DESIGN OF A SPLIT HOPKINSON BAR APPARATUS FOR USE WITH FIBER REINFORCED COMPOSITE MATERIALS

by

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ABSTRACT

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The main focus of this work was to design a Split Hopkinson Bar apparatus to determine the dynamic compressive behavior of fiber reinforced composite materials. Graphite epoxy laminated composites have compressive failure strengths of 100 MPa. Dynamic compressive testing shows that the failure stress has increased to 260 MPa, an increase of approximately
230%. This testing shows that material properties are a function of the rate at which they are loaded.
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The main focus of this work is to design a Split Hopkinson Bar apparatus to determine the dynamic compressive behavior of fiber reinforced composite materials. Graphite epoxy laminated composites have compressive failure strengths of 100 MPa. Dynamic compressive testing shows that the failure stress has increased to 260 MPa, an increase of approximately 230%. This testing shows that material properties are a function of the rate at which they are loaded.
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CHAPTER 1

OVERVIEW OF THE SPLIT HOPKINSON BAR TEST

1.1 Introduction

For many years tests have been developed to determine the strength of materials under static loading conditions. However, there was little research on the effect that the loading rate had on tabulated material properties until about 50 years ago. Starting in the 1950s and 1960s there was a spike in interest relating to the study of high loading rate mechanical behavior. This rise in interest was driven by military research dealing with ballistics defense applications and the aerospace industry interests in meteorite impact on satellites and bird ingestion in jet engines.

Prior to this research, material properties were measured using hydraulic or screw type testing machines that were only capable of obtaining a maximum strain rate on the order of 0.1 s\(^{-1}\) [1]. Since then other test fixtures have been developed that can achieve a strain of approximately 100 s\(^{-1}\). These types of tests include, but are not limited to, pendulum impact tests, such as Charpy tests, and drop impact testing. However, these tests do not yield a complete dynamic stress-strain curve.

There are several ways to determine dynamic material properties but the most common and widely used method is the split Hopkinson pressure bar apparatus. The split-Hopkinson pressure bar was first suggested by Bertram Hopkinson in 1914 [1]. His design consisted of a long steel bar, a short steel billet (test specimen), and a ballistic pendulum. Hopkinson would impact one end of the steel bar by means of an explosive charge which would generate a compressive wave that would travel through the bar and into the steel billet. The idea was to generate pressures in the bar that would resemble pressures seen in an impact. From these
experiments Hopkinson was able to generate pressure-time curves that would describe an impact event [2].

In 1949 Kolsky added a second pressure bar to Hopkinson’s original design. Instead of putting a billet at the far end of the bar he sandwiched it in between the bars. This _split_ bar system is how the Hopkinson split bar apparatus got its name. This design has become the most common and widely used technique to determine dynamic material properties [2].

This design was used by the University of Cape Town’s Blast, Impact, and Survivability Research Unit when they presented a paper in the Latin American Journal of Solids and Structures on the effects of strain rate on material properties. Their research demonstrated the response that strain rate has on the stress-strain curve for annealed mild steel. The results of their research shows that for a static tensile test, tested at a strain rate of 0.001s\(^{-1}\), the annealed mild steel has an ultimate strength of approximately 400 MPa. For a dynamic tensile test, tested at a strain rate of 2000 s\(^{-1}\), the annealed mild steel has an ultimate strength of approximately 600 MPa [3].

1.2 Literature Review

1.2.1 General Split Hopkinson Bar Testing

The original Split Hopkinson Bar was designed to characterize compressive material behavior. It is the most widely used method to determine dynamic material response. However, other Hopkinson Bar schemes have been devised to load samples in uniaxial tension, torsion, and to test fracture toughness of notched samples. Although there is no universal standard design for the Split Hopkinson Pressure Bar test apparatus, all Split Hopkinson Pressure bars share common design elements [4]:

- Air cannon/compressed gas gun that fires a projectile
- Sensing device to determine the projectile’s velocity
- Two long symmetric pressure bars
- Bearings and alignment tooling to allow the pressure bars to move freely
- Strain gauges mounted on both pressure bars
- Test specimen
- Instrumentation to record stress, strain, and strain rate information.

The compressed gas gun consists of a reservoir of pressurized stored gas, a quick opening valve, a launch tube, and sensors to measure the projectile’s velocity. The projectile, also called the striker bar, travels down the launch tube and impacts one of the long pressure bars. The impact of these two bars produces a wave in the pressure bar that is then transmitted into the test specimen which is sandwiched in between to the two pressure bars [5].

The two pressure bars are termed the incident bar and the transmitted bar. The incident bar is the bar that the striker bar comes into contact with once it exits the launch tube. The transmitted bar is the located on the other side of the test specimen. The elastic displacements measured in these bars are in turn used to determine the stress-strain conditions at each end of the sample [5]. A schematic of the Hopkinson pressure bar is illustrated in Figure 1.1.

