Virtual Manipulatives in the Classroom and Resulting Articles and Lesson Plans

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MMATH PROJECT

Research Study

Virtual Manipulatives
In the Classroom
And Resulting Articles
And Lesson Plans

Prepared by: Cheryl Juliana
August 13, 2010
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Abstract
Is the Knowledge Gained Working through Math Manipulatives worth Taking Time out of Book Study for the General Population of Junior High School Mathematics Students?

Cheryl Juliana

Master's Project

Upon coming across mathematical manipulatives generated and produced by Utah State University, as a math teacher, I conducted a classroom teaching experiment in three pre-algebra classes with students of various achievement levels. After teaching the entire year using no manipulatives in the classroom, I tested my students with a general, end-of-year, core criterion, or cumulative test. Their scores were noted. The students in the study group were then given opportunities to try several manipulatives offered on the “National Library of Virtual Manipulatives,” both as a class, and alone, and then retested. The following paper gives the parameters of the study, the statistical data surrounding the scores, responses from the students participating in the experiment, and the information found concerning the data collected.

Following the experiment, after analyzing the data collected, I began making a collection of activities, web-sites, and journal reviews for use in my classroom as a math teacher. I read journals that reiterated my research findings. My personal reviews are also included in this project. I
reviewed one journal article similar to my own study for a group of math teachers at a fraction conference. The power point I used is included in the software portion of this report. I also made new and more comprehensive activities incorporating virtual manipulatives both “home-made” and web-site based which are included. I also began a web-site for virtual manipulatives for future reference list which I refer to on occasion when I am looking for virtual manipulatives, or technology for certain topics. I have included this also.

Teachers in classrooms everywhere are faced with the difficult task of taking 30-40 various junior high-aged students, at various educational levels, and teaching them over the course of a year, a large amount of mathematics mandated by the state, all at the same time, with very little, if any time to spare. It is a gamble at most, to include virtual games and activities because they take extra time to learn from, and are not traditional. Research shows that using virtual manipulatives benefits, but most teachers are reluctant to use them. Most mathematics teachers today believe the virtual manipulatives and technology use more time, create more problems, or are hard to use. They don’t really know the research that accompanies using virtual manipulatives. There are some studies available to the public that show virtual manipulatives actually teach mathematics in a more exciting, interactive, visually stimulating way. Some teachers wonder, Is the time taken away from teaching mathematics in a
more traditional way to teaching using virtual manipulative helpful? Are the benefits worth deviating from the hurried, fast-paced "Here's how you do it. . . now you try it," type of teaching that goes on in most classrooms?

After researching manipulatives found on the internet, I decided to take this question to my classroom, created a research project to gather some data on this subject, and then use the outcomes to base future activities and teaching strategies in a more effective way.

Technology has the potential of being a major part of learning and making sense of mathematics for students everywhere. It has already played a major part in teaching mathematics with the use of calculators, graphing calculators, and other computer programs. Every day there are more and more tools created and made available to math teachers through the internet. Do they benefit the students in such a way that make it important to use them to enhance learning?

First, it is important to understand what constitutes a virtual manipulative: "Virtual manipulatives are interactive, visual representations of a dynamic object that present opportunities for construction mathematical knowledge (Moyer, Bolyard and & Spikell 374). Some studies have shown that "students who use appropriate technology persist longer, enjoy learning more, and demonstrate gains in mathematics performance" (Goldman, 4). There are many technology tools available on the World Wide Web that may be used by students everywhere with no cost, and have been
developed with common errors that many teachers see again and again in their classrooms in mind. The manipulatives are designed to target student errors that are common and even exasperating for teachers to teach over and over again, sometimes with no success.

Although many virtual manipulatives exist on the World-Wide web, my experiment focuses on some specific National Library of Virtual Manipulates available through Utah State University. They were chosen because they were easy to use and free to everyone. The manipulative applets I used are located on the following web site:


The applets used in this project were meant to target some common errors and mathematical misunderstandings that were observed in students' work throughout the year. They were general and across-the-board, not focusing on any one problem, but generally targeting many problems common to junior high students. The manipulatives were chosen in three areas: Number and Operations, Measurement, and Geometry. In the area of number and operations, “Circle-O,” “Fraction-Comparing,” “Algebra Balance Scales,” and “Function Machine,” were used. In the Geometry area, “Geo-board Coordinate” and “Pythagorean Puzzles” and finally, in measurement, “Converting Units,” “Fill and Pour,” and “How High” were used. They were chosen to fill a broad spectrum of common problems with junior high pre-algebra students.
The applets involved in this study all have one thing in common; they are interactive. Each applet chosen included tutorials designed to teach mathematics through instructions and activities. Many studies show that using the virtual manipulatives helps students explore mathematical relationships in a visual and symbolic way. The tutorials were important to the study in this way; hopefully, they would guide students through the mathematics so that they are able to explore the idea fully.

To help me find an idea for the project, I read many articles that furthered the idea that using visual and interactive models to teach mathematics helps the students explore and develop mathematical ideas in their minds. It has been shown that visual models are very useful in students’ ability to understand and practice new mathematical topics.

Students understand mathematics better when represented in a variety of situations, pictures, graphs, and symbols. The tutorials that accompany the applets provide lessons and representations that directly apply to those found to be successful in student learning in math. My question was, will these applets build any bridges in the students’ conceptual mathematical storage banks, and help them more fully understand mathematical topics better than what I am already doing? Are virtual manipulatives actual pieces making up their mathematical knowledge puzzle that is necessary for them to move into their next level of math with competency?
SUBJECTS

Each year students are placed in my math classes by a computer scheduling program. They are placed based on schedule only, and not by ability, or merit. Because I wanted to see what would work better for my students at my school where I teach, I based my research project on the condition that each year students come from the same demographic area and are placed in my classes in the same way year after year. During my research study, there were three teachers at the school who taught pre-algebra, with six classes between them. During the study, I taught three of the six classes, or 50 percent of the school's pre-algebra students. The students were in either seventh, eighth, or ninth grade, with the majority of them being seventh graders. Whatever distinctions were made between students placed in my math class versus another teacher's math class were have been in place for three years. I felt it was safe to assume that whatever my study indicated for my students this year, the same would apply the next few years.

STUDY

One class was the entire control group. It was selected as the control group because it was my "middle" class in the teaching set up. It was third period, and the study groups were second and fourth. Both groups were given the same or similar on-site instruction and modeling of mathematics throughout the year. While the study group classes were given a two-week period of
45-minute instruction and lesson plans using virtual manipulatives near the end of the year, the control group continued on with the same type of teacher-based instruction as was given throughout the year.

Both groups were tested with two separate standardized cumulative tests, one before the two week period, and one after. Though the tests had different questions, they were two separate tests with similar form and content, covering the state core, objectives and standards taught throughout the year with similar yet different questions.

For the two weeks between testing, the control group was given similar lessons, constructs, and activities to the study group. Evidently, the same subjects were taught; though, the study group learned them using virtual manipulatives, the control group learned lessons and did practice problems on worksheets. The control group finished the instruction with a few days to spare; they were given a cylinder activity to fill the time in class available while the study group finished their virtual manipulatives.

Each day of instruction for the study group included a virtual manipulative. The students were given instructions on how to use the applet, were given task sheets to work through involving the applet, though much of their virtual manipulative work involved the tutorials included with the applets. Few of them were done as a class, while most were done individually in the computer lab. When finished with the day's activity, students were allowed to visit other manipulatives and work through their tutorials.
RESULTS

There were two sources for data gathered throughout the study. First, data was collected through test results. Students were pre-tested with a sample core criterion test, covering state objectives and standards taught throughout the year. These results were recorded and set aside. After the two week period of instruction, they were then post-tested with the same type of test, covering the same topics. These scores were recorded right next to the pre-test scores. They were compared and their averages or means were taken. Their standard deviations were compared, as well as their r-scores and r-squared scores of both tests.

The average pre-test score for the study group was 64.0 percent while the post-test average was 68.2 percent. The average or mean score for the study group was four percent higher after the two-week period of virtual manipulative activities. The average or mean score for the control group was .5 percent higher after the two week period of normal instruction. It seems that the study group had a larger learning jump with use of the manipulatives for learning activities.

