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Capstone Mathematics and Technology: A Collection of Mathematical Technology Enhanced Activities for Students and Teachers

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Capstone Mathematics and Technology: A Collection of Mathematical Technology Enhanced Activities for Students and Teachers

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Utah State University, 2007

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Abstract

The purpose of this project is to provide an introduction to how technology can be used in the mathematical classroom to enhance students' learning of mathematics, while at the same time leading students to a richer and deeper understanding of those mathematical concepts. The topics were selected based on their relevance to the Utah State Core Curriculum for middle and secondary mathematics courses. It was intended that each lesson plan would challenge a pre service mathematics educator to build relationships between different areas of mathematics and/or to create deeper understandings of specific mathematical concepts. At the same time many of the lesson plans can be used at the high school level to teach mathematical ideas. The ideas are not too complex for the secondary level, but their extensions that will hopefully inspire a pre service mathematics teacher to search for deeper understandings. It is hoped that these lessons will promote a desire in the students who work out the activities to create their own lesson plans, plans which relate activities, mathematical topics and technologies together for deeper understanding.
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Heidi Jean Eastman
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Introduction

Pre service middle and secondary school mathematics teachers are prepared throughout their college experience to be able to act and react to situations that they may encounter when they are in the classroom. Pre service teachers, upon graduations, should be prepared to manage a classroom and to convey the content of their subject area to the students in their classes.

Specifically, it is imperative that a mathematics teacher have a deep understanding of not only those courses and ideas that they teach but also those ideas that are the framework for those courses. Usually understanding of those framework courses implies a deeper understanding of the base course. With this deeper understanding the instructors will be better prepared to direct their student in learning, because they are able to see the bigger picture and know where the students should begin and in which direction they should precede with the information. If the students are instructed in a direction that encourages them to move forward with the information, the students will be better prepared to make connections between what they are learning and what they will learn in the future and how all the information can be applied and useful. Those who instruct mathematics should strive to be able to employ knowledge that they have gained from all the mathematics courses that they have completed. Once knowledge can be employed by an instructor, the instructor should analyze it so that it may be interrelated with other mathematical concepts. This knowledge should also be integrated into real life application of mathematics concepts.

Most mathematics classrooms have recently been and continue to be enhanced with access to some type of technology. The teaching of technology with mathematics can be, if used correctly, a great addition and a wonderful enhancing tool to assist both the teacher in teaching and the student in learning. The benefit of the use of technology in the classroom depends on how and for what purpose it is implemented.

The major objective of this project is to create a framework for a course that will be used to help to prepare pre service middle and secondary school mathematics teachers
to solidify mathematical connections and ideas for themselves. These connections will come about through the completion of activities that are enhanced with technology for further and deeper mathematical understanding. This course, Capstone Mathematics and Technology for Teachers, has been recently added to the requirements for all Mathematics Education majors at Utah State University.

It is not intended that these exact activities will necessarily be used in every classroom. Each class and each student bring with itself individual needs which require individually catered lesson plans. As I created and brought together ideas for the following lesson plans, I focused mainly on my individual needs. I do not know which ideas each mathematical student has, but I do know ideas that I have struggled with and still struggle with. As stated before, it is intended that the following ideas and lesson planes are a launching point, off which each instructor may depart on an adventure of their own. An adventure that leads their students to a deeper understanding of mathematics, how it is used in the world in which they live and how technology can help them to expand that knowledge.
Course Description
Math 5010 Capstone Mathematics and Technology for Teachers

Students will employ knowledge gained from prior courses to analyze and interrelate mathematical concepts and to identify real-life applications of content in the secondary school mathematics curricula. They will learn to employ technologies available for teaching and learning mathematics and use these technologies to make connections between mathematical ideas.

Goals
Goal 1: Pre-service mathematics teachers will use the depth of their understandings of mathematics from prior courses (Discrete Mathematics, Modern Geometry, Foundations of Analysis, Introduction to Algebraic Structures, History of Mathematics and Introduction to Number Theory, and Introduction to Probability) to analyze mathematical content from secondary school curricula (to include Pre-Algebra, Classical Algebra, Geometry, Trigonometry, Calculus, Statistics, and Discrete Mathematics) so they (a) interrelate topics, (b) construct critical concepts, (c) discover why relationships exist, (d) discover why certain algorithms work, (e) assess the value of various topics, and (f) apply useful topics to address problems that are perceived to be real-life by pre-adolescent and adolescent students.

Goal 2: Pre-service mathematics teachers will learn to employ computer-based technologies for analyzing mathematical content from secondary school curricula. These technologies will include software for learning and teaching mathematics (e.g. Geometers' SketchPad), equation editors (e.g. LaTeX), software for numerical and symbolic manipulations (e.g. Maple), and web bases instructional resources (e.g. The National Library of Virtual Manipulative).

Goal 3: Pre-service mathematics teachers will design and assess the effectiveness of electronically enhanced mathematical learning activities for secondary school students.
Curriculum

Mathematics literacy is essential and the need for it is universal. (Utah Secondary Mathematics Core Curriculum)

Teachers and pre service teachers alike must understand the goals and requirement, for middle and high school students, set forth by the state. To create lesson plans for students the instructor must understand what it is that the student should take form the class or course. Therefore it is requisite that a course such as Capstone Mathematics and Technology for Teachers assist pre service teachers to obtain this knowledge.

The introduction to the Utah Secondary Mathematics Core Curriculum states, “The goal of the Core Curriculum is to develop mathematical proficiency in every student by building a conceptual base and developing mathematical fluency. Students who understand mathematics will be able to communicate their reasoning, use multiple representations, and think logically. They will develop positive attitudes toward mathematics, solve problems, and think creatively while connecting mathematics to other disciplines and to life. Students will use mathematical tools, such as manipulative materials and technology, to develop conceptual understanding and solve problems.”

Students who graduate from high school encounter a complex, technological, and changing world. Students who wish to participate in the changing complexities must be prepared with mathematical skills and knowledge and be able to understand the uses of these skills and knowledge. Students are responsible to obtain knowledge themselves. Teachers, parents and society act as support in the accumulation of knowledge. An understanding of mathematics and its applications comes through active participation in learning. Enhanced knowledge comes as the students learn to make connections to prior learning and other disciplines.
Teaching and Learning

*Students want to make sense of their worlds. Mathematics becomes part of that world when it is seen both as sensible in itself and as a tool for making sense out of otherwise confusing situations.*

(Mathematical Sciences Educational Board)

Traditionally mathematics has been taught through lecture and memorization. This teaching method is not the only method to teach and learn mathematics. In fact, it may be found that it is actually in contrast to the method in which most students learn mathematics best. An alternative is that mathematics can be learned through activities that allow exploration and understanding. Active involvement is one way in which learning mathematics can be enhanced. A teacher should strive to find and provide opportunities to students that will help them to become engaged in the learning process. Activities if chosen and implemented correctly give students an opportunity to create the math for themselves and more than often attract the attention of the students. As students learn with activities they learn that mathematics is a real part of the world and not just part of their classroom. Mathematics comes alive for students when it is learned through experiences that are personally meaningful and valuable.