![Figure 1.1: Schematic of split Hopkinson pressure bar](image)

The bars used in a Split Hopkinson Pressure bar setup are traditionally constructed of a high-strength metal such as 4340 steel or a nickel alloy such as Inconel. These materials are used because the yield strength of the pressure bars determines the maximum stress attainable...
in the deforming specimen. Inconel bars are typically used in high temperature Hopkinson bar testing because its elastic properties change very little up to temperatures of 800°C [4].

The length ($L$) and the diameter ($d$) of the pressure bars are chosen to meet a number of criteria for test validity and to obtain the maximum strain rate desired in the specimen. Some of these criteria are stated in the ASM Handbook, volume 8. One criterion states that the length of the pressure bar must ensure one dimensional wave propagation for a given pulse length; for most engineering measurements the propagation requires the length to be approximately ten bar diameters. Another criterion states that for clarity in strain measurement and oscilloscope readings each bar should exceed a length to diameter ($L/d$) ratio of approximately 20. The third criterion states that the pressure bars must be at least twice as long as the incident wave [4].

For proper Hopkinson bar operation, and to ensure one dimensional wave propagation the bars must be physically straight and free to move without binding. The bars must be careful assembled and aligned to ensure that they do not bind. If the bars cannot move freely they produce noise in the measurements and make determining stress-strain relations difficult. Typically the bars are centerless ground along their length to ensure uniform diameter and straightness. A paper written by the University of North Carolina walks through a detailed process of how to properly align the pressure bars using a laser mounted sighting device [6].

The striker bar is normally fabricated from the same material and of the same diameter as the pressure bars. The diameter and the velocity of the striker bar are chosen to produce the required total strain and strain rate within the specimen. When the striker bar impacts the incident bar it generates a one dimensional longitudinal wave. Once this wave travels through the incident bar and reaches the test specimen interface, part of the longitudinal wave is transmitted through the specimen, while the rest of it is reflected back into the incident bar. The strain rate in the specimen is directly proportional to the amplitude of the reflected wave.
The reflected wave is integrated to obtain the strain in the specimen and is plotted against stress to develop a dynamic stress strain curve [4].

1.2.2 Compression Split Hopkinson Bar Theory

The configuration in Figure 1.1 is the typical set up for compression split Hopkinson bar testing. Just before the striker bar impacts the incident bar it is in static equilibrium, seen in Figure 1.2.

**Figure 1.2:** Pressure bar differential element

Just after impact the differential element is put into compression due to forces $F_1$ and $F_2$ seen in Figure 1.3 [2]. The forces $F_1$ and $F_2$ can be written in terms of the stress acting on each face of the element by:

$$\sigma_1 = \frac{F_1}{A_0} \text{ and } \sigma_2 = \frac{F_2}{A_0}$$

(1.1)

where $A_0$ is the cross sectional area of the bar. The stress in the element can be related to the strains by Hooke's law:

$$\sigma = E \varepsilon = E \frac{dU}{dy}$$

(1.2)

Combining equation 1.1 and 1.2 and rearranging to solve for $F_1$ and $F_2$ yields:

$$F_1 = EA_0 \frac{dU_1}{dy} \text{ and } F_2 = EA_0 \frac{dU_2}{dy}$$

(1.3)

These forces acting on the differential element can be seen in Figure 1.3.

Summing the forces acting on the element and setting them equal to mass times acceleration, Newton’s second law, yields:
By using the Taylor Series expansion it can be shown that the displacement $U_2$ can be written in terms of $U_1$ and the change of $U_1$. Thus the displacement $U_2$ is:

$$U_2 = U_1 + \frac{dU_1}{dy} dy$$  \hspace{1cm} (1.5)

Differentiation of $U_2$ with respect to $y$ yields:

$$\frac{dU_2}{dy} = \frac{dU_1}{dy} + \frac{d^2U_1}{dy^2} dy$$  \hspace{1cm} (1.6)

It is known that the one dimensional wave velocity $C_o$ for free vibration is [7]:

$$C_o = \sqrt{\frac{E}{\rho}}$$  \hspace{1cm} (1.7)

where $\rho$ is the bars density and $E$ is the elastic modulus. Equation 1.7 can be rearranged and solved for $\rho$ and substituted into equation 1.4.
Substituting equation 1.6 into equation 1.8 and simplifying yields the governing equation for one dimensional wave propagation:

\[ EA_0 \frac{dU_1}{dy} - EA_0 \frac{dU_2}{dy} = A_o dy E \frac{d^2 U}{C_o^2 \frac{dt^2}{dy^2}} \quad (1.9) \]

\[ C_o^2 \frac{d^2 U_1}{dy^2} = \frac{d^2 U}{dt^2} \quad (1.10) \]