The standard deviation for each group was found to be quite high, though roughly the same. The standard deviation for the study group pre-test was 21.73 and the post-test was 21.59. The standard deviation for the control group was 19.81 for the pre-test, and 21.15 for the post-test. They averaged around 21 percent each. I believe the deviation was so high,
while similar, because of the way students are placed in classes at the junior high level. There is no system in place to put students in classes according to ability or merit. Therefore, there would normally be a large variance in mathematical ability and learning styles, therefore, the students’ test scores should deviate. This was consistent for both groups and their scores used in the study.

I also took the r-score and the r-squared score for both the study group and the control group to see the correlation between the two test scores for the classes. I found the R-score for the study group to be .876 or 87.6% and the R Squared Score to be 77%. For the control group, the R score was .70 or 70%, and the R squared Score was .70 or 70%. The correlation scores were greater in the study group. With the correlation statistic higher in the study group, it seems that the greater change was present in the group experiencing the virtual manipulatives.

The second type of data collected throughout the study was student responses. Students were asked to write down what they learned by completing the activity, and what they thought the applet was trying to teach them. This was a different type of data collection, but well-recorded and interesting. Though I heard very different responses to the several different manipulatives used in the study, the overall reaction was positive. They all seemed to be helpful in one way or another to the students. The responses were very positive and they seemed to learn from each
manipulative. Most of the students responded in a similar way to this one, "I thought it was fun. It helped me refresh on how to do equations correctly and take all the right steps to find the right answer. There was nothing I didn't like about it. I thought it was fun." Or, "This game was easy. I learned how to problem solve. I liked solving the equations." I think from the student responses that the visual aspect present with the manipulatives really deepened their understanding of the different mathematics concerning the manipulatives. The graphics created with the manipulatives helped the students engage with the learning. They understood the abstract mathematical ideas better. They seemed to learn the mathematics more deeply, and recorded it in their responses. They were obviously focused on making decisions, reasoning, and problem solving as they worked through the virtual manipulatives and used the technology.

CONCLUSION

Does my study conclude that we as teachers present a better learning experience for our students if we teach using virtual manipulative applets instead of constant and direct instruction and modeling? The results in my classroom study show a direct difference.

As a teacher, involved in this study, I noticed that as my students used the virtual manipulates, some very key knowledge began to develop in my students. They understood better and more deeply because they experienced mathematics visually through the manipulative. For example,
after experiencing the balancing beams manipulative on the NLVM site, I noticed fewer of my students try to add the integer twice to one side. There is no longer a need to teach students to draw a long vertical line through the equal sign to help them see the two sides. They experienced it visually with a balancing scale on the virtual manipulative. Later on in the year, teachers can refer to these ideas and students visually are reminded of the idea of an equation. The time spent on manipulatives is very valuable. This concludes my study.

Works Cited


Data Collected:
Test Results
Student Response
Data Collected

Test Statistics

Pre-And Post-Tests
The first type of data I collected was test score data. After collecting all of the scores for the pre-test, I calculated the mean. I did the same for the post-test. The results are as follows:

<table>
<thead>
<tr>
<th>Study Group</th>
<th>Control Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean/Average</td>
<td>Mean/Average</td>
</tr>
<tr>
<td>Sample A Core Test: 64</td>
<td>Sample A Core Test: 60.3</td>
</tr>
<tr>
<td>Sample B Core Test: 68.2</td>
<td>Sample B Core Test: 60.8</td>
</tr>
</tbody>
</table>

I found it very interesting that there was a 4% difference between pre- and post-test score means in the study group. The control group averages only differed by ½%. This data leads the recorder to believe that the students who participated in the activities with the virtual manipulatives had a greater understanding of the mathematics tested on the post cumulative test.
Next, I calculated separate standard deviation scores for both post and pre tests, in both the control group and the study group. I found that both the study group and the control group had a very large standard deviation score. There was a lot of variability in the individual scores. They were not tightly clustered. This makes sense to me because of the way the classes are formed at Sand Ridge Junior High. There was no ordering in ability when students were placed into classes. The classes were formed only by computers programmed to build schedules, and not on individual mathematical ability. When looking at the data range, on average, about 4 student scores were below the range, while 2 were above the range, meaning 6 outliers existed. The scores were quite diverse.
### R and R squared Scores

<table>
<thead>
<tr>
<th>r-score and r-squared score between before and after tests in study group</th>
<th>r-score and r-squared score between before and after tests in control group</th>
</tr>
</thead>
<tbody>
<tr>
<td>I took the r-score and the r-squared score correlating the Sample Core tests before and after the manipulative applets were completed</td>
<td>I took the r-score and the r-squared score correlating the Sample Core tests before and after the additional in-class work was completed</td>
</tr>
<tr>
<td>R score was .876, or 87.6%</td>
<td>R score was .838, or 83.8%</td>
</tr>
<tr>
<td>R squared Score was .77, or 77%</td>
<td>R squared Score was .70, or 70%</td>
</tr>
</tbody>
</table>

I wanted to see the relationship between the two test scores, post- and pre-test, particularly with the lower-scoring students in order to see if any of them really learned through the interactive and visual manipulatives better. I looked at their over-all scores, down the line, and saw that there wasn’t that big of a change for those who were in the study group, but wanted some type of statistic to prove this. Most of them improved a bit, but very few dramatically improved. Usually, their post test scores were better; yet only sometimes they were worse. This was quite baffling to me. Especially since the same patterns existed with the control group. I thought a correlation statistic might help me interpret the results a little better.

I found the correlations to be quite high, as posted above. Usually the students, who scored well on the pre-test, scored well and even a little better on the post test in the study group. The students scored roughly the same on both tests in
the control group. There were a few students who didn’t follow this trend, but in most cases, the trend was a good way to summarize the test scores.

Having r scores correlating the scores on the two tests at 87.6% in the study group and 83.8% in the control group lets the recorder know that the data is strongly related. There was a little, but still a greater variance experienced in the study group than in the control group. Those students who were involved in the virtual manipulatives activity had a slightly larger difference in scores. They did score 4% higher on the post test. Furthermore, having an r squared score of 77% for the study group and 70% in the control group means there was a slightly larger variance for the study group when compared to the control group. The variance for those not involved in the virtual manipulative activities had 7% less variance in their r squared scores. This was not significant, but still worthy of mentioning.
Data Collected

Student Responses
Besides testing the students before and after the virtual manipulatives lessons, I collected another type of data. I asked the kids to write a paragraph on the manipulative describing what they learned, liked, and didn’t like about the manipulative. Finally, in the end, I asked them which one of the manipulatives was their favorite, and why. I also asked them to explain what they learned from each activity. This was the “student response” portion of my research. I learned a lot from this feedback. I will summarize the students’ responses here for each manipulative.

**Summary of Responses to Manipulatives:**

First of all, I learned that the Fraction Manipulatives were valuable to the kids. They said that they “learned to compare fractions,” and that it was “a fun way to learn fractions.” I also read that “I like being able to see the little picture of the fractions colored in.” The student responses echoed the research I found on the subject. The valuable part of virtual manipulatives is the visual representation and the immediate feedback that the manipulative affords. The students learned a lot about fractions using the manipulative, and also from the visual feedback. There were some students who said “they didn’t learn anything” but I found that more than 80% had a positive response to this manipulative.

The Circle O manipulative the students used had a positive-sound in the student responses. They really became good at adding and subtracting integers. They liked how they got really good at doing them in their heads, yet many couldn’t figure out the puzzle. In summary, they “learned a lot about adding and subtracting integers, but it was too frustrating.” Another student responded, “I learned that it is hard to get three numbers to equal 0. I liked how it made you think. But, I didn’t like that it is hard.” I think they learned more about adding and subtracting integers, but that it was so frustratingly hard, they gave up
easily. I did learn from the responses, though, that the activity really made them understand negative and positive numbers deeply. That was important information. I counted a 75% positive response.

The Algebra Scales manipulative was helpful to students. The responses were very positive. They seemed to learn from this particular manipulative. Most of the students responded in a similar way to this one, "I thought it was fun. It helped me refresh on how to do equations correctly and take all the right steps to find the right answer. There was nothing I didn’t like about it. I thought it was fun." Or, "This game was easy. I learned how to problem solve. I liked solving the equations." This one was very well-liked with the students, and the students seemed to really learn the process of solving equations from it. I think from the student responses that the visual feedback that is given with this manipulative really helped the kids see what happens as they work through solving equations. I found a 85% approval rating with the student responses on this manipulative.