It is important that students are able to use mathematics to model real life phenomena. If students do not make meaningful connections between the concepts and the ideas, it diminishes the purpose of learning mathematics. They should be able to use mathematics just like they do when they study the sciences. Students should be able to pose problems and advance hypotheses after they have examined a situation for the patterns and relationships they contain. The Mathematical Sciences Educational Board suggests, “Mathematics is a science. Observations, experiment, discovery, and conjecture are as much a part of the practice of mathematics of any natural science. Trial and error, hypothesis and investigation, and measurement and classification are part of the mathematician’s craft and should be taught in school.”

For mathematics to be useful and understandable, a student must learn to apply its ideas. A student must reach the construct a concept level; they attain this level when they are able to use inductive reasoning to distinguish between examples and non examples of
a mathematical concept. It is also imperative that relationships are discovered by students for their understanding to be enhanced. A relationship is discoverable if reasoning or experimentation can be used to find out if the relationship exists. Students should also learn to apply the ideas from different areas of mathematics together, for example algebra, geometry, statistics and discrete mathematics can be combined together for further exploration and connection.

The National Council of Teachers of Mathematics (NCTM) suggests, “the idea of mathematics as a thought process, not just a body of knowledge. This is not possible unless students take an active role in the learning of mathematics, and teachers provide situations that allow meaningful engagement with the material. Mathematical communication, problem solving, reasoning, connections, and representation go beyond the math content itself. To teach or understand these concepts, students must be comfortable with their conceptions of mathematics, and they must be able to take ownership of them. The process standards are not something that can be taught via pure lecture. Teachers must provide learning opportunities that allow inquiry and student involvement.”
Technology

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.
(National Council of Teachers of Mathematics)

There is no doubt that technology is changing the ways in which mathematics is used and taught. There are mixed opinions on the importance of using technology in the mathematics classrooms. Proponents, of technology, believe that technology should be embraced in all classrooms. This opinion is based on the fact that the society in which we live, demands that we be able to use a calculator and a computer. It is suggested that using technology helps students to cognitively reorganize mathematical knowledge. While on the opposite end of the argument, critics suggest that relevance is a major concern for using technology in the classroom. It is proposed that students’ learning is hindered as a result of becoming reliant on the technology, which in turn impedes the understanding of the underlying mathematical concepts of the students. Critics of technology see it as a crutch or replacement for learning and understanding.

Powerful computing technologies are making math an experimental science, yet it still has a need for observation and inquiry. Students should be provided with an opportunity to learn basic functions and uses of technology and in addition the students should know how to do basic math by hand. Ideally, students should learn how and why the particular technological application is useful in a given situation, students should be provided with meaningful opportunities to learn to use technology in context.

Technology is changing the content of mathematical programs. The fields of mathematical studies continue to expand and increase because of the introduction of new technologies. The instructional methods for teaching mathematics must also be flexible enough to incorporate the changes that are occurring because of technology. In years past the major concern dealt with the availability of technological resources for the students, this concern is fading quickly as technology is becoming more readily available to all students both inside and outside the classroom. The NCTM recommends that technology receive increased emphasis in mathematics curriculum. This emphasis is especially
important to teachers' professional development. The NCTM makes the following recommendations:

- Every student should have access to an appropriate calculator.
- Every mathematics teacher should have access to a computer with appropriate software and network connections for instructional and non-instructional tasks.
- Every mathematics classroom should have computers with Internet connections available at all times for demonstrations and students' use.
- Every school mathematics program should provide students and teachers access to computers and other appropriate technology for individual, small-group, and whole-class use, as needed, on a daily basis.
- Curriculum development, evaluation, and revision must take into account the mathematical opportunities provided by instructional technology. When a curriculum is implemented, time and emphasis must be given to the use of technology to teach mathematics concepts, skills, and applications in the ways they are encountered in an age of ever increasing access to more-powerful technology.
- Professional development for pre service and in-service teachers must include opportunities to learn mathematics in technology-rich environments.

If technology is taught for technology’s sake only it deprives students of access to critical content knowledge. Technology in the mathematical classroom should not merely enhance the students’ ability to push buttons. Instead, technology can be a powerful tool for teaching mathematics, but using it does not guarantee a pathway for teaching or learning significant mathematics. Students will only gain value from technology if it is being used to enhance and supplement the classroom while at the same time being used to teach math. Technology should be used as a way to enhance discoveries and connections made by students. Using technology appropriately provides a means whereby teachers can effectively guide students to deeper mathematical thinking, reasoning, problem solving, and understanding. A simple example might lead students to a deeper understanding of what it really means to revolve a given function around some axis, by exploring an applet that provides a visual representation of this idea. Using
instructional technology appropriately is integral to the learning and teaching of mathematics.

It is important that the correct technologies are chosen to assist teaching. If chosen correctly technology can be used to extend the students' understandings and applications of mathematics. Students should be able to make connects between the mathematical concepts and the technology. It is important to note that not all computer programs are designed to be used to assist mathematical explorations, nor will all programs motivate the students or provide meaningful learning. Further exploration is needed to understand extensive impact that technology does and will have on the instruction of mathematical concepts, skills and applications.

The Utah Secondary Mathematics Core Curriculum states, “The purpose of technology is to enhance the investigation and modeling of a wide variety of mathematical concepts and engage students in the learning process. Technology must be integrated in the curriculum and used appropriately as part of mathematical instruction and assessment. Technology facilitates the organization and analysis of data, and efficiency and accuracy in computation and, used appropriately, has been shown to be a tool that can support the development of flexibility in the use of various representations. It is used to provide visual images leading to understanding of mathematical ideas and concepts.”

Historically, people have developed and used manipulative and mathematical devices to help them understand and develop mathematical skills. Historical manipulatives include fingers, base ten blocks, geoboards and algebra tiles. Historical mathematical devises include protractors, coordinate systems, and calculators. The mathematics teacher cannot be replaced by technology just like it was not replaced by manipulatives or mathematical devises. The primary role of technology in the classroom is to change the tasks we do not merely amplifying them but also by reorganizing the operating of our minds.
Modular Arithmetic

Technology Used: Internet Applet, Calculator

Objectives:
1. Internet Applet
   a. Students will discover the relationship between clock addition, the train chain game, and modular addition.
2. Calculator
   a. Students will learn how to use the mod feature on their calculator.
2. Modular Operations
   a. Students will make connections between clock addition and modular addition.
   b. Students will explore multiple modular operational problems.
   c. Students will use additive and multiplicative inverses.
3. Teaching
   a. Students will explore the extensions to modular division.
      (Multiplication by the multiplicative inverse)

Activity Plan:
Part I: Motivation
Use the Math Train Chain Game (described below) to introduce modular addition to the students.

Part II: Investigation
Students will work in groups. Students will discuss and answer questions that deal with modulus operations and their applications and extensions to the Math Train Chain Game.

- What happens if you start with the same train cars but reverse their order?
- Will a train always loop back to the beginning, or can you have a train that never repeats?
- How short can the train be?
- How long can the train be?
- How many different starting pairs of train cars are there?
- How many different trains are there?

Part III: Technology Connections
The students will be introduced to the applet found at http://www.shodor.org/interactivate/activities/ClockArithmetic/
The students will explore the different options available on the applet that relate to modulus addition.
Students will explore the modular features on their calculators.