The solution to equation 1.8, for the incident bar, with free boundary conditions can be written as:

\[ U = f(y - C_o t) + g(y + C_o t) = U_i + U_r \quad (1.11) \]

where \(f\) and \(g\) are functions describing the incident and reflected wave shapes, and \(U_i\) and \(U_r\) represent the displacement of the incident wave and reflected wave respectively. By differentiating the displacements in equation 1.11 with respect to \(y\), the strains in the incident bar are:

\[ \epsilon = \frac{dU_i}{dy} + \frac{dU_r}{dy} = \epsilon_i + \epsilon_r \quad (1.12) \]

Differentiating equation 1.11 with respect to time yields,

\[ \dot{U}_i = C_o (-\epsilon_i + \epsilon_r) \quad (1.13) \]

Differentiating the displacement in the transmitted bar yields:

\[ u = h(y - C_o t) = \epsilon_t \quad (1.14) \]

\[ \dot{U}_r = -C_o \epsilon_t \quad (1.15) \]

where \(\epsilon_t\) is the strain in the transmitted bar and \(h\) is functions describing transmitted wave shapes. The strain rate in the test specimen can be expressed as:
\[ \dot{\varepsilon} = \frac{(U_1 - U_2)}{L_s} \]  

(1.16)

where \( \dot{U}_1 \) and \( \dot{U}_2 \) are the velocities at the incident bar specimen and specimen transmitted bar interface, respectively and \( L_i \) is the instantaneous length of the test specimen [4]. An enlarged view of the specimen interface with the incident and transmitted bar can be seen in Figure 1.5.

Substituting in definitions of \( \dot{U}_2 \) and \( \dot{U}_1 \) into equation 1.16 the strain rate in a test specimen can be expressed as:

\[ \dot{\varepsilon} = \frac{C_o(-\varepsilon_i + \varepsilon_r + \varepsilon_t)}{L_s} \]  

(1.17)

![Figure 1.5: Expanded view of incident bar specimen and transmitted bar interface](image)

After applying the conservation of momentum to the striker bar, incident bar, test specimen, and transmitted bar, the strain rate in the test specimen can be approximated by:

\[ \dot{\varepsilon} = \frac{V}{L_s} \]  

(1.18)

where \( V \) is the velocity of the striker bar prior to impact with the incident bar.

The engineering stress in the test specimen is calculated from the force divided by the test specimen’s original area [4]. Since the specimen is considered to be an incompressible solid, the volume prior to impact with the striker bar can be related to the volume after impact by:

\[ A \times l = A_s \times l_s \]  

(1.19)
where $A$ and $l$ are the cross sectional area and length of the specimen prior to impact and $A_s$ and $l_s$ are the instantaneous cross sectional area and length of the specimen after impact. Then engineering stress can be calculated from a strain gauge measure of the transmitted force divided by the instantaneous cross section area.

$$\sigma = \frac{AE\varepsilon_t}{A_s}$$  \hspace{1cm} (1.20)

This is termed a one wave stress analysis because it only uses the values recorded from the transmitted wave. A more accurate representation of the stress can be calculated by a two wave method at the incident bar/specimen interface [4]. This method is defined as:

$$\sigma = \frac{AE}{A_s} (\varepsilon_i + \varepsilon_r)$$  \hspace{1cm} (1.21)

However, this method is not valid at the early stages of the test because of the transient effect that occurs when loading starts at the incident bar/specimen interface.

### 1.2.3 Research Related to Composite Materials

Traditionally composite materials have lower impact energies than metal structures. Composite have lower impact energies because they have low transverse and interlaminar shear strength, and they do not deform plastically. Unlike metals, which can undergo a plastic deformation and still retain structural integrity, composite materials do not undergo plastic deformation, once they reach a certain stress level they are permanently damaged. This can result in local or structural weakening [8].

There are five basic failure modes that can occur in a composite reinforced structure after elastic deformation:

- Fiber failure or fracture
- Resin crazing, micocrakcing, and gross failure
- Debonding between fiber and matrix
- Delamination of adjacent laminates
- Fiber pull out from the matrix

When composites are tested statically they tend not to fail by delamination. However, when they are subjected to impact type loads delamination tends to be the governing mode of failure [8].
CHAPTER 2

OBJECTIVES

- Build a Compression Split Hopkinson Bar Apparatus to validate theory and prove concept
- Write a program to measure the strain and strain rate of the materials being tested
- Perform dynamic compression testing
CHAPTER 3

DESIGN OF THE SPLIT HOPKINSON BAR APPARATUS

3.1 Design of the Launch System

The launch system is composed of a pressurized tank, valves, striker bar, and the launch tube. Typically the launch tube is bored out from a solid bar of steel, holding the tolerance on the inside diameter to ± 0.001 inches. The striker bar is usually centerless ground to fit very snugly inside the launch tube. Such a tight fit is usually required to ensure that no pressure is lost around the outside of the bar when it is fired. However, due to the cost associated with machining with this tight of tolerance on both the launch tube and striker bar a different approach needed to be taken.