The Function Machine seemed almost too elementary, if you look at all of the student responses. In summary, it was geared more toward elementary-aged students, though the concept of functions is more junior high-aged. One student response stated, "This was also an easy game to do. You just have to figure out the pattern. I learned how to solve patterns. I liked having to figure out the different patterns. "Another student responded, "It refreshed my memory on functions. I liked that it gave you the first 4 numbers so you could get an idea of the pattern. What I didn’t like about it was it was too simple and after a while, it got really lame." Oddly enough, to summarize the student responses for this manipulative, it was “too easy” and became “boring” quickly. Hopefully, they understood the output and input relationship a little more fully, though not one student responded that way. There was no evidence that any mathematical relationships were learned with this activity. I might want to change the worksheet. I counted a 10% approval.
According to their responses, the Geo-board manipulative was well-liked by the students as a manipulative, but answering the questions was hated. They enjoyed learning how to make the shapes, and creating the shapes. They even liked moving the shapes, but nothing was learned by answering the questions. Others enjoyed learning how to create lines and slopes, but hated figuring out the shapes. Some said it was too hard, others said it was quite fun. Most concurred that they didn’t like answering the questions. There were a lot of diverse responses on this manipulative. I found a 55% approval rating for this manipulative.

The Pythagorean Puzzles were mostly given a “they’re ok” response, in summary. Most didn’t know how to figure them out, and a lot of guidance was necessary while they worked through the rigors of the manipulative. Overall, most responses were that this applet was very challenging. Most said they didn’t get past the first puzzle. They liked the building part, and how the shapes snapped into place, but had a hard time understanding what they were doing. They had a great feeling when they put the shapes in the right spot. But, they didn’t like how hard it was to understand, even with some help. There was only a 15% approval rating on this manipulative. I will use this one, but will need to come up with a new worksheet, and change my teaching technique.

The Converting units manipulative was hard for the students at first. They needed a lot of coaxing and help, but after a while, they started to like it. Some comments that were more universal were “There were some problems that I could solve just like that. others took me a while” and “At first I did not get this. After a few tries it was a lot easier. I didn’t really like this game however yet I still learned how to convert things to other things.” They found it was time consuming. From the student responses, I could see that the learned a little about converting units. In summary, they learned a little, but felt that it was really hard to do. I only counted 22% of the responses that really enjoyed this manipulative.
The Fill and Pour game was similar in responses to the Converting Units Manipulative. According to the student responses, it was hard to do at first, and took a lot of work to figure out. Unfortunately, as problems got harder, it became too difficult for them to accomplish. One student responded, “On this one, the first 2 or 3 problems were really easy after that though, I couldn’t figure it out. I really liked this game because you can just click around but it was pretty hard and that sucked.” I didn’t read many student responses that made me believe they learned a whole lot from this one. I felt that the responses indicated that they weren’t interacting with the learning portion of the manipulative, but just clicking things to move on. Again, I think I need to give different instruction and worksheets to help with deeper student learning.

How High was similar with the responses. It was hard for them and they didn’t learn a whole lot about volumes. One student remarked, “This one was fun yet hard. I like that all you have to do is guess. Yet almost all of my guesses were way off this one is probably a favorite of mine. I just need to try harder.” This manipulative had a 30% approval rating, but I never really read any responses indicating that anything new was learned or understood.

Final Favorites:

When I asked for a response on their favorite game, “Hamlet Happens” was the winner. The one they said they learned the most from was the Balancing Scales. The student responses were most favorable on Balancing Scales, and oddly enough, Hamlet Happens became a favorite on its own. The students were able to explore on their own in order to find games they wanted to play after they were done with the day’s activity. The word spread, and Hamlet Happens became an overnight success.

Overall: What I Learned From Student Responses:

I learned from the student responses, which virtual manipulatives to use the following year. Some were too easy, others were too hard, and still others
didn’t work in teaching those things that I wanted them to learn. A lot of the success might have been due to the teaching technique of the teacher, as well as how the worksheet is written. The manipulatives that taught the most, as per the student responses, I used the following year. The following year, I used the Balancing Scales, Hamlet Happens, and the Fraction Manipulative.

The most important thing I learned from the student responses was that the research I was gathering and reading about using virtual manipulatives in the classroom was true. The ideas brought about from my research were modeled in my classroom. The manipulatives helped my students learn some mathematical ideas that they had somehow missed through the year. They were very valuable in that they gave immediate feedback, were visually stimulating, and engaged the students in learning in ways that were difficult to produce otherwise. Students learn from the visual and interactive aspect of the virtual manipulatives, and that they see the changes and feedback immediately.

The manipulatives were exciting and engaged the students. They learned because they were interested. It was quite a learning experience for all of us. The articles I read with similar research seemed to conclude the same type of ideas. After reading through the student responses where some claimed they finally understood certain ideas from the manipulatives, I feel I can’t leave them out of my teaching activities. It is easy to see what students learn by their responses. They learned some ideas more deeply, through greater engagement, because of the manipulatives. For this reason, I will use them more.

One last thing I learned from the manipulatives was that my worksheets needed to be more engaging to the student. They needed to involve more learning. They needed to guide the student through the learning process, with the aid of the manipulative. For this reason, I generated stronger worksheets, even generated them differently. This, I am sure, is very noticeable.

I have learned a lot from the student responses, for both the learner and the teacher. My teaching has changed dramatically through this process.
Original Worksheet

Examples
Computer Activity

Please go to the following site:

http://nlvm.usu.edu/en/nav/vlibrary.html

Click on grades 6 – 8 in the “Algebra” row.

Go to the following game: “Algebra Balance Scales”

Read the directions and then play the game at least 10 times through.

When you are done, go back to the games and click on “Function Machine.”

Read the instructions and play the game at least 10 times through.

When you are done, write up a little paragraph on each game and what you learned, liked, and didn’t like below:

Algebra/Scales. This game was easy. Mostly because I understood it. I knew how to solve it. I learned how to solve. I liked solving the equations.

Function/Machine. This was also an easy game to do. You just had to figure out the pattern. I learned how to solve patterns. I liked having to figure out the different patterns.

If you have time you can play Pentaminos.
Computer Activity

Please go to the following site:

http://nlvm.usu.edu/en/nav/vlibrary.html

Click on grades 6 – 8 in the "Measurement" row.

Go to the following game: "Converting Units"

Go through the instructions and then go through about 10 problems.

When you are done, go back to the games and click on "Fill and Pour."

Follow the directions, and play the game.

When you are done, go back to the games and click on "How High."

Follow the directions, and play the game.

When you are done, write up a little paragraph on each activity, answer questions, and explain what you learned, liked, and didn’t like below:

**Converting/Units**

At first I did not get this. After a few tries, it was a lot easier. I didn’t really like this game however. Yet I still learned how to convert things to other things.

**Fill&Pour**

On this one, the first 2 or 3 problems were really easy. After that though, I couldn’t figure it out. I really liked this game because you can just click around. But it was pretty hard and that sucked.

**HowHigh**

This one was fun yet hard. I like that all you have to do is guess yet almost all of my guesses were way off. This one is probably a favorite of mine. I just need to try harder.

If you have time you can play with the different Transformations.
Computer Activity

Please go to the following site:

http://nlvm.usu.edu/en/nav/vlibrary.html

Click on grades 6 – 8 in the "Geometry" row.

Go to the following game: "Geoboard Coordinate"

Go through the instructions and then do the activities. Click on the activity button in order to get to more activities.

When you are done, go back to the games and click on "Pythagorean Puzzles."

Follow the directions.

When you are done, write up a little paragraph on each activity, answer questions, and explain what you learned, liked, and didn't like below:

Geoboard/Answers

All I, yes, I learned how to make shapes, certain shapes.
I liked making the shapes. I also liked answering the questions. I didn't like.

Pythagorean/Puzzles

I, yes, I learned how to move the pizes. I didn't.

If you have time you can play with the different Transformations
Computer Activity

Please go to the following site:

http://nlvm.usu.edu/en/nav/vlibrary.html

Click on grades 6 – 8 in the “Number and Operations” row.

Go to the following game: “Circle O”

Read the directions and then play the game at least 5 times through.

When you are done, go back to the games and click on “Fraction – comparing.”