Part IV: Extension
Student will experiment with the Math Train Chain Game and applets that use different mods to create a deeper understanding of how modular operations work. They will be introduced to directions for computing modular addition. Students
will integrate inverses into their understanding of modular operations. They will answer questions which will help them to expand on the generalizations related to modular operations.

For a given modulus, what is the length of the longest loop?
How is the length of the longest loop related to the modulus?
What are the lengths of all the orbits, and how many are there of each for a given modulus?
How are the lengths of all the orbits related to the length of the longest orbit?

Part V: Assignment
Students will use the applet, calculators and the connections that they have made to complete a worksheet. (See attached assignment)

Part VI: Discussion
Discuss what was learned, advantages/disadvantages of software, extensions, what was liked/not like, etc.
GROUP ACTIVITY
Math Train Chain
Introduction and rules

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]

1. Pick a first and a second train car from the list above. They can have the same number.
2. The third train car will be selected by adding the numbers on the first and second cars. If the sum is more than 9, subtract 10 from the sum.
3. The fourth train car will be selected by using the same method. Add the numbers on the two previous train cars.
4. Keep going until you get back to the first and second beads, in the same order.

Example
- Choose 4 and 2 for the first and second train cars
- The third train car is 4+2=6
- The fourth train car is 2+6=8
- Then 6+8=14, so 14-10=4
- Then 8+4=12, so 12-10=2
- Note that the last two beads are the same as the first two so they connect back to make a loop.

Try it in your groups, choosing your own numbers to start!
After you have found a few loops, answer the following questions with your group.

What happens if you start with the same train cars but reverse their order?

Will a train always loop back to the beginning, or can you have a train that never repeats?

How short can the train be?

How long can the train be?

How many different starting pairs of train cars are there?

How many different trains are there?

12
FURTHER APPLICATION

Instead of having 9 train cars to choose from at the beginning let's change the option to 12. Now because we have 12 options let's change the rules a bit. If the sum of the train cars is more than 11 then subtract 12 from the sum. For example, suppose you select train car 11 and train car 3 to be your first and second cars, to calculate the number for the third car do the following: $11 + 3 = 14$, 14 is greater than 11 so, $14 - 12 = 2$. The third car will be the number 2.

Try it out using 12 train cars.

What train cars will you select for your first two cars? __________ and __________

After you have found a successful loop, go to the interactive applet located at [http://www.shodor.org/interactivate/activities/ClockArithmetic/](http://www.shodor.org/interactivate/activities/ClockArithmetic/)

Use the numbers that you have written in the blanks above and use the first one as “starting time” in the applet and click on “set start time.” Input the second number into the “elapsed hours” box and click on move forward.

This kind of addition is called modular addition. If we use a clock size of 12, we are doing mod 12 addition. To write the sum of 11 and 3 for a clock of size 12 we would write: $11 + 3 = 2 \mod 12$.

• Discuss in your group how this applet, is similar to the last activity.
• Use the applet to explore different clock sizes.
• Look in the index of your calculator instruction manual or online for ‘mod’ to learn how to use this feature on your calculator.
• In your groups discuss the following directions for modular addition, and decide if you agree with them and if you like them.
  1. Do regular arithmetic and get the regular answer.
  2. Divide the answer by the mod and find the remainder.
  3. The remainder is the answer to the problem.

Answer the following questions with your group.

For a given modulus, what is the length of the longest loop?

How is the length of the longest loop related to the modulus?

What are the lengths of all the orbits, and how many are there of each for a given modulus?

How are the lengths of all the orbits related to the length of the longest orbit?
ASSIGNMENT

Complete the following task. Use both the applet and the mod feature on a calculator to help you solve the problems. If neither the applet nor a calculator will work for a particular problem find another method to solve.

Do the following arithmetic mod 5. Do the following arithmetic mod 6.

1. \(3 + 2 = \_\mod 5\) 11. \(4 + 2 = \_\mod 6\)
2. \(3 + \_ = 1 \mod 5\) 12. \(3 + 5 = \_ \mod 6\)
3. \(3 + 4 = \_ \mod 5\) 13. \(3 + 2 = \_ \mod 6\)
4. \(\_ + 2 = 1 \mod 5\) 14. \(2 + \_ = 0 \mod 6\)
5. \(5 + 1 = \_ \mod 5\) 15. \(2 + \_ = 2 \mod 6\)
6. \(2 + \_ = 4 \mod 5\) 16. \(2 \times 4 = \_ \mod 6\)
7. \(2 \times 4 = \_ \mod 5\) 17. \(3^2 = \_ \mod 6\)
8. \(3 \times \_ = 1 \mod 5\) 18. \(3 \times \_ = 1 \mod 6\)
9. \(\_ \times 2 = 3 \mod 5\) 19. \(2 \times \_ = 2 \mod 6\)
10. \(3^2 = \_ \mod 5\) 20. \(4 \times \_ = 3 \mod 6\)

Find the additive and the multiplicative inverses, if they exist. Organize your answers in some sort of chart.

21. 0, 1, 2, 3, and 4 mod 5
22. 0, 1, 2, 3, 4, and 5 mod 6
23. 0, 1, 2, 3, 4, 5, and 6 mod 7
24. 0, 1, 2, 3, 4, 5, 6, and 7 mod 8
25. 0, 1, 2, 3, 4, 5, 6, 7, and 8 mod 9

26. Does a number always have an additive inverse? If not, come up with a conjecture of when it does have one.

27. Does a number always have a multiplicative inverse? If not, come up with a conjecture of when it does have one.

28. Give an example of when and how modular addition, subtraction, multiplication and division would be used in real life.

29. How would modular division be performed?
The Exponential Function

Technology Used: Excel, Calculator, Internet

Objectives:

1. Calculator
   a. Students will learn how to access the finance program in their calculator and explore with how to use it.

1. Excel
   a. Students will explore the solver functions that are available in Excel.
   b. Students will be able to program equations in Excel.
   c. Students will program a mathematical formula in Excel that will be able to calculate numerical values if initial information is used as input.

2. The Number e
   a. Students will review interest rate formulas, with monthly compounding.
   b. Students will explore multiple definitions of the number e, and how it was derived.
   b. Students will be able to explain the connection between a compounded interest rate formula and an exponential model.

3. Teaching
   a. Students will create sample questions that will test a pre-calculus student on their understanding of the interest rate formula.
   b. Students will formulate an activity that high school students could do, that would demonstrate the connection between compound interest and exponential growth.

Activity Plan:

Part I: Motivation
Look at examples of how interest affects the lives of students. (Student loans, credit cards, savings accounts, etc.) Guide students to use the finance program in the calculator.

Part II: Functions
Show the students the available functions for Excel’s solver. (Highlight cell, Shift F3) Demonstrate an easy example of how to assign a formula to a particular cell and how to apply the formula to multiple cells. (=‘formula’)

Part III: Investigation
Students will work in groups. If necessary, students will review the compound interest formula, either on the internet or in a mathematical text book. Using Excel, students will create a spreadsheet that will calculate answers to the following questions (and/or similar)

1. How much is a common interest rate for a car or truck in this area? (Use this answer in your calculations)
2. How much would your monthly payment be? (Don’t forget to include sales tax.)
3. How much money total will you pay for the car?
4. How much of the total that you pay for the car will go to interest?
5. How much of your first payment goes toward principal?
6. How much would you save over all if saved more money and paid a $2000 down payment?
7. What other variables could you change to help you save money? How much impact would this have?