The launch tube was designed to use a 4340 high strength seamless tube with the inner diameter tolerance of ± 0.008 inches. The cost to purchase this tubing was significantly less expensive than the precision machine work. The striker bar was also made of 4340 high strength steel turned to a diameter that was slightly smaller than the inner diameter of the launch tube to ensure it wouldn’t bind when it was fired.

Typically a fast acting solenoid valve is used to release the pressure from the tank to accelerate the striker bar. A fast acting solenoid valve is used to help generate an instantaneous release of pressure. However, due to the cost of a solenoid valve, a ball valve is used instead. Although the ball valve cannot be opened as quickly as the solenoid valve, good results can still be obtained.

When firing the striker bar some pressure is lost because of flow around the bar due to the fact that it is slightly undersized. A possible redesign of the striker bar would allow for a seal to be made between it and the launch tube.
Typically the striker bar is sized so that its diameter matches the diameter of the incident and transmitted bar and is accelerated fast enough to produce the desired strain rate. To accelerate the bar the system is pressurized by compressed air or nitrogen.

In general there are two approaches to determine the pressure required to accelerate the striker bar to a given velocity: Newton’s method and the Energy method. The assumptions and results of both methods are discussed in depth in the following sections.

3.1.1 Striker Bar Velocity: Newton’s Method

Newton’s second law will be used to relate the forces acting on the striker bar to the bar’s acceleration. It is assumed that the force acting on the striker bar is constant until it leaves the launch tube (i.e. the pressure is constant), and that the effects of friction and air resistance are neglected. The force acting on the striker bar can be written as the pressure acting over the area of the bar, and can be seen in Figure 3.1. Starting with Newton’s second law:

$$\sum F = ma$$  \hspace{1cm} (3.1)

where $F$ are the forces acting on the bar, $m$ is the mass of the bar, and $a$ is the acceleration of the bar. Substituting in the force acting on the striker bar yields:

$$F = P \cdot A_o = ma$$  \hspace{1cm} (3.2)

Equation 3.2 can be written in terms of the striker bars velocity by relating acceleration to the time derivative of velocity:

$$F = P \cdot A_o = m \frac{dV}{dt}$$  \hspace{1cm} (3.3)

where $V$ is the velocity of the striker bar, $P$ is the pressure acting on the bar, and $A_o$ is the cross sectional area of the bar. Separation of variables in equation 3.3 and integrating yields:

$$\int_0^t P \cdot A_o \cdot dt = \int_0^V m \cdot dV \hspace{0.5cm} \rightarrow \hspace{0.5cm} P \cdot A_o \cdot t = m \cdot V$$  \hspace{1cm} (3.4)
Equation 3.4 can be written in terms of the striker bars position by relating velocity to the time derivative of position:

$$P \cdot A_o \cdot t = m \frac{dy}{dt} \quad (3.5)$$

The striker bars position, $x$, can be solved for by separating variables and integrating:

$$\int_0^t P \cdot A_o \cdot t \cdot dt = \int_0^y m \cdot dy \quad \text{yields} \quad P \cdot A_o \cdot t^2 = m \cdot y \quad (3.6)$$

Rearranging equation 3.6 and solving for time yields:

$$t = \sqrt{\frac{2 \cdot y \cdot m}{P \cdot A_o}} \quad (3.7)$$

Plugging equation 3.7 into equation 3.4 and solving for velocity yields:

$$V = \sqrt{\frac{2 \cdot P \cdot A_o \cdot y}{m}} \quad (3.8)$$

Assuming that the striker bar is made of 4340 steel with a density of .28 lb/cubic inches, is 15 inches long, and has a diameter of 1.125 inches, Equation 3.8 yields the velocities shown in Figure 3.2.

In order to obtain a strain rate of 2500 s\(^{-1}\) the velocity of the striker bar needs to approximately 115 ft/s (35 m/s). For safety reasons the maximum pressure in the gas gun cannot exceed 200 psi.
3.1.2 Striker Bar Velocity: Energy Method

The conservation of energy method will now be used to relate the potential energy of compressed air to the kinetic energy of the moving striker bar. In this solution it is assumed that the effects of friction and air resistance are neglected, that all of the potential energy of the gas is transferred to the striker bar, and that the gas undergoes an isentropic expansion.