Read the instructions and play the game at least 10 times through.

When you are done, write up a little paragraph on each game and what you learned, liked, and didn’t like below:

Circle O: It is fun but it was a little bit hard. I learned that you have to add some and have to subtract some numbers in the game to get a 0 all the way. I liked to have fun when I was learning how to play the game. I didn’t like the puzzle because you have to guess where it goes.

Fractions/comparing: It is hard because you don’t get a lot of clues and directions how to do it. I learned how fractions can be added to make the same fraction. I like the part where you added up the fractions to get it. I didn’t like the part where there was little clues to help you out.
Related Research Article Reviews
Article Reviews

When I first started my project, I was amazed to find so many virtual manipulatives on the internet that could be valuable in teaching mathematics. As I was introduced to virtual manipulatives through various classes I took, I became more and more intrigued, and wanted to learn more about how valuable the manipulatives could be in a classroom setting. I started looking up research articles to see what they had to say about the manipulatives and their use in classrooms. I couldn’t find very much at first. I had a hard time even finding the manipulatives themselves.

As time went on, and I was more involved in my research, I continued to find more and more articles and became more intrigued with the ability to teach mathematics with the technology tools available. Every time I came across an article that was interesting to me, I reviewed it myself, and wrote out a little summary. I then tried to apply it to my teaching of mathematics. I included some of the more important articles that I found in my project folder, along with some of my reviews. I want to be able to look through them from time to time, to remind myself of the value of technology and virtual manipulatives in the classroom. I reviewed them in hopes that I will read the reviews and remind myself more often, than the entire articles. Eventually, over the past two or three years, much more research has been done in this field. I have been able to add a lot of articles just in this past year alone.

One thing I came across as I researched, which still boggles my mind, is our current President’s ideas for the future of education. This was the best thing I found throughout the past three years. He wants far more technology in the classrooms, and even has been involved in making a lap top for every student, under $100. He also wants 70% of the American population to graduate from college. This alone was very interesting. Through my project and the research I did, I have grasped the idea of where education is moving. That was very valuable in and of itself.
What are Virtual Manipulatives?

This is one of the first articles I came across after looking into virtual manipulatives. It was basically the article I have had in the back of my mind that introduced me to virtual manipulatives and defined them. It says that “Virtual Manipulatives may very well be the most appropriate mathematical tool for the next generation (Moyer, Bolyard and Spikell 377). The article gives the first definition for virtual manipulatives. Virtual manipulatives are defined as “... an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard and Spikell 372). In this article, the world of virtual manipulatives is introduced to the world. At this time, as the article points out, millions of virtual manipulatives are being developed and introduced on the world wide web. It also states that the wireless classroom will probably be a big part of the future.

It states that there are unlimited possibilities of using the virtual manipulatives, and that teachers can use the sites to instruct their students. It distinguishes between static sites and dynamic sites. Static sites show a visual representation and then ask the user to manipulate the object. Dynamic visuals can be manipulated in some way and therefore are interactive. There are a lot of great positives for this type of a manipulative, for example, they are free, and require no clean up time.

This was one of my most valuable articles because it gave me the idea to do the study I did for my project. I wanted to see what kind of affect these manipulatives had on student learning, so I went to the web, and started my project. It would have been great to have more information about research projects once I started, but I still learned about the power of manipulatives.

For me as a teacher, this was something that was very new and very exciting for me. I don’t see a “wireless” classroom available in the next 10 years or so, but I do see the power of using the virtual and interactive manipulatives in my classroom.

Young Children’s Use of Virtual Manipulatives and Other Forms of Mathematical Representations

This article reiterated the benefits of using virtual manipulatives in the classroom. It gave useful background about virtual manipulatives, saying that they are very strong creations that generate the great benefit of opportunities for technology to be used in the elementary classroom. Usually, technology is a little more complex than what a kindergartener or second grader would be interested in, so this study researched the use of two virtual manipulatives with very young learners, kindergarteners, and second graders. There were many people involved in the study including teachers and educated observers. The teachers involved themselves in first teaching the mathematics to the students, then using the concrete manipulatives, and then the virtual manipulatives. The observers extracted, and then, recorded the data. The virtual manipulatives involved were “Virtual Pattern Blocks” and “Virtual Base-Ten Blocks.” After the teaching and activities were done, data was gathered on how the students actually used the technology, and how it helped them in their learning processes. (P. N. Moyer)

They found that the process of using virtual manipulatives was very successful. The virtual manipulatives helped the students both explore and test their own mathematical ideas. They found that students were able to experiment on their ideas through using the virtual manipulatives, and explain their ideas through their use of the manipulatives. Their mathematical thinking was also communicated and demonstrated to others, which demonstrates a very high cognitive level of learning, through the use of the manipulatives. Observers found that students were also able to strongly relate the idea of concrete to virtual, and use symbols to represent math, which is also a very strong teaching technique, high on the cognitive learning ladder.

In my classroom, with my study, I found the same thing, but on an older level. The virtual manipulatives allow my kids to clarify their ideas in their head, and because of the interactive use of the manipulative, they are automatically interacting with the math. Though this research was concerning much younger students than I teach, I still learned a lot from this article. I have seen similar results with my students, though they are in an older learning cycle. I still see my students clarify their thinking of mathematical ideas through the manipulatives, and explore their own ideas. I see a lot of connection with their ideas and topics related by the manipulatives. This confirmed my study.
"Best practices in teaching k-12 online: Lessons learned from Michigan Virtual School teachers"

This article is about the best practices in teaching online courses, researched through at The University of Florida, and uses as subjects, Michigan Virtual School teachers. There are virtual schools already formed in The United States, and this article points out that there are unfortunately not very many research-based investigations into the teaching and learning process at this level. The research in this paper was done to find the best practices for virtual teachers. It was interesting in that it stated that the best virtual teachers incorporate the skills of both an interaction facilitator and an instructional designer into their role. A lot of teachers may think that a virtual school would eliminate the need for teachers. The article said quite the opposite. It seemed to say that there are a different set of skills needed for teaching in virtual learning environments, but that the techniques a teacher implements during a course to support student achievement of learning outcomes need more exploring. The teacher has to be extremely flexible, and is needed more than ever in a virtual setting. The research techniques were extremely sound, and in the following I will highlight those things that I will need to do in the future in order to be a virtual teacher.

I might add here, that I think a really great virtual teacher needs to be the same type of teacher that a really great regular teacher should be. Great teachers are great teachers, no matter if they are virtual teachers, or not. The article points out that great virtual teachers go the extra mile to support student learning, are skilled with the uses of technology, and are interested in exploring new technologies that have potential value for virtual school environments. The biggest thing I
noticed is that they need to be flexible with their time. They need to be aware of different learning techniques, and have good organizational skills. They need to understand the impact of pacing and course design on the strategies they use, and continually extend their content and technological knowledge. I basically think that they need to be everything great teachers already are, with the added element of making sure they are using as many of the technology sources available as possible. This is an added element, because just in the past 10 years, so much more has become available through technology, that it is almost unthinkable. The technology that has come about just in the past 10 years leads us to double our content/teaching strategy skills. We have to know so much more, research what is out there so much more, and do so much more to be the best we can be.

Just adding the technology aspect to class doubles your work load. The only real solution is to get a lot of things ready during the summer months. I will search and prepare a lot of online activities so that I have them ready to implement come next year. This way, I can keep up with the grades, learning styles, 504’s, and most importantly, roll (just kidding) throughout the school year!
"The Technology Principle" Article Review

The message of this article is that technology is a very essential part in teaching mathematics, and in learning mathematics. Technology helps in teaching mathematics, influencing the concepts that are taught, and enhances the students' learning. The article makes a point that students can focus on mathematics more deeply with the appropriate use of technology. Technology's greatest asset is the ability to generate, easily, visual models. These models are so powerful as teaching tools, and many students find it hard to come up with them on their own.