Part IV: Extension
Look at several definitions of the mathematical constant e.

Part V: Assignment
Students will explain, using the definition of e, how the derivation of the mathematical constant e is connected to the compound interest formula and why these ideas are usually introduced together.
Students will formulate ideas that can be used in a classroom to inspire students to solidify and connect ideas dealing with the compound interest formula and exponential growth and decay. (see attached assignment)

Part VI: Discussion
Discuss what was learned, advantages/disadvantages of software, extensions, what was liked/not like, etc.
GROUP ACTIVITY (Adapted from work by Shane Goodwin)

- Write a program using Excel that you could use to help you grade the following assignment/test. Write it in such a way that if you input only the car/truck value and the loan rate it will in return calculate the answers to problem numbers two through six.

Sample Assignment/Test

Car / Truck Loan Analysis
Find an advertisement for a car or truck that you might someday be interested in buying. Be realistic, and search for a car that you could use now in High School or one that you could take to college, this means that you should search for an affordable car. It can be used or new.

- The advertisement should include the price of the car or truck.
- This advertisement should be turned in with this assignment.

Answer the following using (your interest rate program in your calculator):

Let's say that you want to buy this car now, and have it paid off in 3 years and that you have $800 now to use as a down payment.

1. How much is a common interest rate for a car or truck in this area? (Use this answer in your calculations)

2. How much would your monthly payment be? (Don't forget to include sales tax.)

3. How much money total will you pay for the car?

4. How much of the total that you pay for the car will go to interest?

5. How much of your first payment goes toward principal?

Explore other options:

6. How much would you save over all if you saved more money and paid a $2000 down payment?

7. What other variables could you change to help you save money? How much impact would this have?
ASSIGNMENT

• Come up with three other questions that could be asked to the students to test their knowledge on interest rates while encouraging them to use technology to obtain their solutions.

• In most text books the exponential function is introduced either in the same section as interest rate, or in the subsequent one. Why is this? Explore the applet found at http://www.ies.co.jp/math/java/calc/exp/exp.html to answer this question. Pay attention to what happens to y as x is increased.

• Find at least two definitions for the mathematical constant e and explain in your own words what they mean. It may help to include a brief description of where the definitions came from.

• Explain using one of the definition from the previous question, how the constant e is related to the interest formula. Specifically address what it means to compound continuously.

• Come up with some sort of very brief activity that a class could participate in that would make a transition from the section on interest rates to the section on the mathematical constant e.
Pascal’s Triangle and Graph Theory

Technology Used: Excel, Internet

Objectives:

1. Excel
   a. Students will explore the solver functions that are available in Excel.
   b. Students will be able to program equations in Excel.
   c. Students will use Excel to generate Pascal’s triangle.

2. The Triangle
   a. Students will review sequences, patterns and combinatorics.
   b. Students will explore mathematical proofs.
   c. Students will make the connection between the grid and Pascal’s triangle.
   d. Students will be able to explain the connection between Pascal’s triangle and binomial coefficients in the binomial expansion.

3. Teaching
   a. Students will create an activity to use in the classroom that uses Pascal’s triangle.
   b. Students will find a pattern within Pascal’s triangle that they can teach other students.

Activity Plan:

Part I: Motivation
Students will look at an example of how Pascal’s triangle could be used to simplify a more confusing and complex problem. Students will be provided with an activity that can be solved in more than one way, but the simplest way results from recognizing the pattern of Pascal’s pattern. Student’s will be given a grid and will work in groups to determine how many different paths there are from the corner of the grid to another point.

Part II: Pascal’s Triangle
Guide students to explore different patterns that occur within and using Pascal’s triangle.
Remind the students of some of the available functions for Excel’s solver.
(Highlight cell, Shift F3) Demonstrate an easy example of how to assign a formula to a particular cell and how to apply the formula to multiple cells.
(=" formula’)

Part III: Investigation
Students will work individually to generate Pascal’s triangle on a spreadsheet.
The previous activity should have reminded the students of the patterns involved.

Part IV: Extension
Students will add information to the spreadsheet that will help them to analyze the number patterns. This information could include columns that contained information such as sum of the entries of the rows and sum of the squares of the entries of the rows.
Part V: Assignment
Students will explain, in written form, using the information obtained in the investigation, ideas that include patterns, sequences, combinatorics, proofs, binomial coefficient's, Pascal's triangle and teaching applications. (See attached Assignment worksheet)

Part VI: Discussion
Discuss what was learned, advantages/disadvantages of software, extensions, what was liked/not like, etc.
GROUP ACTIVITY

Find how many different paths there are from A to B. You can only move down and to the right along the edges in the grid. *Hint: It may be helpful to solve for the points that are closest to A first and look for a pattern.*

Discussion

*After you have completed the activity above discuss in your groups the following.*

There is more than one way to come up with the answer. What are some of the original ideas that you came up with as a group? What worked with these theories? What did not work? Did you find one method that seemed to be easier? More efficient? Better overall?

What sort of patterns do you see? Have you seen something similar to this before? Can you attach a name to the pattern?
ACTIVITY

Excel has some pretty useful features. One of the features allows you to type in a brief description of what you would like to do and then it will tell you step by step instructions on how to perform this feature on the spreadsheet. A box will open and ask you to search for a function. You type in the function that you are looking for and then the program will find the ‘best fit’ function for your description.

Let’s try an example of a search. Imagine that we want to find the average of ten students test scores. We first input the scores into excel. Go ahead and try it using the data found below.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>89</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>97</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>89</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>73</td>
<td></td>
</tr>
</tbody>
</table>

Next click on some empty cell, let’s say B5, now that it is highlighted hit the Shift button followed by F3 (without releasing Shift). A box should open that asks you to search for a function. Since we want to average the scores, type the word average into the box. The program will return in order the functions that it thinks match best your description. AVERAGE is highlighted at the top of the box. Now click on ‘Help on this function.’ It is blue letters at the bottom of the box. This brings yet another box which gives us directions on how to use the function.

Following the directions, use the AVERAGE function to average the 10 scores that are found in cells A1 to A10. The answer that you get should be 80.1.

Find another function in the search box that you are unfamiliar with and try it out to become familiar with how the commands work.

You will find that if you want excel to perform a mathematical operation you need to start with an equal sign (=)

Another useful tool in excel is the click and drag feature. If you highlight one cell and then click on the bottom right hand corner of that cell and drag the highlighted portion down or to the right, excel will copy the information to the next cell, whether it be a given number or a formula. Try it to see how it works.
ASSIGNMENT

1. Generate Pascal’s triangle using excel. Do not just type in all of the numbers. Use some of the features built into the program to save you some time and to become familiar with them. Go ahead and expand it until it fills up the screen. (I would suggest that although we often see this triangle represented pointing upward, you might try to make yours point toward the upper left hand corner of the spreadsheet. This will allow you to more easily compare it to your previous ‘group activity.’