Conservation of energy can be expressed as:

$$\sum P.E. = \sum K.E.$$  \hspace{1cm} (3.9)

where P.E. and K.E. are potential energy and kinetic energy, respectively. The potential energy of a stored gas can be calculated by modeling it as an isentropic blow down. The potential energy of a system can be expressed as:

$$P.E. = \delta Q + \delta W_{\text{reversible}} - \delta W_{\text{irreversible}}$$  \hspace{1cm} (3.10)

where $\delta Q$ is the energy lost to heat, $\delta W_{\text{reversible}}$ is the reversible work done, and $\delta W_{\text{irreversible}}$ is the irreversible work done by the system. Assuming an isentropic expansion it is assumed
that $\delta W_{\text{irreversible}} = 0$ and $\delta Q = 0$. The reversible work can be modeled by assuming the pressure undergoes a reversible volume expansion, $dV$.

$$P.E. = \delta W_{\text{reversible}} \rightarrow W_{1-2} = \int_{V_1}^{V_2} P\,dV$$  \hspace{1cm} (3.11)$$

where $W_{1-2}$ is the work done by a pressure ($P$) expanding from volume $V_1$ to volume $V_2$. Using the molar form of the Ideal Gas Law the work done can be expressed as:

$$W_{1-2} = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \rho R_s T\,dV$$  \hspace{1cm} (3.12)$$

where $\rho$ is the density of the gas, $R_s$ is the specific gas constant, and $T$ is the temperature.

By using the isentropic relations for an ideal gas the temperature $T$ can be expressed as:

$$\frac{T}{T_1} = \left(\frac{\rho}{\rho_1}\right)^{\gamma - 1}$$  \hspace{1cm} (3.13)$$

where $\gamma$ is defined as the heat capacity ratio and can be written as:

$$\gamma = \frac{C_p}{C_v} = 1.4$$  \hspace{1cm} (3.14)$$

Rearranging equation 3.13 solving for $T$ and substituting it into equation 2.12 yields:

$$W_{1-2} = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \rho R_s T_1 \left(\frac{\rho}{\rho_1}\right)^{\gamma - 1} \,dV$$  \hspace{1cm} (3.15)$$

Simplifying equation 3.15:

$$W_{1-2} = \int_{V_1}^{V_2} \rho R_s T_1 \left(\frac{\rho}{\rho_1}\right)^{\gamma - 1} \,dV = \left(\frac{R_s T_1}{\rho_1^{\gamma - 1}}\right) \int_{V_1}^{V_2} \rho^\gamma \,dV$$  \hspace{1cm} (3.16)$$

the density $\rho$ can be expressed as the mass $M$ divided by the volume $V$.

$$W_{1-2} = \left(\frac{R_s T_1}{\rho_1^{\gamma - 1}}\right) \int_{V_1}^{V_2} \left(\frac{M}{V}\right)^\gamma \,dV = \left(\rho_1 R_s T_1\right) \left(\frac{M}{\rho_1}\right)^\gamma \int_{V_1}^{V_2} \left(\frac{1}{V}\right)^\gamma \,dV$$  \hspace{1cm} (3.17)$$

$$\left(\frac{M}{\rho_1}\right) = V_1 \text{ and } (\rho_1 R_s T_1) = P_1$$  \hspace{1cm} (3.18)$$
\[ W_{1-2} = P_1(V_1)^\gamma \int_{V_1}^{V_2} \left( \frac{1}{V} \right)^\gamma dV \] (3.19)

Integration and simplification of equation 3.19, see Appendix A for details, yields:

\[
P.E. \approx W_{1-2} = \frac{P_1 \cdot V}{\gamma - 1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \right]
\] (3.20)

where \( P_1 \) is the initial pressure, \( P_2 \) is the venting pressure (atmosphere), \( V \) is the volume of compressed air, and the heat capacity ratio \( \gamma = 1.4 \) for air. The kinetic energy of the striker bar can be expressed as:

\[
K.E. = \frac{1}{2} m \cdot V^2
\] (3.21)

where \( m \) is the mass of the striker bar and \( V \) is its velocity. Combining equation 3.20 and 3.21 and substituting into equation 3.9 yields an expression for the total amount of energy in the system.

\[
\frac{P_1 \cdot V}{\gamma - 1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \right] = \frac{1}{2} m \cdot V^2
\] (3.22)

This result can be rearranged to solve for the velocity of the striker bar in terms of initial pressure and volume.

\[
V = \sqrt{\frac{2}{m} \cdot \frac{P_1 \cdot V}{\gamma - 1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \right]}
\] (3.23)

Again assuming that the striker bar is made of 4340 steel with a density of .28 lb/cubic inches, is 15 inches long, and has a diameter of 1.125 inches yields the velocities shown in Figure 3.3.
In order to obtain a strain rate of $2500 \text{ s}^{-1}$, the velocity of the striker bar needs to approximately $115 \text{ ft/s} (35 \text{ m/s})$. According to Figure 3.3, one gallon of air compressed to a pressure of $30 \text{ psi}$ will produce a velocity of $35 \text{ m/s}$. This solution is optimistic in the fact that it does not account for any head losses in the system.