As a future math teacher, this article was very important! This research study I performed in my own math classes showed me first-hand what kind of power technology tools had. From this study, I too learned that I must use technology in order for the concepts I am teaching to take hold in the students' mind! This has been a very interesting study to me since my students' test scores after being introduced to the technology tools were higher on average by 4 percent. I use technology in my classroom, but know that I can do a lot more with it than I currently do. In my own classroom, I have noticed that the students learn more mathematics more deeply if I bring in the appropriate technology. Another thing I have already noticed in teaching is that the applets that are found on the internet generate an immediate feedback system for the students. If you find the right applet, the feedback is instant, and the students are not allowed to move on unless they do the math correctly. The feedback in a teacher only set up, is definitely not that fast! As a teacher, it is important for me to test the technology first, and come up with worksheets, or guided experiences for the students in order for them to get the most out of the technology. This way, technology will aid not only in assessment, but also allows the students to investigate, and come up with results.

Because of the many benefits of using the technological tools, the students have a greater parameter of learning available to them. They are more able to solve and even model complex problems, which would create obstacles without the technology.

I agree with this article, but I do have a problem with the final page, in a little way. Students develop differently, and I think even technology will blow them away if it is introduced to them before they are ready to understand the concepts and principles underlining certain sophisticated ideas and use of algorithms. If they don't understand first how to graph themselves, the technology is not as valuable to them. I get frustrated with students who are not able to generate points on a graph, because they would rather just type the equation into a calculator, every time. I think students need to know that the points on the graph are generated through an equation, not by the calculator god. After they are clear on that, sure! Graph the equation on the calculator! I think they need to have a good understanding of concepts first. I don't think, even with technology, students should be asked to work at higher levels of generalization or abstraction. But, I think it is good to introduce them to it. I just don't think they should be asked to master the concept if they are not able to master all parts of the concept.

I agree with the article in every other way because I have watched these claims unfold right in front of my eyes through this research study! The power of technology in a classroom is awesome! The visual models and the technological tools which allow students to learn concepts can be taught in no easier way. Go technology!
Third-Graders Learn About Fractions Using Virtual Manipulatives: A Classroom Study

After I conducted my research project in my classroom, I came across this study. It was set up similar to mine, but better. I didn't have as many resources nor brainpower, as seem apparent by reading the article. However, the study was a very similar to mine in some ways, and it had a similar outcome. Things I observed in my own study were brought about by this study. Reading this research article helped me see what I could improve on if I were ever to do such an endeavor again. I wanted to keep a copy and a review of it in my file to refer to in order to use in the future.

Some of the data seemed about the same as mine. This study was more concentrated involving one subject, fractions. This seemed to be a far better way to study the effects of virtual manipulatives since it is much easier to generalize data having just one topic. The researchers found similar results to my study. They pre-tested and post-tested. Their post-test averages were quite a bit higher, even higher than mine were. I believe this was because mine involved very broad mathematical topics; also, my study materials were very broad, involving a few topics instead of just one. The results showed the students' knowledge in fractions increased, which was obvious when looking at specific scores. The data gathered was somewhat similar to mine. The kids liked the manipulatives because they generated immediate feedback. They knew immediately whether or not they were right or wrong. They liked the computer mouse, and being just a click away from the answer, opposed to using pencils and writing on paper. The students in this study, like mine, enjoyed the virtual manipulatives a lot more than regular instruction. One difference I did notice is that I used standardized tests, and they used teacher generated tests.

Basically, this study turned out to be something that was a little, not much, but a little, similar to mine, but was conducted on a higher research-based level. Mine seemed a little red-neck in comparison. It made me want to start all over again with new kids the next year, and create a better project study. I learned a lot from my red-neck version, though. Though I was on my own and made a few poor, or naive decisions throughout my study, I still confirmed some things that I originally set out to.

My students tested higher after they were introduced to and interacted with the manipulatives. This was very exciting to me. I saw the same thing with this higher level and more concentrated study. I saw as a teacher first hand, as well as the excitement and enjoyment generated by using the manipulatives. Students really try harder to understand the mathematical idea or relationship so that they can get positive feedback. Their learning is enhanced as a result of the
interaction, and the visual representation provided by virtual manipulatives. The manipulatives use pictures, words, and symbols that help students better understand concepts. These are integrated to provide a deeper learning experience.

I was glad I came across this article. It was a thorough study, backing up and deepening my understanding and knowledge of the benefits of using virtual manipulatives in the classroom.

This article starts out recognizing a very prominent theme from the National Council of Teachers of Mathematics, empowerment. That students should feel empowered to do meaningful mathematics is a very strong theme for math teachers throughout the country. The role of the teacher in promoting this theme is as a facilitator of learning. We as teachers need to provide our students with the ability to be a private investigator, and to allow them to perform tasks repeatedly. This is something that mathematic manipulatives can provide for teachers easily. There are at least four roles in using virtual manipulatives in the teaching and learning of mathematics. They are first, mathematical concept and skill development, second, mathematical problem solving, third, mathematical reasoning, and fourth, mathematical communication. These roles can be very important if you as a teacher want to further the National Council of Teachers' of Mathematics' theme of empowerment.

The message for me as a future math teacher was pretty big. I understand the need for technology in classrooms, and how the applets and manipulatives available can be very instrumental in students' learning of mathematics through my research project. The visualization in the calculus applet in this article was very motivating and powerful. I will admit that I will look for any type of virtual applet as I teach the geometry lesson for the day in order to help with the visualization. My own son Jake was struggling with his geometry teacher and her style of teaching—which seems to include no technology. The fact that I took him to our computer and showed him a diagram applet of a 30-60-90 triangle that I use in class, dragging the triangle's corners around and pointing out how the measurement of the sides change in relation to specific ratios helped him understand a lot—far more than what his teacher did (or so he said) "she read the theorem and told us the assignment." Along with this, the research I did in my own classes proves that the visualization enhances my teaching, and I have used it on many occasions, even with my own children.

I agree with the visualization effect, and the ability for students to focus on the process of problem solving instead of the computational aspect. I also know that virtual manipulatives help students focus on realistic problems instead of being restricted to made-up problems which have nice, whole-number answers. Students have a really hard time with that, and even question their own ability to do the math problems if their answers don't turn out to be a perfect whole number. They are also able to be introduced to interesting problems associated with the subject matter. It is interesting to see the real-life problems that they run into as opposed to the manufactured problems that seem to be duplicated over and over again in math books. Also, the ability to present mathematical ideas both orally and in writing is really important and enhanced by technology. Students have such a hard time transferring their writing skills from English to Math. The mathematical concepts that can be presented by visual applets, because they are visual, are so much easier for the kids to express or explain in plain English. I did not find anything in this article that I didn't really agree with, or went against my research.
modeling to change it. I would even like to be involved in it. I think Obama’s goals are a little out of hand, tough. I think this not because we are a stupid nation involved in a teaching strategy that is wrong, but because of the psychological aspect. Many of the kids who fail, fail not because they are not fitting into the current system of education. They fail because of social issues, and there are many. A new system of education will not provide them with food, a home, and freedom from violence or issues that take their focus from their education.
Taxonomy of Software for Mathematics Instruction

The message of this article is that schools have spent an increasing amount of money for computer labs with computers for student use. Because they have put so much money into providing computers, teachers have a great resource available to them. They get to use mathematics software to enhance student thinking and development in mathematics. This article breaks the software into five categories: reviews and practice, general, specific, environmental, and communication. Each of the categories has advantages and disadvantages. The paper gives reasons to use the applets to enhance students' knowledge of mathematics, and then gives examples, benefits, and disadvantages to each of the five categories of tool-based mathematics software that can be used in the classroom. They can focus students on what can be done above and beyond using just a pencil and paper to do math. The article seems to really focus on communication software, and the advantages of using it in the classroom.

Then, it gives reasons to introduce teachers to the software and benefits, claiming that it helps teachers want to use the tool-based software more than if they are never introduced to it through classes or instruction.

For me, as a math teacher, the implications are what I expected. I think that tool-based teaching is really great. Students are very prone to learn using technology. They live in a very technological world. If we as math teachers use technology to teach, our students will most likely learn more from our lessons. They are wired that way, and very used to it. I have never thought of different categories of tools, however. The paper did not see many benefits from the "review and practice tools" category. I like them. I get my daughter at home to play a math game on the internet a lot that has to do with division. It is just a review type applet, but she is now the top in her class in multiplication. She is the fastest multiplier in her class by far! I think some skills are rooted into students by these types of tools. I should, as a math teacher, think about drawing from each of the types of categories as I teach math. As a math teacher, I need to be on the internet more, finding virtual manipulatives and using them in my classroom. I should take more technology classes that introduce me to more tools with technology. More and more tools are created each year, and can really benefit our math teaching. We as teachers need to become familiar with the tools and then implement them into our teaching schedules. I have done this to a point with this project, and will need to continue it in the future.