2. Explain how to create Pascal’s triangle. (This is a question dealing with the math and patterns, not a question that asks how a spreadsheet can create the triangle)

3. a) Discuss the patterns that occur in the sequences of the sum of the entries of the rows and the sum of the squares of the entries of the rows. b) Discuss where the sums of the squares of the entries of the rows occur within the triangle. c) Find and discuss one more pattern, of your choice, that occurs within or using Pascal’s triangle.

4. Explain how the construction of Pascal’s triangle is related to the binomial coefficients in the binomial expansion of \((x + y)^n\). (Include more than a one line response; a thorough explanation may include examples.)

5. Prove: For any nonnegative integer n and any integer k between 0 and n
\[
\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}
\]
Briefly explain how this relates to Pascal’s triangle.

6. Describe an activity that you can use in a math classroom using Pascal’s triangle.

7. Create a spreadsheet that could be used to record and calculate grades for a class with the following criterion.

   Grades will be calculated according to the following:
   8 Quizzes @20 points
   3 Mid Term Exams @100 points
   1 Final Exam @ 200 points
   Total: 660 Points
Unit Circle and Trigonometric Functions

Technology Used: Internet Applets

Objectives
1. Internet Applet
   a. Students will relate trigonometric functions to the coordinates of the unit circle.
2. Unit Circle / Trig Functions
   a. Students will make connections between the unit circle and trigonometric functions.
   b. Students will create graphs of both the sine and cosine functions using modeling of real world examples and by transferring information from the unit circle.
3. Teaching
   a. Students will explore interactive applets that teach trigonometric principles and decide what features they like on them and explore how they would use them in a classroom.

Activity Plan:
Part I: Motivation
   Students will explore in groups the applet found at http://www.colorado.edu/physics/phet/simulations/massspringlab/MassSpringLab2.swf.

Part II: Investigation
   Using the idea of a strip chart, students will model the behavior of oscillating springs friction. Students will expand their conclusions to include what would happen if a spring had no friction affecting it. Students will make connections between the graphs of frictionless examples and trigonometric functions.

Part IV: Extension
   Students will do an activity that will lead them to make connections between the unit circle and trigonometric functions. More specifically the connection between the x coordinate and the cosine function, and the y coordinate and the sine function.

Part V: Assignment
   Students will use the applets and the connections that they have made to complete a worksheet. (See attached assignment)

Part VI: Discussion
   Discuss what was learned, advantages/disadvantages of software, extensions, what was liked/not liked, etc.
GROUP ACTIVITY

Explore the applet found at:

http://www.colorado.edu/physics/phet/simulations/massspringlab/MassSpringLab2.swf

A strip chart is a long roll of paper that slides along underneath a movable pen. As the pen moves, it leaves a trail on the paper. Imagine that there is a pen attached to the bottom of each spring that is in the applet and that there is a strip chart moving behind the spring. Select three different weights and alter the variables in the green box in any way that you like. Then draw on the graph below what the motion of the spring would report on a strip chart. (All three can be drawn on the same chart, you may want to create a legend to identify which graph goes to which spring weight) Make sure to label the graph

Displacement from starting position

Time

Discuss the following questions with your group. For all questions assume that the friction is zero on the spring.

- How long will the spring oscillate if there is no friction on the spring?
- How would the graphs above change if there was no friction on the springs?
- If the frictionless function were modeled by a polynomial what would happen at each x intercept of the graph?
- Are there other functions that have the same general shape as this graph?
- What function would model the graph for the spring without friction better than a polynomial?
Draw the unit circle. Be sure to label the angles in degrees and radians and also label the x and y coordinates where each angle intersects the circle. Fill in the chart.

Make an angle vs. x coordinate graph and an angle vs. y coordinate graph below. Label the graph.

<table>
<thead>
<tr>
<th>θ</th>
<th>sin(θ)</th>
<th>cos(θ)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

Make an angle vs. x coordinate graph and an angle vs. y coordinate graph below. Label the graph.

- Discuss: What similarities exist between the two graphs and the graphs of the oscillating springs?

- Check out http://www.ngsir.netfirms.com/englishhtm/SpringSHM.htm does this applet demonstrate the results that you have shown. Be sure to look at the graph and the circular motion options.
ASSIGNMENT

1. Determine whether the values of the following trigonometric functions are positive or negative in the specified quadrant

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>Sine</th>
<th>Cosine</th>
<th>Tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>III</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Determine if the three trig functions above are odd, even or neither. Is there a connection between the information found in the chart and whether the function is odd or even? If so explain.

3. How can the x and y coordinates be calculated on the unit circle? Are there functions that model the values of the coordinates?

4. Using the applet found at http://www.ies.co.jp/math/java/samples/graphSinX.html compare the motion of the point on the unit circle as it goes around with the point on the graph area. Explain what you see.

5. How could you use the unit circle to introduce the tangent function?

6. Find an applet that you think would be useful to teach some trigonometric idea to a group of high school students.

7. Go through the tutorial found at http://catcode.com/trig/index.html, that starts with the link ‘sticks and shadows.’ Would you use this applet to teach trigonometric concepts to high school students? What parts do you like? What parts do you dislike?
Law of Sines and Cosines

Technology Used: Internet Applets

Objectives:
1. Internet
   a. Students will learn how to use a mathematical applet that is found on the internet.
   b. Students will find an applet on the internet.
2. Law of Sines and Cosines
   a. Students will review trigonometric functions and identities along with their applications.
3. Teaching
   a. Students will find an applet that they can use as a teacher in the classroom that will teach students about the law of sines or the law of cosines.
   b. Students will share with other members of the class the applet that they have selected.

Activity Plan:
Part I: Motivation
Students will complete the review trigonometry activity worksheet (See below). Students can use the internet or textbooks to look up information that they may need to complete the activity, such as trigonometric identities.

Part II: Investigation
Students will explore an applet dealing with the law of sines and two others that demonstrate the law of cosines. All three applets are found at the following address: http://www.ies.co.jp/math/products/trig/menu.html. Students will start with the applet named, ‘Law of Sines,’ followed by ‘Law of Cosines (1),’ then ‘Law of Cosines (2).’ Students should take time to understand what the applet is trying to teach them about the relationship between the Laws and the graphics. Students can discuss with others.

Part III: Law of Sines and Cosines
Students will explore the applications of the laws of sines and cosines and determine its’ uses. Students will investigate a visual proof involving the law of cosines. Students will find out where the laws of sines and cosines come from through explorations with their proofs.

Part IV: Extension
Students will find an applet on the internet that they think would be useful to teach others about the laws of sines or cosines. Students will explain why they prefer the applet to the one shown in class and why they do not prefer it to the in class example. Then they will share the applet with another student.
Part V: Assignment
   Students will explore the uses trigonometric functions using applets and will solve
   problems and complete proofs that are related to the applets. (See Attached)

Part VI: Discussion
   Discuss what was learned, advantages/disadvantages of software, extensions,
   what was liked/not like, etc.
ACTIVITY

1. The baseball player in center field is playing approximately 320 feet from the television camera that is behind home plate. A batter hits a fly ball that goes to the wall 420 feet from the camera (see figure). The camera turns 6° to follow the play. Approximate the distance the center fielder has to run to make the catch.