### 3.1.3 Striker Bar Velocity Summary

As can be seen in the previous two sections, the two methods used to determine the velocity of the striker bar yield different results. The calculations based upon Newton’s method are less accurate than the energy method because the Newton method assumes that the force acting on the striker bar is constant when it is not. As the striker bar travels down the launch tube the pressure in the tank is decreasing because the overall volume of the system is
increasing. The assumptions made in the energy method approach more accurately represent the physical conditions than the assumptions made in Newton’s method.

3.2 Design of the Pressure Bar System

The bars used in a typical Split Hopkinson Bar apparatus are constructed from a high strength structural steel or a nickel alloy such as Inconel. For this system AISI-SAE 4340 steel has been selected because of its high yield strength. The yield strength of the bars determines the maximum allowable stress within the system because the pressure bars must remain in a state of elastic deformation.

The length of the pressure bars are chosen to ensure one dimensional wave propagation for the given material. The length of the bars must be long enough that the data acquisition system will be able to detect the incident wave and reflected wave. If the bars are too short, the data acquisition system will not be able to distinguish the end of the incident wave and the start of the reflected wave. It is suggested in Volume 8 of the ASM Handbook that each of the bars should have a minimum of length to diameter (L/d) ration of 20 [4]. The incident and reflected bars used in this system are 5 feet 3 inches long, see Appendix A for pictures.

To ensure one dimensional wave propagation the incident and reflected bars must be free to move in the axial direction. To ensure this motion is enforced each of the bars is supported by three adjustable mounting brackets, which contains an oil impregnated bronze bushing for the bars to slide through. The oil impregnated bronze has a very low coefficient of friction which allows the incident and transmitted bars to slide with minimal resistance. Alignment of the mounting brackets is performed by mounting a laser in the center of the launch tube. The mounting brackets are individually adjusted until the laser passes strait through the center of the bushing.
CHAPTER 4
DATA ACQUISITION AND INSTRUMENTATION

4.1 Striker Bar Velocity Measurement

It was shown in Chapter 1 that the strain rate can be approximated as:

\[ \dot{\varepsilon} = \frac{V}{l_s} \]  

(4.1)

where \( V \) is the velocity of the striker bar prior to impact with the incident bar, and \( l_s \) is the length of the specimen. This means that the velocity must be measured every time the Split Hopkinson Bar is used to ensure proper strain rate was achieved. The velocity measurements are made by using 2 photo-gate sensors installed in the Split Hopkinson Bars launch tube as seen in Figure 4.1. Once the striker bar passes the first sensor a timer is started, and once it passes the second sensor the timer is stopped. The velocity can then be calculated by

\[ V = \frac{x}{\Delta t} \]  

(4.2)

where \( x \) is the distance between the two sensors and \( \Delta t \) is the time recorded by the two sensors.

A curve was generated to relate pressure stored inside the gas gun to velocity of the striker bar as it exits the launch tube, Figure 4.2. This curve is then used to estimate the fill pressure required to obtain the desired strain rate for testing.

\[ \text{Figure 4.1: Photo gate sensor placement} \]
4.2 **Stress and Strain Measurement**

When the striker bar impacts the end of the incident bar it generates a longitudinal compressive stress wave termed incident wave. Once the wave reaches the test specimen incident bar interface, part of the incident wave is reflected back into the incident bar and the rest is transmitted into the test specimen, seen in Figure 4.3. These waves are termed reflected and transmitted waves [2].
The time required for these waves to travel in the bars and their associated magnitudes are recorded by electrical resistance strain gauges mounted on both the incident and transmitted bar. A quarter bridge type II strain gauge configuration is used to measure the axial waves produced in the bars. The type II configuration uses an active strain gauge to measure axial strain and one dummy gauge to allow for temperature compensation. The active gauge is mounted in the direction of strain and the dummy gauge is mounted within close thermal proximity [9]. Figure 4.4 shows a quarter bridge type II configuration, R4 is the active strain gauge and R3 is the dummy gauge [10].

In order to minimize any residual strain caused by bending of either the incident bar or the transmitted bar, two strain gauges are wired in parallel and mounted on opposite sides of the bars. By mounting the gauges on opposite sides of the bar, when the bar bends one gauge will have an increase in resistance and the other gauge will have a decrease in resistance. This produces a cancelation of the strain caused by bending and allowing the strain gauge to measure pure axial strain.

