equation B11 becomes:

\[
\text{Var}(M^*) = \text{Cov}(T_2 - \frac{1}{2}T_1 + \frac{1}{2}T_2, T_2 - \frac{1}{2}T_1 + \frac{1}{2}T_2)
\]

Following the constant rule, equation B12 becomes

\[
\text{Var}(M^*) = \text{Var}(T_2) - \frac{1}{2} \text{Cov}(T_2, T_1) - \frac{1}{2} \text{Cov}(T_2, T_2) - \frac{1}{2} \text{Cov}(T_1, T_2) - \frac{1}{2} \text{Cov}(T_2, T_2) + \\
\frac{1}{4} \text{Cov}(T_1, T_1) + \frac{1}{4} \text{Cov}(T_2, T_2) + \frac{1}{4} \text{Cov}(T_2, T_1) + \frac{1}{4} \text{Cov}(T_2, T_2)
\]

Cancelling redundant terms leads to:

\[
\text{Var}(M^*) = \frac{1}{4} \text{Var}(T_1) + \frac{1}{4} \text{Var}(T_2) - \frac{1}{2} \text{Cov}(T_1, T_2)
\]

\[
\text{Var}(M^*) = \frac{\text{Var}(T_1) + \text{Var}(T_2) - 2\text{Cov}(T_1, T_2)}{4}
\]

Therefore, in the LM model, the variance of the method factor is also identified by a function of the difference in variance between the TMU factors and the covariance between them.

The identification of the covariance between trait and method factors can be derived by substituting the definitions of the trait and method factors and using covariance algebra:
\[ \text{Cov}(T, M^*) = \text{Cov}\left[\left(\frac{1}{2}T_1 + \frac{1}{2}T_2\right), (T_2 - \left(\frac{1}{2}T_1 + \frac{1}{2}T_2\right))\right] \]  
(B15)

From equation B15, applying the sum rule gives:

\[ \text{Cov}(T, M^*) = \text{Cov}\left[\left(\frac{1}{2}T_1 + \frac{1}{2}T_2\right), T_2\right] + \text{Cov}\left[\left(\frac{1}{2}T_1 + \frac{1}{2}T_2\right), -\left(\frac{1}{2}T_1 + \frac{1}{2}T_2\right)\right] \]

\[ \text{Cov}(T, M^*) = \text{Cov}\left(\frac{1}{2}T_1, T_2\right) + \text{Cov}\left(\frac{1}{2}T_2, T_2\right) + \text{Cov}\left(\frac{1}{2}T_1, -\frac{1}{2}T_1\right) + \text{Cov}\left(\frac{1}{2}T_2, -\frac{1}{2}T_2\right) \]  
(B16)

From equation B16, applying the constant rule and cancelling out redundant terms gives:

\[ \text{Cov}(T, M^*) = \frac{1}{2} \text{Cov}(T_1, T_2) + \frac{1}{4} \text{Var}(T_2) - \frac{1}{4} \text{Var}(T_1) - \frac{1}{4} \text{Cov}(T_1, T_2) - \frac{1}{4} \text{Cov}(T_2, T_1) - \frac{1}{4} \text{Var}(T_2) \]

\[ \text{Cov}(T, M^*) = \frac{1}{4} \text{Var}(T_2) - \frac{1}{4} \text{Var}(T_1) \]  
(B17)

\[ \text{Cov}(T, M^*) = \frac{\text{Var}(T_2) - \text{Var}(T_1)}{4} \]

Therefore, in the LM model, the covariance between the trait and method factors is identified by a function of the variances of each of the trait method units, when the method loadings for \( T_1 \) are negative and the method loadings for \( T_2 \) are positive. The identification of each portion of the structural model for the LM and LD models from the parameters of the TMU model is shown in Table B1.
### Table B1.
Identification of structural variance parameters from TMU model for LD and LM models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation in LD model</th>
<th>Identification in LD model</th>
<th>Notation in LM model</th>
<th>Identification in LM model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of trait variable</td>
<td>$\text{Var}(T_1)$</td>
<td>$\text{Var}(T_1)$</td>
<td>$\text{Var}(T)$</td>
<td>$\text{Var}(T_1) + \text{Var}(T_2) + 2\text{Cov}(T_1, T_2)$</td>
</tr>
<tr>
<td>Variance of method variable</td>
<td>$\text{Var}(M)$</td>
<td>$\text{Var}(T_1) + \text{Var}(T_2) - 2\text{Cov}(T_2, T_1)$</td>
<td>$\text{Var}(M^*)$</td>
<td>$\text{Var}(T_1) + \text{Var}(T_2) - 2\text{Cov}(T_1, T_2)$</td>
</tr>
<tr>
<td>Covariance of trait and method variable</td>
<td>$\text{Cov}(T_1, M)$</td>
<td>$\text{Cov}(T_1, T_2) - \text{Var}(T_1)$</td>
<td>$\text{Cov}(T, M^*)$</td>
<td>$\frac{\text{Var}(T_2) - \text{Var}(T_1)}{4}$</td>
</tr>
</tbody>
</table>

*Note. LD=Latent Difference; LM=Latent Means; TMU=Trait-method unit; $T_m=$Trait-method unit for method $m; M=$Method variable for the LD model; $T=$Trait variable for the LM model; $M^*=$Method variable for the LM model.*