2. Find the height of the kite. (Not the length of the string.)
ASSIGNMENT

I. Explore the applets named, Law of Sines and Law of Cosines (1 and 2) found at the bottom of the following page:
http://www.ies.co.jp/math/products/trig/menu.html

II. Explain in your own words what you think the applet is trying to teach. In other words, why does this applet exist?

III. The law of sines can be used to find the unknown sides and angles of an oblique triangle. What information must be known about a triangle in order to use the law of sines? (There are 2 cases.)

IV. Explain in words what is happening in the 'visual proof for the law of cosines.' (Found at: http://www.ies.co.jp/math/products/trig/menu.html Law of Cosines (2), the visual proof)

V. Consider a triangle that has three acute angles, as shown in the figure. Note vertex B has coordinates (c,0). Therefore, C has the coordinates (x,y). In this case \( x = b \cos A \) and \( y = b \sin A \). Note, that \( a \) is the distance from vertex C to vertex B. Prove the following. \( a^2 = b^2 + c^2 - 2bc \cos A \)
VI. Prove the following. If $ABC$ is a triangle with sides $a$, $b$, and $c$ then

$$ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} $$

VII. Search the internet and find an applet that you think would be useful to teach others about the laws of sines or cosines. Explain what features you prefer of the new applet over the one used in class and which you do not. Explain why you chose it and share the applet with someone in the class. Explain to them how you would use it in a classroom.
Graphs of Derivatives and their Functions

Technology Used: Maple, Internet Applets

Objectives
1. Maple
   a. Students will learn how to use the Maple tutorial and have a brief introduction to what Maple has to offer.
2. Applet
   a. Students will review the relation between position, velocity, and acceleration.
3. Derivatives
   a. Students will review differentiation techniques along with their uses and names.
   b. Students will practice obtaining a graph of a derivative given a graph of its function and obtaining a graph of a function given a graph of its antiderivative.
4. Teaching
   a. Students will discuss

Activity Plan:
Part I: Motivation
Students will review the definitions, uses and applications of several differentiation rules and create examples of each including constant, identity, constant multiple, sum, difference, product, quotient, power, chain, integral, exponential, natural logarithmic, trig, hyperbolic trig, arc trig, and arc hyperbolic trig techniques.

Part II: Investigation
Student will use an interactive tutorial found in Maple to practice choosing which differentiation techniques or rules to use in various cases. Students will also create functions whose derivatives use multiple techniques solve.

Part IV: Extension
Students will review the relation between position, velocity and acceleration. Students will make observations that relate the graphs of a function and its derivative using two similar applets found at http://www.walter-fendt.de/ph14e/springpendulum.htm and http://www.walter-fendt.de/ph11e/pendulum.htm
Students will use their observations to practice drawing the graph of a derivative given a graph of its function and of drawing a graph of a function given the graph of its antiderivative using the applet found at: http://www.ltcconline.net/greenl/java/index.html#Calculus

Part V: Assignment
Students will complete a worksheet. (See attached)
Have the students discuss with one another the answers to questions nine and ten before they are submitted.

Part VI: Discussion
Discuss what was learned, advantages/disadvantages of software, extensions, what was liked/not like, etc.
GROUP ACTIVITY  Do this activity with a partner. For each of the differentiation techniques below, state the definition, when each is used, give an example of a function that uses the rule when its derivative is computed and find the derivative of the function. Some rules have more than one definition. (Just state 1)

<table>
<thead>
<tr>
<th>RULE</th>
<th>DEFINITION</th>
<th>WHEN</th>
<th>FUNCTION</th>
<th>DERIVATIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant Multiple</td>
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<td></td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Product</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Quotient</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Power</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chain</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Integral</td>
<td></td>
<td></td>
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<tr>
<td>Exponential</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Natural Logarithmic</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Trig</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hyperbolic Trig</td>
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<tr>
<td>Arc Trig</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>ArcHyperbolic Trig</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Open Maple 11. A start up box will appear. Under Help Resources select Maple Tour option. Select the Education, Assessment, Maple T.A. which is located near the bottom of the list. Choose the Calculus student package. Select Step by Step Differentiation.

One of the drop down options is Rule Definition. You can compare your definitions in the chart you just filled out with those found in the drop down option.

There is a sample problem located in the function box at the top of the page. Decide which techniques should be used to find the derivative of the given function. Located on the bottom left hand corner of the screen are various differentiation techniques. Select those that apply. (Sometimes order matters in the selection.) Continue making selections until you have found the derivative.

Now try some of the functions that you previously created. Choose two of the example functions form your chart and type them in the function box then select the techniques to find the derivative. Create three other functions that need multiple techniques to differentiate. Write them below. Trade papers with your partner and use the maple applet to find the derivatives of your partner’s three functions.

1.  
2.  
3.  

Take time to discuss with your partner how position, acceleration and velocity are related to one another and how they are obtained from one another. Record the highlights of your discussion below.

Explore the following applets:
http://www.walter-fendt.de/ph14e/springpendulum.htm
http://www.walter-fendt.de/ph11e/pendulum.htm
In particular compare the graphs of position (the applet refers to it as elongation), velocity and acceleration. Jot down any observations that you see that are consistent between the graphs.

Once you think that you have a good grasp on how the graph of a function and its derivative are related. Follow the directions and try the activities called Try to Graph a Derivative Given a Function and Try to Graph an Antiderivative Given a Function found at: http://www.ltcconline.net/greenl/java/index.html#Calculus. If you are correct your green line will match the red line on the screen. Practice until you have completed at least ten in each section or until you fill you understand the concept.
ASSIGNMENT

State which technique(s) need to be used to find the derivatives of the following functions.

1. \( y = \frac{x^4 + \frac{5}{6}x^3 - x^2 + 9x - 4}{4} \)

2. \( f(x) = \frac{9(x^2 - 3)}{x^3} \)

3. \( f(x) = \sin^3(4t) \)

4. \( y = \frac{5 + 7x - 8\sqrt{x}}{x} \)

5. \( F(x) = \int x \sin t \, dt \)

6. \( y = \ln(\sqrt{2x + 3}) \)

7. \( F(x) = [x \sinh x - \cosh x] \)

8. \( y = 3^{2x} \)

9. Do you think that it is important for students to know the names of the techniques that are used for differentiation? Why or why not?

10. Recall a method that was used in your calculus class that was particularly effective in teaching some principle. Briefly describe the method below. Why do you think that it was effective?
11. For each of the six graphs below sketch, on the same plane, the graph of both the derivative (in one color) and the antiderivative (in another color).
Geometry, Set Theory and Maximum Volume

Technology Used: Geometer’s Sketch Pad

Objectives:
1. Sketchpad
   a. Students will explore the features of Geometer’s Sketchpad.
   b. Students will explore the sample documents in Geometer’s Sketchpad.
2. Geometry
   a. Students will develop geometric thinking.
   b. Students will develop better understanding of how to construct with straight edge and compass techniques (no measurements).
   c. Students will create definitions of geometric ideas.
3. Set Notation
   a. Students will review set theoretic definitions of geometric ideas.
   b. Students will explore propositions that will test them on their understanding of the set theoretic definitions that they created.
4. Teaching
   a. Students will explore some sample documents that are part of Geometer’s Sketchpad and discuss how they could be used to help teach math in secondary education.

Part I: Motivation
Working in groups students will develop set theoretic definitions of words used in geometry.

Part II: Sketchpad
Students will explore the features available in Geometer’s Sketchpad.