![Figure 4.4](image.png)

**Figure 4.4:** Quarter bridge type II strain gauge configuration

### 4.3 Acquired Strain Signal

The output voltage of the active strain gauge R₄, in Figure 4.4, must be amplified in order for the data acquisition system to accurately measure the signal. To accomplish this, a Wheatstone bridge circuit needs to be constructed, Figure 4.5 [10].
Figure 4.5: Wheatstone bridge

Resistors \( R_1 \) and \( R_2 \) are half bridge completion resistors, \( R_3 \) is the dummy gauge, and \( R_4 \) is the active strain sensing gauge. The half bridge completion is performed using a National Instruments 9237 strain module. To convert the output voltage, \( V_{ex} \), of the Wheatstone bridge into units of strain, the following equation is used.

\[
strain \ (\varepsilon) = \frac{-4V_r}{GF (1 + 2V_r)} \left( 1 + \frac{R_L}{R_g} \right)
\]  \hspace{1cm} (4.3)

where \( GF \) is the gauge factor of the strain gauge, \( R_L \) is the lead wire resistance, \( R_g \) is the nominal resistance of the gauge, \( V_r \) is the voltage ratio and is defined as [10]:

\[
V_r = \frac{V_{ch}(strained) - V_{ch}(unstrained)}{V_{ex}}
\]  \hspace{1cm} (4.4)

4.4 Developing a Dynamic Stress Strain Curve

When the striker bar impacts the incident bar it generates a wave termed the incident wave. Once this wave has traveled the length of the incident bar and meets the incident bar specimen interface part of the incident wave is transmitted into the specimen and part of it is reflected back into the incident bar. The wave that is reflected back into the incident bar is termed the reflected wave and the wave that is transmitted into the specimen is termed the transmitted wave.
All three waves of the waves (Incident, reflected, and transmitted) are measured using strain gauges. When the measurement of the strain gauge is writing to an output file it also writes the corresponding time associated with the value. Since all three waves initiate at different times each has a different time stamp associated with it.

In order to develop a dynamic stress strain curve all three of these waves must be used. To get a measure of the strain experience in the part the incident wave magnitude must be subtracted from the magnitude of the reflected wave. Since both waves are occurring at different times the initiation of the wave is reset to zero and the time scales are adjusted accordingly. Now that the two waves a time scale that matches their magnitudes of the waves can simply be subtracted from one another to determine the strain in the specimen as a function of time.

The same procedure of resetting the zero value also needs to be applied to the transmitted wave. Once this waves time has be reset to zero we can develop a dynamic stress strain curve by using the transmitted wave to calculate the stress, as defined in Chapter 1:

\[ \sigma = \frac{AE\varepsilon_t}{A_S} \]  

(4.5)

This stress is a function of time and can now be plotted against the difference in magnitude in between the Incident wave and the reflected wave to generate a dynamic stress stain curve.
CHAPTER 5
RESPONSE OF MATERIALS TO DYNAMIC LOADING

5.1 Dynamic Stress Strain Curves of Balsa Wood

In order to ensure the Split Hopkinson bar and the associated data acquisition system were functioning properly balsa wood samples were tested and the results compared to the results published by the California Institute of Technology [11]. The balsa wood used in testing had a density of approximately 175 kg/m^3. Figure 5.1 shows the dynamic stress strain curve which can be compared to the results publish by California Institute of Technology, Figure 5.2.

By using Hooke’s Law \( E = \sigma/\varepsilon \) the dynamic compressive secant modulus of elasticity can be calculated to be 1,654 MPa, compared to the static compressive modulus of 460 MPa [12]. This is an increase of almost 350 %, Figure 5.1.

The secant modulus of elasticity can be calculated by dividing the yield stress by the strain at the yield stress \( E = S/\varepsilon \) seen in Figure 5.3.

![Figure 5.1: Dynamic stress strain curve balsa wood tested at USU](image-url)
Figure 5.2: California Institute of Technology dynamic stress strain curve of balsa wood

Figure 5.3: Secant Modulus of Elasticity
5.2 Dynamic Stress Strain Curves of Polystyrene

Further testing was conducted to determine the effect strain rate had on toughened polystyrene. Figure 5.4 shows the dynamic stress strain of polystyrene tested with USU Split Hopkinson Bar. The dynamic yield stress is approximately 100 MPa and the apparent compressive secant modulus of elasticity is approximately 0.57 GPa.

Static material properties for polystyrene are as follows: yield strength – 40 MPa and modulus of elasticity – 3 GPa [13]. Comparing these values to the dynamic material properties generated at USU shows a significant difference. The yield strength has almost doubled while the modulus of elasticity has decreased by a factor of six.

![Figure 5.4: Utah State University’s Dynamic stress strain curve polystyrene](image)

5.3 Dynamic Stress Strain Curves of Graphite Reinforced Composite

A graphite epoxy composite laminate with a stacking sequence of [0/0/90/90/0/0] was tested using the split Hopkinson bar at USU. It can be seen in Figure 5.5 that the dynamic compressive failure stress is approximately 260 MPa in the 2 direction. GY70/934 Graphite epoxy unidirectional prepreg is known to have a static compressive failure strength of
approximately 100 MPa [14]. The dynamic compressive stress has increased by approximately 230% from the published data for graphite composite laminates.