I agree that the technology in all areas is helpful in each student's mathematical education. I agree that it needs to be used, and the kids need to be familiar with it. I love to teach with Geometer's Sketchpad and Geogebra. I build slopes and circles and quadrilaterals all of the time with it, and use my "homemade applets" a lot. My kids are familiar with them. But, I don't have communication software. I don't agree that all of the benefits the article lists for communication software are applicable in junior high school teaching. Students this age are too immature to gain from a communication site. They are more worried about what others have to say about them, than what should be learned by the activity. There are probably a few [one or two] who are mature enough to benefit from this, but mostly this is a technique that college or upper high school students would benefit more from. I have put work up on the board, and circulated a paper in groups so that everyone had to make a comment on the problem verbally on the paper. This is sort of close to what a communication software site would provide, but is monitored more directly so students are a little more discretionary as to what they write down. I use test questions and student responses for these, and it helps bring home messages as to what is right and good on a test answer.

All in all, a great article! It was very supportive of my project and taught me more about being a "rounded" teacher.
Obama's sanctioned 21st Century Model of Learning Powered by Technology Article Review

I really enjoyed this article so I am including my review in my project write up. It changed my current focus of teaching with virtual manipulatives.

This article was about a 21st Century Model of Learning Powered by Technology, and actually gave steps in order to implement the model in our country. It also quoted Obama's speech about The United States of America’s educational goals, and why technology is important to education. It says that with the rapidly changing demands of the sciences, economy, and education, there is no reason why we should not act on this knowledge and understanding and move forward with it to change the face of education as we know it. Technology provides access to more learning resources because there are several resources all over the world just waiting to be tapped in to.

It even brought out the challenges that the system faces. Today's economy is a huge problem. There isn't money available to put this system in place. The article states that relevant data must be made available to the right people at the right time and in the right form in order to implement this system. It mentioned that educators needed to be involved in all levels of the learning process, providing support, tools, and training to help manage the assessment process, data, and take the appropriate action. It states that the technology is available now to run the program, but not all of the conditions that are necessary to leverage it are.

Right now, the teaching staff currently in place across the country is not necessary at ease with using technology. This is a big problem if they are to implement this great model. Another big issue is that it would require all 50 states to buy into it. It would be a great challenge to involve them all. Education has been given to the states to manage, not the nation. Education is a state responsibility, and this model involves the entire country. I loved the goals and recommendations section. It was nice to see that the authors had physically thought this through, and made recommendations.

This system reminds me of the futuristic replications of education on television where the learning all takes place on some fancy technological machine in the living room. I think that is great, but a lot of work, time and money will be needed. I would really love being a part of it. I was a little leery of President Obama's goal to have 60 percent of all Americans graduate from college. When I see what goes on in the classroom, I just don't see 60 percent with a desire enough to finish college. So many of the kids drop out of school at some point before college— and with such diverse reasons. I believe that like most things, this will cost a lot of money up front. But, once it is in place, it will save a lot of money. I liked the grand challenge problems. It was interesting to think about myself as a teacher in this system. Most teachers would be involved in the research section. All learning experiences should be designed specifically by teachers. They should be working together with the technological designers to make up the systems and design the learning experiences. They would need to be efficient and effective. What an awesome responsibility and goal it is for 60% of the population to be college graduates! He says that our educational system fails a lot of students currently.

Well, I agree that our system of education is somewhat outdated and needs to be increasingly improved. I like the idea that we all work together as a nation and implement new technology and
Works Cited


Resulting Lesson Plans
Lesson Plans

As I reviewed the data collected from my research project, I found that even a little exposure to the virtual manipulatives helped deepen my students' understanding of mathematics. I really liked the 4% higher average in scores I saw, just with 8 days of manipulative use and instruction. I thought I could get this higher if I used the manipulatives and technology in general on a regular basis throughout the year while teaching mathematics.

So, I started implementing the technology into my teaching. I ran into a lot of problems though, and it has been a slow-going endeavor. Sometimes I had computer problems, and sometimes I set my sights too high. Also, as I became familiar with different programs and sites, I found some most assuredly disappeared!

I first began using the original worksheets that I made up to implement my project. They were very rudimentary, and did not focus the students' attention to the ideas and relationships that were expressed in the virtual manipulatives. So, I dug deep, and started preparing more worksheets that required the students to be very interactive with the web site, virtual manipulative, or technology used. The kids had to think harder, use the manipulatives better, and answer more involved questions with my new worksheets. I found others and included them here because they were so valuable in the project. They are really, the complete outcome of my research. They are not complete yet, however.

I change them a little each year, at least it seems so. They are fun, though some are a little tough for the crowd I teach. I change them a little every once in a while. I seem to only be able to add a few each year, and most of my work on them happens in the summer. They are quite time consuming to build. But, they are fun, and the kids enjoy them. They are also great teaching tools. Most of them are used in Geometry.

I plan to build a library of lesson plans using technology each year, until I have one for every unit. I am almost there. This has been the best outcome of this project.
Solving Equations

Overview: Students will solve one-step and two-step equations, as well as equations with variables on both sides, using a balancing beam applet.

Objectives:
- Students will solve one step equations.
- Students will solve two-step equations.
- Students will solve equations with variables on both sides.
- Students will investigate solving equations with negative and positive coefficients.

Connection to Core Curriculum: Algebra, standard 1, Students will expand number sense to understand, perform operations, and solve problems with real numbers. Objective 2 c. Compute solutions to problems, represent answers in exact form, and determine the reasonableness of answers.

Technology: Math applet found at: http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html?open=instructions&from=category_g_4_t_2.html

The Role of Applet: The applet will facilitate investigation of various equations in order to assist with solving them with one-step, two-steps, or variables on both sides.

Activity Plan:
- Pass out worksheet.
- Demonstrate the applet with the students via the projector.
- Discuss questions 1–5 as a class.
- Each student is to work through the worksheet alone.
- Each student is to finish the rest of the activity as homework.
Solving Equations

Name: ______________________ Date: _____ Period: _____

First, represent the equation on the balancing scales with positive x and unit boxes as positive and balloons and negative. The scale will balance when you are finished, as both sides are equal. Then click on “continue.” Don’t try to click continue until your equation is properly represented. Then, begin clicking on the appropriate operation in the boxes, typing in the variable or number that you wish to operate with. When we are finished with solving the equation, we will have a variable on one side, and a value on the other. Please answer:

1. What do the blocks represent?
2. What do the balloons represent?

Please go to:
http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html/open=instructions&from=category_g_4_t_2.html

After exploring this applet for a little while, please answer the following concerning the first equation you tried:

1. What equation were you given? ______________________

2. Now add 5 to both sides. What happened to your equation? ______

3. Now subtract 5 from both sides. What happened to your equation? ___

4. What should we do instead, as the first step to solving this equation? __________

5. What should you always end up with on each side of the equation? ________

6. What did you end up with your equation? __________
Solving Equations

Name: ___________________  Date: _____  Period: _____

Final Activity: Solve 10 equations. Please fill out the following table as you solve the equations.

<table>
<thead>
<tr>
<th>Equation Given</th>
<th>Text Box</th>
<th>End result</th>
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<tbody>
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<td>10.</td>
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</tbody>
</table>
Circle O

Overview: Students will solve equations using a balancing beam applet designed to help teach solving skills.

Objectives:
- Students will compute expressions involving three integers.
- Students will estimate reasonably, expressions involving rational numbers.
- Students will investigate solving equations with negative and positive coefficients and integers.

Connection to Core Curriculum: Algebra, standard 1, Objective 2
Compute fluently and make reasonable estimates with rational and irrational numbers.
Objective 5 Evaluate and simplify numerical expressions containing rational numbers and square roots using the order of operations.

Math Applet: Math applet found at:
http://nlyvm.usu.edu/en/nav/frames_asid_122_g_4_t_1.html/open=instructions&from=category_g_4_t_1.html

The Role of Technology: The applet will facilitate investigation of summing integers in order to assist with solving expressions involving simple integers in their head. It is a fun puzzle that will help students estimate expressions with rational numbers, as well as simplify numerical expressions containing rational numbers.