Part III: Investigation
Students will create a rectangle with specific dimensions in Sketchpad. They will cut movable corners out of the rectangle, so that if they were folded up, a box would be created.

Part IV: Assignment
Students will review set theoretic definitions and explore geometric propositions. Students will use Geometer’s Sketchpad to create a box of max volume.

Part V: Extension
Students will explore a Sample Sketchpad document and discuss how it could be used to teach some concept in mathematics secondary education.

Part VI: Discussion
Discuss what was learned, advantages/disadvantages of software, extensions, what was liked/not like, etc.
ACTIVITY (Adapted from work by Jim Cangelosi)

Begin to develop a set theoretic Euclidean geometry. Read and complete the following working in groups.
Comprehend the following notes and definitions.
1. “Point” is an undefined word.

   Space=\{points\}

   Given \( A, B \in \{\text{points}\} \) , (“\( AB \)” is read “the distance from point \( A \) to \( B \)”).

   “Distance” from one point to another is an undefined word.

   Given \( A, B \in \{\text{points}\} \) , \((AB \in [0, \infty))\).

   Given \( A, B \in \{\text{points}\} \) , \((AB = 0 \iff A = B)\).

   Given \( A, B \in \{\text{points}\} \) , \((AB = BA)\).

2. What are the attributes of a “point”?

3. Explain the following, specifically why is the set equal to 3? Given \( A, B, C \in \{\text{points}\} \) , \( \exists \|\{A, B, C\}\| = 3 \).

4. Given the following definition, decide what \( A - B - C \) means.

   Given \( A, B, C \in \{\text{points}\} \) , \( \exists \|\{A, B, C\}\| = 3 \) , \((A - B - C \iff AB + BC = AC)\)

5. Develop a definition for “collinear points”
6. For each of the following, 1) discuss and agree on the concept of the idea and 2) develop a set theoretic definition for the idea.

- Given $A, B \in \{\text{points}\} \ni A \neq B$, ("$AB$" is read "segment $AB$")

- Given $A, B \in \{\text{points}\} \ni A \neq B$, ("$\overrightarrow{AB}$" is read "ray $AB$")

- Given $A, B \in \{\text{points}\} \ni A \neq B$, ("$\overline{AB}$" is read "line $AB$")

- Given $A, B, \text{ and } C$ are non collinear points, ("$\triangle ABC$" is read "angle $ABC$")

  Note: "$m\angle ABC$" is read "the degree measure of angle $ABC$"

  Also, in Euclidean geometry (unlike trigonometry) $0 < m\angle ABC < 180$

- Given three non collinear points $A, B, \text{ and } C$, ("$\triangle ABC$" is read "triangle $ABC$"
INVESTIGATION ACTIVITY

Open Geometer's Sketchpad. Open a new sketch (document) and maximize both windows. Practice using the features located on the right edge of the screen. From top to bottom you will find the following tools: Selection Arrow Tool, Point Tool, Compass Tool, Straightedge Tool, Text Tool, and Custom Tool. Practice using these features.

1. Make 2 distinct points on the sketch with the Point Tool.
2. Highlight both points using the Selection Arrow Tool, with the mouse button still depressed after selecting the second arrow move the arrow around the screen. (What happens?)
3. Select the Selection Arrow Tool and click on a blank spot on the screen. (What happens?)
4. Make a line segment with the Straightedge Tool.
5. Make a circle with the Compass Tool.
6. Type some text using the Text Tool.
7. Type ctrl and z at the same time. (What happens?)
8. Make a triangle using the Straightedge Tool.
9. Select one of the vertices of the triangle with the Selection Arrow Tool and with the mouse button still depressed move the arrow around. (What happens? How could you move the whole triangle without changing the length of the line segments and angles?)
10. Take 2 minutes to create some geometric shapes and designs using the tools.
11. Right click on a point.
12. Explore the options that are available. Try to figure out what each one does.
13. Create a triangle using three lines with the Straight Edge Tool
14. Highlight only one side of the triangle.
15. Select Measure then Length from the drop-down bars. Notice what happens.
16. Do this for all three sides of the triangle that you created in step 13.
17. Select Measure again, then Calculate.
18. Add the three triangle side lengths together in the calculator, do not enter in the numerical measures but instead create a code that adds the lengths up by selecting each measurement by clicking on the values obtained in steps 15 and 16, then hit OK. (example: in the calculator box it might read, “m \( AB \) + m \( BC \) + m \( CA \)”)
19. Right click on the new box with information that was just created.
20. Select Properties then Label.
21. Type in the word “Perimeter” then select OK
22. Select one of the point of the triangle and drag it around, notice what happens to the numerical measured values.
BOX VOLUME ACTIVITY (Directions for activity created by Bryan Bornholdt)

In Geometer’s Sketchpad go to the “Help” tab located at the top of the sketch. Select “Sample Sketchpad Documents.” Next select the “Calculus” folder and choose “Box Volume.” Get ready to create this box yourself.

1. Open a new sketch in Geometer’s Sketchpad.
2. Click on the Point Tool to enable creating points. Place a point very close to the top of your sketch left of the center of the sketch area. This is the upper left hand vertex of the rectangle. Right click on this point and Show Label. It should be labeled point A. If not, edit the label to make it “A”.
3. Click Selection Arrow Tool to disable creating points.
4. Because we want to calculate side lengths and the volume of the box, we will employ the *translate* feature. Notice that the initial point is already highlighted. Under *Transform*, select *Translate*. Notice that the point to be created is faintly shown in its proposed position.
5. Using either *polar* or *rectangular* vectors, locate a second point 12 cm right of the initial point. The second point is permanently fixed relative to the initial point. (Try moving either of the two points and see what happens.) Right click on this point, go to *Properties* and label the point $B$. DO NOT select *Show Label*.
6. Locate the remaining two points 9 cm below the first two points using the *Translate* feature. To do this, only one point can be highlighted at a time. Again, move any point and verify that the vertices form a fixed rectangle. Label so the points $A$, $B$, $C$, and $D$ are in a clock-wise order.
7. We will now complete the rectangle by constructing the sides. Highlight two adjacent points, say $A$ and $B$, and under *Construct*, select *Segment*.
8. De-select the segment and points by clicking in a blank area and repeat the process with adjacent pairs of points until the rectangle is complete. Only label $A$ is to be displayed.

**Constructing the Inner Rectangle**

The next phase is to construct the adjustable inner rectangle $A'B'C'D'$ formed by cutting squares from each corner. All of the squares must adjust symmetrically. We will create the inner rectangle with a single adjusting point, $A'$, located at the upper left-hand vertex.

We continue by constructing the adjustable point $A'$.

Note: The inner rectangle adjusts with point $A'$ moving along the bisector of angle $A$ of the fixed rectangle. The adjustable point $A'$ must not be able to slide beyond the perpendicular bisector of side $AD$. This corresponds to the domain of the square’s side length, $x$.