The compressive failure stress of a graphite laminate in the 2 direction is approximately 100 MPa [15]. The dynamic compressive failure stress has increased by approximately 230% from the data published by Hyer.

The apparent dynamic secant modulus of elasticity in the 2 direction is approximately 3.25 GPa. This is a decrease of approximately 50% compared to the GY70/934 Graphite composite which has a modulus of elasticity in the 2 direction of 6.4 GPa [14]. The dynamic modulus decreases approximately 75% compared to the data published by Hyer which has an elastic modulus of 12.12 GPa in the 2 direction [15].

![Graphite Composite [0/0/90]s](image)

**Figure 5.5:** Dynamic stress strain curve for graphite reinforced composite laminate
CHAPTER 6
CONCLUSIONS

6.1 Summary and Conclusions

A compression split Hopkinson bar apparatus was constructed to characterize how material properties change with respect to the rate at which they are loaded. The split Hopkinson bar setup was developed through a series of iterations of experimental as well as analytical designs. After several modifications to the design and data acquisition system, experiments to characterize the dynamic response of different materials were conducted.

The split Hopkinson bar was used to with the following materials: balsa wood, polystyrene, and graphite composite laminates. Each of these different materials experienced a shift in mechanical properties when tested dynamically instead of statically. In testing the different materials it was noted that they all experience a large shift in yield strength. Further studies should be conducted to determine if metals experience the same shift in mechanical properties.

Since tabulated material properties are the starting block for the design of most structures it is important to have accurate material properties. It is shown by the information presented in Chapter 5 that the loading rate at which materials are tested affects the mechanical properties. Having accurate material properties is the key in producing analysis representative of real life situations.

6.2 Future Work

The following section outlines possible suggestions to improve upon the split Hopkinson bar at Utah State University.
6.2.1 Launch System

As outlined in Section 3.1 the launch tube and striker bar were fabricated on a low budget. As a result of the budget the dimensional tolerances of the launch tube and striker bar allow leakage of gas pressure - when the system is fired. This means that the pressure tank must be charged to a higher pressure to produce the appropriate velocity.

The launch tube should be fabricated in such a way that allows the inner diameter to be held to a very tight tolerance level. Possible manufactures to produce the launch tube would be companies that specialize in producing rifle gun barrels.

The striker bar should be sized such that it fits inside the launch tube with minimal clearance. This can be performed by machining the striker bar and polishing the surface to minimize friction.

Another approach would be to incorporate a nylon sealing features such as O-rings imbedded into the striker bar.

6.2.2 Pressure Bar System

As outlined in section 3.2 the alignment brackets for the pressure bar system were built with adjustability in them to ensure that a one dimensional wave would be produced in them. These mounting brackets were made in the Utah State University student prototype fabrication lab. These brackets tend to come loose and become misaligned after each time the system is fired. These brackets should be re-designed and fabricated by a professional machinist to ensure that it is produced to specifications.

The mounting brackets should also be designed so different sized incident and transmitted bar can be used. Different sized bars can produce different strain rates in materials. So to have a system able to test a wide variety of materials it needs to be able to incorporate different size bars.
The pressure bars themselves should be manufactured in such a way that they are perfectly straight and polished or coated with a low friction coating. This will ensure that a one dimensional wave is produced in system.

6.2.3 Data Acquisition System

The current data acquisition system used to measure the strain in the bars is performed using a NI 9237 strain module for use with a Compac DAQ system. The current settings in the data acquisition system are set to sample the voltage signal produced by the strain gauges as fast as it will allow. However these settings are barely adequate to capture the strain produced in these bars. This system works great for static testing but is not designed to handle dynamic loading conditions.

The current strain module does a variety of things at once internally. It samples the voltage produced by the strain gauge, amplifies the voltage measurement, reads in the gauge factor and converts the strain gauge voltage to strain, writes the strain measurement to an output file. All of these operations that the data acquisition system performs slow down the rate at which the strain voltage can be sampled.

The data acquisition system should be redesigned to be capable of performing dynamic data operations. The new data acquisition system should only sample the strain voltage and save/write the voltage to a file. The module to sample the voltage should be capable of sampling at least 100,000 samples per second.

The voltage amplification should be done by an external Wheatstone bridge. The strain voltage output should be converted to strain in a post processing calculation and the stress strain curves can then be produced by this data. By moving these data operations to a post processing application will significantly improve the rate at which the signal can be sampled.
WORKS CITED


APPENDICES
Appendix A: Split Hopkinson Bar Pictures