Activity Plan:
- Pass out worksheet, Circle o.
- Demonstrate how to use the applet on the projector.
- Work through solving one Circle O puzzle on the projector.
- Give students time to fill in the puzzle’s circles on their paper.
- Each student is to finish the rest of the worksheet in the writing lab, finishing at least 2 puzzles.
Circle O
Activity Sheet

Name: ____________________ Date: _____ Period: ____

Please pay attention to the teacher as she explains how to use this applet.
Each circle must independently add to zero. The hint that I like to give is to start with the circle containing zero. It should be the circle on the right side. Try opposite integers in the “O” circle first (remember that opposite numbers are numbers that add to 0, such as 2 & -2, or -10 & 10. Try a few to give them an idea of how the circles must add to zero.

Please open the applet at:
http://nlvm.usu.edu/en/nav/frames_asid_122_g_4_t_1.html?open=instructions&from=category_g_4_t_1.html

After completing the puzzle in class, please fill in the circles below:
Please finish as many puzzles as you can. Time yourself, and record the final puzzle below:

Puzzle 1: Time: 
Puzzle 2: Time: 
Puzzle 3: Time 
Puzzle 2: Time:
**Geogebra Special Quadrilaterals Activity**

**Overview:** Students will use Geogebra to investigate the following structures: rectangles, parallelograms, rhombuses, and squares. They will make conjectures about their similarities and differences after manipulating structures on a Geogebra applet.

**Objectives:**
- By manipulating parallelograms and measuring segment lengths and angle measures, students will find special relationships between diagonals in rhombuses, rectangles, and squares.
- Students will investigate and compare special quadrilaterals and their properties.
- Students will articulate common quadrilaterals, manipulating their angles and side measurements in order to see similarities through the visual applet provided by Geogebra.
- Students will create the figures using a Geogebra applet designed specifically for that purpose.
- Students will explain what properties exist in special parallelograms, in relation to each other.

**Connection to Core curriculum: Geometry, Standard 1, Objective IV**

Students will use examples and counterexamples to classify subsets of quadrilaterals. Students will prove properties of quadrilaterals using triangle congruence relationships, postulates, and theorems.

**Technology:** Homemade Geogebra Applet made using the following directions:

**The Role of Technology:** Facilitate investigation of manipulating parallelograms and measuring segment lengths and angle measures in order to find special relationships between diagonals in rhombuses, rectangles, and squares.

**Activity Plan:**
- Discuss the applet in class and the use of its functions to manipulate the parallelograms.
- Each student needs to have use of Geogebra in order to manipulate the applet.
- Discuss the exploration of special quadrilaterals and their diagonals, discussing terms such as segments, angles, diagonals, measurements, etc.
- Give the students time to explore the applet, and complete their worksheet.
Exploring Graphs Activity Lab

Go to the following web site: C:\Documents and Settings\chjuliana\Desktop\Web site\index.html

Access the Geogebra applet entitled "Quadrilaterals."

This is a Geogebra applet that already has a parallelogram constructed. If you are going to construct it, please do the following (after opening the Geogebra program):

- Draw line segment AB using the tool “Segment between 2 points”
- Make a point, C, off of the line
- Click on Parallel line in tools menu
- Click on line segment then Point C
- Make another segment between 2 points, AC
- Make a parallel line through 2 points in the tool menu
- Click on line AC then on B
- Make a new point in tool menu
- Click on intersection of AB and its parallel line to make the segment AD, parallel to AC
- Make the segment BD in the same way
- Hide all lines, take them off. If you are missing any segments between the points then finish them by “Segment between 2 points”
- If there are any labels on the sides of the parallelogram, hide them also. They will get in the way
- Measure the side lengths
- Measure the angles
- Make segments as diagonals

Have the students do the following, and answer any of the following questions on the exercise section:
Diagonals of Parallelograms

Activity:

Using all that you know about squares, rectangles, parallelograms, and rhombuses, please play around with the quadrilateral on the Geogebra Applet entitled, "Quadrilaterals" making all of the above-mentioned figures.

1. Manipulate the parallelogram to get a RECTANGLE. Make note of what appear to be any special properties of the diagonals of a rectangle. Manipulate the rectangle to check whether the properties hold.

2. Manipulate the PARALLELOGRAM to get a square. Make note of what appear to be any special properties of the diagonals of a square. Manipulate the square to check whether the properties hold.

3. Manipulate the parallelogram to get a RHOMBUS. Make note of what appear to be any special properties of the diagonals of a rhombus. Manipulate the rhombus to check whether the properties hold.

Exercises:

1. Make as many conjectures as you can about each of the following:
   a. the diagonals of rectangles
   b. the diagonals of rhombuses
   c. the diagonals of squares

2. Manipulate the diagonals so that they are perpendicular. Make a conjecture about the type of parallelogram that is determined by perpendicular diagonals.
3. Manipulate the diagonals so they are congruent. Make a conjecture about the type of parallelogram that is determined by congruent diagonals.

4. Manipulate the diagonals so they bisect the angles of the parallelogram. Make a conjecture about the type of parallelogram that is determined by diagonals that bisect the angles.

4. Construct a trapezoid and then its two diagonals. Manipulate the trapezoid until the diagonals are the same length. Make a conjecture about the type of trapezoid that is determined by congruent diagonals.

5. If a parallelogram has one right angle, what can you conclude about its other angles?

Make a list of your conjectures here:

<table>
<thead>
<tr>
<th>#</th>
<th>Figure</th>
<th>Conjecture</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>
Overview: Students will construct geo-board models and make conjectures to discover conditions when a quadrilateral is a parallelogram.

Objectives:
- Students will investigate and compare methods of identifying special quadrilaterals.
- Students will articulate parallelograms and compare slopes of the sides.
- Students will derive the slopes of different sides of parallelograms.
- Students will explain what conditions are present in a parallelogram, rhombus, rectangle, square, kite, and trapezoid.

Connection to Core Curriculum: Geometry, Standard 1, Objective 4: Analyze characteristics and properties of polygons and circles. Use examples and counterexamples to classify subsets of quadrilaterals.

Technology: Applet found at: http://nlvm.usu.edu/en/nav/frames_asid_282_g_3_t_3.html?open=activities&from=category_g_3_t_3.html

The role of technology: Facilitate investigation of various methods of identifying a parallelogram, rhombus, rectangle, square, kite, and a trapezoid.

Plan:
- Discuss the activity in class, and the problems with identifying special quadrilaterals.
- Each student needs to find the correct applet on the computer.
- Discuss question 1 as a class.
- Discuss the definition of bisect. (Each segment divides the other segment in half)
- Give students time to work on the remaining questions.
By definition, a quadrilateral is a parallelogram if both pairs of opposite sides are parallel. You can use a geo-board or geo-paper to explore other conditions that seem to force a quadrilateral to be a parallelogram.

Open the applet at
http://nlvm.usu.edu/en/nav/frames_asid_282_g_3_t_3.html?open=activities&from=category_g_3_t_3.html

For the following problems, use the applet.

Using the applet, you will explore quadrilaterals. After the applet is open, do the following:

1. On the geo-board, grab rubber bands and make pairs of segments that are congruent and parallel. Please look at the following and make sure your geo-board looks just like it:
   a. Imagine the endpoints connected to form a quadrilateral. Does the quadrilateral appear to be a parallelogram?
   b. The picture at the left shows other pairs of congruent parallel segments. Copy the picture using elastic bands forming the quadrilaterals. Look at each quadrilateral separately. Do they each appear to be parallelograms?
2. Now make pairs of segments that bisect each other. Please see the following to make sure your geo-board looks the same.

   a. For the pair you have made, imagine a quadrilateral wrapped around them. Does the quadrilateral appear to be a parallelogram?

   b. Now make the picture in figure 4. Does your geo-board show four more quadrilaterals wrapped around diagonals that bisect each other? Does each quadrilateral appear to be a parallelogram?

   c. Make a conjecture about a quadrilateral whose diagonals bisect each other.

3. Now look at the chart. It shows a 2-by-2 geo-board and a 3-by-3 geo-board at the top. Complete the chart, listing in each cell the number of different quadrilaterals of the indicated type that you can find on each size geo-board. Two quadrilaterals are different if they are not congruent. One cell has been completed for you.