**Constructing the adjustable point $A'$**

9. Highlight segment $AD$ (but NOT the endpoints) and under *construct*, select *Midpoint*.
10. With the midpoint already highlighted, highlight segment $AD$ as well and under *Construct*, select *Perpendicular Line*.
11. We next want to construct the angle bisector of angle $A$. Actually, we must think of the angle as either $\angle DAB$ or $\angle BAD$. Select the points in the order $A,D,B$ and...
then construct the angle bisector. What happens? Type CTRL-z to undo the incorrect bisector and click a blank region to de-select the points.
12. Highlight the points in a correct order, say D,A,B, and construct the angle bisector.
13. Construct the intersection point of the angle bisector and the perpendicular bisector of segment \( AD \). There is more than one way to do this.
15. Highlight the newly created intersection point and point A and construct the segment between them.
16. Select the Point tool left of the sketch area and create a point on the new segment along the bisector (the segment is already highlighted). De-select all objects.
17. Highlight the segment and intersection point and hide them.
18. Label the newly created point on the hidden bisector segment \( A' \) so the label appears left of the point. Move \( A' \) around. Where can it move? How far can it move?

Next we construct the squares to be cut from each corner.

Considerations: Point \( A' \) moves as desired. We want the other three vertices of rectangle \( A'B'C'D' \) to move with and symmetrically with point \( A' \).

19. Highlight point \( A' \) and segment \( AD \) and construct a perpendicular line. De-select all objects.
20. Highlight point \( A' \) and segment \( AD \) and construct a parallel line.
21. Construct the point of intersection of the perpendicular line and segment \( AD \).
22. Construct the point of intersection of the parallel line and segment \( AB \).
23. Hide the perpendicular and parallel lines that you just created.
24. Create dashed segments between point \( A' \) and the newly created points of intersection on segments \( AD \) and \( AB \). Once you specify a dashed segment, each subsequent segment will be dashed. Move point \( A' \) to verify that the new segments adjust as desired.

Identifying points and segments for the remaining corners being “cut”
The squares being cut have adjustable sides between the two rectangles. We will construct these sides around the figure. In reality, we want dashed lines corresponding to the cut square on the outer rectangle and solid lines on the inner net edge.

25. Highlight the upper endpoint of the dashed segment parallel to segment \( AD \) and then highlight point B. Under Transform, select Mark Vector. A dotted line should flash from the initial endpoint to point B.
26. Highlight the initial upper endpoint, segment and point \( A' \). Under Transform, select Translate to translate this segment onto segment \( BC \). De-select all objects.
27. Highlight the left endpoint of the segment perpendicular to segment \( AD \) and then highlight point D. Under Transform, select Mark Vector. A dotted line should flash from the initial endpoint to point D.
28. Highlight the initial endpoint, segment and point $A'$. Translate this segment onto segment $CD$. You may want to move point $A'$ to see what happens.

**Constructing the remaining squares:**

29. De-select all objects. Highlight the dashed segment and its endpoints on side $BC$. Double click the lower endpoint to select the point about which to *Rotate* the segment. Under *Transform*, select *Rotate*. View the proposed new segment to make certain that it lies inside the rectangle parallel to segment $CD$. Adjust the angle of rotation if necessary. Click on *Rotate* to perform the rotation.

At this point in the construction, you should be able to repeat the rotation steps to complete the remaining net having dashed sides representing the edges of the squares cut from the sheet and solid lines representing the outer edges of the net to be folded.
ASSIGNMENT (Created by Jim Cangelosi)

Suppose that \((A, B,\) and \(C\) are three non collinear points \(\wedge D\) is a point such that \(\wedge D \in \overline{AB}\)). Under that supposition, determine whether the following propositions are true or false and why.

1. \(p \Rightarrow \triangle ABC = \triangle CBA\)
2. \(p \Rightarrow \triangle DBC = \triangle ABC\)
3. \(p \Rightarrow \triangle BCA = \triangle ABC\)
4. \(p \Rightarrow \angle ACB = \angle CBA\)
5. \(p \Rightarrow AB \subseteq \triangle ABC\)
6. \(D \in \triangle ABC\)
7. \(D \subseteq \triangle ABC\)

Suppose that \(q : (A, B,\) and \(C\) are three non collinear points \(\wedge E\) is a point such that \(A-E-B \wedge F\) is a point such that \(C-F-B \wedge G\) is a point such that \(E-G-F\)).

8. Under that supposition, determine whether or not the following proposition is true. \(q \Rightarrow G \in \triangle ABC\)

Note: Each triangle is associated with at least two numbers: \(\triangle ABC\) is associated with its perimeter (i.e., \(AB+BC+AC\)) and also the area of its interior region.
When responding to the following prompts, assume that the universe consists of a plane $P$; thus, all points referred to in the problem are coplanar. Draw the following or explain why it is impossible for such a set to exist.

9. Given $A$, $B$, and $C$ are three non collinear points.
   - $AB \cup BC \cup AC$

10. Given $A$, $B$, and $C$ are three non collinear points so that $AB>BC$ and $m\angle BAC > m\angle BCA$.
    - $AB \cup BC \cup AC$

11. Given $A$, $B$, and $C$ are three non collinear points
    and $I = \{X: Y \in BC \land Z \in AC \land Y - X - Z\}$.
    - $I$

12. Given $A$, $B$, and $C$ are three non collinear points and $C$ is a point such that $A-C-B$.
    - $\angle DBA - CA$

13. Given $A$, $B$, and $C$ are points such that no three of them are collinear.
    - $AC \cup AB \cup DC \cup DB$

14. Given $A$, $B$, and $C$ are points so that no three of them are collinear.
    $CD \cap BA \neq \emptyset$ and $BD \cap CA = \emptyset$.
    - $AC \cup AB \cup DC \cup DB$
ASSIGNMENT

Construct the folded box:

Continuing on with the box created in the Box Volume Activity, use the translate feature to create a representation of the folded box that adjusts as you move the point $A'$ on your own sketch.

Finally, your completed GSP file should actively display the volume of the box changing as the point $A'$ is moved. Also, all points except for point $A'$ should be hidden.

Shade the sides similar to the example provided in the Box Volume Problem under “Samples” provided by Geometer’s Sketchpad and construct a scaling bar beside the net for rescaling the box and hiding point.

Indicate somewhere on your sketch the dimensions of the box of max volume. Also indicate how these dimensions can be found using calculus.

Teaching:

Explore a different example found under the Sample Sketchpad documents. Write a paragraph about how you would use the document to teach some mathematical principle to a secondary education class. Be sure to mention the mathematical principle.
Conclusion

The lesson plans found in this project are methods to assist students, who are future mathematical educators, to understand higher level mathematical ideas, connect ideas across different mathematical fields, and also to offer a brief introduction to the students of how technology can be integrated into the mathematical classroom to assist with learning. Because not every student has a deep understanding or misunderstanding of the same ideas, the suggested lesson plans offer a base upon which teachers can build and expand as they encounter special needs for their own students. It is hoped that these lessons will promote a desire in the students who work out the activities to create their own lesson plans, plans which relate activities, mathematical topics and technologies together for deeper understanding.
References


Dildine, James P. Technology Intensive Instruction with High Performing and Low Performing Middle School Mathematics Students. 1999.


Lampert, Magdelene. When the Problem is Not the Question and the solution is Not the Answer: Mathematical Knowing and Teaching. American educational Research Journal. 1990.


Niess, M. L. Guest Editorial: Preparing Teacher to Teach mathematics with Technology. Contemporary Issues in Technology and Teacher Education. 2006