<table>
<thead>
<tr>
<th>Parallelogram</th>
<th>1</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>rhombus</td>
<td></td>
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</tr>
<tr>
<td>rectangle</td>
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<tr>
<td>square</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>kite</td>
<td></td>
<td></td>
</tr>
<tr>
<td>trapezoid</td>
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</tbody>
</table>
4. How many different squares can you find on a 4-by-4 geo-board?

5. The vertices of a cube form a 2-by-2 lattice in space. How many different squares can you find whose vertices are points of this 2-by-2 lattice?

6. How many different squares can you find in a 3-by-3 lattice?
Surface Area Activity

Overview: Students will use a graphing calculator to investigate how the surface area of a square prism with fixed volume changes as the length of each side of a base changes.

Objectives:

- Students will investigate and compare surface areas of rectangular prisms.
- Students will articulate surface areas of prisms by using a calculator.
- Students will derive the surface areas of different sizes of prisms.
- Students will explain volumes of square prisms in order to find the dimensions with the smallest surface areas yet greatest volume.

Connection to Core curriculum: Geometry, Standard IV, Objective 1: Students will use algebraic, spatial, and logical reasoning to solve measurement problems. Find measurements of plane and solid figures. Determine perimeter, area, surface area, lateral area, and volume for prisms.

Technology: TI 83, TI 84, or TI 89 Graphing Calculator

The Role of Technology: Facilitate investigation of various dimensions of prisms, and to find the largest and smallest surface area of prisms of specific dimensions.

Plan:

- Discuss the activity in class and the use of surface area on prisms to aid advertising.
- Each student needs to have use of a TI graphing calculator.
- Discuss the exploration of volume and surface areas of prisms.
- Discuss the exercises as a class.
- Give students time to work on the remaining exercises and the extended exercises with their graphing calculators in groups.
At room temperature, 1 L, 1000 mL, and 1000 cm³ all represent the same amount of water. Thus, one type of model for a liter is any square prism that holds 1000 cm³. The best model, perhaps, is a 10-cm cube as shown in the diagram to the right. But, there are many others.

A graphing calculator could be used to study how height, \( h \), and surface area, \( S.A. \) of a 1-L square prism change as the length, \( s \), of each side of a base changes.

The volume of a prism equals the area of a base times its height \( (V = Bh \text{ or } V = s^2h) \). You can solve for \( h \) in each equation to find the height:

\[
h = \frac{V}{B} = \frac{V}{s^2}.
\]

The surface area equals two times the area of a base plus four times the area of a face, or:

\[
S.A. = 2B + 4sh = 2s^2 + 4sh = 2s^2 + 4s \frac{V}{s^2} \quad \text{Substitute.}
\]

\[
= 2s^2 + \frac{4V}{s} \quad \text{Simplify.}
\]

Below are lists of commands that show on the screens of your graphing calculator to generate lists \( L_1, L_2, \) and \( L_3 \), for the side, height, and Surface Area, respectively. The fourth screen shows what the lists should look like:
You may need to walk them through the process. 2\textsuperscript{nd} function list, ops, 5 os where sequences are found. To store them into the lists, you must use the store button.

\begin{verbatim}
seq(N, N, 1, 50, .2) \rightarrow L1
{1 1.2 1.4 1.6 ... }

1000/L1^2 \rightarrow L2
{1000 694.44444... }

2*L1^2 + 4*1000/L1 \rightarrow L3
{4002 3336.2133... }
\end{verbatim}

Generate the lists (demonstrated above) on your graphing calculator. Scroll down to study them. Then do the following exercises and answer them. (I would use larger increments, 1, not \( .2 \))

**Exercises:**

1. How small can the surface area be? How large can it be?

2. Which dimensions give a very large surface area?

3. Which dimensions give the smallest surface area?

4. How do \( s \) and \( h \) compare in the prism with the smallest surface area?

5. What is the shape of the prism that has the smallest surface area?

6. If a square prism must have a volume of 100 cm\(^3\), what dimensions would give the smallest surface area?
7. A cereal manufacturer is designing a cereal box that has a capacity of 3000 cm³. Surface area should be large enough to provide space for advertising. What else should be considered for the box design? Use a graphing calculator as needed to support your conclusions.

8. Substitute 1000 for the Volume and graph the function:

\[
S.A. = 2s^2 + \frac{4V}{s}
\]

on your graphing calculator. Please look at the graph and then summarize the pattern of the relationship between the side length and the surface area.
Reference List
For Virtual Manipulatives
Reference List

As I began my project, I began researching various web sites and virtual manipulatives on the World-Wide Web, I found some that I really liked, and even began using them now and again throughout the year. Eventually, I began to need to revisit certain web sites, so I starting copying useful URL addresses together on a reference list to refer to now and again when I am looking for more virtual manipulatives for lesson plans.

As I ran into something that was cool and informative, I copied the URL address, and listed the things available at the web site. This list was stored in my computer at work. I still have it and add to it every once in a while. It has been quite handy. Usually, while looking for something, I find something else that I know I can use in another class or lesson plan, so I reference it for later use.

I have also e-mailed the list to other teachers. They tend to enjoy it, or so they say. It does take some time to become familiar with the list though. Because I generated the list, I know what I am looking for when I refer back to it. In this way, it is a quick reference for me.

My list grows all of the time, and is very handy. A lot of web sites make you pay money to get plans or ideas from them. This is not an option for me and my budget, so knowing where the free ideas are is very nice! My list of free-bees has helped a lot when I am making new lesson plans. My list is a short cut to great manipulatives, at a glance. I will use it a lot next summer when I am building more lesson plans using technology and manipulatives.

The only thing I need to do is clean the list out every once in a while. When I pull up something I want to use and it is no longer available, I need to remember to delete it and not just move on to something new.
Virtual Manipulative Web-Site Ideas

TeAchnology – This comprehensive online teacher’s warehouse offers free lesson plans for use in high school math. TeAchnology also features worksheets and rubrics.

The Math Forum – The Math Forum is a huge math site that provides math resources for teachers of all grades. The site can be used by high school teachers to find lesson plans, activities, games, manipulatives, books, calculators, and more.

Math-Videos-Online.com – Teachers can use this site to find educational teaching videos for explaining difficult concepts in algebra, geometry, probability, and more.

National Library of Virtual Manipulatives – The National Library of Virtual Manipulatives offers web-based concept tutorials to help with math teaching. These concept tutorials or virtual manipulatives allow your student to actually work with and visualize relationships.

Illuminations – Illuminations is an excellent site for finding activities, lessons, and standards for your math classroom. The site also features links to more than 700 math resources on the web.

Mathematics Lesson Plans – This lesson plan site features downloadable lesson plans for K-12 students. The lesson plans for high school math cover geometry and algebra.

FREE – FREE (Federal Resources for Educational Excellence) offers over 100 resources for teaching algebra, geometry, data analysis, and number operations. Within this site, you will find activities, modules, lesson plans, advanced methods, and instructional units.

TheTeachersCorner.net – TheTeachersCorner.net provides math lesson plans, games, and worksheets for high school math. Teachers can also use this site to connect with others through message boards, bulletin boards, and a teacher’s lounge.

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Mathway – With over a million problems already solved, Mathway offers a way for your students to compare and understand solutions to difficult math problems. Within this free beta site you can find solutions and graphs for basic math, pre-algebra, algebra, trigonometry, pre-calculus, and calculus.
**NCTM** – NCTM (National Council of Teachers of Mathematics) offers lessons, practice problems, and teaching tips for high school math teachers. Teachers can also find professional development resources, journals, books, and news for educational mathematics.

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**Math-Play.com** – Math-Play.com features free online math games for algebra and geometry. This site also provides interactive, logic, and elementary games.
Wired Math – Wired Math offers free online games, exercises, and answers that teachers can use with their freshman students. The games are provided for number sense and numeration, linear relations, geometry, and algebra.

Wolfram Mathworld – The Wolfram Mathworld site is a comprehensive mathematics resource for teachers to find examples of concepts in algebra, calculus, geometry, probability and statistics, and topology.

Math.com – Although this site was designed for students, there are plenty of free resources for math teachers including worksheets, lesson plans, and formulas.

Guest post from education writer Karen Schweitzer. Karen is the About.com Guide to Business School. She also writes about online-degree-programs for OnlineDegreePrograms.org.

http://skyview.billings.k12.mt.us/departments/math/riehl/geogebra/factoring/factoring.html

Four functions

The End